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Concept-based instruction: Improving learner performance in mathematics through conceptual understanding

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The study examined the impact of concept-based instruction on the teaching and learning of mathematics. Functions is the topic that was implemented in order to identify the improvements in learners' understanding. A total of 35 Grade 11 learners from a South African township school in Limpopo province participated in this study. Data were collected using tests, questionnaires, and semi-structured interviews. The constructivist learning theory underpinned the exploration. All the 35 participants wrote a test twice, completed a questionnaire and only 6 of them were interviewed. The study employed the sequential explanatory research design because of its richness in data findings as it incorporates both quantitative and qualitative research methods. The initial stages of data analysis started with emerging themes from data being coded and categorised. To identify changes in learners' performance in the administered pre and post assessment tests, a dependent *t*-test was carried out, while questionnaires and interviews were used to assess learners' attitudes and perceptions. The findings from the inquiry indicated positive gains in deep understanding, critical thinking, long-term retention of information, transferable skills, engagement and integration, which are all directly linked to conceptual understanding, resulting in enhanced performance. Both numerical and descriptive analyses confirmed that concept-based instruction enables learners to construct their own knowledge and enhance their conceptual understanding which in the end improves their performance in mathematics. The findings confirm that learners responded positively to concept-based instruction and suggest its broad adoption in mathematics education to address issues of conceptual gaps.

Contribution: The study provides teachers with an opportunity to implement innovative teaching and learning approaches such as the concept-based instruction to improve learners' conceptual understanding.

Keywords: concept; concept-based instruction; conceptual understanding; procedural understanding.

Introduction

Mathematics is a subject that is regarded as pivotal to a learner's progress in life especially with careers in science, technology, and engineering. The subject is indeed fundamental in most educational systems around the world (Chand et al., 2021), and its inclusion in the curricula is widespread (Shimizu & Vithal, 2023). That is the reason why in most countries it is a compulsory subject in schools (Jacob et al., 2020; Nitzan-Tamar & Kohen, 2022). However, the level of its compulsory nature and level to which it must be studied varies by country (Bocconi et al., 2022; Keller et al., 2022). Mathematics provides learners with a distinctive arsenal for rational analysis, problem-solving and abstract thinking (Andamon & Tan, 2018). Mathematical understandings influence decision-making in all areas of life consciously or unconsciously. Mathematics is crucial in most fields of human endeavour, be it private, social or civil, and is also a gateway to most scientific and technological fields.

Despite the role played by mathematics in people's lives, the subject is feared by most learners in their educational journeys (Ablian & Parangat, 2022; Saputri et al., 2024). Learners perform badly in Mathematics: the results for the subject trail behind almost all subjects in most schools at all levels (VaraidzaiMakondo & Makondo, 2020). In South Africa it is extremely difficult for learners to score above 40% in exams (Mabena et al., 2021; Naidoo & Kapofu, 2020). According to the diagnostic report from the Department of Basic Education (DBE, 2024) for 2023 matriculates, the highest number of learners' marks always fall between 20% and 40% for each year. This poor performance in Mathematics badly affects learners in many ways. Some of the learners are prevented from progressing to the next grade, some from doing the courses that they want at tertiary level and the others from getting careers that they want in life.

Poor performance in Mathematics is not only a problem of South African learners. Globally, learners have serious challenges in achieving tertiary entry levels (Alemu et al., 2021). Moreover, the situation seems to be worse in South Africa as stipulated by most researchers. According to Jojo (2019) and Fleming (2020), low achievements in Mathematics continue to be worrisome in South Africa. This is also witnessed every year when the matric results for Mathematics lag behind those of other subjects (Machisi, 2023). To worsen the situation, the number of learners registering for Mathematics is going down with a remarkable number of learners opting to do Mathematical Literacy instead of pure Mathematics. The subject is feared by most learners to an extent that at high school learners wait for a chance to drop it. In most cases learners who end up writing Mathematics examinations do so because the subject is a prerequisite for the courses they want to do at varsity or college.

According to Chand et al. (2021) and Yong et al. (2018), several factors contribute to poor performance in Mathematics: the subject is perceived as too complex, requires memorisation of numerous formulae, fails to engage learners due to lack of understanding of its relevance, is time consuming, and learners often lack fundamental knowledge which leads to an inability to achieve the expected. It seems as if teachers, subject advisors, district managers, policymakers and researchers are failing to come up with strategies to overcome these challenges in Mathematics. The reason for all the stated challenges is mainly lack of conceptual understanding as informed by other studies like the research done by Chinn (2020) and Verschaffel et al. (2020), which highlights that learners often struggle with Mathematics because they focus on procedural skills rather than understanding underlying concepts. If learners have deep understanding of concepts, all of these challenges will not be faced or will be greatly reduced.

The solution can come from improving teaching and learning approaches. If educators can teach learners to grasp concepts and avoid memorisation and use of algorithms, learners will be in a position to recall, connect and apply their prior knowledge to new situations and be in a position to solve not only mathematical problems but also real-life problems (Akben, 2020; Maryani & Widjajanti, 2020; Verschaffel et al., 2020). Realising the vital role played by conceptual understanding, the researchers of this study aimed to explore whether concept-based instruction could enhance Mathematics teaching and learning, thereby improving learners' understanding of concepts and boost their performance.

Literature review

Numerous studies have explored how mathematics is taught and learnt with focus on different instructional approaches. Nevertheless, there is not much on how concept-based instruction specifically influences mathematics education and whether it could be a transformative solution for improving poor performance. The study is going to briefly give a review of the related literature to provide a grounding theory to reveal the impact of concept-based instruction on understanding of concepts by learners as well as their performance in tests and examinations.

According to Mwakapenda (2004), the most important traits associated with educational goals is understanding of concepts, which encompasses mastering specific knowledge, acquiring new skills and developing critical thinking skills. Understanding is when connection or relationships between known ideas can be identified, and understanding concepts in this context is crucial because the foundation for achieving the goals is based on it. Maynard (2019) supports that understanding is the ability to link ideas, rather than just knowing isolated facts. Concepts are intricate ideas that are essential in influencing effective teaching and learning (Darling-Hammond et al., 2022), serving as key drivers that shape instructions and facilitate the learning process (Cheung et al., 2021; Stewart, 2012). Concepts are indeed fundamental to learning as they represent categories or abstractions that help individuals organise and interpret information (Guizzardi et al., 2021). Concepts provide a framework for understanding new information through linking it to existing knowledge (Antunes & Pinheiro, 2020), and the ability to connect and apply concepts is crucial for problem-solving and deep learning. Leggett (2011) defines concepts as accepted collections of attributes related to objects, events, situations or conditions. Leggett means that concepts are formed by grouping and categorising occurrences or events with shared characteristics beyond any individual observation. For effective learning, learners should gain conceptual understanding but on the other hand conceptual understanding cannot be easily attained by learners.

When learners are being taught, it is extremely important to make sure they understand what they are being taught as understanding enables memory transfer, greatly influences attitudes and beliefs, and breeds confidence and engagement while not understanding demotivates learners (Han et al., 2021). When teaching, one has to be sure of the kind of understanding one wants the learners to gain depending on its impact on learners. Procedural understanding and conceptual understanding are the two main forms of mathematical understanding (Rittle-Johnson & Siegler, 2022).

Understanding of procedures (procedural understanding)

Understanding of procedures entails mastery of rules, symbols, and algorithms (Hechter et al., 2022). This kind of understanding is about mastering computational skills without questioning the reasons or mechanisms behind how things work. Procedural understanding results in learners memorising rules and algorithms and reciting of facts with minimal or extremely limited grasp of the fundamental concepts (Duarte & Robert Noguera, 2019). Mutsvangwa (2016) observed that this kind of understanding is not transferrable and is difficult to retain. Procedural understanding is meant for specific situations, lacks generalisability, puts emphasis on rules and facts and is, therefore, not adequate for conceptualisation. Procedural understanding has little value because of lacking underlying meanings and does not equip learners with problem-solving skills.

Conceptual understanding

Conceptual understanding refers to knowledge that entails a deep comprehension of the fundamental concepts underlying the algorithms used in mathematics (Andamon & Tan, 2018). Rizvi and Lawson (2007) describe conceptual understanding as knowledge of abstract ideas. Kharatmal (2009) views conceptual understanding as the ability to connect mathematical ideas. Therefore, a learner with conceptual understanding knows more than isolated facts and is in a position to describe, explain, and use the same concept in various ways across different situations. Rittle-Johnson and Siegler (2022) view conceptual understanding as possessing knowledge of core principles and their relationships. They then depict that conceptual understanding as knowledge of relationships and patterns between concepts that assists learners to make sense of mathematical ideas.

Concept-based instructional approach

Concept-based instruction (CBI) is an instructional approach for teaching and learning that was proposed by Hilda Taba in the early 1960s with the opinion that learners should construct their own knowledge through organising and categorising information. Concept-based instruction brings learners understanding and meaningful learning and improves their thinking skills (Erickson, 2012; Saez-Lopez et al., 2019). Ross and Myrers (2017) see CBI as facilitating connections and promoting higher levels of thinking in learners leading to improved retention of information. Skemp (1976), Giddens (2019), and Higgins and Reid (2017) add that CBI helps learners grasp how and why each idea or relationship functions as it does. Concept-based instruction equips learners with the ability to identify how things are the same and also how they differ. Therefore, with CBI, learners interpret facts, connect and use them in various contexts. Concept-based instruction also moves learners towards deep conceptual understanding because there is comprehension of operations and relations. Through collaboration of ideas and exploration of concepts, CBI fosters mathematical understanding and confidence in learners (Akben, 2020). As suggested by Romey (2021), learners engaging with CBI become active contributors to knowledge by asking questions, exchanging ideas with each other, explaining the reasons behind their work, and evaluating similarities and differences. With CBI learners should be able to connect the dots, communicate, justify their thinking processes and link content knowledge to real-life situations, making them view mathematics as doable, accessible and enjoyable. Therefore, CBI is an innovative teaching and learning approach that can be used to meet challenges of the world that learners live and work in.

Other studies on teaching and learning approaches

There have been a lot of studies on concept-based and procedural (traditional) teaching and learning approaches. The researchers were exploring the impacts of concept-based and/or procedural teaching. Some of the researchers are Al-Qatawneh (2012), Borji et al. (2021), Chappell and Killpatrick (2003), Chinofunga et al. (2023), Chirove and Ogbonnaye (2021), Erickson (2012), Ghazali and Zakaria (2011), Lee and Paul (2023), Nahdi and Jatisunda (2020), Ntow and Hissan (2021), and Mosimege and Winnaar (2021).

In their study on concept maps as a resource to enhance teaching and learning of mathematics at senior secondary school, Chinofunga et al. (2023) used a survey and semi-structured interviews to investigate how concept maps serve as a valuable resource for teachers in linking junior and senior mathematics concepts. They established that teachers believe that concept maps significantly enhance mathematics teaching and learning by linking new and prior concepts.

Lee and Paul (2023) reviewed pedagogical approaches for improved engagement and learning outcomes of mathematics. Their findings highlighted that the traditional lecture-based approaches led to passive learning environments. They also established that while innovative methods are effective, there is great need for teacher preparation and need to work on motivating learners.

Ntow and Hissan (2021) conducted their study on the impact of CBI on senior high school students of the circle theorem. They used two different groups of students to investigate the effectiveness of CBI against traditional teaching methods on enhancing performance in circle theorems. They used *t*-tests for independent samples which indicated a significant difference in the achievements of learners from the two different groups. They, therefore, established that CBI may be effective in teaching circle theorems.

Chirove and Ogbonnaya (2021) examined Grade 11 learners' procedural and conceptual knowledge of algebra. They discovered that learners possessed low procedural and conceptual knowledge. However, their procedural knowledge was better than their conceptual knowledge and they recommended teachers to be aware of and implement strategies that promote conceptual understanding.

Borji et al. (2021) investigated the importance of procedural and conceptual teaching on learners' mathematics performance over time. The findings deduced that the conceptually taught group performed better than the procedural group. Their study concluded that conceptual teaching could create deeper understanding and sustain mathematics learning.

Al-Qatawneh (2012), Chappell and Killpatrick (2003), and Ghazali and Zacharia (2011) all examined the effectiveness of concept-based teaching methods compared to procedural approaches. Their findings were similar, pointing out that concept-based taught learners outperform those taught using procedural approaches. The studies also established that concept-based teaching significantly enhanced understanding and retention of information.

Nahdi and Jatisunda (2020) carried out an investigation to determine the impacts of procedural and concept-based teaching. Their results indicated that learners do well when they are taught using a combination of procedural and concept-based methods. Their conclusion was that both methods are needed for developing good knowledge in mathematics.

Erickson (2012) also conducted a study on CBI and discovered that it increases motivation, encourages learner-centred teaching and learning, and gives learners an opportunity for meaning making. Another study was carried out by Mosimege and Winnaar (2021) on teachers' instructional strategies and their impact on learner performance. In their findings they discovered that problem-solving with direct teacher guidance and teacher-learner interactions were significantly associated with learner performance.

The studies indicate that learners exposed to concept-based approaches perform better than the ones taught using procedural teaching and learning approaches. In the studies that used two groups, procedural and concept-based, there was consistently a notable disparity in the achievements of the learners from the procedurally and conceptually taught groups. Conceptual teaching and learning approaches were found to promote learners' understanding much better than procedural approaches. One of the studies noted that combining the two approaches produces better results; in fact, having conceptual knowledge improves one's procedural skills.

Kishpaugh (2021) noted that educators often rely on ineffective, fast-paced instructional methods to help learners pass assessment tests rather than preparing them to tackle real-world problems. Mostly, mathematics educators concentrate on completing tasks according to curriculum assessment plans at the expense of deeper learning. The teachers adhere to prescribed procedures and objectives instead of adapting to learners' needs, fostering deeper understanding of the subject matter. The work prepared in the annual teaching plans does not spare time for active construction of knowledge but only allows them to memorise and cram in preparation to get better marks in formal tasks. Both teachers and learners focus attention on getting correct answers for marks and not on how the answers are obtained.

A lot of research has also been done on teaching and learning approaches in several disciplines like English, Nursing and Physics. From literature, on the part of research in mathematics, innovative teaching and learning approaches help learners to develop conceptual knowledge and are linked to performance improvement. However, many of the studies carried out aimed to compare conceptual teaching and procedural teaching. There was little or no emphasis on the extent to which CBI could be used as an intervention for addressing poor performance in mathematics.

Educators, researchers and policymakers worldwide are struggling to understand the changes that are needed to improve educational outcomes (O'Dwyer et al., 2015). With the elucidated gaps from the literature reviewed, the researchers for this study felt the need for more research on teaching and learning approaches as a way of trying to come up with informed decisions on instructional approaches that will improve learners' conceptual understanding and hence improve their performance in mathematics. This research focuses on examining the impact of CBI on mathematics teaching and learning which can be a way of equipping learners with 21st century skills. Concept-based instruction focuses on understanding broad concepts rather than just memorising facts. It aligns with and promotes the development of the four Cs which are: critical thinking, collaboration, communication and creativity. By emphasising deep understanding of concepts, CBI encourages learners to think critically, work collaboratively, communicate effectively and apply creativity, thereby improving their performance in mathematics.

Theoretical framework

Learning theories are vital to educators and researchers as they help them to understand what affects learners in the classrooms. This study utilised the constructivism learning theory to elucidate the role of CBI in mathematics teaching and learning. According to Miranda (2011), constructivism is deeply rooted in the works of Dewey, Piaget, Vigotsky and Bruner. The quartet collectively see new knowledge as based on prior knowledge. To the constructivists, knowledge is gained through a process of reflection and active construction in the mind (Misra, 2020; Zajda & Zajda, 2021). This depicts that learners build knowledge on past experience, personal views and cultural background instead of passively receiving new knowledge.

Constructivism learning theory was chosen because of its ability to develop: critical thinking skills, analysis, evaluation and making connections. Constructivists shift responsibility of knowledge construction from the teacher to the learner (Watling & Ginsburg, 2019). Constructivists believe that learners develop mathematical concepts through adaptive and cognitive active processes (Vintere, 2018). Constructivism supports learner-centred approaches with active participation of learners highly prioritised. Learners acquire mathematical knowledge by constructing concepts internally using their own thinking abilities and not through internalising rules enforced from outside (Piaget, 1973). Constructivists emphasise the understanding of underlying concepts and the meaningful use of procedures rather than rote memorisation of rules and tricks. While the constructivists critique teaching that solely focuses on rote learning, they do not entirely oppose procedural learning. Constructivists advocate for integrating procedures with conceptual understanding so that learners grasp the 'why' behind the methods they use (Hwa, 218).

The constructivists believe that the learner should be an information constructor while the educator is a facilitator supporting the learner's constructive processes. According to Mutsvangwa (2016), constructivism stands out as one of the most effective teaching and learning theories due to its capacity to enhance conceptual understanding. It can be seen that constructivism emphasises the stance of shifting responsibility of learning from educators to learners.

Concept-based instruction and constructivism

The constructivists do not encourage traditional teaching and learning approaches which have learners as content receivers (Mutsvangwa, 2016). The intention of the constructivists is to move from traditional teaching that involves direct instruction and go for teaching and learning approaches that have educators as facilitators. Hwa (2018) proposes implementation of constructivist-based teaching and learning approaches to support construction of understanding by learners themselves. Concept-based instruction aligns with this by encouraging learners to explore mathematical concepts deeply rather than following rote memorisation.

Concept-based instruction and constructivism enable learners to foster or improve their thinking skills and also allow them to make transfers, links, connections and applications to other situations, enhancing their overall performance. Constructivists argue that learners do not start a new level of education as blank slates; instead they bring with them knowledge from prior experiences (Hinduja, 2021; Matsumoto, 2021). They believe that learners already possess some understanding and should not passively wait for educators to provide knowledge. Instead, learners should build their own knowledge. Following the views of Hinduja (2021), traditional approaches regard learners as empty vessels needing knowledge to be transmitted by the educator, which opposes the goals of CBI and constructivist principles. The traditional teaching and learning approaches are in contrast with the objectives of CBI and the views of the constructivists.

Current reforms are highly in favour of conceptual understanding. Active learning, a key feature of the constructivist theory, improves retention as learners achieve deeper understanding when they actively engage in constructing their own knowledge (Vaz de Carvalho & Bauters, 2021). The constructivism theory of learning excels in promoting deeper understanding, meaning making and high levels of retention (Osei, 2019). This aligns with the concept-based teaching and learning approach, which focuses on fostering deeper comprehension of overarching concepts rather than just memorising facts. The researchers of this study think that adopting a constructivist approach to teaching and learning could enhance the performance of learners. Although, the constructivists offer valuable insights into how learners build knowledge through experience and active engagement (Misra, 2020; Zajda & Zajda, 2021), it is important to recognise that different theories can be effective depending on the educational context and objectives.

While constructivists advocate for CBI that fosters deeper comprehension of material and critical thinking, procedural teaching approaches play an important role in developing foundational skills and fluency in mathematical processes. Importantly, procedural understanding is enhanced through conceptual understanding as a strong grasp of concepts provides the framework for mastering procedures. Therefore, a balanced approach that integrates both concept-based and procedural-based methods can offer a more comprehensive educational experience with each reinforcing the other.

Methodology

The study employed a sequential explanatory mixed-methods design. For gathering and analysing data, the study utilised quantitative and qualitative methods. Data were collected from a Grade 11 class of 35 learners. A one-group pre-test-post-test experimental framework was utilised in order to see changes before and after intervention without disadvantaging other learners by taking them as a control group (Pardede & Ramadia, 2021). The one-group design was used to make sure learners benefit from the intervention which is particularly important in educational settings where fairness and equal access to resources are paramount (Tornivuori et al., 2023). The single group provided valuable insights (Attia, 2020), with complementary evidence to build more comprehensive understanding of the effectiveness of the intervention, gathered from questionnaires and follow-up interviews (Hampson & Izem, 2023). The one-group design offers educational value by providing a basis for reflection and further development (Laupichler et al., 2022), as it helps educators and researchers to refine their approaches which is a crucial step in the iterative process of improving teaching practices (Khoiriyah et al. 2023).

Tests, questionnaires and interviews served as data sources for this study. All the 35 participants wrote the test twice, completed the questionnaire and were taught using CBI, of which only 6 learners were interviewed. At the beginning, a test was written for pre-assessment as a way of determining participants' level of concepts and rewritten after three weeks of intervention. The test was written for the second time to test and compare level of understanding of learners before and after intervention, and also to ascertain changes brought about by CBI. After writing the test for the second time, a questionnaire that consisted of closed and open-ended questions was administered to give participants a chance to voice their opinions, provide suggestions and express their feelings about CBI. Lastly, there were interviews to follow up on learners' conceptual understanding. The habituation of the sequential explanatory design was to make a way to thoroughly examine the impact of CBI on the mathematics teaching and learning.

The study used the same test before and after intervention to specifically measure the changes in learners' understanding that could be attributed to CBI. To some extent, over-time improvement might occur regardless of the instructional method; however, with the calibre of learners that were selected, after some time they would be found not to remember most of the things. According to Tetzlaff et al. (2021), using the same test was valid to allow direct measure of learners' progress and understanding the specific concepts targeted by CBI. The same test provided a clear baseline and follow-up comparison in the same context which helped in ensuring that the observed changes are attributable to the intervention (Riley-Tillman et al., 2020). The study included additional intervention, qualitative feedback from learners through questionnaires and interviews. The gathered qualitative insights through learners' feedback were to further understand the impact of CBI beyond what was captured in the test scores (Gaber, 2020). There was also a statistical analysis that was conducted to determine if the observed improvements were statistically significant (Walker & Kettler, 2020). The analysis helped to ensure that the changes in the test scores were not due to random variation, but specifically associated with the intervention (Gaber, 2020). Involving multiple data sources aimed to ensure that improvements observed could be more accurately attributed to CBI rather than a general trend of improvement.

The 35 Grade 11 learners who participated in this study were purposively and conveniently selected from a Limpopo province township school. The school had an enrolment of about 1500 learners in a coeducational setting. Out of the 248 Grade 11 learners, 158 of them were taking Mathematics and the rest were registered for Mathematical Literacy. The school that was used is where one of the authors of this study was working which meant easy access to learners and also reduced financial costs. The 35 selected learners were from one of the six Grade 11 classes at that school. The class was chosen because it had serious challenges in Mathematics. The selection was done with the intention to clearly check if CBI could improve learners' conceptual understanding and performance. Using that class would mean that changes brought about by the implementation of CBI could be clearly identified. Using low-performing learners would also give them a chance to improve academically.

The study started with all the 35 participants writing a pre-test to identify the level of understanding at which the learners were operating. An intervention that involved teaching learners using the concept-based approach was implemented. The intervention concentrated on the Grade 11 Functions topic. This topic was chosen after realising that learners had poorly answered questions on that topic in the previous end-of-term test and also because of the fact that it is a topic that is crucial in Paper 1 in the Further Education and Training (FET) phase of South Africa. The intervention focused attention on the following functions: parabola of the form $y = a(x + p)^2 + q$; hyperbola of the form $\frac{a}{x+p} + q$; and the exponential of the form $y = a.b^{x+p} + q$. For these functions, learners were taught to sketch and interpret graphs, determine the effect of the parameters (a, p and q), determine average gradient between two points, and effects of reflection. The fact that learners had an average of 20.26% in the test before intervention indicates that, although learners had been exposed to the topic before, they were having challenges, which aligns with the findings by Chimhande et al. (2017) and Dlamini et al. (2017) who discovered that learners' understanding on a topic can be characterised by vague definitions and memorisation of procedures resulting in them having difficulties in linking different representations of concepts.

According to the National Curriculum Statement (NSC 2011), learners are expected to acquire these skills: sketching graphs, exploring, analysing, interpreting, comparing, illustrating, discussing, detailing and evaluating. During the intervention, the CBI that was employed focused on deeper understanding and connections between different aspects of functions. This included extensive exploration of function properties, real-world applications and problem-solving strategies which were not involved in the traditional approach that had been used before. Learner engagement was possible for all these mentioned skills. Moreover, the topic involves a lot of aspects that are linked to other topics for Mathematics, meaning that if a learner does not grasp the functions concepts well then they will not be able to perform well overall.

In this study pre and post testing were done to evaluate the impact of the intervention on learners' understanding of functions. The participants had previously been taught functions and had written an end-of-term test that included functions so the researchers set a test on functions for this study. The test questions included application and synthesis of function concepts. The study pre-test and post-test were meant to measure changes in performance. The same test which was administered before and after intervention had 13 questions. The questions were set in line with the NSC (2011) for South Africa. Various questions from past examination papers and Grade 11 textbooks were incorporated. The questions were meant to test learners' knowledge, comprehension, application, synthesis and evaluation. As in Appendix 1, questions 1, 2, 4, 9, 10, and 11 involved application of procedures and required basic information and facts while 3, 5, 6, 7, 8, 12 and 13 tested learners' ability to analyse, interpret, evaluate and express opinions.

The one-hour lessons for the intervention went on for three weeks. The lessons involved activities that allowed participants to take more active roles in their learning journey. During intervention, the participants were given a chance to come up with their own solutions mostly. At times they were guided to discover and build their own knowledge.

The intervention was designed to enhance conceptual understanding and to expose learners to scenarios where their incorrect information would be ineffective (Cusack et al., 2018). Participants worked on solving given problems using a variety of strategies. During intervention lessons, follow-up questions were posed to seek clarification and deepen conceptual understanding. Participants were invited to ask questions if they required additional clarification. They were given ample time to analyse and justify their reasoning. Participants employed a range of illustrations primarily, without help from the educator. The lessons mainly aimed to enhance conceptual understanding.

Descriptive and inferential analysis were done on pre-test and post-test data. The results from the two tests were summarised to compare performance changes before and after intervention. There was computation of the arithmetic mean, standard deviation, five number summary and range of data for the two tests results, to describe and explain the effects of CBI. An dependent *t*-test was run to examine changes in performance of the participants. In addition to the inferential analysis, descriptive analysis was performed on the qualitative data collected through questionnaires and interviews. The questionnaires provided a summary of participants' attitudes, perceptions, and experiences with CBI, using both closed and open-ended questions to quantify trends. The interviews offered more detailed, narrative insights, giving participants the opportunity to elaborate their learning experiences and perceived changes in understanding. Qualitative analysis was meant to identify common patterns and variations in responses, offering a broader perspective on the impact of CBI, which complemented the quantitative findings from the tests.

Results

The guiding research question for this study was: *What was the effect of CBI on learners' performance in Mathematics?*

The study's intention was to investigate whether CBI could improve performance in Mathematics. While the researchers acknowledge that there is no single solution to poor performance in Mathematics, this study focuses on exploring the potential benefits of CBI as one of several strategies that could contribute to enhancing mathematical understanding. By examining the effects of this approach, the research seeks to add to the broader conversation on the effective teaching methods which may include factors such as self-efficacy, metacognition and motivation. Eight subthemes were thematically identified to direct presentation and analysis of collected data. These were the subthemes: comparative analysis of data from the two tests, learners' emotions towards concept-based teaching, grasping of concepts, effectiveness of educator' explanations, interactions between educator and learners, learners' understanding of concepts, learners' views about the teaching and learning approach for the intervention, and freedom to express oneself.

In the excerpts **T** represents teacher and **L** represents learner.

Comparative analysis of pre-test and post-test results

The marks achieved by participants in the pre test and post test were analysed. The computations included minimum, maximum, mean, median, standard deviation, range, lower and upper quartiles for the two tests. The presentation of these measures of central tendency and dispersion are presented in Table 1.

Looking at the minimum and maximum marks for the two tests there is greater variation indicating improvement in learner performance. The average score for the pre test (20.26%) was significantly lower than on the post test (70.23%). The range of scores for the two tests was approximately 50%. The lowest 25% of the participants scored below 8% in the pre test, while in the post test the bottom quarter had a maximum of 66%. The lower half of the participants scored under 18% in the pre test whereas in the post test the minimum score for half of the participants was 75%. Three-quarters of the participants in the pre test scored below 29% whereas 66% was the minimum mark for three-quarters of the participants in the post test. Moreover, the top quarter achieved at least 85% in the post test. The highest marks for the pre test and the post-test were 52% and 100% respectively. There was a considerable variation in scores between the two tests, as indicated by high standard deviations and ranges, reflecting substantial difference in performance in the two tests.

It is reasonable for one to expect that marks would go up with the additional instruction (CBI). However, the significant improvement observed from an average score of 20.26% to 75.23% highlights the effectiveness of CBI approach in addressing conceptual learning gaps, thus enhancing understanding beyond what was achieved with the traditional teaching approach.

To illustrate the success of the intervention, a bar graph was used to visually depict the significant improvement in performance from pre test to post test for the learners' scores. Figure 1 illustrates the overall performance for learners in the two tests.

Three-quarters of the learners scored 0% and 29% in the pre test, with only 10% of the learners scoring above 40% while in the post test 80% of the learners scored more than 60%.

TABLE 1: Measures of central tendency and variability.

Test	Minimum	Maximum	Range	Mean	Standard deviation	Q_1	Q_2	Q_3
Pre test	2	52	50	20.26	14.12	7.5	17	28.5
Post test	38	100	62	75.23	16.73	66	75	88

The dependent *t*-test was conducted to compare means for the two tests at 95% level of significance. The null hypothesis (H_0: $\mu_1 = \mu_2$) posited that there was no significant difference between the two means while the alternative hypothesis (H_1: $\mu_1 \neq \mu_2$) suggested a significant difference. In this presentation μ_1 and μ_2 represent means for the pre test and post test respectively. The results of the *t*-test showed a t-value of 33.16 with 34 degrees of freedom. The obtained *p*-value of 0.0001 ($p < 0.05$) was much lower than the significant levels of 0.005 resulting in the rejection of the null hypothesis, and opting for the alternative one. This indicates a statistically significant difference between the pre-test and post-test means. The implication was that the intervention led to significant improvements in learner performance as evidenced by the substantial increase in post-test scores compared to pre-test scores.

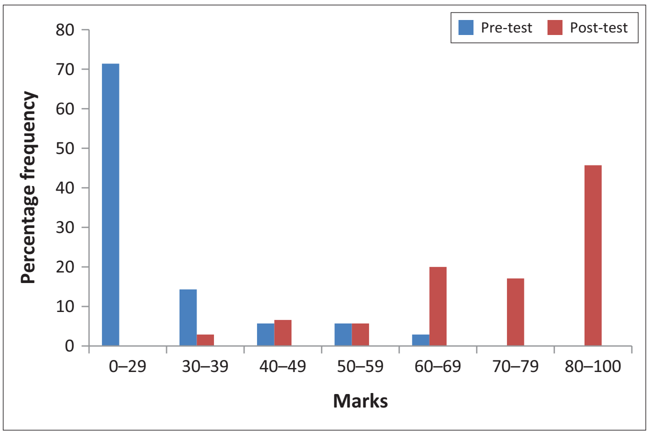


FIGURE 1: Results for pre and post tests.

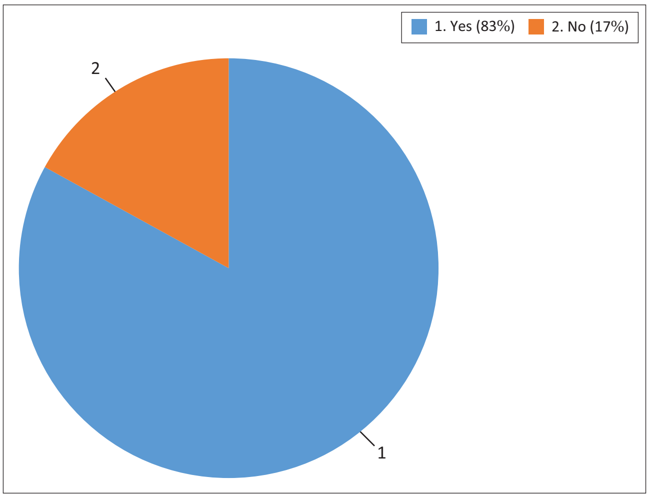


FIGURE 2: Learners' enjoyment of concept-based instruction.

Learners' feelings about the intervention instructional approach

One part of the questionnaire required participants to indicate what they felt when they were learning functions during intervention time in terms of enjoyment. From the findings, it was deduced that 29 out of the 35 learners enjoyed the intervention lessons. The responses for enjoying or not enjoying the lessons are illustrated in Figure 2.

Enjoyment according to Bourne (2018) and Schukajlow (2015) is associated with positive attitude in Mathematics, confidence and willingness to perform activities. In the questionnaire there was a section for learners to express themselves in their own words as shown in Excerpt 1.

Excerpt 1: Learners' responses from the questionnaire

L5: I truly enjoyed the fun and engaging lessons. Before this, I had difficulties in understanding most of the things that we were taught.

L11: It was absolutely fantastic. I appreciated that the teacher began with the basics and progressed to the complex.

L5 associated having fun during intervention lessons with increased lesson engagement. Bordbar (2019) supports this, stating that positive emotions related to activities, like enjoyment, enhance learners' curiosity, effort and involvement. L11 complimented the manner in which the lessons were delivered, that is, from known to unknown things. The CBI that was used during intervention likely contributed to learners' enjoyment and the development of their understanding.

Learners were given more time to work on their own, then use the other part of their lesson to present and discuss their findings. The teacher would first give instructions explaining what learners were supposed to do then ask them to start working on the given tasks individually and at times in pairs or groups. In the interviews, learners expressed their feelings as indicated in Excerpt 2.

Excerpt 2: Interview responses

L5: The teacher provided us with sufficient time to grasp the basic concepts and she clearly gave explanations to guide us. The process of building knowledge was simplified as we were able to follow along easily with many examples given. I now know more about asymptotes and axes of symmetry for given functions.

T: So do you now understand what an asymptote is?

L5: Yes, I am extremely happy I now know that it is the set of points that the graph approaches but will never touch.

T: Which functions have asymptotes when you sketch them?

L5: I know we have the hyperbola and the exponential.

T: That's great. And which functions have axes of symmetry?

L5: The parabola and the hyperbola.

T: Can you give anything of special interest concerning in the axes of symmetry for those two functions that you have mentioned.

L5: Yes, the hyperbola has two lines as axes of symmetry, one with negative gradient and the other one with positive gradient, the point of intersection for the asymptotes is also the point of intersection for the axes of symmetry. As, for the parabola, it has one line of symmetry that passes through the turning point. I now know this well mam. You can ask more questions if you want.

Before the intervention it can be seen that the learner who was interviewed did not know much about asymptotes and axes of symmetry. The learner's responses reflect a positive reaction to CBI, highlighting key elements of the teaching approach that contribute to understanding. After intervention, the interviewee's responses display that he had more knowledge about asymptotes and axes of symmetry. The responses suggest that the CBI positively impacted the learner's understanding. The sufficient time, clear explanations, guided support, and practical examples helped the learner build a deeper understanding of mathematical concepts. This is an indication that CBI, when applied well, can enhance learner engagement and comprehension

Grasping key concepts in learning

When participants were requested to provide explanations of how they arrived at their solutions, interesting responses were given. There was a learner who could not answer the question that required distance between two functions but got it right in the post test. The learner's explanation in the interview is shown in Excerpt 3.

Excerpt 3: Interview responses

T: The question required you to find an expression for the distance between R and T (RT). Initially your answer was 2 but you got a different answer was in the second test. Can you please explain how you determined the expression in the second test?

L11: Because RT connects the 2 graphs and runs parallel to the *y*-axis, RT stands for the *y*-values difference for the two graphs which in this instance is represented by g(x)–h(x).

L11 demonstrated a clear understanding of the key concept by recognising that distance RT, which runs parallel to the *y*-axis, is the vertical distance between the *y*-values of the two graphs. This is an indication that the learner gained conceptual understanding. Therefore, the learner's knowledge building was enhanced by CBI.

After intervention learners were asked in the interviews how they had understood what was taught. From their responses it was established that the teaching and learning approach that had been used helped a lot in building their knowledge. The narration in Excerpt 4 can confirm what transpired.

Excerpt 4: Interview responses

T: How were the lessons on functions overall?

L24: They were improved because each lesson allowed time for reflection explanation on our understanding. Explaining concepts to peers made them clearer and I was able to come up with additional examples that clarified what we had learnt.

L24's response illustrates the effectiveness of CBI in deeper processing of material, reinforced understanding and demonstration of ability to generalise and apply concepts. Overall, the learner's journey from confusion to clarity, supported by time for reflection and interaction with peers, underscores the value of the instructional approach in fostering a more thorough and transferrable understanding of complex mathematical concepts like the distance between two points on different graphs.

Effectiveness of teacher's explanations

In the questionnaire there was a section that allowed learners to evaluate the effectiveness and clarity of the explanations that the educator provided during the intervention. Out of the 35 participants, 29 stipulated that the explanations from the teacher were clear. Figure 3 indicates learners' responses.

Most of the participants indicated that the educator's explanations were clear which aligns with the findings by Chinn (2020). The studies highlight that providing support is crucial for understanding mathematics. Following Hussain and Cambria's (2018) arguments, explanations are vital for developing conceptual understanding which is essential in mathematics education. Explanations facilitate the integration of prior and new knowledge.

Another learner in the questionnaire indicated this (Figure 4).

The learner acknowledged that the explanations from the teacher were clear which enabled him to enjoy and understand what was being taught. Explanations enhance motivation and concept building.

Interactions between educator and learners

To show that there was notable improvement in learners' understanding, let us start by considering one of the learner's solutions for the pre test (Figure 5) and post test (Figure 6) then go on to the same learner's interview responses.

The learner's solution indicates that the learner had no idea of what domain and range were before intervention then in the post test the learner seems to have gained conceptual understanding as shown in Figure 6.

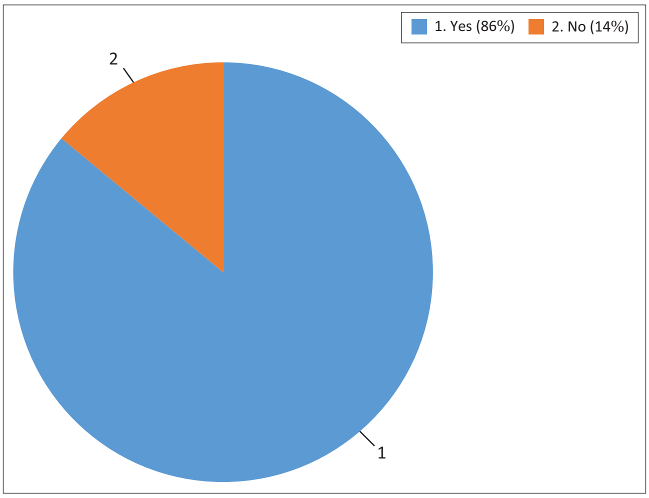
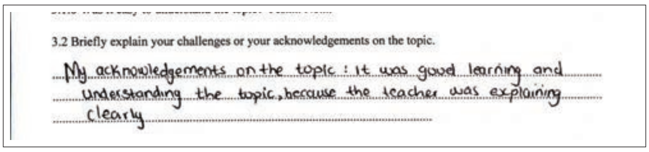


FIGURE 3: Learners' assessment on educator's explanations during intervention.


3.2 Briefly explain your challenges or your acknowledgements on the topic.

My acknowledgements on the topic : it was good learning and understanding the topic, because the teacher was explaining clearly

FIGURE 4: L10's response in the questionnaire.

After intervention, the solution by L13 shows clear enhancement of conceptual understanding. This change demonstrates the effect of the intervention in addressing the learner's needs.

The change in the learner's understanding demonstrates the effect of the intervention in addressing the learner's needs. The learner's performance after intervention is supported by Assefa (2016) who found that lack of conceptual understanding greatly affects learners' performance in interpreting mathematical procedures. Let us also check what was deduced from L13's responses in the interview.

Excerpt 5: Interview responses

T: I would like to review your answers for question 2 from both tests. You were unable to determine range and domain for given functions in the first test, then you managed to get correct answers in the second test. How did the educator assist you with this?

L13: I can say the questions she asked me during the process of learning functions for the second time, helped me create a mind-map.

T: What problems did you encounter in the first test?

L13: Honestly speaking I did not know how to determine domain and range.

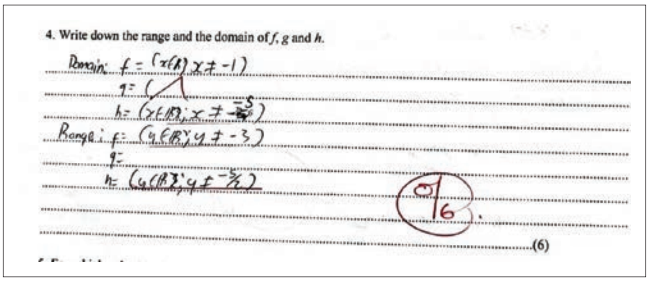
T: Were you not taught about range and domain before the pre-test?

L13: We were taught, but I did not understand.

T: So, what helped you to come up with correct answers in the second test?

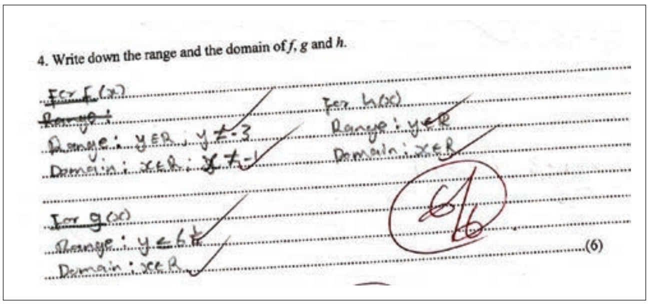
L13: It became easier after the teacher explained what to look at when determining range and domain. The teacher also asked us questions giving us a chance to explain according to what we had understood. What was interesting is that I did not know the effect of the turning point on range of a parabola. Now I know that if I know the y-value of the turning point and also if the point is minimum or maximum then I can state the range of the parabola.

L13's responses indicated that questions and explanations are effective in concept building. The teacher's questions promoted conceptual understanding by requiring the learner to develop answers rather than just recalling and applying algorithms. At some point, L13's response seems to imply that maybe learners were not taught about the domain and range. However, the learner meant that before intervention they did not understand the concepts. This lack of understanding which showed initial struggle with concepts led to lower scores in the pre test. Nevertheless, the purpose of the intervention was to address these gaps in understanding.


4. Write down the range and the domain of *f*, *g* and *h*.

(6)

FIGURE 5: L13's solution on domain and range.


4. Write down the range and the domain of *f*, *g* and *h*.

(6)

FIGURE 6: L13's solution after intervention.

Therefore, the improvement in marks from pre test to post test reflects not only the learners' unfamiliarity with concepts but also the effects of the instruction in helping them to grasp the concepts.

The interviewee was able to find range and domain for the given functions after the intervention. According to the interviewee's response, the range of the parabola was made easy through linking maximum and minimum of a graph to highest or least distance respectively. In this regard, long-term memory, cognitive, physical and social development were nurtured by enjoyment and it helped learners to grasp concepts as well as building their conceptual understanding. The responses of L13 highlight the following key effects: clarity in understanding, active engagement and specific concept mastery. The responses indicate that CBI improves learners' conceptual understanding and problem-solving abilities by focusing on core concepts and promoting active engagement.

Educator-learner interactions during intervention played a crucial role in addressing misconceptions and deepening understanding of functions. The interactions facilitated immediate feedback, and guided learners through complex concepts. There was enough time for scaffolding. The questioning, explanations and interaction were not just general teaching techniques but were tailored to enhance both content and comprehension. Learners were encouraged to actively participate, asking open-ended questions. There was scaffolding, clarification of misconceptions, encouraging collaboration, tailored instruction, positive reinforcement and reflective practices.

Evidence of learners' understanding of concepts

Interviews carried out provide evidence that learners had gained conceptual understanding. The solution displayed in

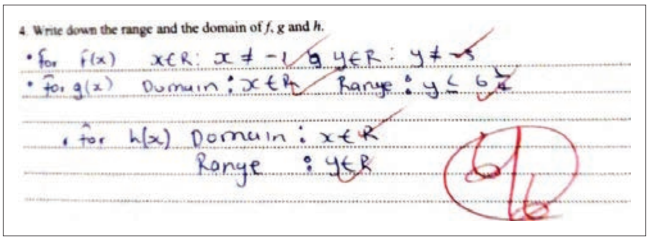


FIGURE 7: L35's solution in the post test.

Figure 7 also confirms that the learner knew what the question wanted.

The pre test was designed to identify baseline knowledge and misconceptions while the post test assessed how well the facilitated learning was. The results show progress made in understanding domain and range.

L35 was then interviewed and the following explanation was proffered about one of the test answers:

Excerpt 6: Interview responses

T: Referring to your Question 4 solutions in the two tests, initially you provided wrong answers for domains and ranges for all the functions but in the second test you got them all right. In the first test what was the issue?

L35: In that test I could not remember what range and domain were.

T: You mean to say you did not know what range and domain meant?

L35: Yes, I had totally forgotten.

T: Can you now provide a brief explanation of what domain and range are.

L35: Domain is a collection of all input values and range is the set of all output values.

T: So, if I give you function now, can you determine its domain the range?

L35: Of cause yes, for I now know what to concentrate on in order to determine them.

T: What do you mean?

L35: What I mean is, for the range of a parabola, I determine its turning point which gives me be minimum or maximum y-values, domain is given by all real numbers. Then for a hyperbola, I exclude the asymptotes from all real numbers for both domain and range.

T: Great, can you give an explanation for how to determine range and domain for the exponential?

L35: The exponential is easy, domain is given by all real numbers and for the range the asymptote plays an important part. The values are either greater or less than the asymptote.

Even if the learners had been exposed to domain and range before intervention, the pre test revealed significant gaps and misconceptions. This indicates that the previous teaching did not fully address these concepts or were not sufficiently clear. The fact that learners struggled in the pre test suggests that the previous instructional approach was not effectively applied. The need for CBI intervention arose from the identified gaps. All in all, exposure to a topic does not equate to mastery of concepts. The intervention was crucial in bridging the gaps and improving understanding. In the discussion, L35 presented his arguments logically. L35 at one point mentioned that she precisely knew what to take into consideration when determining the domain and range of function. L35 was able to integrate and relate her knowledge, allowing justifications of her answers. The responses by L35 indicate deeper understanding of concepts after intervention. The other learner responded as follows:

Excerpt 7: Interview responses

T: Describe the sudden improvement in your ability to find the parabola axis of symmetry between the first test and the second test.

L24: In the first test I was not sure of what to write, which is why I did not answer the question. Nevertheless, because of the teacher's clear explanations when we were learning for the second time. I understood that the axis of symmetry is a line that divides an object into two symmetrical halves. So for a parabola I realised that the axis of symmetry is parallel to the y-axis and passes through the turning point, making x = turning point x-value the equation of the axis of symmetry.

L24 demonstrates a thorough comprehension of axis of symmetry. During the interviews learners displayed plausible knowledge on functions which is a sign that they understood concepts that were taught during intervention. To the researcher this meant concept-based teaching provides learners with sufficient understanding of concepts.

The fact that L24 did not attempt to answer all questions in the pre test is a sign of a significant gap in understanding. It is crucial to take note that leaving blank spaces is a good sign to distinguish between prior experience and actual understanding or ability to effectively apply known concepts. The blank spaces depict that exposure to a topic does not guarantee mastery of what is taught. The blank spaces imply that despite any teaching and learning, L24 was unable to effectively apply or articulate the understanding of axes of symmetry. Leaving a question unanswered can be a significant sign that the previous instructional approach might not have been sufficient. This could also be a result of incomplete explanations, lack of practice or insufficient examples.

The CBI aimed to address learners' challenges in mathematics including the above-mentioned by providing more targeted instruction. Learners' gaps indicated lack of understanding and the intervention then focused on helping learners to develop a more accurate and comprehensive understanding of functions. The improvement that was displayed in the learners' responses following the intervention demonstrates that CBI effectively addressed the identified gaps. The fact that learners provided correct answers on the post test is an indication that CBI successfully enhanced learners' comprehension. The goal of the intervention was to build on existing knowledge and fill the gaps. The results of the pre test provided a baseline for where learners were struggling, while the post-test results reflect the progress made because of CBI.

Views of learners about the recently used instructional approach

During the intervention lessons, learners got the chance to independently build their knowledge alone or collaboratively with peers. The questionnaire had a section that asked learners if they had the opportunity to describe and explain mathematical phenomena. From the responses, 83% of the learners indicated that they got a chance to do so. During interviews, the researchers also inquired how learners viewed the concept-based approach.

Excerpt 8: Interview responses

T: How do you perceive the teaching and learning approach that has been used recently in your lessons for Mathematics compared to the other methods that have been used before?

L11: It was fine and great. Many different activities were involved. We understood many things which we did not understand in our first learning of functions.

L13: Teacher must continue teaching us with these methods. It was good.

L23: We were taught slowly as if we were playing. It was enjoyable. I had time to get answers on my own.

L35: The teacher taught us patiently giving us activities to work alone and discuss to come up with solutions.

Taking the interview responses into consideration it can be concluded that the participants valued the concept-based instructional approach that was implemented during intervention. The participants' feedback aligns with findings from other researchers like Schnitzler et al. (2021), who highlighted that learners might feel overwhelmed or lose interest if lessons are poorly presented. L23's responses indicate that there was increased understanding, enhanced engagement and motivation. Therefore, learners recognised the benefits of concept-based teaching and learning.

Freedom to express oneself

The following is from the interview with L23 who admitted that the educator gave them an opportunity to articulate and clarify mathematical phenomena:

Excerpt 9: Interview responses

T: After the intervention, did you notice any changes in the way you grasp concepts?

L23: Yes, I realised that I could connect many concepts.

T: What enabled you to connect those concepts?

L23: The reason was that the teacher allowed us to work independently, giving us enough time to explore and identify relationships. We were not provided with direct answers this time.

T: Oh that is great. What then do you suggest should be done to help learners grasp mathematical concepts?

L23: Teachers should avoid simply providing us with answers and making us memorise rules and theorems for Mathematics. Relating Mathematics to our daily experiences and activities can make it easier for us to understand concepts.

The above interview excerpt reveals that participants had an opportunity to articulate and clarify mathematical phenomena. There was a reduced focus on memorising learnt stuff. The participant indicated how he felt after realising he came up with his own solutions instead of depending on the teacher. The responses from L23 suggest that prior teaching might not have been effective. The pre test having a very low average mark of 20.26% indicates poor initial understanding of functions, despite previous experiences. This does depict that the previous instruction may not have effectively conveyed the concepts, which is a valid concern. It is possible that aspects of effective teaching like clear explanations, engaging question techniques, or sufficient practice opportunities were not fully utilised. Context-based instruction was implemented to address gaps in understanding that were identified from the pre test. The aim was to provide a more comprehensive approach to teaching functions by promoting deeper understanding through CBI.

Discussion

An investigation into the impact of CBI on mathematics teaching and learning was conducted with the guiding research question: What was the effect of CBI on learners' performance in Mathematics?

Data were gathered from a group of 35 participants from one high school through questionnaires, tests and interviews. The data were illustrated using tables, diagrams and excerpts, and were statistically analysed. Effects of CBI were highlighted using the gathered data. Positive effects of the CBI approach were observed from the way learners exceled in the post test. Statistical evidence indicated that CBI led to improved learner achievements. The improvement in participants' performance suggested a better understanding of concepts.

It is important to recognise that attributing improvements to CBI alone can be misleading: prior instruction might have had limitations like lack of sufficient time allocated to finish content to be tested. However, this does not negate the possibility that prior instruction might have lacked effectiveness or that CBI might have contributed to the observed changes. The pre-test results indicated that learners struggled with concepts which points to potential gaps in the previous instruction. This highlights the importance of evaluating and improving teaching methods to ensure they effectively support learners' understanding. While CBI was associated with improved performance, it is one of the several factors that influence outcomes.

According to Borji et al. (2021), conceptual understanding plays a significant role in sustaining performance in mathematics. However, it is important to recognise that performance sustainability is influenced by a combination of factors including motivation, self-efficacy and learners' disposition. Learners enjoyed the topic functions, particularly the way it was taught. From the questionnaire and interviews, learners indicated that they appreciated the concept-based teaching and learning approach. The enjoyment also helped to nurture long-term memory and improve learner engagement. Ainley and Ainley (2017) suggest that improvement in learners' performance is an indication of enjoyment of lessons. This means the intervention brought cognitive, social and physical positive changes in learners that aided their conceptual understanding. In the pre test there was clear indication of lack of conceptual understanding in learners. In the post test, it seems learners had acquired knowledge which led them to achieve higher marks. The researchers associated learners' enjoyment with understanding of concepts.

Mastering of concepts as suggested by Permana et al. (2019) contributed effectively to learners' critical thinking skills. All responses in the interviews confirm deeper understanding of functions. There is also evidence that the teacher engaged well with learners, explaining, guiding and emphasising concepts. Chauraya and Brodie (2018) claim that explanations capture learners' interest, encourage them to make connections between information, enhancing retention levels and reinforcing their learning. Schnitzler et al. (2021) adds that explanations help in clearing misconceptions. The teacher identified some of the learners' problems and cleared misconceptions. Learners were fully engaged during intervention. They had the teacher asking them questions as a way of guiding them to discover and build concepts. Interaction between the teacher and the learners gave learners a chance to be more active in their learning. The teacher's probing questions and instructions led to improvements in concept building.

There was enough justification of solutions from the interviews to indicate that learners had managed to build their conceptual knowledge. The gained conceptual knowledge enabled them to make connections and applications as they tackled given questions. The CBI teaching and learning approach gave learners a chance to evaluate their capability in solving problems on their own. The teacher was a facilitator and mediator, guiding learners to build their own conceptual understanding. Using the CBI approach during intervention proved advantageous as highlighted by the benefits that have been mentioned. Consequently, the researcher concluded that the CBI approach is a more effective than the traditional teaching and learning approaches.

The findings indicate that learners taught with the concept-based instructional approach found mathematics aspects more comprehensible than previously. They had ample opportunity to actively engage in their learning. This aligns with Davadas and Lay's (2017) observation that learners exposed to a concept-based approach view mathematics as an active, inquiry-based field. The improved scores on the post test depict conceptual understanding facilitates meaning making and the construction of meaningful systems, suggesting that learnt material was refined (Chiphambo, 2018). Learners were in a position to build their personal knowledge and skills, leading to a better grasp of concepts and improved information retention.

The comparison of pre-test and post-test scores shows significant gain in scores. Interview responses indicate a change in participants' thinking levels. According to Summermann et al. (2021), transformation is a result of learners integrating ideas and facts, and engaging in synthesis, generalisation, explanation, hypothesis formation or conclusion drawing. The CBI approach enabled participants to think critically and coherently present their arguments, as reflected in their interview feedback. Through the questionnaires, participants noted that being allowed to articulate mathematical concepts widened their understanding. This opportunity of self-expression in class fostered open-mindedness, flexibility, curiosity and motivation, boosting their confidence and ability to debate and deliver facts in an organised manner. The benefits of using the CBI approach during intervention highlight its effectiveness.

The intervention aimed to address areas of difficulty and reinforce understanding through CBI. To draw valid conclusions about teaching effectively, it is crucial to consider a comprehensive evaluation of both the intervention and the prior instruction. The experience with CBI provides valuable insights into effective teaching practices. It underscores the need for ongoing reflection and professional development to integrate successful elements from various instructional approaches and continuously enhance teaching quality. The study suggests implementation of the CBI approach in the teaching and learning of mathematics, emphasising the need for educators to be aware of the need for learner support in concept building. The study also recommends that educators participate in development programmes to gain expertise in concept-based teaching and learning approaches.

Conclusion

The study intended to identify a teaching and learning approach that could help learners to develop conceptual understanding and enhance achievements. Pre-testing and post-testing, questionnaires and interviews were the data sources of this study. Learners' poor performance in Mathematics was the main worry of the researchers. Poor performance in Mathematics can result from various factors, and lack of conceptual understanding is just one potential cause. Other factors that play significant roles are: instructional quality, motivation, study habits and external influences. The study then banked on lack of conceptual understanding as having a significant impact on learners' Mathematics performance. Based on review of existing literature and observations in the teaching field, lack of conceptual understanding was linked to ineffective teaching and learning approaches. Consequently, the researchers investigated the impact of CBI on mathematics teaching and learning.

Better outcomes in Mathematics are achieved when the underlying aim of teaching is to promote conceptual understanding among learners. The findings from this study indicate positive gains in performance and concept building. The study established that CBI also had positive effects on learners' attitude, beliefs, motivation and knowledge development. From the findings, it was established that CBI has a lot of valuable properties that can aid or replace traditional teaching and learning approaches. Concept-based instruction encourages learners to build their knowledge, understand concepts, think critically, permit higher-order thinking and to justify their thinking.

In conclusion, CBI significantly enhances learners' conceptual understanding and improves their performance in mathematics by promoting deeper understanding of underlying principles rather than rote memorisation. By focusing on fundamental concepts and their applications, this approach encourages learners to make connections between different mathematical ideas leading to improved problem-solving skills and greater retention of knowledge. Consequently, learners are better equipped to tackle complex problems and apply mathematical concepts more flexibly across various contexts. Overall, CBI supports more meaningful learning experiences and fosters greater mathematical proficiency.

Hence, the researchers suggest that CBI should be adopted more broadly in mathematics education to address issues of conceptual gaps and to support better learning outcomes. Additionally, the study recommends ongoing professional development and reflection for educators to effectively implement this approach. To ensure ongoing effectiveness, it is valuable to continuously evaluate and refine teaching methods.

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Competing interests

The authors declare that they have no financial or personal relationships that may have inappropriately influenced them in writing this article.

Authors' contributions

M.N. and K.L. did the contextualisation. M.N. did the collection of data. M.N. did the analysis and drafting. K.L. edited the article.

Ethical considerations

An application for full ethical approval was made to the University of South Africa College of Education Review Committee and ethics consent was received on 14/11/2018. The ethics approval number is 2018/10/17/48809969/57/MC.

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Data availability

The data that support the findings of this study are openly available from the corresponding author, M.N., upon reasonable request.

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Appendix starts on the next page →

Appendix 1

Grade 11 mathematics test: Functions

Learner code: ....

Date: ....

Instructions

1. T: 1 hour.
2. Answer all questions.
3. Show all working where necessary in the spaces provided.
4. The use of an electronic calculator is allowed.

Consider the graphs defined by $f(x) = \frac{2}{x-1} - 1$, $p(x) = \frac{-1}{2}(x + 1)(x - 2)$ and $q(x) = -2x + 1$

1. Draw f, g and h on the same system of axes. Clearly label all asymptotes and intercepts with the axes as well as the coordinates of any turning points.

(10)

2. For which value(s) of x *is* $\frac{-1}{2}(x + 1)(x - 2) > 0$?

....

....(2)

3. For which values of x is $(x).q(x) < 0$?

....

....(3)

4. Write down the domain and the range of f, p and q.

....

....

....

....

....(6)

5. For which value(s) of x is $f(x) = q(x)$

....

....(2)

6. If K (2; 1) a point on function f is reflected in the axis of symmetry with positive gradient, determine the coordinates of its image.

....

....

....

....(5)

7. Express g in the form a $(x-p)^2$ + q. Hence or otherwise write down the equation of the axis of symmetry and the coordinates of the turning point of g.

....
....
....
....(5)

8. MN is a line parallel to the y-axis, with M on p and N on q.

8.1 Determine a simplified expression for the length of MN.

....
....
....(2)

8.2 Hence find the maximum length of MN.

....
....
....
....
............................(4)

9. If $f(x)$ is translated 2 units to the right and 1 unit up, state the new equation in the form y =...................... .... (2)

10. If $q(x)$ is reflected in the y-axis, state the equation of the reflected graph in the form y = ...(1)

11. If $p(x)$ is translated 3units to the left and 2 units down, determine the equation of the translated graph in the form $y = ax^2 + bx + c$. ....(2)

12. Determine $f((q(-2))$. ....(3)

13. Determine the average gradient of $f(x)$ between: $x = 1$ and $x = -2$ ....(3)

Total: 50 Marks