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Quantitative Reasoning: What's Math Got To Do With It?

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Abstract

This keynote address explores the history and role of college math requirements with a focus on ensuring math courses serve to expand students' horizons, rather than serve as gatekeepers. It discusses the advent of general education math courses, which brought more students into math departments, which ultimately contributed to broadening the scope of the courses to align with more students' interests and majors, since their purpose was to advance quantitative reasoning, not mathematics skill per se. It also examines several practices to address calculus' gatekeeping role: revising placement practices and prerequisites, redesigning courses, and updating instruction and assessment practices. Lastly, it considers the role of college admission requirements in perpetuating an overemphasis on calculus coursetaking.

Keywords

quantitative reasoning, mathematics curriculum, general education, pedagogy

Cover Page Footnote

Pamela Burdman, a policy analyst and strategist on equitable college access, readiness, and success, is the founder and executive director of Just Equations, a policy institute focused on reconceptualizing the role of mathematics in education equity. She began her career as a reporter for the *San Francisco Chronicle* and first focused on math equity issues as a program officer at the William and Flora Hewlett Foundation, working with the early developers of new college statistics pathways. She is the author of numerous reports and articles on math opportunities that have influenced policy changes in K–12 and postsecondary math education in California and beyond. She earned a bachelor's degree in philosophy and East Asian studies from Princeton University and a master's in business administration and master's in Asian studies from University of California, Berkeley.

About Me[1]

I am not in a math department. I don't have a math degree. I'm a policy analyst with a focus on education equity, and especially the relationship between math education and education equity. As such, I likely can't tell this audience things you don't know, at least not about math and maybe not even about math education. But I do hope to stimulate your thinking by sharing observations from my journey researching and writing about math education.

As you know, sometimes a person observing from the outside can have a clearer—or at least a different—view from that of folks who are on the inside. It's that perspective I hope to bring.

The overarching theme that I want to address is the evolution of postsecondary math requirements, also known as quantitative reasoning requirements. I will also inquire into how best to align those requirements and design the courses to maximize meaningful math learning and support college success while minimizing gatekeeping.

But first, let me address how I, a nonmathematician, came to be exploring this topic. Early in my career, I was an education reporter for the San Francisco Chronicle. More recently, I worked in philanthropy. As a program officer, my focus on college success pointed to the role math requirements often played in delaying or deterring students' progress toward a degree, particularly at broad-access public institutions, including community colleges. I started writing about math education policies about 10 years ago.

To shed light on the patterns I had observed, I began looking for resources on why we had the math requirements that we had at the time.

Exploring College Math Requirements

College Algebra was extremely prevalent then; it was perhaps the primary college math requirement. Things have changed a bit since then. But I wondered, how did this course and other calculus prerequisites become the dominant math requirements, given that they are generally requirements for STEM, and most students do not major in STEM?

I didn't find any resources that laid out the background for these requirements. I had to consult surveys. Every five years or so, the Conference Board of the Mathematical Sciences conducts a survey of college math departments.[2]

[1] An earlier version of this paper was presented as a keynote address at the annual meetings of the National Numeracy Network on October 20, 2023.
[2] https://www.ams.org/profession/data/cbms-survey/cbms-survey

Those surveys produced some interesting discoveries. In 1970–71, the earliest available survey, university math chairs were asked whether a mathematical science course (defined as including mathematics, statistics, or computer science) should be a college graduation requirement for every student. Only 10 percent answered yes. That's right. Only 10 percent. I found this fascinating, because there was another, more recent survey, from 2010, that showed that nearly 90 percent of colleges and universities had a math requirement at the time (Schield 2010). This was quite a change over a 40-year period.

I also looked at enrollment in mathematical science courses overall and found a huge increase. From 1960 to 2015, enrollments nearly quadrupled. There was also a dramatic increase in college enrollment over the same period, but math enrollments grew even faster. Interestingly, I also discovered a drop in upper-division enrollments as a proportion of overall mathematical science enrollments—from 17 percent to just 10 percent. So the increase in overall enrollment was clearly not related to more undergraduates becoming math majors or going into fields that required advanced math.

At the same time, the surveys showed rapid growth in statistics enrollment. Statistics, as a percent of overall enrollments in mathematical sciences, went from around 3 percent to 17 percent, nearly a six-fold increase.

So then I tried to unpack why these things were happening. It appeared there were several factors at play. First of all, there was a push at a national level to excel in science, lead the world in STEM, and compete with the Soviet Union. At the same time, STEM majors were becoming attractive as paths to lucrative careers.

General Education Expands

Another key factor was the growth of general education in higher ed. This growth seems to have been related to the expansion of public higher education in this country through the GI Bill. As I noted, far more students were coming into higher education overall. In conjunction with that development arose the idea of adopting some minimum requirements to get a college degree.

There was this notion that, with far more students coming in, clearer standards were needed—quality controls, in effect. Math requirements were among the enforcers of standards through general education.

At the time, math departments tended to offer traditional math courses. They weren't initially thinking about offering service courses or survey courses for nonmajors that were newly entering their departments. That did eventually come about. But early on, not many math departments were thinking creatively about it. They were focused on the sequence of courses taken by math and other STEM majors. In surveys, department leaders voiced concern that students were entering less prepared to succeed in these courses than previously.

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While the survey reports never said so, it seems likely that this impression was driven by the fact that students who previously wouldn't have taken math in college—either because math hadn't previously been required for all students or because those students wouldn't have been going to college at all—were showing up in math departments. So accompanying the growth of general education, there was an expansion in entry-level math enrollments, and due to concerns about preparation, an expansion in remedial mathematics courses and sequences.

Then there is the role of statistics. Computers are ubiquitous. Big data is all around us. It's not surprising that stats enrollment has been increasing. In fact, it would be surprising if it weren't. At times, statistics has had an uneasy coexistence with mathematics at universities. Though statistics is often housed within math departments, at many research universities, the two are in different departments, with math faculty perhaps looking down on statistics as a discipline.

There is not great data showing this, but it's certainly my interpretation that another genesis for the expansion of statistics and other nontraditional math courses is the demand that resulted from more students being required to take a math course. For those outside of STEM pursuits, these courses were more relevant. Majors such as social sciences even started requiring them instead of traditional math courses.

Still, this begs the question: What is the purpose of these courses beyond saying we need gen-ed math courses because we have gen ed in other subjects? What's the purpose of them, and what content should they cover?

A Broader View of Math Requirements

Calculus is considered appropriate preparation for most STEM fields, along with a few other fields, such as economics. Other advanced topics can support student interest in a broader range of fields. Besides statistics, these include data science, mathematical modeling, and discrete math.

We needed to start thinking more broadly about how to prepare students mathematically. Not only is calculus not necessary preparation for all fields, but data science, statistical reasoning, and mathematical modeling are arguably essential to every—or nearly every—field of study, and also simply to living in the 21st century.

In addition to being increasingly relevant to contemporary careers, these courses can lend themselves to the kinds of student-centered teaching approaches that evidence suggests are more effective than traditional instruction. This is not to say that one can't teach calculus in a student-centered or effective way, but some of these other courses are naturally suited to doing so—and more obviously relevant for more students.

While there's been a lot of focus on calculus as the pinnacle of high school math achievement, math associations have actually pushed back on more traditional approaches to content and teaching. They are insisting that students need a broader spectrum of mathematical preparation, in addition to the math content that will enable them to pursue their field of study in college.

Along with this, associations have also noted the need to make math courses more engaging, to sustain students' interest in STEM pursuits. I'm using *math* in the broad sense. Here are some examples of what they say:

> We challenge department chairs to incentivize innovation for the sake of their students and the health of our discipline.
>
> —Joint Policy Board for Mathematics, 2014

> The educational offerings of typical departments in the mathematical sciences have not kept pace with the large and rapid changes in how the mathematical sciences are used in science, engineering, medicine, finance, social science, and society at large. This diversification entails a need for new courses, new majors, new programs, and new educational partnerships.
>
> —National Research Council, 2013

> Introductory mathematics courses often leave students with the impression that all STEM fields are dull and unimaginative, which has particularly harmful effects for students who later become K–12 teachers. Reducing or eliminating the mathematics-preparation gap is one of the most urgent challenges—and promising opportunities—in preparing the workforce of the 21st century.
>
> —President's Council of Advisors on Science and Technology, 2012

> There is a call to provide mathematically substantive options for students who are not headed to calculus. These entry courses should focus on problem solving, modeling, statistics, and applications. Current college algebra courses serve two distinct student populations: (1) the overwhelming majority for whom it is a terminal course in mathematics, and (2) the relatively small minority for whom it is a gateway to further mathematics. Neither group is well served by the traditional version of the college algebra course. There is a mismatch between a curriculum designed to prepare students for calculus and the reality that only a small proportion of these students subsequently enroll in calculus. We acknowledge the need to focus on the calculus sequence and ensure that pathways to it remain a high priority, as calculus is central to most further study in the mathematical sciences, but it behooves us to develop curricula effective for the majority of the population as well.
>
> —Mathematical Association of America, 2015

Math or Quantitative Reasoning?

In this context, how do we think about general-education math requirements? I looked at a number of definitions and noted that the word *math* often doesn't appear in the title or description of the requirement.

For example, Princeton University has a quantitative and computational reasoning requirement. According to the university's website:

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> Quantitative and computational reasoning is used to some degree in almost every area of learning. A strong foundation in quantitative reasoning helps students think clearly and apply quantitative methods to a wide range of projects and equips them to critically evaluate statistical claims.

Here is another, from the University of Washington:

> This requirement is intended to ensure that students have the opportunity to improve your capacities for and abilities to critically evaluate and effectively use information utilizing symbolic and/or numeric methods, or the theoretical study thereof.

Note that both of these talk about critically evaluating information. The California State University's Quantitative Reasoning Task Force (2016) adopted a similar approach, defining quantitative reasoning as:

- The ability to read, comprehend, interpret, and communicate quantitative information in various contexts in a variety of formats.
- The ability to reason with and make inferences from quantitative information in order to solve problems arising in personal, civic, and professional contexts.
- The ability to use quantitative methods to assess the reasonableness of proposed solutions to quantitative problems.
- The ability to recognize the limits of quantitative methods.

The argument for taking math, if we are going to call it math, has not been that all students need to know math, as defined in narrow terms. The argument has been that quantitative reasoning is important.

We see mentions of reasoning with quantitative information, making inferences, assessing reasonableness, recognizing limits, and interpreting and communicating quantitative information. What we don't see are the words *math* or *cosine* or *tangent* or *function*. None of those are mentioned because they are not the purpose. Rather, learning has a higher purpose, which math serves.

I note that this approach does challenge the association of traditional mathematics with a concept known as *rigor*. It requires thinking anew about rigor in mathematics, a topic I've blogged about.[3] The group Achieve the Core described rigor as, "deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades."[4] Rigor also means giving equal emphasis to "conceptual understanding, procedural skills and fluency, and application."

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[3] https://justequations.org/blog/getting-rigorous-about-mathematical-rigor

[4] https://achievethecore.org/page/1090/rigor#:~:text=Rigor%3A%20in%20major%20topics%20pursue,and%20application%20with%20equal%20intensity.&text=Students%20must%20be%20able%20to,of%20mnemonics%20or%20discrete%20procedures.

Diversifying Math Requirements

The thinking about quantitative reasoning has influenced higher ed institutions, particularly over the past decade or so. For example, in collaboration with the Charles A. Dana Center at the University of Texas at Austin, dozens of states have adopted diversified postsecondary math pathways that reflect the range of goals students pursue. Some of these offerings lead to careers in STEM. Others, such as quantitative reasoning courses, are well aligned with majors in humanities and communications. Statistics is often seen as being particularly important in the social sciences.

Offering multiple pathways is a good strategy for ensuring that more students complete their math requirements, but a statistics pathway should not be seen as a detour out of STEM. We don't want to direct students out of STEM or be indifferent about whether students are successful in STEM.

Some of this diversification was already happening quietly before it became a movement. For example, psychology departments were already offering statistics courses, but then, through the development of math pathways, these practices became part of a more official policy.

Math departments play a central role in some of the pathways work. Courses including statistics, quantitative reasoning, and the calculus sequence are typically offered in math departments. But if our focus is quantitative reasoning, we shouldn't be limited to math departments. Other departments have an important role to play, as is suggested by numerous examples of courses that meet general education requirements from universities across the country:

- **Biological Anthropology,** Macalester College
- **Surface/Groundwater Hydrology,** Macalester College
- **Cognitive Neuroscience,** Macalester College
- **Infections and Epidemics,** San Diego State University
- **Galaxies and the Universe,** Yale College
- **Game Theory and Political Science,** Yale College
- **Strategic Visual Thinking,** University of Georgia
- **Human Language as Computation,** University of Southern California
- **Foundations of Data Science,** University of California, Berkeley
- **Quantitative Reasoning for Civil Engagement,** San Francisco State University
- **Primitive Navigation and Wayfinding in the Natural World,** University of Michigan

The growing role of other departments is important. It means that students are learning the content in a context that is relevant to them, whereas taking a straight math course may not be as meaningful to every student.

The involvement of other departments opens up many more opportunities. It also raises questions about traditional math education and one of its features—or perhaps I should call it a bug.

Calculus and Gatekeeping

Despite the range of options that more and more schools are offering, gatekeeping remains common in traditional math courses. By that I mean math courses for STEM majors, courses such as calculus. By gatekeeping I mean using the courses to rank and sort students and decide whether they're allowed another opportunity, more than to provide the students a real foundation for future learning. I mean using them as weed-out courses.

Though it is a slogan, there is actually evidence that calculus functions as a weed-out course.[5] Women and students from minoritized backgrounds are not proportionally represented among STEM graduates (President's Council of Advisors on Science and Technology 2012). We know from research examining why students leave STEM majors that women and students from minoritized backgrounds disproportionately leave those majors, and that calculus is highly associated with their departures (Seymour and Hunter 2019).

My colleagues and I looked at various approaches to addressing calculus' gatekeeping challenge (Burdman et al. 2021). One is revising placement practices and the prerequisite pathways that lead to calculus.

Placement Practices and Prerequisites

More and more at community colleges, and some four-year universities, we're seeing a growing emphasis on placing students based on their transcripts, not simply their test scores. Say a student has taken and passed Algebra II in high school. Transcript-based placement means that that student is not going to be placed into Intermediate Algebra to repeat that curriculum, at least not if they earned a particular grade, as designated by the college.

Sometimes College Algebra repeats that content as well. Instead, the focus is on what the student has done, paired with the use of corequisite courses, rather than prerequisite remedial courses. Corequisites are approaches for helping students brush up on prior material as they learn new material—rather than relegating students to remedial courses.

But our research highlighted another category of prerequisites: courses that aren't considered remedial. They are credit-bearing courses, but they are prerequisites to calculus. Typically, these prerequisites don't count toward the requirements for STEM majors.

[5] https://www.mathvalues.org/masterblog/launchings20200501

We have pointed to the need to examine more deeply what these prerequisites look like, so that we understand how to optimize student success when it comes to these college-level prerequisites. When we looked at the 23-campus California State University system, for example, we found something very interesting. There were many different versions of the prerequisites that were required.[6] Depending on which campus a student attended, the prerequisite pathway to calculus for a student with the lowest possible placement was anywhere from one to four semesters.

We initially thought that range might reflect the fact that some of the campuses have less-prepared students who actually need—or are perceived to need—more semesters to get them prepared. But that wasn't the case. What we saw was that the proficiency of the student population based on placement test score data was not correlated with the pathway lengths.

We also wondered whether varying numbers of units for courses might explain the variation. Again, the units were all over the map. The number of units in the full sequence ranged from 3 units at two campuses to 17 units at one campus. That's the campus with the four-semester prerequisite path. Not every student at that campus had to take four semesters—only the least-prepared ones, but it's clear that we don't know enough about how to optimize student success if these practices are diverging so widely.

Most of the research we have seen suggests that the shorter the pathway, the more you are going to optimize student success. That is, even if the pass rate in a particular course goes down a little bit when more students access the course, the success in terms of the absolute number of students ultimately completing the sequence and getting into the next course goes way up because they're given access. Research has shown this time and time again. (Though of course there are limits to this. No one is recommending putting fourth graders in calculus classes.)

Lastly, we observed differences across campuses in the placement scores that were being used. Again, this illustrates the importance of collecting data and conducting research to ensure that these prerequisite approaches are effective in promoting success to create the shortest possible sequences that support success for students seeking entry to STEM fields.

Course Redesign

Redesigning calculus courses is another strategy for improving success. We observed three related types of redesigns:

1. Contextualization
2. Focus on big ideas and conceptual learning
3. A modeling approach

[6] https://justequations.org/resource/staying-the-course-examining-college-students-paths-to-calculus

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UCLA's life sciences department designed a math-for-life-scientists course. It focuses on modeling dynamical systems, something that biology and life sciences students need to do.

CSU East Bay pursued a focus on big ideas and conceptual learning. The resulting course dramatically increased the success of students, particularly students of color, by changing how students are engaged.

Macalester College has had a modeling approach in its introductory calculus course for many years. One of its design advantages is that it serves students who are new to calculus, as well as those who took calculus in high school. This way, all students are experiencing new material.

A disadvantage is that some of these innovations may face challenges within public systems, where students transfer between institutions, since students' progress within the calculus sequence wouldn't align unless all institutions adopted the redesign.

What strikes me about these approaches is that they share a great deal with the conversation about quantitative reasoning when we talk about numeracy. We're talking about how students can actually reason using mathematical and statistical modeling, but that raises questions when it comes to redesigning calculus courses. Namely, is there a risk that changing them amounts to watering them down in ways that jeopardizes student success in STEM courses or fields? That is certainly a risk, but it is not at all a certainty. In fact, I'd like to propose that it's also possible that the opposite could be true: that modeling approaches could actually result in deeper understanding that supports success in STEM fields.

One illustration is UCLA's math-for-life-scientists course. When it was first developed, there was huge pushback from the math department, which said that students in life sciences are required to take physics and chemistry and wouldn't be able to pass those courses without a traditional calculus course as a prerequisite. The pilot said otherwise. It turned out that the students taking the life sciences course did better in physics and chemistry than students taking a more traditional calculus class. Today, most life sciences majors at UCLA take the new course.

Another example comes from the Association of American Medical Colleges. A decade or so ago, when they outlined the math skills needed for medical school students, their focus was much broader than a list of math topics. It featured dynamical systems, extracting relevant information, making inferences, interpreting data, and quantitative numeracy, along with applying traditional algorithmic approaches and principles of logic.

Instruction and Assessment Practices

Of course another important strategy is the adoption of effective instruction and assessment practices. These emphasize pedagogical approaches such as interactive classrooms, active learning, group work, inclusive classrooms, and student networks. Interestingly, such approaches also align with many of those used in quantitative literacy, courses focused on numeracy, and the truth is, they should be integrated into all courses, even math courses with a capital M.

The point here is that quantitative reasoning is not just for non-STEM majors. This is why I am a mild critic of the nomenclature of three pathways I discussed earlier. If we talk about calculus, statistics, and quantitative reasoning pathways, we risk losing sight of the centrality of quantitative reasoning to all of those paths, and I don't think quantitative reasoning should have the reputation of being "math for poets" or something watered down, because all students need to reason quantitatively.

Math and College Admissions

Another issue with college math is the signals colleges send to high school students about preparation for college. My organization, Just Equations, conducted some research with the National Association for College Admission Counseling. We surveyed college admissions officers to understand how they think about the role of math in admissions (Anderson and Burdman 2022).

We confirmed that—outside of schools such as MIT, CalTech, and Harvey Mudd—calculus is rarely a requirement for admission. About 21 percent of colleges responding to the survey did expect calculus for technical majors, but in general, calculus is not an across-the-board requirement for all students.

At the same time, we found that calculus weighs heavily in admissions. It is seen as giving applicants an edge. There's a perception that students who take the course are more likely to succeed in college. Seventy-nine percent of respondents held that perception.

There are two concerns with this emphasis. First of all, not all students have access to calculus in high school. Black and Latinx students are far more likely to attend schools that don't offer it—or to be tracked out of the calculus pathway in middle school. Secondly, it means an unnecessary de-emphasis on other areas of quantitative reasoning because students who want to get into a more selective college may feel that they have to put their interests aside to prove that they can take calculus and succeed at it.

Such practices reinforce what I call the prevailing architecture of math opportunity, which begins with misconceptions about math ability and who can do math and what it means to do math (Burdman 2018). These practices narrow our perceptions rather than broaden them to realize that everyone can do math and needs to do math. Existing inequities in access to educational resources reinforce this narrowing of opportunity.

Those inequities include things like implicit bias and stereotype threat. We also have a cultural tendency to treat math as a form of pedigree, with mathematical opportunities begetting future opportunities. The most advanced math options are reserved for a privileged few, and that enables them to have further opportunities, regardless of whether those opportunities truly require those math skills.

After admissions, what happens to students who took calculus? About 19 percent of them take Calculus II or a higher course. According to David Bressoud's (2016) analysis, the remaining students are either repeating Calculus I (with some of them earning a C or lower), taking a lower-level course that leads to calculus, taking a course for which calculus wasn't even a requirement, or not taking math at all. That was me. I took AP calculus and took no math in college.

In addition to surveying admissions officers, we also surveyed high school counselors (Burdman and Anderson 2022). What was interesting to us was that counselors were even more likely than admissions officers to emphasize calculus. As one of them said, "It's like an arms race for taking the higher-level courses, and the more everybody takes, the more everybody takes."

Our overall advice is not to use calculus as a shortcut, especially if it's not relevant to a student's major—and to evaluate students' math backgrounds in the context of the courses available to them. I'm not a mathematician, but if I were, I wouldn't want my discipline being used primarily to rank and sort students.

We also recommend ensuring transparency and awareness about math requirements, and at the high school level, it's important to expand access to calculus, particularly for students who traditionally have lacked access, and to other advanced math courses as well.

My overall message is that, rather than thinking about math in terms of a list of math competencies or algebraic algorithms or algebraic procedures or any kind of procedures, our compass should be quantitative reasoning, regardless of the specific content students are expected to master. After all, developing quantitative reasoning is the justification that many universities historically have given for having these general education math requirements. The goal of that broader definition is to ensure more students develop not just procedural knowledge, but also conceptual understanding and the ability to apply their math learning in meaningful ways.

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