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Review Article

Effects of the theory of didactical situations' application in mathematics education: A metasynthesis

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Conceptualized in France during the sixties, the Theory of Didactical Situations [TDS] is a pivotal framework for developing mathematics teaching and learning processes. Despite the increasing qualitative studies over the last ten years, there remains a dearth of analysis of its effects on mathematics education. Through a Grounded Formal Theory approach, this paper presents the metasynthesis of 28 studies screened from four research databases— ProQuest, EBSCO, SCOPUS, and JSTOR. Data extraction was performed to enumerate the author/s, country, publication year, and connected theoretical approaches with TDS. Subsequently, the studies were categorized into three axial codes based on the studies' goals of applying TDS: lesson sequence, teacher development, and learning innovations. The synthesis underscores TDS' capacity to improve the sequence of mathematics lessons, particularly geometry and number patterns; develop teachers' pedagogical practices in teaching elementary, high school, and college students; and be a practical tool in innovating didactical resources for learning mathematics. TDS was also associated with other theoretical approaches, such as constructivism, realistic mathematics education, gamification, and technology-based instruction. Most of these effects were rooted in the challenges experienced during the COVID-19 pandemic and the adaptation of TDS to the digital era, including distance and online learning. While the European and Western Asian countries have been at the forefront of TDS integration, the paper advocates for broader global adoption of this theory to enrich mathematics education worldwide.

Keywords: Theory of didactical situations; Mathematics education; Metasynthesis; Grounded formal theory; Lesson sequence; Teacher development

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1. Introduction

Effective mathematics education is contingent on both the development of mathematics pedagogy and the observance of learning through the actions of students outside academic grounds. From the French perspective, pedagogy and learning are effective if students can autonomously demonstrate mathematical tasks and integrate these into real-life scenarios (Hersant & Perrin-Glorian, 2005). Such perspective in mathematics education brought the development of the Theory of Didactical Situations [TDS] by Guy Brousseau and other French mathematicians, such as Christine Mangiante-Orsola and Marie-Jeanne Perrin-Glorian, in the late sixties (Artigue, 2014; Lupu, 2017). The central notion of TDS is the development of didactical situations wherein

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students have the opportunity to learn independently and take responsibility for their own learning when given the necessary learning resources (Brousseau, 2002). It offers a model for determining better teaching strategies in mathematics as a subject and as an area where students are most challenged. Since mathematics has relied chiefly on psychology to explain human mathematical thinking, the need to establish its own theorization was deemed essential by mathematicians and educators (Artigue et al., 2014). Now, it has developed as a framework and continues to develop, applying different didactical tools in teaching and in research, such as the notions of didactical situation, a-didactical situation, didactical contract, milieu, devolution, situations of action, formulation, validation, and institutionalization.

Didactical situations are characterized by a typical classroom setup where there is an exchange of communication between the teacher and students (Brousseau, 2002). As opposed to this situation, a-didactic situation describes the absence of teacher intervention in students' learning experience and (s)he observes around them. The didactic contract and the milieu are two essential components of didactic and a-didactic situations. The former pertains to the implicit rules that regulate mutual expectations between the teacher and the students regarding the mathematics lesson being tackled. The latter pertains to tangible or intangible learning resources that students interact with and, in return, provide feedback for their actions. A milieu, either symbolic or material, must be able to support students, especially during the a-didactical situation. The situations of devolution, action, formulation, validation, and institutionalization are considered the five phases of TDS, which elaborate the stages of learning and the transition from didactical to a-didactical when developing teaching-learning situations (Brousseau, 2002; Hersant & Perrin-Glorian, 2005). An exemplar of didactic situations and the basis of most TDS research studies is the game *Race to 20*:

> "The game is played by pairs of players. Each player of a pair tries to say "20" by adding 1 or 2 to the number given by the other. One of the pair starts by saying "1" or "2" (for example, "1"); the other continues by adding 1 or 2 to this number ("2" for example) and saying the result (which would be "3" in this example); the first person then continues by adding 1 or 2 to this number ("1" for example) and saying the result (which would be "4" in this example); and so on." (Brousseau, 2002, p.3).

Before the game starts, the *situation of devolution* allows the teacher to introduce the game and the rules, which serve as part of the *milieu*. This prepares students for the succeeding tasks the teacher assigns them to do, including working with a pair or group to solve the task. During the *situation of action*, the teacher refrains from intervening, and the pairs start to play, adhering to the rules of the game. It is acceptable that, at this stage, students play with the goal of winning (i.e., competing with each other) and may try as many rounds as possible. Throughout many rounds and trial-and-error, students are expected to formulate or realize a winning strategy to arrive at the number 20 first. This situation is now called the *formulation* stage, where students formulate initial solutions or hypotheses for the given task. As the students apply their winning strategy to validate its correctness, they have progressed to the *situation of validation*. This will now be corroborated further by the teacher in the *institutionalization phase*, where the strategy is decontextualized around a formal knowledge of mathematics (Artigue et al., 2014; Brousseau, 2002).

The didactic situation described above is just one way of applying the principles of TDS in mathematics. In Artigue's (2014) article 'Potentialities and Limitations of TDS,' she explains the complexity of the theory and the variety of ways one can integrate it into a mathematics lesson or in the design of resources. Another approach is to connect its framework to other theoretical approaches, such as the anthropological theory of didactics, the theory of semiotic mediation, and constructivism. Since its emergence in the late sixties, there has been a research gap on how the theory shaped mathematics education, including questions on other theoretical approaches resembling connections with the theory and even countries that applied its principles in their mathematics education program since its conceptualization in France. In addition, there are few research syntheses conducted regarding the theory, one of which is the analysis of three research papers by doctoral students during the eighties (Artigue, 2014). As such, the goal of the study is to

comprehensively understand the effects of TDS in mathematics education through a Grounded Formal Theory [GFT] approach. GFT was specifically selected among other approaches of metasynthesis as it is characterized by a constant comparative approach, ensuring a robust synthesis or generalization that is reflective of the individual studies (Finlayson & Dixon, 2008). The study's objectives encompass the selection of 28 qualitative studies that have utilized TDS as a guiding framework within mathematics education research. Additionally, it seeks to delineate the diverse applications of TDS, other theoretical approaches associated with it, and other countries' mathematics education that were influenced by the theory. To achieve these goals, we formulated our overarching research question, how does the application of TDS affect mathematics education?

2. Review of Related Literature

This literature review section is divided into two parts: the theory of didactical situations and the grounded formal theory as an approach to metasynthesis. The first part provides an overview of TDS, detailing its origins in Europe and the subsequent trends in its publication and application. The second part focuses on conducting metasynthesis, particularly emphasizing the selection of GFT for synthesizing research studies in mathematics education.

2.1. The Theory of Didactical Situations

TDS has been extensively applied in education, specifically in mathematics. Previous studies have documented that it can be applied at the elementary level (Mangiante-Orsola et al., 2018), secondary level (Bos et al., 2020; Hersant & Perin-Glorian, 2005), and even tertiary level (González-Martin et. al., 2014) as a teaching methodology aimed at equipping students to adapt to a milieu essential to their learning. The seminal work of Guy Brousseau, "Theory of Didactical Situations in Mathematics Education," published in 2002, laid the foundation of the theory and explained the need for mathematics to have its own set of theories explaining students' mathematics learning (Lupu, 2017). This is possible through the following components ranging from the devolution stage to the institutionalization stage. In addition, it can also aid teachers in improving their teaching practices, as Mangiante-Orsola et al. (2018) described it as "a tool to understand and develop mathematics teaching practices." Futhermore, TDS as a tool can also be used to answer research questions regarding regular teaching practices, the development of teaching resources, and develop teachers' teaching practices (Arsac et al., 1992; Mangiante-Orsola et al., 2018). Teachers can use TDS to specify teaching situations and may control their effect on their students (Arsac et al., 1992). For example, a milieu that is effectively designed by the teacher would lead students to the intended outcome of the lesson even without the teacher's interference during the lesson (Brousseau, 2002).

In the early 1900s, not long after the conceptualization of TDS, the notion of didactical engineering came to light based on the theoretical principles of TDS (Artigue, 2009; Brousseau, 2002). It has been a tool in developing teaching resources in mathematics through a didactical process, such as the work of Hortelano and Lapinid (2024), which emphasized the extent of the use of TDS in the field. This period also began the theory's dissemination beyond France, as it started gaining international attention. Publications during this time began appearing in broader educational and mathematics education journals in French and English. Research indicates that in the 2000s, TDS was used to design classroom mathematics curricula and develop students' problem-solving skills through the adidactical situation, as documented by Margolinas et al. (2005). Through adidactical situations, students could work and have a better reflection on their solutions and what works best in certain scenarios. Aside from grade level, TDS has been utilized in different branches of mathematics, such as algebra and geometry (Laborde, 2005), and integrated with some research frameworks, such as lesson studies (Clivaz, 2015).

Later on, the framework of TDS was adapted for the design and development of technology integration into didactical situations and cross-disciplinary research, applying TDS in the digital age. Hoyles and Noss (2003) highlighted the potential of integrating technology with TDS to create

enriched learning experiences that support student autonomy and engagement. With this, Maracci et al. (2013) stated that while the core principles of TDS are broadly applicable in teaching through technology, adaptations are necessary to address specific educational contexts and cultural aspects that concern cross-disciplinary research. Nonetheless, the ability of TDS to be adapted and modified based on a specific educational context is another reason for the increase in its application. Despite its contributions, TDS has faced critiques. Some scholars argue that the theory's emphasis on autonomous student discovery may not adequately account for the diverse needs of all learners, particularly those requiring more guidance (Sierpinska, 2004). In addition, Trouche (2005) states that implementing TDS in diverse classroom settings can be challenging due to varying teacher expertise and resource availability. These support Artigue's (2014) statement on the limitation of the theory and its adaptability with other theoretical frameworks, such as conceptual fields and the anthropological theory of didactics (Clivaz, 2017; Maracci et al., 2013; Lagrange & Psycharis, 2014). Nonetheless, TDS has remained an effective tool in classroom mathematics and research, and there remains a gap in its application at present (Bessot, 2024).

2.2. Grounded Formal Theory as an approach to Qualitative Metasynthesis

Grounded Theory [GT] is a modus operandi for establishing theories through planning, comparing, collecting, and analyzing empirical sets of data (Strauss & Corbin, 1994; Vollstedt & Rezat, 2019). Its application is mainly associated with the social sciences, such as psychology and education, to understand social interactions and develop generalizations explaining them. Qualitative research synthesis, termed Qualitative Metasynthesis [QMS], is the counterpart of Quantitative Meta-analysis (e.g., Juandi et al., 2022), typically in a non-positivist research paradigm. Unlike mere literature reviews, the goal of QMS is to perform an exhaustive review and appraisal across qualitative research studies with the objective of comparing different findings (Kozikoğlu, 2019; Sandelowski & Barroso, 2007). The systematic procedure of selecting articles to be reviewed in QMS is defined through inclusion-exclusion criteria and parameters, such as topical, temporal, population, and methodological parameters (Thunder & Berry III, 2016). Similar to GT, it offers generalizations or even the development of theories by observing trends and integrating findings as a result of the synthesis. Qualitative studies in mathematics education have increased over the past decades to describe students' mathematics learning and teachers' pedagogy (e.g., studies concerning the theory of didactical situations). However, there is a scant analysis across these studies in the field, unlike QMA. Thunder and Berry III (2016) agreed with the dearth and made a commentary on the benefits and procedures of doing QMS, particularly in mathematics education.

There are four approaches in metasynthesis (Finlayson & Dixon, 2008). Each differs by the complexity of procedures and the required appraisal: meta-ethnography, grounded formal theory, cross-case analysis, and metastudy. Finlayson and Dixon (2008) recommended the use of GFT as an extension of GT when planning to develop a theory or generalization from the synthesis of qualitative studies. GFT is interpreting qualitative findings using constant comparative analysis and going beyond the conclusive remarks of these primary qualitative studies, eventually developing a novel theory or generalization. The term 'formal' in GFT denotes that the synthesis should not be limited to a single human interaction but encompasses or influences broader areas of interest. In addition, GFT, as an approach to metasynthesis, adheres to the systematic procedure of carefully appraising qualitative studies through open, axial, and selective coding (Eaves, 2001; Kearney, 2007; Strauss & Corbin, 1994). Open coding is the analytic process of breaking down data from reading articles and labeling them based on conceptual similarities and differences. It is the first step in GT that allows researchers to explore and discern the data. Axial coding establishes connections between the labeled data and groups them into categories (e.g., phenomenon, strategies, and consequences). These categories are also compared against the data for scrutiny. Lastly, selective coding combines all the categories formed in the previous step to form a core category. This will form part of the new theory or generalization at the end of the study. While

most synthesis studies that employed GFT approach are in the field of nursing (e.g., Kearney, 2001; Finfgeld, 1999), the current study carefully integrates the approach in synthesizing studies in mathematics education.

3. Method

3.1. Research Design

Thunder and Berry III (2016) stated that there needs to be more QMS studies in mathematics education given the increase in qualitative studies in the field. As such, this study utilizes the inductive technique of GFT to metasynthesis. That is, formulating a generalization out of the findings of the selected qualitative studies after exhaustive reading and comparative analysis (Finlayson & Dixon, 2008), which formed part of a response to the overarching question. Moreover, this design uses qualitative research articles as objects of study, and exhaustive reading of the articles was essential for constant comparison.

3.2. Data Collection

Based on Sandelowski and Barroso's (2007) steps in doing QMS, we conducted a comprehensive search of previous literature using four research databases: ProQuest, EBSCO, SCOPUS, and JSTOR. These databases index high-quality journals and peer-reviewed articles that contribute to a strong recall of relevant articles (Sandelowski & Barroso, 2007). Since our metasynthesis is about TDS, our topical parameter was set by using the keyword "theory of didactical situations" in the search engines to determine the totality of the study; the temporal parameter was set from 2013 to 2022 (i.e., ten years) for the latest trends and developments in TDS; the population parameter included studies that employed both students and teachers in elementary, secondary, and tertiary levels as participants; and, the methodological parameter included qualitative methodologies such as case study, grounded theory, and phenomenology. Applying these parameters during the search was expedited by the advanced filter features of the different databases, and some of these parameters were included in the list of inclusion-exclusion criteria. We organized the flow of the process of exhaustive search literature in Figure 1.

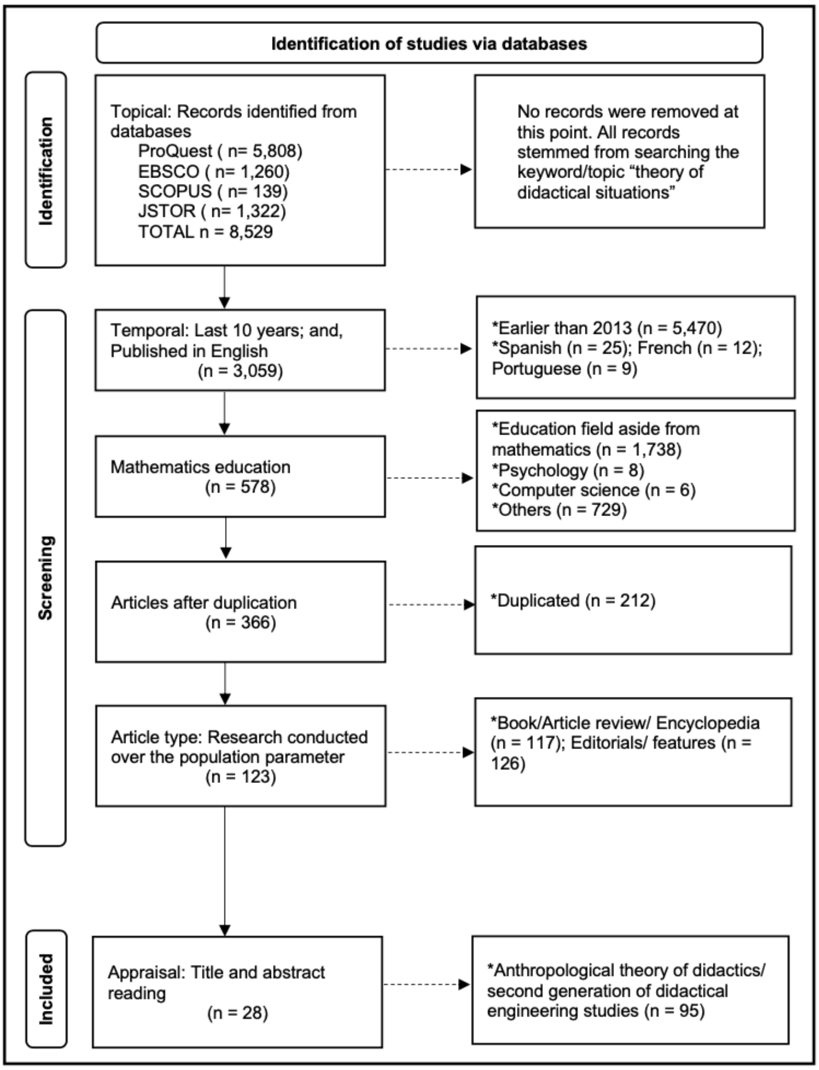
Aside from the four parameters, included as exclusion criteria are language, subjects outside the scope of mathematics education, book or article reviews, editorials, and duplicated files. Studies that were not published in English were difficult to understand, more so, to synthesize. In addition, subjects such as science education (i.e., biology, chemistry, and physics), psychology, and others do not contribute to the research question. Lastly, book reviews and editorials are not research reports or articles conducted over the target population parameter. Admittedly, we checked duplicated files during the third stage of the screening process, which is unusual from most metasynthesis and meta-analysis studies. We found it easier to check duplications after we narrowed down the vast number of articles through advanced filters. Other criteria not mentioned were the bases for the inclusion and selection of the articles for appraisal.

After the screening, we determined the final set of articles to be included in the metasynthesis. The appraisal was conducted by reading the title and the abstract and considering how each article could contribute to the overarching question. Articles that employed solely the Anthropological Theory of Didactics or the second generation of didactical engineering, which are two related theories of TDS, were excluded. In addition, our knowledge about TDS also guides us in appraising the quality of studies to be included. Finally, we were able to select 28 qualitative research articles for the study, which underwent thorough analysis.

3.3. Data Analysis

Given that the QMS adheres to GFT, we utilized the associated coding paradigm, which includes open coding, axial coding, and selective coding (Strauss & Corbin, 1994). During the open coding phase, data extraction was systematically conducted. Articles were organized in Microsoft Excel to detail the authors, publication year, journal name, country, research goals, and theoretical

Figure 1
Flowchart of the comprehensive literature search



frameworks used or associated with TDS. To guide the coding process, we developed sub-questions: *(1) How did researchers apply TDS in mathematics education? (2) What problems did TDS seek to address in mathematics education? (3) What other theoretical approaches are connected to TDS?* Note that these sub-questions helped identify relevant sections of the articles for highlighting and extracting open codes and not necessarily part of the research question.

Following the generation of open codes, axial coding was employed to group similar open codes into broader categories. Finally, selective coding was used to synthesize these categories into a generalization that contributes to the overarching question of the QMS, thus conceptualizing the findings for the goal of the metasynthesis. To establish the validity of the synthesis, the results were presented to a research adviser and a group of doctoral students for review. They approved the synthesis, and no contradictions or concerns were raised.

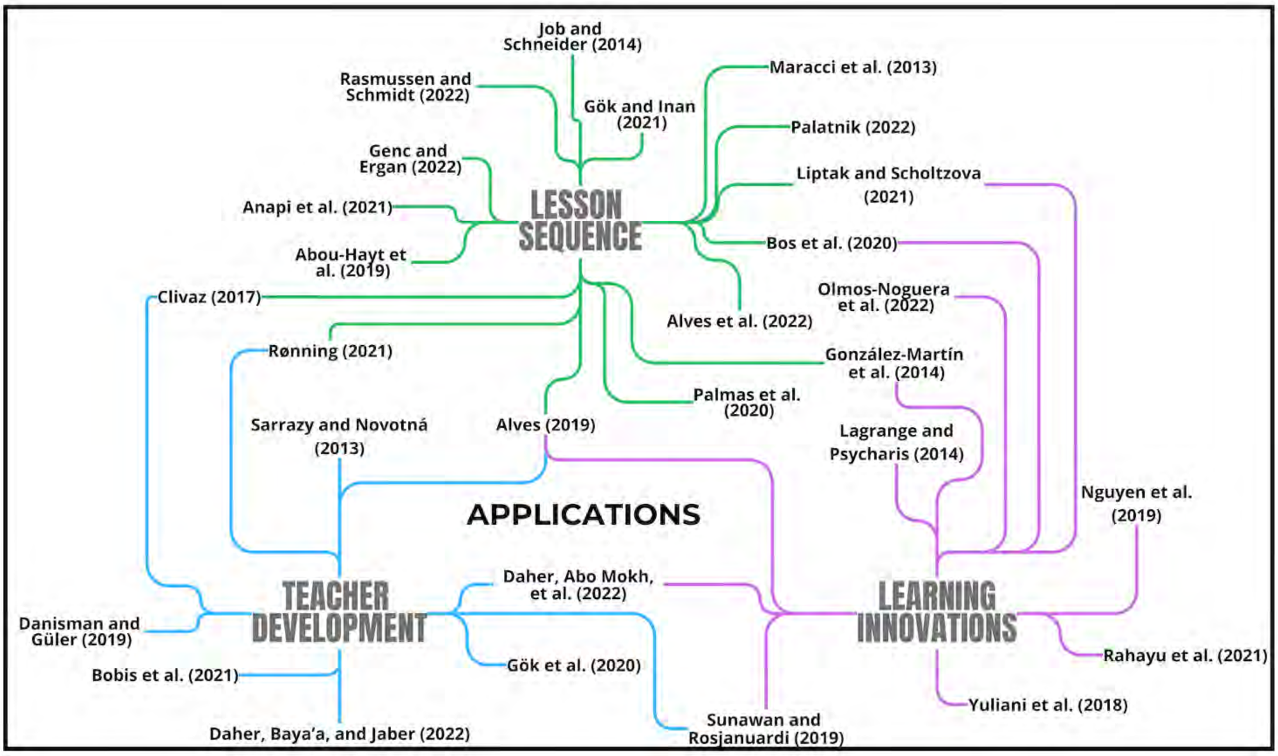
4. Results

In this section, we outlined the results of the metasynthesis. First, we presented the applications of TDS derived from the coding process. Then, the data extraction results were included, organized by author/s, journal, and group as shown in Table 1, the country in Figure 3, and the year in Figure 4. Subsequently, we elaborated on the formed codes.

4.1. Applications of TDS

Through careful reading and rereading, we formed 31 open codes based on how researchers apply TDS in their respective studies vis-à-vis goal/s. The codes exceeded the number of articles as some studies outlined two or three goals of conducting TDS with their target population. From the open codes, three axial codes were formed based on similarities of the open codes: lesson sequencing, teacher development, and learning innovations. As these three resemble the researchers' applications of TDS in mathematics education, our first sub-question was addressed on this point. We also used our axial codes to group the articles and establish connections across studies. These groupings also pertain to the applications: application 1- TDS for lesson sequence, application 2- TDS for teacher development, and application 3- TDS for learning innovation. In Figure 2, we formed a conceptual map of the applications, illustrating the relationship of each study to their respective groups. Application 1 involves research papers that applied TDS to sequence mathematics lessons using the phases from devolution to institutionalization. Application 2 involves research papers that employed teachers as participants and explicitly discuss teacher developments in their findings. Application 3 involves studies that applied TDS to develop learning and teaching resources relevant to a specific challenge in mathematics education.

Figure 2
Conceptual map of the 28 studies based on three TDS applications



4.2. Data Extraction

This section outlines the outcomes of the synthesis conducted. First, the 28 studies are organized in Table 1, with author/s, journal, and group as headings. The categorization into groups stemmed from addressing the first sub-question concerning the researchers' application of TDS, thereby facilitating the arrangement of studies into three distinct axial codes. Subsequently, the results of the data extraction based on country and year are presented.

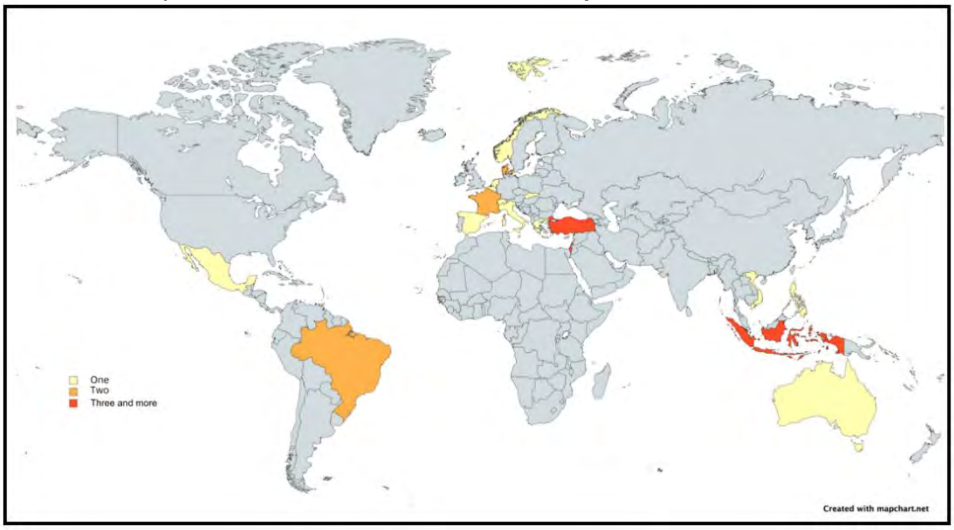
Table 1
Articles included in the metasynthesis, their journal, and group

Article	*Journal*	*Group/Axial code*
Abou-Hayt et al. (2019)	Proceedings of the 47th SEFI Annual Conference	Lesson sequence
Alves (2019)	Acta Didactica Napocensia	Lesson sequence; Teacher development; Learning innovation
Alves et al. (2022)	Acta Didactica Napocensia	Lesson sequence
Anapi et al. (2021)	Turkish Journal of Computer and Mathematics Education	Lesson sequence
Bobis et al. (2021)	Mathematics	Teacher development
Bos et al. (2020)	Journal of Mathematics Behavior	Lesson sequence Learning innovation
Clivaz (2017)	Education Studies in Mathematics	Lesson sequence; Teacher development
Daher, Abo Mokh, et al. (2022)	Sustainability	Teacher development; Learning innovation
Daher, Baya'a, et al. (2022)	Mathematics	Teacher development
Danisman and Güler (2019)	Inovacije U Nastavi (Teaching Innovations)	Teacher development
Genc and Ergan (2022)	Acta Didactica Napocensia	Lesson sequence
Gök and Inan (2021)	Journal of Research and Advances in Mathematics Education	Lesson sequence
Gök et al. (2020)	Ilkogretim Online- Elementary Online	Teacher development Learning innovation
González-Martín et al. (2014)	Research in Mathematics Education	Lesson sequence; Learning innovation
Job and Schneider (2014)	ZDM Mathematics Education	Lesson Sequence;
Lagrange and Psycharis (2014)	Technology Knowledge and Learning	Lesson sequence; Learning Innovation
Liptak and Scholtzova (2021)	European Journal of Contemporary Education	Lesson sequence
Maracci et al. (2013)	Educational Studies in Education	Lesson Sequence
Nguyen et al. (2019)	Journal of Physics: Conference Series	Learning innovation
Olmos-Noguera et al. (2022)	Mathematics	Lesson sequence; Learning innovation
Palatnik (2022)	Journal of Mathematics Behavior	Lesson sequence
Palmas et al. (2020)	Teaching Mathematics and its Application: An International Journal of the IMA	Lesson sequence
Rahayu et al. (2021)	Journal of Physics: Conference Series	Learning innovation
Rasmussen and Schmidt (2022)	International Journal of Educational Research Open	Lesson sequence
Rønning (2021)	ZDM Mathematics Education	Lesson sequence; Teacher development
Sarrazy and Novotná (2013)	ZDM Mathematics Education	Teacher development
Sunawan and Rosjanuardi (2019)	Journal of Physics: Conference Series	Learning innovation; Teacher development
Yuliani et al. (2018)	Journal of Physics: Conference Series	Learning innovation

4.2.1. Country

The map in Figure 3 illustrates the global distribution of TDS studies, highlighting the varying levels of research activity in different countries. Countries shaded in yellow indicate where a single study was conducted, which includes the Philippines, Australia, the Netherlands, Switzerland, Belgium, Greece, Slovakia, Italy, Vietnam, Spain, Mexico, and Norway. In these nations, there has been some interest and initial exploration into the application and implications of TDS in mathematics education. Countries marked in orange have two studies included in the metasynthesis, specifically Brazil, Denmark, and France. This suggests a growing academic interest in these regions. However, in the case of France, although TDS has been an established framework, more published studies warrant attention. Lastly, three or four studies have been carried out for countries shaded in red, such as Israel, Turkey, and Indonesia, indicating a high level of research activity and an academic focus on TDS. Overall, Europe emerges as the most prominent continent in conducting TDS studies.

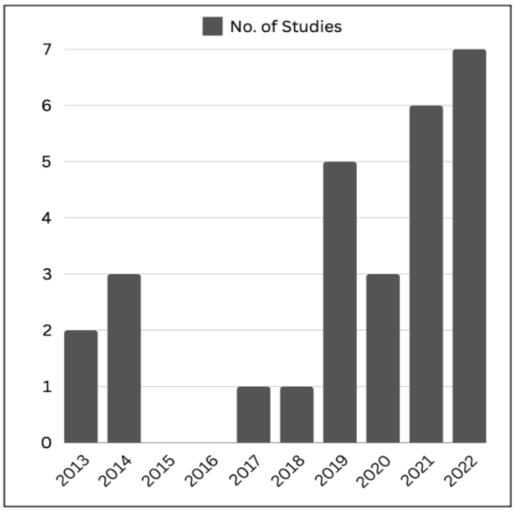
Figure 3
Distribution of the 28 studies based on the country



4.2.2. Year

The temporal distribution of studies on TDS from 2013 to 2022, highlighting a pattern of increasing research activity over these years, as shown in Figure 4.

Figure 4
Number of studies in the temporal parameter each year



In 2013, research on TDS started with 2 studies. This increased to 3 studies in 2014. However, no studies were recorded in 2015 and 2016 based on the parameters. In 2017 and 2018, research resumed with 1 study each year. An increase in the collection happened in 2019, with 5 studies, 3 in 2020, 6 in 2021, and 7 in 2022. This trend indicates a growing academic interest in TDS over time. Note that these counts, including the country, were based on the exhaustive search of the study, which was conducted according to specific parameters set for the metasynthesis.

4.3. Axial Code 1: Application of TDS for lesson sequence

Applying TDS in practice is methodical. As such, this group is the collection of studies incorporating the five phases and concepts under TDS to develop lesson sequences: devolution, action, formulation, validation, institutionalization, didactical contract, and milieu. Variables such as participants, problems (i.e., mathematics lesson addressed in the study), *milieu*, and connected theoretical approach are considered aspects in this section.

As the formal education setting or basic education level (i.e., elementary and secondary level) is prominent in educational research as well as the majority of the TDS studies, Abou-Hayt et al. (2019) have utilized college engineering students and Palmas et al. (2020) involved low-schooled adults in their countries as participants. Regarding the mathematics lessons being addressed and observed by the researchers, geometry and number patterns were two of the most studied in mathematics, mainly geometrical elements, shapes, and number sequences. These lessons are stipulated in the national mathematics curriculum of each country, wherein recommended teaching strategies are also established. Higher mathematics lessons were also considered, such as calculus topics on the concept of limits and the slope of a curve. Prior to the conduct of each study, these lessons were seen as the areas where students struggle most in mathematics, forming the problem being addressed in the studies.

All the papers have explicated the significance of a *milieu* in the five phases. It was present in the a-didactical situations where students take the responsibility of learning and solving the tasks on their own or with a pair. Most of the *milieus* were problem-solving, accompanied by the rules and instructions necessary to help students learn independently. Aside from word problems, GeoGebra and other technological software (Alves, 2019; Alves et al., 2022; Clivaz, 2017; González-Martín et al., 2014; Maracci et al., 2013; Olmos-Noguera et al., 2022; Palmas et al., 2020) were also seen as helpful interactive tools in mathematics. Games were creative ways to deviate from traditional teaching and offered another perspective on learning mathematics (Gök & Inan, 2021; Liptak & Scholtzova, 2021). Lastly, constructivism, technology-based instruction, theory of semiotic mediation, anthropological theory of didactics, and problem-based learning were the most prominent theoretical approaches connected with TDS among the studies in this group.

Some studies have implicitly discussed the methods they used in some of the phases of TDS. We uncovered these details from the findings by carefully reading, rereading, and returning to the definitions to give explicit descriptions. Thus, we present in Table 2 the central ideas of the 18 studies included in this group as to how the phases were conducted.

Table 2
Central ideas from the 18 studies on the five phases of TDS

Devolution	*Action*	*Formulation*	*Validation*	*Institutionalization*
Preparing students for the a-didactical stage	Students work in groups using their prior knowledge and the milieu	Students start to form answers to the problem by sharing ideas and utilizing provided tools	Validating results either by groups or with the teacher, other situations, or software.	Formalizing knowledge

The common trend in all the studies, from devolution to institutionalization, was to start by introducing the task students will work on during the a-didactical stage and end with formalizing knowledge. The *milieus were* utilized well at the start of the a-didactical situation, which is the *action stage*, to explore the problem and to assist them in building connections using their prior knowledge related to the current topic. In the *formulation stage*, most studies explained that students start to form conjectures or hypotheses as initial solutions to the given problem. For the studies that employed pair and group tasks, such as Genc and Ergan (2022) and Rasmussen and Schmidt (2022), their formulation stage was done within or between group discussions. Furthermore, interacting with tools and software was another way to deal with the formulation stage so students could continue without the teacher or researcher's intervention. However, given the nature of the a-didactical stage, Rønning (2021) involved teacher and researcher intervention at this stage to guide students along the process, departing from the principle of TDS. Nonetheless, these approaches preceded the *validation stage*, where students formalized their conjectures, and teachers slowly involved themselves in the process. Most of the students presented their answers to be assessed by teachers or researchers. Some students used tools like calculators and GeoGebra to validate results and continue the discussion among their classmates. On the other hand, Abou-Hayt et al. (2019) gave students a final task to determine their knowledge gained at the end of the lesson, and unlike the 17 studies, they performed the validation last in the process instead of the *institutionalization phase*.

4.4. Axial Code 2: Application of TDS for teacher development

Teacher development is a shared research interest in TDS aside from its goal to improve the lesson sequence of a particular mathematics lesson. Three studies utilized the participation of pre-service mathematics teachers, three studies on elementary teachers, one for mathematics and science teachers, and three on students. Bobis et al. (2021) studied the instructional practices of Foundation level to Grade 2 teachers in Australia and employed TDS to explain these practices. Using TDS for such a purpose, the researchers deduced that the teachers employ didactical and a-didactical situations in their teaching strategies. Their differentiated instruction approach was for the benefit of the teachers who are teaching diverse students at the primary level.

Daher, Abo Mokh, et al. (2022) and Daher, Baya'a, et al. (2022) are studies conducted in Israel concerned with task design, and both approached TDS with a technology-based instruction. The former involved mathematics and science teachers in task design necessary for distance learning (i.e., online activities) during the COVID-19 pandemic. TDS was used in the study to evaluate these tasks, especially during the devolution and institutionalization stage. Unlike the former, the latter involved pre-service teachers majoring in mathematics. The task design was on Scratch programming as a valuable tool to be utilized by teachers in their pre-service years of teaching the subject

Danisman and Güler (2019) and Gök et al. (2020) are Turkish studies that employed a constructivist approach and the integration of games in delivering mathematics lessons. They stated that the teaching of mathematics lessons should go beyond the traditional setting and let students explore problems with adequate resources such as prior knowledge and the *milieu*, for instance. In these studies, the teacher participants took the role of students and were guided on delivering game-based learning using the phases of TDS. These were executed through demonstrative activities (i.e., taking the role of students and being active learners), which contributed to developing their practice as future mathematics teachers. In the development of teachers' mathematical knowledge, Clivaz (2017) investigated how TDS can be influential in developing their content knowledge, pertinence, and choice of teaching strategy.

In studies that employed students as participants, teacher development comes as a consequence of developing the students' mathematical capabilities. Teachers involved in the studies were able to reflect better in their teaching and the learning of their students. Alves (2019) aimed to increase student participation in mathematical competitions in Brazil, eventually increasing teacher

participation in the said competition. Rønning (2021) aimed at students' language development in mathematics, offering teachers insights into their students' use of mathematical language. Moreover, Sarrazy and Novotná (2013) showcased how TDS can be a tool for teachers to foster students' mathematical creativity in problem-solving and generating solutions.

4.5. Axial Code 3: Application of TDS for learning innovation

Learning innovation is defined in this section as establishing developments and changes in mathematics education to benefit learning. As most studies can be categorized under this code, the reviewers focused only on unique innovations in mathematics education as products of TDS and made for novel reasons.

Each study included in this section has a particular reason for developing innovation, such as predicting students' mathematics anxiety using learning trajectories (Yuliani et al., 2018) and creating online learning activities to learn mathematics during emergency situations, such as the COVID-19 pandemic (Daher, Abo Mokh, et al., 2022). Others consider innovating hypothetical learning trajectories for students' active engagement and mathematical representation ability (Rahayu et al., 2021; Yuliani et al., 2018), as well as their engagement in international standards, such as the Trends in International Mathematics and Science Study (TIMSS) to be at par with high performing countries (Sunawan & Rosjanuardi, 2019). Other considerations were providing students with alternative solutions other than what is provided in the book (Bos et al., 2020; Olmos-Noguera et al., 2022; Rahayu et al., 2021), motivating them by providing creative ways to teach mathematics (Gök et al., 2020; Liptak & Scholtzova, 2021), and innovating technology-based milieus in teaching particular mathematics concepts (Lagrange & Psycharis, 2014). Moreover, realistic mathematics education, constructivism, technology-based instructions, and didactical design research are the theoretical approaches connected with TDS in this group, which supported these innovative projects.

5. Discussion

In this section, we return to the overarching question: *How does the application of TDS affect mathematics education?* The following discussion addresses the response and enables us to contribute to Artigue's (2014) remarks on the potentialities and limitations of TDS in education.

5.1. TDS and Its Components for Developing Different Aspects of Mathematics Education

In the different phases of TDS, devolution is performed uniformly by researchers with the initial hypothesis that students' knowledge before learning is unstructured. The aim is to structure this knowledge as students transition from devolution to institutionalization. While the 18 studies did not explicitly state teacher or researcher intervention during the a-didactical stage, Rønning (2021) provided an exception. Even with disruptions in the flow of TDS, it remains the responsibility of teachers or researchers to support students when necessary. Additionally, the validation phase of Abou-Hayt et al. (2019) was performed after the institutionalization phase to measure students' learning at the end of a lesson, serving as the assessment part of lesson planning and offering another approach to performing the phases and developing a mathematics lesson sequence.

In the different phases, devolution is performed uniformly by the researchers with an initial hypothesis that students' knowledge before learning is unstructured. They aim to structure such knowledge as they transition from devolution to institutionalization. While the other 17 studies under axial code 1 did not explicitly state teacher or researcher intervention during the a-didactical stage, Rønning (2021) did otherwise. The explanation is valid even if there was a disruption in the flow of TDS, as it is always the responsibility of teachers, or even researchers, to support students when necessary. Aside from this, the validation phase of Abou-Hayt et al. (2019) was performed after the institutionalization phase to measure students' learning at the end of a lesson. This type of validation was used synonymously as the assessment part of lesson planning, which imparted another way of performing the phases and developing the sequence of a mathematics lesson.

Nonetheless, these TDS components were essential in describing regular teaching practices in a classroom (Mangiante-Orsola et al., 2018).

For the lessons, number pattern is the most frequently studied in TDS, as brought by Brousseau's (2020) exemplar didactic situation Race to 20, followed by geometric and calculus lessons. There were, however, fewer studies in other fields of mathematics, such as trigonometry and algebra. Such choices of mathematics lessons to be studied under TDS were rooted in students' experienced difficulties in learning higher concepts as perceived by researchers.

For teachers, TDS has been an effective didactical tool to support their practice. The approaches employed by the researchers were mostly student-centered, which supports Artigue (2014) and emphasizes the goal of TDS for teachers as a tool to understand and develop mathematics teaching practices (Arsac et al., 1992; Mangiante-Orsola et al., 2018). The studies were cognizant of the development of in-service and pre-service teachers and what the theory can offer in the pedagogy. Pre-service teachers were usually trained by taking the role of students in a classroom, while in-service teachers were guided in the TDS ways. An example is the design of didactical resources and learning innovations that they can use in a normal classroom scenario or during precarious situations, such as the COVID-19 pandemic. These innovations emerged as solutions to problems in mathematics education, made to make mathematics accessible for students, and acted as tools to support didactical practices. Moreover, despite some critiques (Sierpinska, 2004; Trouche, 2005), TDS addresses national and international challenges pertaining to teacher and student performance in mathematics, which widens further its scope and effects in education.

5.2. Adaptability of TDS in the Digital Age and Pandemic Era and Its Global Influence

TDS has proven to be a practical tool during the pandemic. In the transition from pre-pandemic to pandemic-era learning, where technology integration has become essential, studies have utilized GeoGebra and other technological resources as effective milieus. Researchers have encouraged their respective populations to adapt and use these technological resources as practical mediums of instruction during educational crises, particularly when face-to-face classes were restricted. This shift underscores the importance of technology integration in teaching mathematics as a key theoretical approach connected to TDS.

Previous research has also documented non-technological milieus, highlighting the transition from non-technological to technology-based instruction and the evolving application and adaptation of TDS to the digital age (Hoyles & Noss, 2003). In terms of its influence on mathematics education globally, the synthesis illustrated that different countries benefitted from applying TDS aside from France, where it originated (Lupu, 2017). The influence can be classified from small scale (e.g., improving the sequence of mathematics lessons, classroom interaction) to larger scale development (e.g., national mathematics education reforms). Despite its global reach, most studies on TDS are still concentrated in Europe, where it was first developed. This global influence and adaptability in the digital age and pandemic era underscore TDS' potential to address diverse educational challenges across various contexts. Moreover, we have observed an increase in the publication of TDS studies over the last 10 years, from only 2 in 2013 to 7 in 2022. At this rate, we can anticipate more scholarly studies in mathematics education employing TDS.

5.3. Associated Theoretical Approaches of TDS and Some Limitations

To address the connected theoretical approaches, we have observed the various frameworks associated with TDS in the different studies. This represents the "plurality" of frameworks in mathematics education, as Maracci et al. (2013) stated in their study. As such, researchers should be strategic in integrating these frameworks for their effective use as they vary in terms of focus or methodology. Examples of papers on such concern are Clivaz (2015; 2017) and Lagrange and Psycharis (2014), who exemplify how TDS could be carefully associated with other theoretical approaches. Clivaz (2015) used TDS within the context of a lesson study, and her study in 2017 combined multiple frameworks, such as mathematical knowledge for teaching and mathematical pertinence of teacher, and concluded the development of a didactical situation in teaching

multiplication. Lagrange and Psyacharis (2014), on the other hand, recommended double analysis in integrating constructionism and TDS.

Regarding the limitations of the metasynthesis, it is essential to acknowledge that the included studies were published between 2013 and 2022 with the identified parameters. Given the dynamic nature of research, future studies on TDS are anticipated, and conducting subsequent metasynthesis could yield new insights into its effects in mathematics education. In addition, other researchers may explore alternative approaches to metasynthesis, such as cross-case analysis or meta-study, to validate and augment the findings.

Finally, the findings underscore GFT as an effective approach for synthesizing various qualitative studies, aligning with the vision of Finlayson and Dixon (2008). As an extension of GT, GFT expands its synthesis beyond individual human interactions to include multiple facets (Eaves, 2001; Kearney, 2007). Our study incorporated several aspects of TDS, ranging from its practical applications to its interconnected theoretical frameworks. Lastly, this synthesis adds to the limited body of research utilizing GFT in mathematics education, a method initially established in the medical field (Finfgeld, 1999).

6. Conclusion

In this study, we offered a comprehensive review and generalization on the effects of TDS in mathematics education. Our synthesis revealed three overarching applications of TDS within this context. The five phases of TDS emerged as essential components for structuring mathematics lessons, which facilitates the exploration of student performance across didactical and a-didactical phases, especially on how they take responsibility for their learning. Particularly, TDS contributes significantly to teacher development by promoting reflection and introducing varied approaches to teaching mathematics, exemplified by the structured sequence of mathematics lessons. Moreover, the development of learning innovation is one of the essential contributions of TDS, mainly because it has provided effective alternative modes of learning in difficult situations. These innovations are also exceptional as they assist students in learning mathematics easily.

The synthesis also underscores the significance of considering the geographical context and publication year of the studies included in our analysis. This aspect allows us to gauge the extent of TDS' influence as a research framework and educational approach in mathematics beyond its origin in France and from its conceptualization in the sixties. Through this, we gained significant insights into the adaptability and relevance of TDS on a global scale. It has influenced the mathematics education of countries in southwestern Asia, particularly in Turkey and Israel, and it is becoming a growing research interest in southeast Asia. Hence, Guy Brousseau has significantly contributed to the research field of mathematics education worldwide, particularly to the practice of mathematics teachers and the learning of students.

We also highlighted specific studies to underscore their interconnections, arguments, and supporting statements. This approach was adopted to make the synthesis more meaningful, bringing attention to the identified unique TDS approaches. By doing so, we provide readers from the academic community with the latest and insightful understanding of the effects of TDS in mathematics education. To further advance this knowledge, future research endeavors should consider the application of TDS to other mathematics lessons, such as its application in trigonometry, algebra, and statistics classes since our metasynthesis suggests that these remain underexplored. Additionally, studies could test other theories yet to be associated with the TDS framework, such as Culturally Responsive Teaching or Situated Learning Theory. Furthermore, it would be valuable for future research projects to adopt the three general applications of TDS found in this study to improve other countries' mathematics education, especially those that are new or less familiar with the theory.

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