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The development of mathematical argumentation: A case study on two mathematics classrooms

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ABSTRACT

Mathematical reasoning has been a critical concern in Turkey especially since the structure of the student selection examination for high schools changed six years ago. The ability to solve the questions in the new exam requires high level reasoning and argumentation skills. Schools, whether they are public or private, prepare 8th graders for this exam with intense educational programs. They frequently use skill-based questions–similar questions to the ones in the new exam, which require high level mathematical reasoning. This study indicates that students gain better mathematical reasoning skills in learning environments promoting collective discussion and argumentation. Hence, these questions need to be solved in these kinds of environments. In this study, two cases are analyzed to understand the argumentation process in classrooms in depth. Audiotapes of two 8th grade classrooms, one from a public school, another from a private school, are analyzed. The same teaching material–a worksheet including skill-based questions–is used in the classrooms. During analyzing, qualified argumentation pattern–a pattern including students' claims and justifications/evaluations for those claims- is defined. Analysis of classroom audiotapes revealed that that there are significant differences between two classrooms' argumentation structure and type of dialogues emerged in the classroom. In one classroom there is a more qualified argumentation process than in the other classroom. In addition, analysis of the type of the dialogues indicates that two classrooms have different types of dialogues. Overall, the study reveals that despite the differences, both classrooms mostly included teacher-individual interactions and they have little collective discussion.

Keywords: mathematics classrooms, argumentation, collective discussion, classroom talk

INTRODUCTION

Problem-solving has always been a critical issue in mathematics classrooms. Students have difficulty in expressing what the problem is about, what it requires and how to progress to solve it. According to the report published by National Council of Teachers of Mathematics (NCTM, 2000), it is important that students can express their ideas, contemplate on their own thinking and reasoning process, and to evaluate others' ideas, during problem-solving. Ministry of National Education (MEB, 2018a) in Turkey also focuses, in mathematics teaching program, on sharing and evaluating of ideas to construct mathematical concepts and to gain deeper mathematical understanding. In a way, students are expected to participate in the argumentation process by expressing their thoughts and reasoning and they are expected to find the gaps or deficiencies in others' reasoning. The education system not only proposes a mathematics teaching program fostering argumentation, but it also changes the way students are assessed accordingly. The format and structure of the exam, for example, whose results are used to place 8th grade middle school students to a proper high school, changed several years ago. Although there is still an examination requirement for success, the format of the exam has undergone modifications. The change in the mathematics exam format had a significant impact, leading to a sharp decline in student achievement. Analysis reports from the years 2016 and 2017 showed that students could answer nearly half of the 20 questions correctly (MEB, 2016, 2017). However, from 2018 to 2021, after the new exam format was introduced, students consistently averaged around five correct answers out of 20 questions (MEB, 2018b, 2019, 2020, 2021, 2022).

One of the primary factors contributing to this decline is the students' deficiency in reasoning and critical thinking skills (Beyendi, 2018). Previously, the exam primarily assessed a single objective related to a specific topic. Hence, professionalization on the objectives usually brought achievement and access to better schools. In contrast, the new exam aims to evaluate students' abilities about integrating multiple objectives. In some questions students need to be good at several objectives to solve it, not a single objective. Hence the complexity of the question has increased. In addition, questions may have long passages and figures, which require full attention, connection, interpretation, integration and but most importantly they require complex mathematical reasoning (Kablan & Bozkus, 2021; Yuzuak & Arslan, 2021). It is significant to note that these questions are not ill-structured. They are well structured–have only one correct answer–, yet they still require high level reasoning skills. Hence, the schools, whether it is public or private, prepare kids, especially 8th graders, for these types of questions, which are called skill-based questions. Teachers ask skill-based questions within or at the end of units.

This study challenges the way these questions are utilized within classrooms. Given that these skill-based questions demand connections, integration, and reasoning these questions need to be solved in argumentation learning environments (Cho & Jonassen, 2002). In such environments, the evolution of knowledge occurs through the provision of evidence to support claims and the ability to justify them, which, in turn, necessitates the development of students' reasoning skills.

For this study, two classrooms in Istanbul–one from a public school and the other from a private school–have been thoroughly analyzed. The main distinction between public and private schools lies in their sources of funding. Public schools are predominantly funded by the state, whereas private schools rely on parents' financial contributions or donations to address their funding needs. Additional differences emerge because of this financial distinction. Privatization of schools leads to a focus on meeting the individual needs of students and parents and fosters a more competitive educational environment. Private schools are generally perceived as offering higher quality education (Uygun, 2003). It's important to note that this study does not aim to highlight the differences between these types of schools but rather seeks to provide a deeper understanding of these two specific cases.

The literature, I will mention in detail below, indicates that collective argumentation in mathematics classrooms is an essential way of attaining reasoning skills. In this case study, two mathematics classrooms in which skill-based questions are solved will be analyzed in depth. Hence, the study will contribute to the literature in the field of argumentation and mathematics classrooms in Turkey. The research questions for the study are, as follows:

1. How is the structure and development of argumentation move in both classrooms?
2. What kind of argumentation dialogues do the teachers initiate in the classroom discussion?

LITERATURE REVIEW

Argumentation & Learning

According to van Eemeren et al. (2014), argumentation is "communicative and interactional act complex aimed at resolving a difference of opinion" (p. 6). With this definition, it is emphasized that argumentation addresses a disagreement of opinions that is aimed to be solved by interacting and communicating with others. Another definition provided by Krummheuer (1995) is "interactions in the observed classroom that have to do with the intentional explication of the reasoning of a solution or after it" (p. 231). Krummheuer (2007) emphasizes more the interactional aspect of argumentation. Even though some theories and conventional views about argumentation focus on the result, which reveals the persuaded and the persuader, learning theories emphasizes the social aspect of argumentation (Andriessen & Baker, 2014). The important issue about argumentation is not the result, instead it is the process in which participants contemplate their own thinking and reasoning processes, share, and reflect on them, also evaluate other's reasoning (Schwarz, 2009).

Research studies show that participating in the argumentation process contributes to students' learning and achievement (Asterhan & Schwarz, 2016; Ing et al., 2015; Miller et al., 2014; Webb et al., 2023). According to Asterhan and Schwarz (2016), argumentation facilitates learning in two ways: First, students explain their thoughts publicly that can be false and incomplete, and makes it open to an evaluation. Even making explanations about their own thinking and reasoning has a beneficial impact on learning (Chi & VanLehn, 1991; Veenman et al., 2005; Webb, 1989; Webb et al., 2009), not solely through processes like negotiation, criticism, or reflection (Baker, 2009). Second, students have better learning gains when they engage in different viewpoints, which provide them with the opportunity to reflect on their own. For example, in one study, students were divided into three different groups about a scientific text assigned to them: one with no purpose, one to engage in classroom discussion, and one to participate in an argumentative discussion after reading (Miller et al., 2014). The findings of the study suggest that the group anticipating especially an argumentative discussion after reading spend more time on reading and understanding the text, and students in the group anticipating an argumentative discussion has better conceptual understanding. Furthermore, Schwarz (2009) asserts that argumentation positively affects both the social and cognitive aspects of participation, through expanding motivation and engagement, decreasing the cognitive load etc. O'Connor et al. (2017), indicates that there is no relationship between vocally participating and achievement scores of students. The result of their study shows that silent students also have significant learning gains. They attribute this situation to active listening being a settled norm of the classroom culture. Students engage in collective argumentation and learn by actively listening to the discussion, although they do not participate in the discussion vocally.

In this study, problem-solving situations in mathematics classrooms are analyzed. Several studies have highlighted the connection between argumentation and problem-solving skills (Cho & Jonassen, 2002; Chiu, 2008; Jiménez-Aleixandre et al., 2000; Shin et al., 2003; Webb et al., 2023). It is suggested that argumentation plays a significant role in solving particularly complex problems, although there are also studies showing that argumentation may not lead to a statistically significant improvement in students' problem-solving skills (Kardas, 2013; Yildirim, 2017). According to Moshman (2020), argumentation is collaborative form of reasoning. Given the relation between reasoning and argumentation, literature addressing reasoning and problem-solving skills can provide further insights into these concepts. Research also indicates a correlation between reasoning and problem-solving skills of students (Cerbin, 1988; Tajudin & Chinnappan, 2015).

Collective Argumentation in Mathematics Classrooms

Jiménez-Aleixandre (2007) gives the ecosystem as an example to refer complex and inter-dependency of classroom life. A classroom is not a collection of isolated people and materials, instead it has a synergistic life, which affects and is affected by everyone and everything. Learning in this environment needs to have social aspects. One of the main theoretical perspectives of this study is that it adopts a socio-cultural perspective of learning since the classroom is the unit of analysis. (Vygotsky, 1986) Socio cultural perspective provides to understand the classroom as a group whose members contribute to each other's learning. Indeed, Vygotsky (1986) argues that, by referring to zone of proximal development, individuals can do things which are outside of their capabilities by sufficient help of others. Hence, classroom is a socio-cultural environment in which meaning making and knowledge building is co-constructed. Some other researchers also emphasize social aspects of learning (Yackel & Cobb, 1996) In classrooms, students and teachers need to interact and students build their knowledge in these interactions. Hence, classroom talk and discussion, the conversations, and interactions in the classroom, helps to find out the traces of students' learning.

Cobb (1999) searches about classroom discourse and collective mathematical learning in classrooms. Although he argues that mathematics is still an individual endeavor, he approaches mathematical learning with a socio-cultural perspective to mathematical learning. He defines classrooms as a community of learners in which individuals participate in collective discussion through bringing their individual reasoning and creativity. Schwarz (2009) also emphasizes the collaborative nature of argumentation. He suggests that in mathematics classrooms, collective argumentation is not about convincing each other, but rather it is a process in which students share their own thinking and reasoning, reflecting on them, and evaluating the ideas presented in the classroom. Not only Cobb (1999) and Schwarz (2009), but many other scholars emphasize the collective mathematical learning in classrooms in their research (Ayalon & Even, 2016; Krumheuer, 2007; Kramarski & Mevarech, 2003; Inagaki et al., 1999; McCrone, 2005; Yackel & Cobb, 1996; Yackel et al., 1999). For example, Inagaki et. al compare Japanese and U.S mathematics classrooms and they claim that U.S teachers give more individual based responses while Japanese teachers use individual responses to create whole class discussion. Japanese teachers are not insistent on explicit teaching, and they do believe that many type of knowledge can be acquired in classroom environment via discussing, while U.S teachers have more behavioristic approach and believe in explicit teaching of mathematics. They analyze the classroom talk with coding they call as inquiry, response, and feedback. In addition, Ayalon and Even (2016) bring out the same discussion through analyzing two cases. Again, they focus on collective meaning making process and mathematical reasoning in the classrooms. They create an argumentation schema of the whole class discussion with very useful coding. This two research, especially the second one, helped a lot to create the coding for this study. In a longitudinal study (McCrone, 2005) mathematics lessons of a fifth-grade classroom are observed. In the study there is an experienced teacher who pays attention to channeling individual reasoning to collective discussion. The importance of this research is that students are more able to participate in collective mathematical argumentation process over time with the help of significant attempts of their teacher. The study of Ing et al. (2015) also presents findings about the impact of teacher's prompts on student participation. Teacher's probing is a significant factor in increasing student engagement and participation. Johnson et al. (2023) emphasizes interesting results in their study. According to the scholars, instead of seeing students unclear or incomplete ideas as obstacles to communication and considering them ideas need to be fixed, these ideas can be valuable contributions to group mathematical activities. Students' vague and incomplete ideas enable and urge other students to comment on them. The results of this study are significant in the sense that it indicates even unclear and ambiguous ideas of students can be used in a productive way to support collaborative problem-solving in mathematics classroom.

Research carried out in Turkey also contributes to the field. Olkun and Toluk (2004) carry out research on collective argumentation in mathematics teaching classrooms of prospective teachers. They emphasize the questioning strategies used in the lessons. There is some more research on prospective teachers and argumentation (Dede, 2019; Erkek & Bostan, 2018). Although studies in this field demonstrate that argumentation is more effective than traditional teaching and learning methods in mathematics education, research in this area is still limited (Yitmez et al., 2023). Hence it seems that there is still need in the literature in terms of in-depth analysis of collective argumentation in mathematics classroom. This study focuses on collective problem-solving and the discussion process of two classrooms. Assuming that learning is a social phenomenon and collective argumentation is a significant learning process in mathematics classrooms, the argumentation moves in two classrooms will be traced. The argumentation process of the whole classroom talk will be deeply analyzed. The classroom talk in this study does not mean the whole utterances of teachers and students in the class time. In fact, it is the teacher and students' utterances when they are in the argumentation process.

METHODOLOGY

Case study is conducted in this study. Case study provides exploration of a "bounded system (a case) or multiple bounded systems (cases) over time through detailed and in-depth data collection" (Creswell, 2007, p. 73). According to Baxter and Jack (2008), case study offers a means to answer "how" and "why" type of questions. Hence, case study is a suitable methodology to analyze classroom talk, which is a discursive product of the complex and dynamic environment of the classroom. In this study, the talk in two mathematics classrooms is analyzed. This is a collective case study focusing on specific content on multiple cases. (Creswell, 2007) These cases provided me to concentrate on the research questions that I have mentioned above. Furthermore, the comparative analysis of these cases helped elucidating, comprehending, and encoding classroom-specific details. Aim of the study is not to make a generalization about the argumentation structure in mathematics classrooms, yet it is to understand the argumentation structure of mathematical discussions in two classrooms and make contributions to the area of argumentation

Table 1. Coding frame of classroom talk

Argumentation moves	Description of argumentation moves	An example from classes
Request for claim	Request for a statement about a fact, rule, or an idea	What about last row, Lale?
Claim	Providing a statement about, a fact, rule, or an idea	$\sqrt{48}$ & $4\sqrt{3}$ are overlapping & they are equal.
Justification	Providing support or evidence for claim	If we take 4 in square root, 4 times 4, it equals 16. 16 times 3, 48. They are equal.
Request for justification	Request for providing support or evidence for claim	Why are they equal?
Approval & elaboration	Statement of approving a claim or justification & providing extra information when needed	Yes, you are right.
Rejection	Statement of rejecting a claim or justification	No, think about it.
Request for evaluation	Request for evaluation of one's claim or justification	What do you think about Kaan's solution?
Collective evaluation	Providing a collective feedback & evaluation to one's claim or justification	Mr. Serkan: What do you think about Kaan's solution? Is there any other solution? Kerim: I think he is doing it a long way. I would do it.

research in mathematics classrooms. Besides qualitative analysis of two case studies some quantitative information will also be used to analyze the classroom environment deeply.

Participants & Setting

The participants of this study are students of two 8th grade classrooms and their mathematics teachers. The reason for choosing 8th grade classrooms is that there is the high school entrance exam at the end of the 8th grade, thus one of the primaries focuses of the schools and teachers is utilization of skill-based questions in the classrooms, which are like the ones in the exam. Since the aim of the study is analyzing argumentation processes in the classrooms especially in which skill-based questions are solved, 8th grade classrooms can be considered as an appropriate context of the study. In each case, the same skill-based questions are solved by different teachers in different schools and in different 8th grade classrooms. A convenient sampling approach is employed to select the two teachers. Although it is a convenient sampling, teachers being from different school types provides me to compare these schools and get different perspectives about the issue (Creswell, 2005). Mrs. Esra (*all the names used in the paper are pseudonyms*) is a teacher who is working at a private school in İstanbul. She has been working at the same school for more than 15 years and she has 20 years' experience of teaching in total. The classroom analyzed for this study consists of 18 students. The school, like the other private schools in the city, has a diverse cultural and economic background. The teacher claimed that mathematics is one of the most valued disciplines in the school, and students' mathematical literacy and problem-solving skills of students is an emphasized issue in general. Mr. Serkan is a teacher working at a public school in İstanbul and he has 8 years' experience. One of his 8th grade classrooms is analyzed in this study. The classroom consists of 25 students. The teacher describes the school, where he works as a school, where middle-class people send their children, and the students having families with moderate socioeconomic background. Both teachers claimed that they pay attention to teaching in a parallel way with the national curriculum and try to solve skill-based questions in their classrooms as far as possible. No information about the study's context is provided and no training is given to the teachers before the study. They were just informed about problem-solving situations of their classrooms will be deeply analyzed. Conversations with teachers revealed that they had not participated in any training about argumentation. For the study, teachers are expected to record their one class time in which they will solve skill-based questions and carry out their routines in their classroom. Since they are already familiar with solving these skill-based questions, their reaction to the requirements of the study is quite normal.

Data Collection

In this study, classroom audiotapes are used for the analysis. For both cases, audiotape of one-class time–40 minutes–is used in the analysis. Catching the traces of the natural and discursive flow of the classroom life of two cases is the focus of the study. Therefore, audiotapes recorded by the teachers are thought to be instrumental, since participation of the researcher could distract the students and spoil the normal flow of the lesson. For this study a worksheet including some skill-based questions is used. Both teachers solved the same questions, which are about square root numbers. In both classrooms, the worksheet is studied at the end of the unit, where students have learned all the procedural rules and the details about the square root numbers. While analyzing the audiotapes, the frequencies of dialogues are also calculated particularly in the classification of classroom dialogues to determine the type of argumentation dialogue, as elaborated in detail in the data analysis section.

Data Analysis

Data analysis comprises two parts: the initial part involves data coding, while the subsequent part entails the categorization of argumentation dialogues. The classroom talk gathered from classroom audiotapes are transcribed and analyzed according to a coding schema. Ayalon and Even's (2016) study has been an inspiration for the creation of the coding for this study. The researchers look for argumentation moves in the mathematics classrooms they observe, and they create a schema of all argumentations moves in the classroom. The schema they provide is facilitating in the sense that it is uncomplicated to catch the traces of the argumentation moves of the whole class time and the scope of the discussions. Hence, in this study, a similar coding schema (**Table 1**) to Ayalon and Even (2016) proposed will be used and argumentation schemas of the two classrooms will be constructed based on the data coded.

In the coding process, the utterances of teachers and students are the unit of the coding process. They are classified according to their function in the argumentation process. For example, if the teacher requests for a solution, generalization, or explanation about a fact/rule etc. then it is classified as "request for claim". If one student responds to that request and makes a claim about

Table 2. Types of argumentation dialogues

Type	Explanation	Abbreviation
Individual	Dialogue between teacher & only one student.	I
Qualified individual	Dialogue between teacher & only one student in which student give justification for his/her claim.	QI
Collective	Dialogue between the teacher and more than one student which involves whole class discussion, evaluation of others' ideas, elaboration on them and/or co-construction of knowledge	C
Qualified collective	Dialogue between the teacher and more than one student involves whole class discussion, evaluation of others' ideas, elaboration on them and/or co-construction of knowledge, and justification of the claims.	QC
Monologue	Dialogue involving the teacher alone.	M

the issue, then it is coded as "claim". Mostly, it is the teacher who requests for claims and students make claims for teachers' requests in the classroom environment. Furthermore, if the student or teacher give justification for his/her claims it is coded as "justification", and if teachers' requests for claim was followed by contributions, elaborations, and justifications of more than one student then it is coded as "collective evaluation". Other codes and explanations can be seen in **Table 1**. The coding process is carried out with another researcher. Most of the time the decisions about the classification of the utterances were the same, yet there were some disagreements, too. At the end, disagreements were resolved through negotiating and persuasion.

The second part of the analysis includes classification of the dialogues. Although the schema gives clear understanding about whole class work, it would be better to determine the number of different types of dialogues. Since the focus of the study is to observe the collective argumentation in the classrooms, there is the need to analyze how much of the discussion is held collectively. The dialogues in classrooms are classified according to teacher student interactions (**Table 2**). If teacher has a discussion with only one student it is classified as 'individual'(I), if teacher creates a collective discussion or it appears spontaneously it is classified as 'collective (C)', and finally if teacher tries to create a discussion but s/he proposes the claim and its justification by herself/himself then it is classified as 'monologue' (M).

In this part of the data analysis, the focus is also on the qualified argumentation process. By qualified, it means that the argumentation process proceeded with justification or evaluation of students, whether it is in an individual or collective way. In other words, students are actively participating in the justification of the claims. As parallel with the literature, collective argumentation process is the focus of attention, however since individual-based interactions are more common in the classrooms, these interactions also needed to be deeply analyzed. Hence, beside the focus on collective argumentation, Individual type of dialogues is also analyzed to understand the depth of whole class argumentation process.

Collective argumentations involve whole class discussion, evaluation of others' ideas, elaboration on them and co-construction of knowledge. If this process also includes students' justification it is coded as qualified collective argumentation (QC). For the Individual dialogues, qualification is determined in terms of the flow of the argumentation process. If a teacher initiates an argument, requests justifications for the student's claim, and the student provides justifications for his/her claim, and then the dialogue concludes, this process is categorized as Individual qualified argumentation (QI). Here, the significant point is that the teacher asks for justification for students' claims and pushes students to prepare proper arguments through questioning strategies. On the other hand, the dialogues between teacher and one student, which either ended with teachers' claims, justifications or direct approval are not considered as qualified since they could not provoke students' reasoning and whole class discussion. This study will search the traces of qualified collective argumentation as a desired outcome in mathematics classrooms, and whole class talk will be analyzed and classified when tracing the qualified ones.

RESULTS

Argumentation Process in Mrs. Esra's Class

Mrs. Esra's one-class time (40 minutes) is analyzed in terms of students and teacher's utterances. **Figure 1** shows the argumentation schema of the whole classroom talk. Circular shapes represent the teacher's actions, while rectangular shapes signify the students' actions. The arrows show the direction of argumentation moves and the number on the arrows points out the sequences of the direction. It is seen that the teacher has made 23 (18+5) attempts to create a discussion, yet five of them quickly ended with teacher's own claims without providing any justification. An example is shown in **Table 3**. Mrs. Esra tries to create a meaningful discussion through bringing crucial issues of skill-based questions, but she gives her own claims and ends the discussion. 18 out of 23 moves are proceeded with students' claims. Most of the time (10 out of 18), students' claims are approved or made some elaborations on them by teacher. They are not canalized into student or class justification or evaluation. **Table 4** shows an example of a sequence of such kind of an argumentation move. In addition, three out of 18 moves ended with teacher's justification. The teacher herself justified the students' claims, which again could not create a meaningful discussion. Three out of 18 moves have concluded with students' participation and discussion and only one out of 18 ended with collective evaluation and collective discussion. Overall, **Figure 1** shows that there have been many attempts for creating argumentation, yet there are few of them promoting student participation, justification, and evaluation of others' ideas.

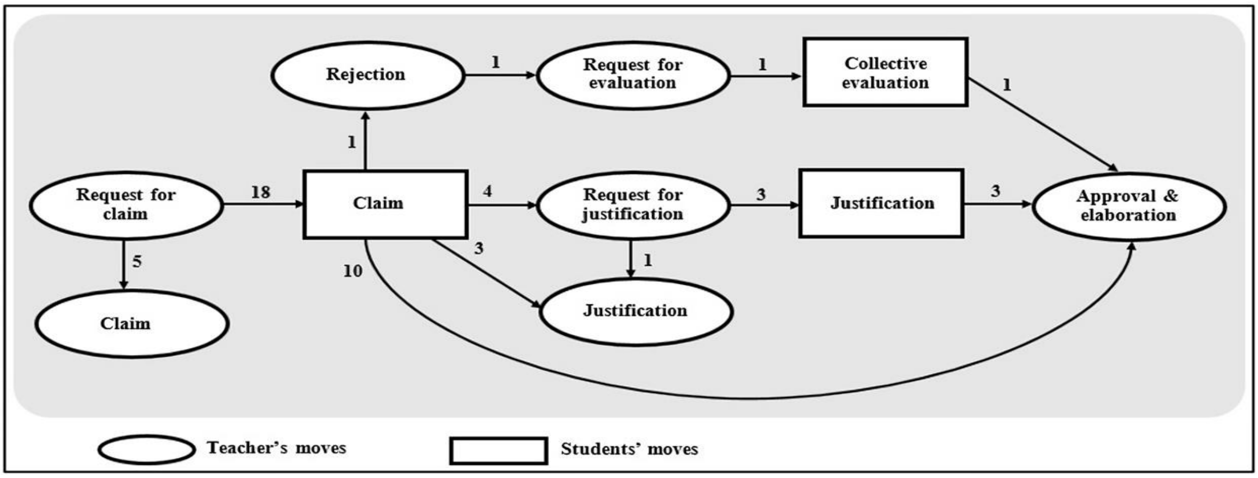

Figure 1. Argumentation schema for Mrs. Esra's classroom (Adapted from Ayalon & Even, 2016)

Table 3. An example of a sequence of argumentation moves-1

Contributor	Utterance	Argumentation moves
Mrs. Esra	In skill-based questions, it is very important to find crucial point of question. How do we determine?	Request for a claim
Mrs. Esra	First one is to read carefully, and the second one is to reason.	Claim

Table 4. An example of a sequence of argumentation moves-2

Contributor	Utterance	Argumentation moves
Mrs. Esra	How should be the square root numbers to obtain a rational number when they are multiplied?	Request for a claim
Student	The same.	Claim
Mrs. Esra	Yes, you are right.	Approval

Table 5. Argumentation dialogues in Mrs. Esra's classroom

Category	Argumentation dialogues		Category	Argumentation dialogues	
	Numbers	Percentage (%)		Numbers	Percentage (%)
IT	15	65	I	12	52
CT	3	13	QI	3	13
M	5	22	C	2	9
Total	23		QC	1	4

Table 5 shows the results of the analysis of classification of classroom dialogues. "IT" in **Table 5** refers to totality of dialogues proceeded in an individual based interaction. They show the amount of all the dialogues which the teacher has initiated with only one student. "QI" refers to the amount of dialogues that teacher initiated an argumentation process in which only one student participated and gave justification for his/her claim. "I" refers to the number of dialogues between teacher and one student, which end with teachers' claims, justifications, or direct approval of the teacher. As mentioned above, they are the type of dialogues in which qualified argumentation could not proceed since they could not provoke students' reasoning and whole class discussion.

The dialogue given in **Table 4** is an example of this type of dialogue. In addition, "CT" is used for the total amount of collective dialogues. "QC "refers only to the number of dialogues in which qualified argumentation is proceeded, and "C" refers to the other collective dialogues in which students could not be involved in justification and reasoning process.

Table 5 shows that in Mrs. Esra's classrooms individual based dialogues (65%) have dominated over the collective ones. 65% of total individual dialogues is composed of 52% I and 13% QI dialogues. Hence qualified individual dialogues are one fifth of total individual dialogues. Moreover, only 13% of the whole class talk proceeded in a collective way and only 4% out of 13% resulted in qualified collective argumentation. Lastly, there is the M type of dialogues in which teachers' argumentation moves end with teachers' own claims or justifications; no one other than teacher participated in argumentation process. **Table 5** also shows the frequency of M type of dialogues. It is seen from **Table 5** that 22% of whole class dialogue is not in fact a dialogue, they are monologue. The teacher's monologue constitutes more one than one fifth of the whole class dialogue. The example dialogue presented in **Table 3** is a Monologue type of dialogue.

Argumentation Process in Mr. Serkan's Class

Mr. Serkan's one class time (40 mins.) is also analyzed in terms of students and teacher's utterances. **Figure 2** shows the direction of argumentation moves and the sequences of them. It is seen that the teacher has made 29 attempts to create a discussion and only one of them quickly ended with teacher's claims without providing any justification. 28 of the moves proceeded with students' claims. 10 out of 28 times, students' claims are directly approved or made some elaborations on them by teacher. They are not canalized into student or class justification or evaluation.

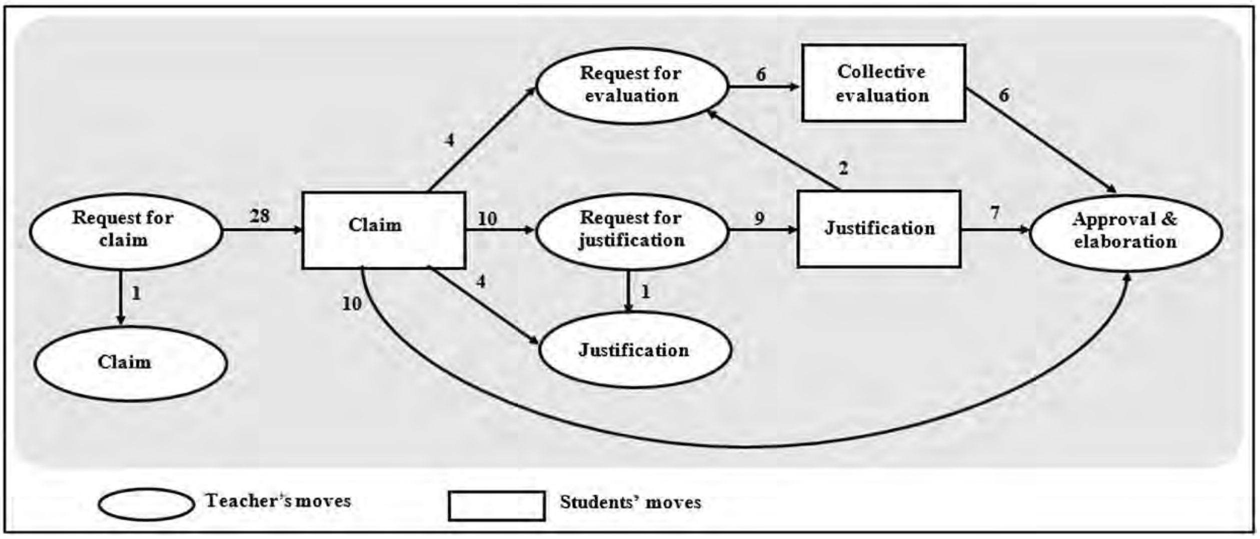

Figure 2. Argumentation schema for Mr. Serkan's classroom (Adapted from Ayalon & Even, 2016)

Table 6. An example of a sequence of argumentation moves-3

Contributor	Utterance	Argumentation moves
Mr. Serkan	(Last part of the question) … Okay. What about last row, Lale?	Request for a claim
Lale	$\sqrt{48}$ and $4\sqrt{3}$ are overlapping and they are equal.	Claim
Mr. Serkan	Why are they equal?	Request for justification
Lale	If we take 4 in the square root, 4 times 4, it equals 16. 16 times 3, 48. They are equal.	Justification
Mr. Serkan	That's right.	Approval

Table 7. Argumentation dialogues in Mrs. Serkan's classroom

Category	Argumentation dialogues		Category	Argumentation dialogues	
	Numbers	Percentage (%)		Numbers	Percentage (%)
IT	19	66	I	12	41
CT	9	31	QI	7	24
M	1	3	C	3	10
Total	29		QC	6	21

Table 8. Comparison for argumentation dialogues in both classrooms

Category	Esra		Serkan	
	Numbers	Percentage (%)	Numbers	Percentage (%)
I+C+M	19	83	16	55
QI+QC	4	17	13	45
Total	23		29	

In addition, four out of 28 moves end with the teacher's giving justification to students' claims. 10 out 28 moves have concluded with students' participation and justification for their claims. **Table 6** shows an example dialogue of one of these 10 sequences. Mr. Serkan requests justification through asking a complex question and the student provides justification for her claim. Lastly, four out of 18 is ended with collective evaluation and collective discussion. Overall, **Figure 2** shows that there have been many attempts for creating argumentation and almost half these attempts succeeded in promoting student participation, justification, and evaluation of others' ideas.

Table 7 shows the details about the type of dialogues in Mr. Serkan's classrooms. It can be seen from **Table 7** that in Mr. Serkan's classrooms, the dialogues are mostly individual based dialogues (66%) and it is almost the same as Mrs. Esra's classroom. 66% of total individual dialogues are composed of 41% I and 24% QI dialogues. Hence qualified individual dialogues are about one third of total individual dialogues and one fourth of total dialogues. In addition, 31% of the whole class talk proceeded in a collective way and 21% out of 31% resulted in qualified collective argumentation. It is seen that qualified collective dialogues are more than half of the collective dialogues and one fifth of the total dialogues. The number of Monologues in Mrs. Serkan classroom is only %3 of total dialogues in the classroom.

Table 8 illustrates a comparison between the classification of dialogues in two classrooms. In Mrs. Esra's classroom only 17% of classroom's total dialogues composed of qualified argumentation process as opposed to 45% in Mr. Serkan's classroom. 83% of the total dialogues in Mrs. Esra's classroom did not include qualified argumentation as opposed to 55% in Mr. Serkan's classroom.

DISCUSSION & IMPLICATIONS

In this study, argumentation processes of two 8th grade classrooms are analyzed. There are both similarities and differences in the argumentation process of Mrs. Esra and Mr. Serkan's classrooms. Mrs. Esra's lesson is a teacher centered lesson and students are not actively participating to the lesson. She has made so many argumentations move, yet argumentation schema of the lesson shows that only four out of 28 moves could create a qualified argumentation process; in other words, only four times students has provided justifications for their claims, or they are collectively evaluated. Sometimes the teacher herself has given the claim and sometimes she justified students' claims. Yet, the most questionable part of the lesson is that almost half of the argumentation moves ended after teacher's approval of students' claims. It is mainly due to her questioning strategies. Questioning strategies are essential tools to be analyzed in argumentation learning environments (Berland & McNeill, 2010) Asking "how" and "why" questions is significant to create classroom discussion (McCrone, 2005). These questions are open-ended and complex questions provoking students' evaluation and reasoning. Mrs. Esra mostly asked simple and procedural type of questions. By the term "simple questions," questions that elicit brief and prompt responses, often taking the form of "yes" or "no" answers, or brief factual explanations are indicated. For instance, "Is 0 considered a rational number?" is a simple question. On the other hand, "procedural questions" are inquiries that seek an explanation of the method or process required to solve a particular problem or issue. For instance, "how do we multiply two square root numbers?" is a procedural question since it looks for an answer, which includes the process of multiplication with square root numbers. When Mrs. Esra asked simple and procedural questions, students were also supposed to give simple and procedural answers. When the students gave the answer, the conversation was finished. Hence, the frequent use of simple and procedural type of questions inhibited to create a qualified argumentation process. In fact, she also asked a few important complex questions about the strategies to solve skill-based questions. She tried to push students to understand the importance of these questions and emphasized the ways to solve them. However, these complex questions were answered by the teacher herself. Complex questions could create qualified argumentation process in the classroom, but it could not proceed with the right moves.

Mr. Serkan's classroom is again a teacher centered class. However, students were actively participating in the class discussion. There was a sincere and friendly atmosphere in his classroom. The number and variety of students who talked in class time much greater than Mrs. Esra's classroom. Students really seemed to enjoy what they are doing in the class. This is probably because of the age difference between teacher and students. Mr. Serkan was also participating in students' out of context conversations and motivating them to make jokes in the classroom.

The argumentation schema of Mr. Serkan's classroom shows that students were active in mathematical discussion, too. The schema of his classroom is more intricate than the other classroom. There are more complex routes in this schema, which may mean that the discussion in this class is more fruitful. Mr. Serkan has also made many argumentations moves in one class time. He also provided a claim for his request, and he justified some of the students' claims. Still there have been 13 qualified argumentation routes. Six of these routes were carried out collectively. In terms of questioning strategies, Mr. Serkan followed a pattern not much different from Mrs. Esra's strategies. He also used too many simple and procedural questions. However, the number of complex questions asked in Mr. Serkan classroom is more than the other. Hence, Mr. Serkan paid more attention to provoke students' reasoning. Yet, some complex questions of Mr. Serkan were answered as if it was a procedural type of question. This is probably because students are not accustomed to being asked these types of questions or the teacher also asked these open-ended questions not on purpose.

In terms of types of dialogues, the two classrooms have different characteristics, too. **Table 5** and **Table 7** show that although the percentage of IT in both classrooms are almost same, the percentage of qualified argumentation processes is significantly different. Mr. Serkan's classroom has more qualified both individual and collective argumentation processes. **Table 8** gives significant quantified data about the comparison of two classrooms. In Mrs. Esra's classroom only 17% of total dialogues included qualified argumentation, on the contrary, in Mr. Serkan's class almost half of the total dialogues included qualified argumentation process. Although Mr. Serkan's class was much better in terms of qualified argumentation process, both classes had poor qualities in terms of collective discussion. Teacher-individual interactions were the dominant type of interactions.

Collective Argumentation in Classrooms

In depth analysis of two different mathematics lessons point out the inadequacy of collective and qualified discussions. The collective argumentation learning environment needed for solving skill-based questions could not be observed intensely in both classrooms, especially in Mrs. Esra's classrooms. Having seen that Mr. Serkan's classroom is much better, the study still indicates that most of the classrooms' talk is individual based and have a poor argumentation structure. Here, a few issues need to be addressed. First, it seems that there is not a significant difference between the classrooms of either public or private school. Indeed, the argumentation structure of the classroom in public school was much better. Since neither teacher has participated in any training about argumentation, the difference may stem from their teaching style, which points to the significance of the role of the teacher. Teachers' role is crucial in creating argumentation learning environments (Jiménez-Aleixandre, 2007; McCrone, 2005). They are the active designers of the classrooms, hence the awareness of sociocultural aspect of learning and importance of different questioning strategies; and being knowledgeable about creating argumentation learning environments is necessary for them. Mr. Serkan is a young teacher and less experienced than Mrs. Esra. Yet, it seems that experience is not a critical factor. It is again an issue pointing to the need for professional development programs for teachers. No matter the years of experience, teachers need to have sufficient training in argumentation. McCrone (2005) indicates in her article that creating collective discussion in the classroom is not a quick and easy process. In her longitudinal study they revealed that it has taken almost one school year that teacher's attempts could provide collective discussion in her classroom. Thus, the research reminds us that this is a process that both teachers and students need to get accustomed to. Teachers as the main actors need to push students to make reasoning, provide justifications for their claims and thereby get involved in the argumentation process after they are prepared with necessary training for it.

To summarize, the results gathered from the analysis of two classrooms indicated that argumentation processes in these two mathematics classrooms have lack of critical aspects of argumentation. Although one classroom has more engagement than the other, overall, the argumentations schemas point to an insufficiency in the number of justifications presented by students and collective evaluations in the classrooms. Dialogue type analyses have also revealed that individual type of conversations dominate in the classrooms. The type of dialogues initiated by the teacher significantly influences the form of interaction and the argumentation process. A high number of individual dialogues may hinder the engagement of the entire classroom in the discussion. In this sense, this study contributes to the literature regarding the critical role of teachers in fostering collective argumentation processes in classrooms.

Limitations of Study & Recommendations for Future Research

This study has been carried out within the framework of several limitations. First, the study's participants consist of two teachers and two 8th grade classrooms. The decision to investigate two teachers and classrooms, rather than just one, was made to analyze and observe a wider range of variables arising in various situations. However, despite this intention, the study has been restricted to this specific group of students and teachers in terms of participant selection. Second, audiotapes of the lessons are used in the analysis. Although careful listening and transcribing enabled us to understand the details of the lesson, it would be appropriate for the researcher to participate in the lesson to feel the discursive flow of the lesson.

The analysis was carried out with only one class of the teachers. For detailed description of the classroom norms regarding the argumentation practices, more classes need to be analyzed. Additionally, teachers' argumentation practices are analyzed through looking at just one classroom. More observations in teachers' classrooms would deepen our understanding about teachers' tools and strategies to develop qualified and collective argumentation in the classroom.

The findings of this case study cannot lead us to generalize that all mathematics classrooms lack enough collective classroom discussion and qualified argumentation process no matter the school type. However, the findings may help us to contemplate the results and find ways to improve the situation. Thus, further studies on collective argumentation in mathematics classrooms is needed to better understand the situation in Turkey. It is evident that mathematical reasoning is a very crucial skill in today's quickly changing world and one important way to attain this skill is through creating collective argumentation learning environments in schools.

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