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# How Productive is the Productive Struggle? Lessons Learned from a Scoping Review

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# How Productive is the Productive Struggle? Lessons Learned from a Scoping Review

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## Abstract

Since its first notable appearance in the *Second Handbook of Research on Mathematics Teaching and Learning,* the phrase productive struggle has garnered support from a few, yet very influential, educational researchers. However, since the phrase was popularized by Hiroko Warshauer in 2011, examinations of the productive struggle have grown substantially within mathematics education. In this article, we present a systematic review of the literature on the productive struggle to quantify the breadth and depth of existing work in the area. Our analysis indicates that many studies: (1) provide a direct implementation of the productive struggle; (2) primarily involve teachers and not students; (3) present primarily qualitative or non-empirical research approaches; and (4) specify the NCTM content area focus less often across studies. Thus, we conclude that there are opportunities for rigorous research designs reporting on observable trends in the implementation of the productive struggle, explicitly targeting underexamined NCTM content and process standards, and focusing on how the productive struggle can support equity in mathematics. We believe that these opportunities should be investigated to provide a broader evidence base for developing meaningful experiences in mathematics for school-aged children that capitalize on the interconnectedness of attribution, meta-cognition, and controlled frustration in mathematics learning.

## Introduction

Conceptual understanding, creativity, and persistence are essential to success in mathematics. Although many mathematics prodigies exist, for most mathematics students and even mathematics educators, experiencing and overcoming challenges in mathematics is critical to our love for mathematics. The following statement best explains the sense of fulfilment one gains from persevering to solve a problem: A great discovery solves a great problem, but there is a grain of discovery in the solution of any problem. Your problem may be modest, but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your means, you may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprint on mind and character for a lifetime. (Polya, 2014, p.17) For many years mathematics educators have lacked an appropriate conceptual framework to characterize the interplay between persistence, creativity, and conceptual understanding. Recently, educators and researchers have begun to conceptualize the interplay between these elements as the "productive struggle". Since its first notable appearance in the *Second Handbook of Research on Mathematics Teaching and Learning,* the phrase productive struggle has garnered support from a few, yet very influential, educational researchers. However, since the phrase was popularized by Hiroko Warshauer in 2011, examinations of the productive struggle have grown substantially within mathematics education. In this article, we present a systematic analysis of literature on the productive struggle in an attempt to quantify the breadth and depth of existing work in the area.

Currently, there are several common definitions of the productive struggle applied within mathematics education. According to the Mind Research Institute (2021), the "productive struggle is the process of effortful learning that develops grit and creative problem solving (p, 11)". While NCTM's Principles to Actions (2014) defines productive struggle as students delving "more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions" (p. 48). Yet another definition of the productive struggle is "developing strong habits of mind, such as perseverance and thinking flexibly, instead of simply seeking the correct solution" (Renaissance, 2020, para. 1). Together these definitions encompass many conceptualizations as well as actualizations of the productive struggle. In the present study we proffer that the productive struggle refers to the process of engaging with challenging tasks or problems that require effort, critical thinking, and persistence to solve. It involves encountering obstacles, making mistakes, and experiencing a level of cognitive discomfort while actively working towards a solution. The definition above will be used to characterize the productive struggle in the present study.

The productive struggle is considered essential for deepening understanding, fostering problem-solving skills, and promoting long-term learning. The productive struggle helps learners move beyond surface-level memorization and encourages them to grapple with complex concepts, thus leading to more meaningful and retained knowledge. Because the productive struggle is essential to mathematics education, it is crucial to understand the characteristics of the literature guiding the operationalization and implementation of the productive struggle in mathematics classrooms. The theory-to-practice conundrum is often a stumbling block for progress in mathematics education. However, the productive struggle is an anomaly in that many teachers have been exposed to and are committed to actualizing the productive struggle in their classrooms. Yet, the absence of a consensus on the most salient considerations and expectations guiding the implementation of the productive struggle can impede progress, reduce efficacy, and ultimately generate undue frustration for students and teachers. To address this challenge, we conducted a scoping review of the literature examining the productive struggle in mathematics classrooms.

## Existing Reviews and Literature Related to the Productive Struggle

To our knowledge, a systematic review of the mathematics productive struggle does not exist. Hence, the importance of the present study. Nonetheless, there have been several reviews of related constructs that can inform the present scoping review. Specifically, we take a moment to consider attribution, meta-cognition, and frustration within the context of mathematics teaching and learning as all these constructs can inform our conceptual understanding of the productive struggle. The concept of the productive struggle involves understanding how individuals engage with challenges, obstacles, and difficult tasks in a way that promotes learning and growth. Therefore, attribution theory, metacognition, and frustration all play pivotal roles in shaping the study of the productive struggle in mathematics. In the sections that follow, we take a moment to consider attribution, meta-cognition, and frustration within the context of mathematics teaching and learning as all these constructs have been examined extensively within the extant literature. Moreover, attribution theory, metacognition, and frustration uniquely and collectively inform our conceptual understanding of the productive struggle. Attribution theory influences how students interpret their struggles, metacognition enables them to reflect and regulate their thinking, and controlled frustration can lead to meaningful learning experiences. Recognizing the contributions of these constructs to our understanding of the productive struggle and providing appropriate guidance can help learners develop strong problem-solving skills and a positive attitude towards challenging mathematics tasks. The following literature review synthesizes related research on these constructs (i.e., attribution, meta-cognition, and frustration) and identifies connections to the productive struggle as well as areas for further investigation.

**Attribution in Mathematics**

One key element of the productive struggle is that students persist and persevere to develop an understanding of mathematics that generates sustained learning and deeper appreciation for the mathematics content. However, understanding to what or whom one attributes success or failure is essential to understanding persistence and perseverance in mathematics. Attribution theory is a psychological framework that deals with how individuals explain the causes of their own behaviours and the behaviours of others (Graham, 1991). According to attribution theory, individuals attribute their success and failures to internal (i.e., based on personal characteristics or abilities—ability or effort) or external (i.e., attributed to outside factors or circumstances—luck or task difficulty) causes (Dweck, 1986; Miller, & Norman, 1979; Powell & Caseau, 2004).

In the context of productive struggle, attribution theory comes into play when students face challenging tasks in mathematics. If they attribute their struggles to internal factors such as lack of ability (e.g., "I'm just not good at math"), they are more likely to experience negative emotions and a decreased willingness to persist. On the other hand, attributing difficulties to external factors like the complexity of the problem or a lack of prior exposure can lead to a more positive outlook and a greater likelihood of engaging in productive struggle. At its core, attribution theory attempts to identify to what or whom a student attributes their success or failure. Thus, attribution theory must be considered in the context of the productive struggle because how a student attributes their success can impact how they receive a mathematics task, which can impact whether the activity reflects the productive struggle.

In a review of attribution in mathematics, Shores and Smith (2010) concluded that the majority of attribution research in mathematics indicates that students attribute their success and failure in mathematics to either internal, external, or a combination of both causes that can be linked to ability, effort, task difficulty, or perceived luck. Hence, understanding the role of attribution in the mathematics productive struggle is essential because attribution studies indicate that when students attribute success to external factors such as task difficulty, they tend to have lower mathematics achievement compared to students who attribute their success to internal factors such as effort. Because perceived task difficulty and effort are important considerations for the development of the productive struggle, the effects of attribution in mathematics cannot be underestimated.

### Meta-Cognition in Mathematics

We posit that meta-cognition is an essential consideration for the productive struggle, because meta-cognition in mathematics tasks is considered one of the most important predictors of mathematics performance (Kuzle, 2018; Ohtani & Hisasaka, 2018), yet few researchers have attempted to impart meta-cognitive enrichment within the context of the productive struggle (see Lemely, Ivy, Franz, & Oppenheimer, 2019). However, we posit that meta-cognition is an essential consideration for the productive struggle. According to Flavell (1976) meta-cognition is “one’s knowledge concerning one’s cognitive processes and products, or anything related to them…[and] the active monitoring and consequent regulation and orchestration of these processes” (p. 232). Metacognition refers to the awareness and understanding of one's own thought processes and cognitive abilities (Zulkiply, 2009). It involves actively monitoring and regulating one's thinking to enhance learning and problem-solving. In the context of productive struggle, metacognition plays a crucial role. Engaging in metacognitive practices allows students to reflect on their struggle, evaluate their problem-solving strategies, and make adjustments when necessary. Through metacognition, learners can recognize when they're experiencing productive struggle, understand the benefits of persevering, and choose appropriate strategies to navigate challenges effectively. As noted earlier one key element of the productive struggle is developing “habits of mind” or ways of thinking about mathematics that resemble the process employed by mathematicians (Cuoco et al., 1996; Mark et al., 2010). Through the productive struggle students are encouraged to think about thinking to promoting preservice and problem-solving practices akin to those implemented by mathematicians. However, habits of mind are just one of the many explicit and implicit meta-cognitive considerations related to the productive struggle.

Yet, another element of the productive struggle is having students think deeply about the mathematics, connections to prior knowledge, and explore creative solutions. Undoubtably, these processes require meta-cognition that is often difficult for teachers to foster due to a lack of the pedagogical knowledge necessary to develop a meta-cognitively rich learning environment (Dignath & Buttner, 2018). We contend that the deep understanding of pedagogies to promote meta-cognition reflect the similar understandings necessary to manage the productive struggle in a mathematics classroom as the teacher must understand how to foster student reflection, exploration, and reconsideration of prior knowledge. All of which are highly meta-cognitive tasks that have been shown to promote mathematics achievement and higher levels of problem-solving effectiveness.

### Frustration in Mathematics

Academic rigor has long been a cornerstone of mathematics teaching and learning, as students are expected to feel both success and challenge daily in the mathematics classroom. However, one longstanding point of contention within the mathematics education community is the appropriate implementation of scaffolding to provide a challenge while avoiding student frustration. Frustration in mathematics can arise when students encounter problems that are perceived as too difficult or when they struggle to grasp certain concepts (Levine, 2012). This frustration can lead to negative emotions, reduced self-confidence, and a reluctance to continue engaging with the material. However, productive struggle suggests that experiencing a moderate level of frustration can actually be beneficial. When students persevere through frustration, they develop a greater sense of accomplishment, resilience, and a deeper understanding of the content. The key is to provide support and guidance during this process to prevent frustration from turning into a barrier that hinders learning. In a review of literature on frustration in mathematical problem-solving, Riegal (2021) concluded that prior researchers suggest that frustration is associated with different cognitive and affective features of mathematics problem-solving, the majority of which but not all happened to be negative. Yet, there are emerging arguments for the importance of the productive struggle or what some have defined as productive failure (Warshauer, 2015). At the heart of the productive struggle construct is the importance of challenging students and encouraging students to struggle. The importance of challenge and frustration is evidenced by the many references to effort, persistence, and perseverance within the active definitions of the productive struggle (Staff, 2022; Townsend et al., 2018). Current definitions of the productive struggle encourage students to move past frustration through the same levels of persistence that are commonplace in sports, gaming, lego-building, and other things students value on a personal level.

Researchers note that overcoming challenges and frustrations within areas of personal interest were considered integral to the personal or recreational tasks, whereas struggling in mathematics has long been considered problematic (Hill Learning Center, 2020). Thus, part of the challenge facing teachers and researchers is translating the importance of frustration to develop passion to the realm of mathematics. Previous theoretical considerations have highlighted the importance of frustration to support or discourage mathematics success. Specifically, Goldin (2000) contends that frustration is the nexus between satisfaction and despair on the affective pathways in problem-solving. According to Goldin, when a student is faced with frustration during problem-solving the proper encouragement can eventually lead to satisfaction. However, a lack of the proper encouragement can promote mathematics anxiety, which can manifest into fear or despair (See Goldin, 2000). Hence, how teachers and students respond to frustration during mathematics tasks is essential to the potential benefits of the productive struggle in mathematics.

In summary, the literature on the productive struggle in mathematics has yet to be reviewed extensively. However, there are several related reviews that inform the current scoping review and justify the need for the present study. Specifically, it is important to understand how attribution, meta-cognition, and frustration are considered within the current literature related to the productive struggle in mathematics as these constructs are essential to the successful implementation of the productive struggle. Unfortunately, the interconnectedness of these constructs has only been alluded to in prior studies thus there is a need to characterize these and other aspects of study considerations through the process of a scoping review.

## Limitations of This Study

As mathematics educators, we are interested in the effect of the productive struggle on the teaching and learning of mathematics. In the present study, we review existing literature to unpack how the productive struggle impacts the teaching and learning of mathematics by examining the characteristics of prior studies. However, we acknowledge that by only examining the "productive struggle" construct within mathematics we may be excluding studies that could inform teaching and learning in general, but not within mathematics education specifically. For instance, the productive struggle is a pedagogical consideration in STEM and non-STEM disciplines (Chen, 2022; Murdoch et al., 2020; Scott, 2012), yet we argue that for the purpose of the present study it is important to place an acute focus on pedagogical content knowledge that is specific to mathematics teaching and learning.

## Problem Statement

Numerous national and international mathematics organizations highlight the importance of persistence in mathematics problem-solving within their policy documents. For example, the 2019 Trends in International Mathematics and Science Study Framework states that "learning mathematics improves problem-solving skills, and working through problems can teach persistence and perseverance" (Mullis & Martin, 2017, p. 13). Researchers have also observed other notable effects of the productive struggle within the biomedical sciences. For instance, engaging in a productive struggle has neurological benefits (i.e., the production of myelin). According to Sriram (2020), challenging tasks spur the production of myelin, a substance that increases the strength of brain signals. This biological link corroborates what mathematics education researchers observe in the classroom (Lemley et al., 2019). Given the mounting evidence surrounding the importance of the productive struggle, mathematics education research productivity has followed suit.

Over the last decade, numerous studies have been conducted examining the productive struggle within mathematics educational spaces (Warshauer, 2015). A simple Google Scholar search of the phrase "productive struggle" yields nearly 3,000 hits. Yet, no one has synthesized the literature on the productive struggle within mathematics education to our knowledge. A scoping review is an important precursor to a systematic review or meta-analysis as it provides a broad overview of the "scope" and depth of available research on a specific topic. This form of literature synthesis is especially pertinent to the operationalization and implementation of the productive struggle in mathematics. Thus, the aim of the present study is to respond to the following two research questions:

1. What scholarly works have been published from 2011 to 2020 in relation to the productive struggle in K-12 mathematics educational contexts?
2. How do these works link the productive struggle to the teaching and learning of mathematics?

The popularity of the productive struggle has grown exponentially across popular and social media outlets. However, for the purpose of the present study we only include peer-reviewed scholarly products. Thus, blogs, Op-Eds, and other commentaries were not included in the present study despite the possible value of these resources to inform our knowledge of how the non-academic audiences characterize the productive struggle in mathematics. However, we argue that the vast variety in audience, scope, and format present across these works would create too much inconsistency across the data and limit the reliability and replicability of the results of the present study.

The productive struggle is a relatively new educational construct, although its theoretical underpinning can be traced back to early research within mathematics problem-solving. According to Leinwand et al. (2014), "Effective mathematics teaching uses students' struggles as valuable opportunities to deepen their understanding of mathematics. Students come to realize that they are capable of doing well in mathematics with effort and perseverance in reasoning, sense-making, and problem-solving" (p. 10). Appropriately, perseverance in reasoning, sense-making, and problem-solving are often what mathematics educators use to characterize productive struggle. This form of persistence supports achievement in mathematics (Roble, 2017). Mathematical understanding is developed, enriched, and extended by persistence in problem-solving (Hiebert & Wearne, 1993). Thus, our scoping review of the productive struggle has practical as well as empirical merits.

## Scoping Review Methods

It is important to explain why a scoping review of the productive struggle in mathematics education is warranted and necessary. Gaps in research can be addressed through a scoping review. A scoping review is a precursor to a systematic review because it serves as the first step toward building a more robust empirical foundation. By quantifying the breadth and depth of existing work, the present scoping review provides a clearer understanding of the current state of research, identifies trends, and pinpoints areas that require further exploration. A scoping review study of the productive struggle in mathematics education is imperative due to the evolving nature of this concept, the existing gaps in research methodologies, the need to align with educational standards, and the potential to enhance equity in mathematics education. By systematically reviewing the literature, this study can lay the groundwork for future empirical research, inform educational practices, and contribute to the development of meaningful mathematics learning experiences that cater to diverse student needs.

Conducting a scoping review of the productive struggle in mathematics education is crucial due to the growing importance of the productive struggle within educational research. The evolution of the productive struggle from its initial mention in the *Second Handbook of Research on Mathematics Teaching and Learning* to its significant expansion since Hiroko Warshauer's popularization in 2011 has increased the utilization and consideration of the construct within mathematics classrooms globally. This expansion indicates a critical need for a comprehensive investigation of the existing literature in this area.

As teachers begin to implement activities and processes to enact the productive struggle within mathematics classrooms, it is imperative that the construct be understood with sufficient depth and breadth. Moreover, if noticeable gaps exist in our understanding of the affordances and constraints of the productive struggle and its implementation in the classroom, the results could be devastating for young learners. Given the vast uptake of the productive struggle within the preservice and in-service teacher communities (Rodgers, 2020), the potential to address underexamined NCTM content and process standards is also an important consideration that can be addressed through a scoping review. Because educational practices are often guided by these standards, it is crucial to align the examination of the productive struggle in the literature with these standards to ensure its broad representation in the extant mathematics education literature. A scoping review can offer insights into how well the existing research aligns with these standards and highlight opportunities for further exploration.

Furthermore, the potential for productive struggle to support equity in mathematics education is an additional consideration for exploration through a scoping review. This is a compelling rationale for conducting a scoping review, as understanding how the productive struggle can contribute to equitable educational experiences is vital for promoting inclusive and effective teaching practices, but this cannot be examined through a systematic review without a representative body of literature to serve as the unit of analysis. By conducting a scoping review of the literature, researchers can examine the feasibility of this and other lines of inquiry. Thus, a scoping review was the method of choice for the present study given the state of the literature in the field and the need for a comprehensive assessment of the depth and breadth of mathematics education research examining the productive struggle.

Scoping reviews are a relatively popular literature synthesis technique within the biomedical sciences. Yet, scoping reviews remain emergent within educational research. Notable scoping reviews have been conducted over the last five years within science, technology, engineering, and mathematics education (STEM) (Denton & Borrego, 2021; Hickmott et al., 2018; Odell et al. 2020).

A scoping review aims to map the existing literature in a field of interest regarding the volume, nature, and characteristics of the primary research (Arksey & O'Malley, 2005). The utility of a scoping review is optimized when previous literature reviews or meta-analyses are absent (Mays et al., 2001), thus, the rationale for the present study. The purpose of this study is to provide an overview of the existing literature on the productive struggle in mathematics education.

Four specific objectives guide this scoping review: 1) conduct a systematic literature search of published studies and grey literature, 2) map the characteristics of studies and the methodologies used, 3) identify limitations and challenges present, and 4) subsequently propose recommendations for advancing the consistency with which the productive struggle is examined and implemented within mathematics education. We borrow our rationale for the present study from Arksey and O'Malley, thus in the present scoping review we seek to examine the extent, nature and range of research activity and to identify research gaps in the existing literature (Arksey & O'Malley, 2005, p. 6–7). In this section, we present the process for searching the databases and explain the process followed and methods used for classifying the studies for analysis.

**Search, Retrieval, and Review Strategies**

The productive struggle is an emergent line of inquiry within mathematics education as the construct is popular amongst both mathematics education researchers and in-service teachers. Over the last decade or so, literature on the productive struggle has expanded beyond the research spaces to popular media outlets used by teachers and administrators to locate instructional resources. Thus, the literature used in the scoping review were both empirical and non-empirical ranging from the years 2011 to 2020 to reflect the full scope of academic resources available. The following keyword combinations were applied during the search: productive struggle AND mathematics ($n$ = 35); productive struggle AND problem-solving ($n$ = 41). Given the need to perform an inclusive rather than exclusive search of the literature, Google Scholar was used as well as EBSCO databases.

The results were limited to scholarly works including but not limited to peer-reviewed journal articles, conference proceedings, dissertations, as well as master's theses. After exporting the results to our shared Mendeley Library, scholarly works that were irrelevant to the productive struggle were excluded and annotated. The results are shown in Figure 1 and can be summarized as follows.

The database search process resulted in an initial pool of 76 documents. Next, duplicates were identified across the different database searches. After removing duplicates, a pool of possible studies for inclusion was identified ($n$ = 69). After the initial search process, the 69 documents selected for consideration were screened using the following inclusion criteria and protocol. Documents were excluded it they did not have an explicit mathematics context or setting. Additionally, documents were excluded if they did not provide adequate explanation or consideration of how the productive struggle was incorporated. This process led to the elimination of 24 documents, leaving a final pool of 45 documents for review (see Figure 1).

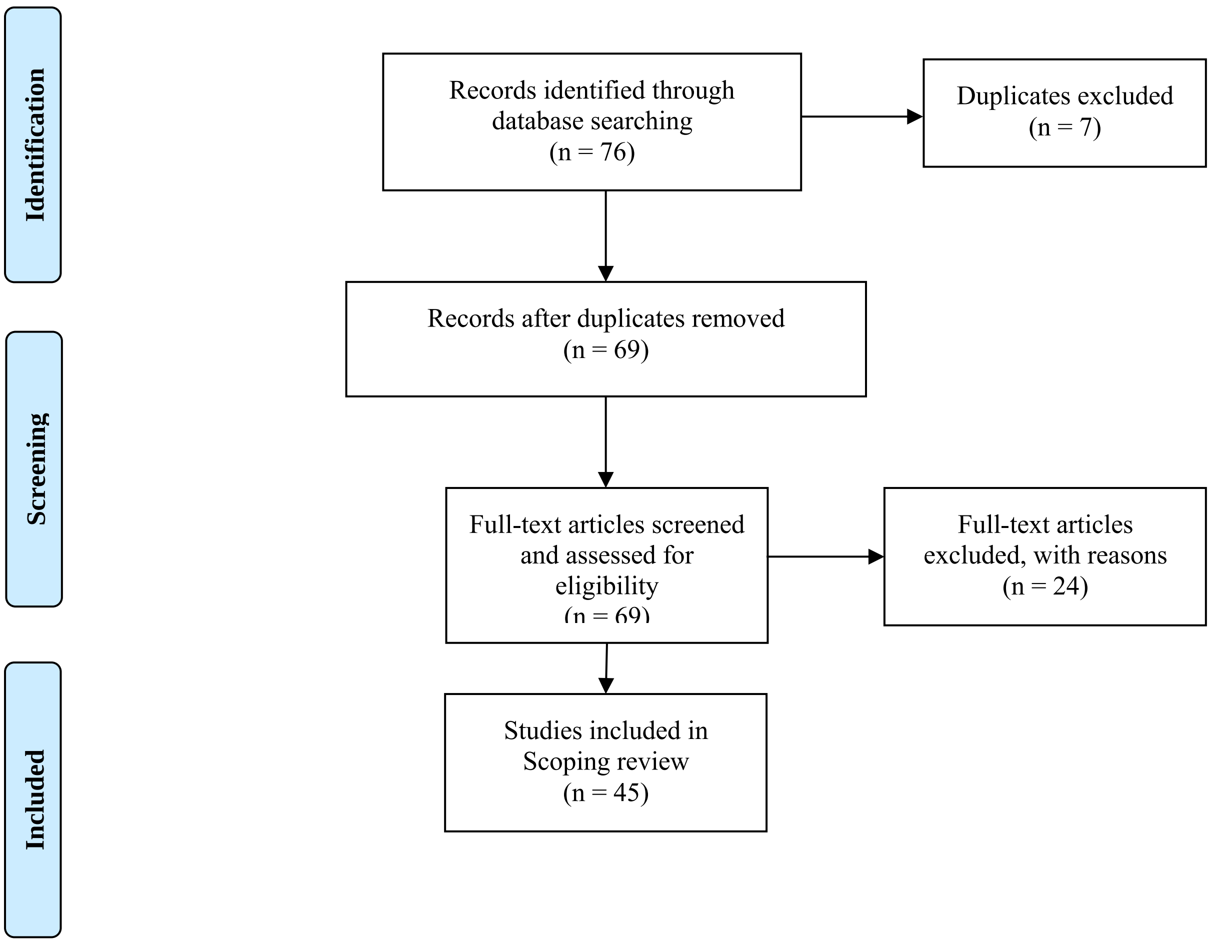


Figure 1. PRISMA Diagram of Search, Retrieval, and Review Process

The document review process took place in three steps. First, studies were classified according to their implementation of the productive struggle (i.e., direct or indirect). Studies that indirectly implemented the productive struggle were those where productive struggle concepts were present, but there was no evidence that the productive struggle was operationalized as an independent or dependent variable under consideration within the study regardless of the methodological approach. While studies with a direct implementation of the productive struggle had explicit connections and operationalizations of the productive struggle as an independent or dependent factor under consideration within the study regardless of the methodological approach. Furthermore, we place particular attention on how the studies attended to attribution, metacognition, and frustration within the context of productive struggle.

Next, the methodological orientation of each study was categorized as either quantitative, qualitative, mixed-methods, non-empirical, or other. Non-empirical works were characterized as works that were mostly theoretical or practical in nature and tended to include practitioner pieces or discussion of how to implement specific aspects of the productive struggle. While the other category was reserved for non-empirical overviews or summaries of the productive struggle. We also characterized the implementation of the productive struggle as direct (i.e., independent or dependent variable) compared to indirect (i.e., moderating variable or other consideration). Finally, the following study characteristics were categorized: participants, grade level, NCTM Content, NCTM Process Standards, and Equity Focus.

## Results

The systematic literature search yielded 45 relevant results to the paper's goal of providing an overview of the existing literature on the productive struggle in mathematics education. A complete list of the characteristics of the included studies is presented in Table 1.

Table 1. Characteristics of Included Studies

| | APA Reference | Year | Purpose | Methodology | Sample | Grade Level | Results |
|---|---|---|---|---|---|---|---|
| 1 | Russo et al. (2020) | 2020 | Examined teacher beliefs about the role of student struggle in the mathematics classroom. | Qualitative | Inservice Teachers | K-5 | Teachers developed resilience indicated that the process of struggling was central to learning mathematics. |
| 2 | Sayster and Mhakure (2020) | 2017 | Explored high school students' productive struggles during the simplification of rational algebraic expressions in a high school mathematics classroom. | Qualitative | Students | 9-12 | Conclude that the struggle is not negative but is critical for student learning. |
| 3 | DuCloux et al. (2018) | 2018 | Examined the process of productive struggle and how to best engage prospective math teachers in productive struggle | Qualitative | Preservice Teachers | K-12 | Pre-service teachers gained tools to approach challenging tasks. |
| 4 | Freeburn | 2017 | Explored ways to | non-empirical | Preservice | NA | Assessment questions provided |

| | APA Reference | Year | Purpose | Methodology | Sample | Grade Level | Results |
|---|---|---|---|---|---|---|---|
| | and Arbaugh (2017) | | determine and examine metacognition. | | Teachers | | access to students' metacognition during challenging tasks. |
| 5 | Baker et al. (2020) | 2020 | Identified instances of the productive struggle with a student learning fractions. | non-empirical | Students | K-6 | Supporting the productive struggle can be difficult at first but is beneficial for both the teachers' deeper learning of their classroom and students' achievement. |
| 6 | Vazquez et al. (2020) | 2020 | Investigated parent perceptions of productive struggle and how they can better facilitate learning math at home. | Quantitative | Parents | K-6 | Policy and programs for parents should increasingly focus on specific activities and behaviors that are supported by evidence like the productive struggle. |
| 7 | Paape (2016) | 2016 | Posed questions that encouraged students to use reasoning or justification. | Qualitative | Students | 7-8 | Providing coded feedback to students and using highlighters with different colors that have different meanings, supports student ownership of their learning a during challenging problem-solving tasks. |
| 8 | O'Dell (2018) | 2018 | Explored instances of the productive struggle with 4th and 5th grade students. | Qualitative | Students | 7-8 | The productive struggle in elementary classrooms affords students an opportunity to experience deeper mathematics understandings. |
| 9 | Warshauer et al. (2017) | 2017 | Examined preservice teachers' understanding of the mathematics that underlie student struggles and the kinds of teacher responses that appear to support the productive struggle. | Qualitative | Preservice Teachers | Post-secondary | Writing assignments support mathematics teachers' understanding of students' knowledge and the identification of mathematics struggles. |
| 10 | Chapman (2016) | 2016 | The study investigated what teacher-related contextual factors can account for differences in student achievement in Algebra-2 content and what the relationship between instructional practices, | non-empirical | Preservice Teachers | Post-secondary | Change requires understanding expectations as well as efficacy especially when faced with challenging tasks. |

| | APA Reference | Year | Purpose | Methodology | Sample | Grade Level | Results |
|---|---|---|---|---|---|---|---|
| | | | mathematical focus, and their students' achievement. | | | | |
| 11 | Jansen et al. (2020) | 2020 | Investigate the impact of teacher education by examining instructional visions of graduates from a teacher education program, 2- or 3-years post-graduation. | Mixed-Methods | Inservice Teachers | K-12 | There are multiple ways to develop mathematics conceptual understanding that reflect subsets of the expectations of the teacher education program. |
| 12 | El-ahwal and Shahin (2020) | 2020 | Identify the effects of the use of a video-Based on Tasks to improve Mathematical Practice and support the productive struggle. | Quantitative | Inservice Teachers | Post-secondary | Video based tasks can support effective mathematics teaching to support the productive struggle. |
| 13 | Gray (2019) | 2019 | Provide a working model of the productive struggle and its influence on students' mathematical understandings. | non-empirical | Inservice Teachers | NA | Develops problem-solving tasks aligned to the productive struggle. |
| 14 | Chapman (2016) | 2016 | Presents four studies on approaches that were used to support mathematics teaching practices for prospective teachers. | non-empirical | Teachers | 7-12 | There are multiple ways to develop mathematics conceptual understanding that reflect subsets of the expectations of the teacher education program. |
| 15 | Livy et al. (2018) | 2018 | Presents examples of challenging mathematical tasks designed to enable students of all abilities to experience productive struggle and examines the lesson structure and teacher's role when implementing challenging tasks. | non-empirical | Students | 7-8 | Novel tasks, time for exploration, guiding questions and time for peer observation, should all be considered when promoting productive struggle. |
| 16 | Lemley et al. (2019) | 2019 | Describes a professional development that was implemented with middle grade | non-empirical | Inservice Teachers | 7-8 | To summarize, the following recommendations are offered for middle school teachers: 1. Select problems that are |

| | APA Reference | Year | Purpose | Methodology | Sample | Grade Level | Results |
|---|---|---|---|---|---|---|---|
| | | | mathematics teachers focused on metacognitive writing as a tool to support productive struggle in the mathematics classroom. | | | | cognitive demanding<br>2. Plan a] opportunities for student reflection<br>3. Welcome mistakes<br>4. Teachers need improved metacognitive skills |
| 17 | Valentine and Bolyard (2018) | 2018 | Examine pre-service teachers' reflections on mathematics teaching and learning as it relates to the practice of supporting productive struggle. | Qualitative | Preservice Teachers | K-6 | College educators should consider features that support productive struggle to support the learning environment and to help pre-service teachers be able to better support productive struggle in their future classrooms. |
| 18 | Edwards (2018) | 2018 | na | non-empirical | Teachers | 7-8 | N/A |
| 19 | Ewing et al. (2019) | 2019 | Examined how to engage English Language Learners in the productive struggle. | Qualitative | Preservice Teachers | Post-secondary | Connecting students with mathematics content and providing access to the content can improve dispositions and lead to the pre-service teacher engaging their students in productive struggle. |
| 20 | Murawska (2018) | 2018 | Explanation of an activity to promote productive struggle with preservice teachers. | non-empirical | Preservice Teachers | K-8 | Tasks that encourage productive struggle can help encourage perseverance and the connection to real life. |
| 21 | Warshauer (2015) | 2015 | Identify the kinds of student mathematics struggles that occur in a classrooms that are visible to teachers. | Qualitative | Inservice Teachers | 7-8 | Emphasizing that doing mathematics involves struggling is a valuable part of learning for understanding. |
| 22 | Roble (2017) | 2017 | Examined students' mathematical creativity within the context of problem solving. | Quantitative | Students | 9-12 | The Hawaii Algebra Learning Project (HALP) model helps foster and develop students' creative potential. |
| 23 | Warshauer (2011) | 2011 | Examined classroom interactions in naturalistic settings and documented how teachers support students who show signs of struggle in | Qualitative | Inservice Teachers | K-12 | Struggle can lead to learning opportunities, thus guidance on creating curricula to help encourage productive struggle is important. |

| | APA Reference | Year | Purpose | Methodology | Sample | Grade Level | Results |
|---|---|---|---|---|---|---|---|
| | | | learning mathematics. | | | | |
| 24 | Granberg (2016) | 2016 | Explores students' struggles during problem-solving while discovering and addressing errors and, more specifically, to what extent their struggle could be described as productive. | Qualitative | Students | 9-12 | Even if a student is struggling on a topic, allow that process to occur is key to learning. |
| 25 | Polly (2017) | 2017 | Provides a mechanism for planning math lessons that involve productive struggle. | non-empirical | NA | K-12 | Using the 5E approach when planning a lesson can incorporate productive struggle. |
| 26 | VanLehn et al. (2019) | 2019 | Explores the productive struggle in the context of using Formative Assessment with Computational Technology (FACT). | Quantitative | Inservice Teachers | 7-8 | The modified Formative Assessment with Computational Technology (FACT) increases the productive struggle. |
| 27 | Lynch et al. (2018) | 2018 | Presents strategies that create access while maintaining the cognitive demand of a mathematics task. | non-empirical | NA | NA | Differentiation is an important consideration in the implementation of the productive struggle in a math classroom. |
| 28 | Kartal et al. (2017) | 2017 | Presents the design and implementation of an activity in which mathematical modeling is the prominent practice. | Qualitative | Students | 9-12 | Struggling through mathematical modeling allowed for learning opportunities. |
| 29 | Roth (2019) | 2019 | Explores the productive struggle, its presence in mathematics education and the role of the teacher. | Qualitative | Inservice Teachers | 9-12 | There were no differences in these episodes from student struggle, responses, and outcomes. |
| 30 | Amidon et al. (2020) | 2020 | Provides suggestions to minimize shame associated with mathematics struggles. | non-empirical | NA | NA | Students who had supportive teachers showed fewer avoidance behaviors than students in classrooms where teachers focused on procedures and correct answers. |
| 31 | Warshauer et al. | 2019 | Examined the development of | Qualitative | Preservice Teachers | K-8 | It is important to carefully weave opportunities |

| | APA Reference | Year | Purpose | Methodology | Sample | Grade Level | Results |
|---|---|---|---|---|---|---|---|
| | (2019) | | preservice teachers understanding the productive struggle. | | | | to develop teaching practices such as support of productive struggle into the content course for teaching. |
| 33 | González & Eli, (2017) | 2017 | Identified teachers' underlying assumptions regarding what should constitute a launch as elements of the practical rationality of mathematics teaching. | Qualitative | Teachers | Post-secondary | Identifying ways to support how teachers can increase their attention to student thinking through teacher education is achieved by understanding how pre-service teachers and inservice teachers conceive the launch of a problem. |
| 34 | Kalinec-Craig, (2017) | 2017 | Discussed how the Rights of the learner can help children and teachers to embrace productive struggle and mistakes as valuable steps in the process of learning mathematics (and learning to teach mathematics). | Qualitative | Preservice Teachers | K-6 | These four Rights of the learner ((1) the right to be confused; (2) the right to claim a mistake; (3) the right to speak, listen, and be heard; and (4) the right to write, do, and represent only what makes sense) can serve as a beginning to a larger conversation about the ways that teachers and teacher educators can implement strategies democracy & education that promote equity in the classroom. |
| 35 | (Keazer & Jung, 2020) | 2020 | Explored how prospective elementary teachers anticipate difficulties with fostering the first mathematical practice (SMP1) of the Common Core State Standards when they decode the language of SMP1. | Qualitative | Preservice Teachers | K-6 | 1. The finding approximately a third of pre-service teachers anticipated difficulties with fostering SMP1 that related to their struggles suggests the importance of mathematics teacher educators supplementing PTs' experiences with opportunities to learn mathematics via SMP1 practices and problem-solving. 2. The interpretation of SMP1 as suggesting that a teacher teach multiple solution strategies, instead of engaging students in productive struggle to develop their strategies, is suggestive of an area for further research to explore the prevalence of this interpretation across pre-service teachers and practicing teachers. 3. This work advances the |

| | APA Reference | Year | Purpose | Methodology | Sample | Grade Level | Results |
|---|---|---|---|---|---|---|---|
| | | | | | | | knowledge within the field of mathematics education by providing insight about specific areas of pre-service teachers' anticipated difficulties fostering engagement in SMP1. |
| 36 | (Lloyd, 2018) | 2018 | Examined early-childhood, elementary and middle-level preservice teachers' beliefs and belief transformations related to the nature of mathematics, how mathematics should be taught, and their abilities to do and teach mathematics. | Mixed-Methods | Preservice Teachers | K-8 | If pre-service teachers experience at least one field experience in which innovative practices appear to be working, they are better able to comply contribute to fulfilling NCTM's vision that all students be prepared for success in college and careers. |
| 37 | (Lloyd & Howell, 2019) | 2019 | Provided insight about where on the continuum preservice teachers' mathematics beliefs are positioned. | Mixed-Methods | Preservice Teachers | K-8 | Therefore, research needs to continue to examine how beliefs are positioned to assess progress toward reform. |
| 38 | (Wickstrom & Aytes, 2018) | 2018 | Illustrated aspects of mathematical modeling that are beneficial to elementary school students in building mathematical proficiency. | non-empirical | Students | K-6 | The Counting and Fair-Sharing modeling task highlights several features of the process of mathematical modeling that are important for elementary school students and the process of mathematical modeling can be an ideal context to promote productive struggle. |
| 39 | (Leitze & Soots, 2015) | 2015 | Described high-level cognitive demand mathematical tasks that would be accessible and interesting to students. | non-empirical | Students | 7-8 | Students exposed to problem-solving tasks over a long duration of time demonstrate greater willingness to try their strategies when a solution path is unclear and greater persistence in willingness to spend longer periods of time developing solutions. |
| 40 | (Tyminski et al., 2019) | 2019 | Examine empirical data collected from K-12 mathematics teachers in order to further develop the emerging Launch Framework to support | other | Inservice Teachers | K-12 | Established a foundation for the development of a framework for effective problem launch practices. |

| | APA Reference | Year | Purpose | Methodology | Sample | Grade Level | Results |
|---|---|---|---|---|---|---|---|
| | | | the examination of high demand tasks. | | | | |
| 41 | (Wilburne et al., 2018) | 2018 | Examines classroom practices and promotes teachers' self-reflection of their teaching. | Mixed-Methods | Inservice Teachers | K-12 | Provides knowledge regarding why teachers are successful or struggle with implementing high-leverage teaching practices providing a rich context for professional development. |
| 42 | (Wolbert & Moss, 2018) | 2018 | Described how students can be introduced to radian measure through a discovery-based lesson with opportunities for active engagement and productive struggle. | non-empirical | NA | 9-12 | Presents opportunities for productive struggle as students reflect on radian measure so that they can build subsequent mathematical understanding on a solid foundation. |
| 43 | (Zeybeck, 2016) | 2016 | Described pre-service middle grade teachers' (PSTs) struggle types in detail as well as to investigate how engaging in a non-routine high-level task (doing mathematics) fosters (or inhibits) productive struggle during instruction. | Qualitative | Preservice Teachers | 7-8 | Recommends that teachers and teacher educators be attentive to learners' struggle types and to be aware of how to support learners to overcome these struggles without diminishing their learning opportunities. |
| 44 | (Angotti & Mudzimiri, 2018) | 2018 | Describes a high cognitive-demand task that requires students to engage and grapple with accurately collecting their own data and then effectively modeling that data using a linear function. | non-empirical | NA | NA | Describes high cognitive-demand mathematical modeling task situated in real-world context that provides students with multiple entry points and different solutions. |
| 45 | (Betts & Rosenberg, 2016) | 2016 | Illustrated the idea of productive struggle, both as a fundamental component of problem-solving ability and as a pedagogic technique. | non-empirical | Inservice Teachers | K-6 | Outlines teacher steps for productive struggle pedagogy. |

The publication dates range from 2011 to 2020. As shown in Figure 2, the density of literature published on the productive struggle in mathematics education sharply increased from 2016 to 2018. The scoping review elucidates the limitations of the existing literature regarding publication type, methods, and participants. For instance, there is a higher frequency of publications directly connected to the productive struggle in mathematics education (*n*=31) which was more than double the frequency of indirectly connected publications (*n*=14) (see Figure 3).

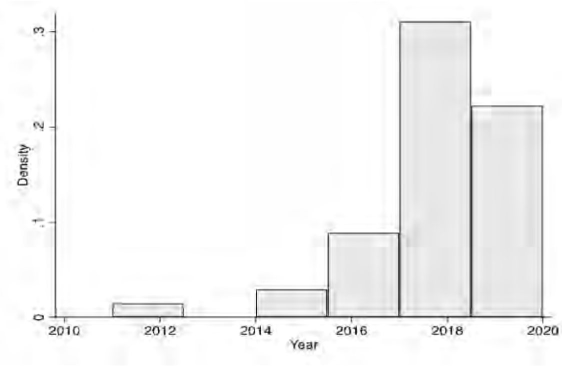


Figure 2. Histogram of Productive Struggle Publication Trends

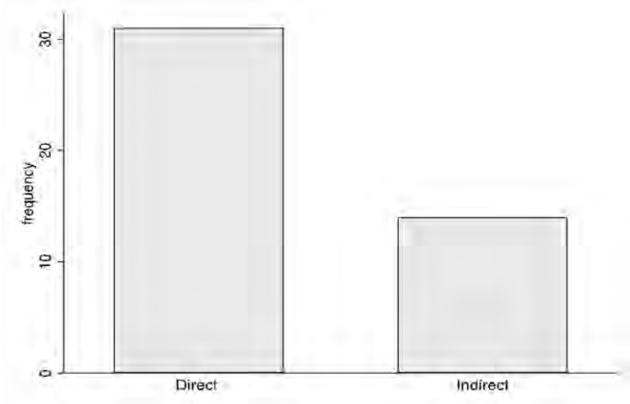


Figure 3. Direct versus Indirect Implementation of the Productive Struggle across Studies

Furthermore, the review includes empirical studies (*n*=26) using primarily qualitative methods and non-empirical methods (*n*=19) (see Figure 3). Although the primary method of investigation for empirical studies is qualitative, Lloyd (2018), Wilburne et al. (2018), Lloyd and Howell (2019), and Jansen et al. (2020) use mixed methods, while Roble (2017), VanLehn et al. (2019), Elahwal and Shahin (2020), and Vazquez et al. (2020) use quantitative methods (see Table 1). Additionally, comparing publications, there is a higher frequency of qualitative methods used in studies directly connected to the productive struggle in mathematics education, while indirectly connected publications have a higher frequency of non-empirical methods (see Figure 4).

The scoping review reveals that the extant literature consists of publications primarily with preservice or in-service teacher participants (see Table 2). Thus, although the productive struggle is an anomaly in that many teachers have been exposed and committed to actualizing it in their classrooms. The data indicate that a slight majority of the productive struggle literature currently focuses on grades 7 and 8 (20%) followed by approximately 18% of the literature residing in the K-6 grades span. It is important to note that only 5 studies or approximately 11% of studies took place in post-secondary classrooms compared to a combined 34 or more than 75% of the studies which took place in K-12 classrooms (note. 6 studies did not specify a grade span).

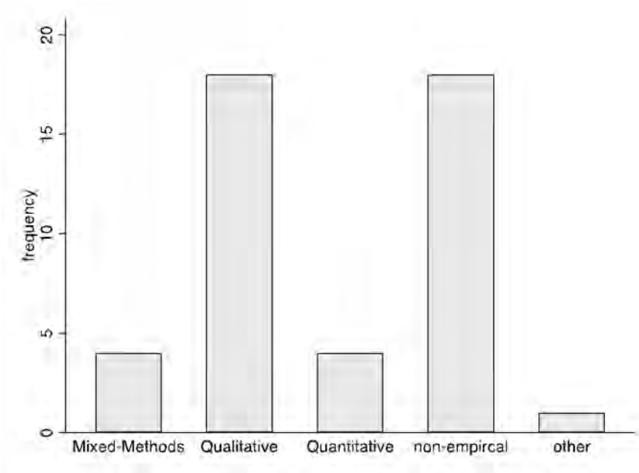


Figure 4. Trends in Methodologies Used to Study the Productive Struggle

Table 2. Frequency of Trends across Study Characteristics

| Category | Frequency |
|---|---|
| Connection | |
| Direct | 31(68.89%) |
| Indirect | 14(31.11%) |
| Participants | |
| Inservice Teachers | 12(26.67%) |
| Preservice Teachers | 13(28.89%) |
| Inservice/Preservice | 3(6.67%) |
| Parents | 1(2.23%) |
| Students | 3(6.67%) |
| Grade Level | |
| K-6 | 8(17.78%) |
| K-8 | 4(8.89%) |
| K-12 | 6(13.34%) |
| 7-8 | 9(20.00%) |
| 7-12 | 1(2.23%) |
| 9-12 | 6(13.34%) |
| Post-secondary | 5(11.11%) |
| Not Specified | 6(13.34%) |
| NCTM Content | |
| Algebra | 4(8.89%) |
| Geometry | 3(6.67%) |
| Measurement | 1(2.22%) |
| Number & Operations | 10(22.22%) |
| Not Specified | 27(60.00%) |
| NCTM Process Standards | |

| | |
|---|---|
| Connections | 2(4.44%) |
| Problem-Solving | 5(11.11%) |
| Reasoning and Proof | 1(2.22%) |
| Combination | 10(22.22%) |
| Multiple | 6(13.34%) |
| All | 4(8.89%) |
| Not Specified | 17(37.78%) |
| Method | |
| Qualitative | 18(40.00%) |
| Quantitative | 4(8.89%) |
| Mixed Methods | 4(8.89%) |
| Non-empirical | 19(42.22%) |
| Cognitive Connections | |
| Attribution | 9(34.62%) |
| Meta-cognition | 10(38.46%) |
| Frustration | 7(26.92%) |
| Equity Focus | |
| Yes | 4(8.89%) |
| No | 41(91.11%) |

Regarding NCTM Content areas, most studies in the data set did not specify a content area (60%). However, of the observed content areas number and operation was identified in 10 studies or 22.22% of the included studies. Of the NCTM content areas, measurement was identified the least often.

Likewise, most studies did not specify any NCTM process standard focus. Specifically, 17 or approximately 38 percent of studies did not provide a NCTM process standard focus. Yet, of the observed studies the majority identified a combination of NCTM process standards (22.22%).

The 26 empirical studies were examined for connections to attribution, metacognition, and frustration within the context of productive struggle. Of these studies, 10 connections between the productive struggle and metacognition were observed, followed by 9 connections to attribution theory, and 7 observations of a connection between the productive struggle and student frustration. It is important to note that most of these observed connections were not isolated, rather 21 out the 26 empirical studies observed multiple connections or shared connection between the productive struggle and three constructs presented.

A Ven diagram is provided in Figure 5 to elucidate these observed shared connections with the productive struggle. Finally, approximately 90% of mathematical examinations of the productive struggle lacked equity considerations or foci. In summary, mapping the literature reveals that there is a lack of robust empirical inquiry regarding mixed and quantitative methods as well as diverse study, content, and participant characteristics. The implications of these findings are discussed below.

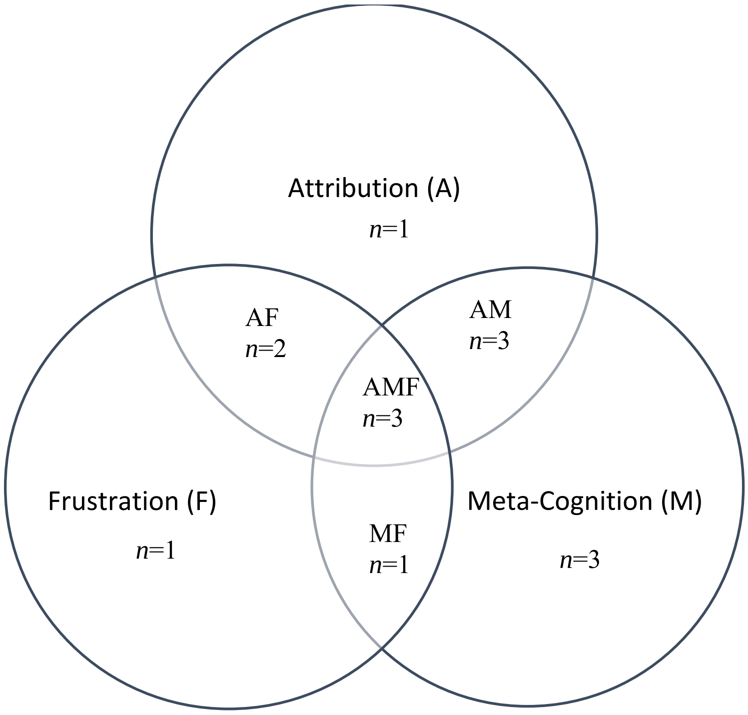


Figure 5. Venn Diagram of the Shared Connections Observed between the Productive Struggle and Three Cognitive Constructs Across Studies

## Discussion

The present scoping review was guided by four objectives: 1) conduct a systematic literature search of published studies and grey literature, 2) map the characteristics of studies and the methodologies used, 3) identify limitations and challenges present, and 4) subsequently propose recommendations for advancing the consistency with which the productive struggle is examined and implemented within mathematics education. After conducting a systematic review of the literature, we located a representative sample of studies including journal articles, conference proceedings, and dissertations/theses. The volume and breadth of the reviewed literature indicate that research on the productive struggle within mathematics education is growing with several key researchers leading the charge through multiple studies examining the productive struggle (e.g., Warshauer and Llyod).

### RQ1: What are the characteristics of the scholarly works published from 2011 to 2020 examining the productive struggle in K-12 mathematics educational contexts?

In summary, the scholarly works published from 2011 to 2020 in relation to the productive struggle in K-12 mathematics educational contexts represent multiple methodological approaches, include primarily Inservice teachers, and lack a consistent reporting of mathematics content foci. Furthermore, there appears to be a lack of sufficient operationalization of the productive struggle construct across studies which is crucial to understanding the meta-cognitive effects of the productive struggle on student learning outcomes. Together these findings suggest that the productive study line of inquiry has evidence of breadth, but lacks depth as it relates to content foci, equity considerations, and measurement as evidenced by the lack of formal instrumentation and operationalization. The lack of operationalization of the construct provides further evidence to explain the absence of quantitative studies, as it is requisite to operationalize the productive struggle prior to measuring it quantitatively.

Since the productive struggle is essential to providing rigorous learning experiences promoting students' development of grit, problem-solving, mathematical creativity, and grade-level achievement (Roble, 2017), it is crucial to understand the characteristics of the literature guiding existing implementation. Due to the absence of a synthesis of the literature on the productive struggle in mathematics education, we conducted a scoping literature review to provide a mapping, elucidating limited empirical inquiry regarding mixed and quantitative methods as well as diverse participant characteristics. Further investigation of the characteristics, effects, and operationalization of this phenomenon will aid in establishing guidance for successful implementation. We contend that these future works will help to develop stronger links between the productive struggle and the teaching and learning of mathematics.

**RQ2: How do these works link the productive struggle to the teaching and learning of mathematics?**

The results of the present study suggest that the productive struggle is linked to the teaching and learning of mathematics through:

1. direct examination of the productive struggle with primarily in-service and pre-service teachers but not students;
2. some consideration of the combined impact of attribution, meta-cognition, and frustration on the implementation and impact of the productive struggle and;
3. limited surface level considerations of mathematics content and teaching processes.

Our mapping of the literature suggests that the productive struggle is characterized by direct observations of the construct and that the primary participants in most of the current investigations are in-service and preservice teachers. The direct observation of the construct is promising as it indicates that the productive struggle is most often considered as an either and independent or dependent variable in the literature. Unfortunately, how the productive struggle is operationalized and measured as a variable remains elusive.

The relationship between attribution theory, meta-cognition, and frustration observed in the extant literature indicates that as we suspected these constructs are recognized most often in combination rather than in isolation. This observation is important because understanding how individuals attribute success and failure (i.e., attribution theory), how they regulate their own learning processes (meta-cognition), and how they respond to challenges emotionally (i.e., frustration) is crucial for fostering an environment where productive struggle can thrive. Unfortunately, the current dependence on teachers rather than student participants is once again problematic because to unpack the interconnectedness of attribution, metacognition, and frustration within the context of the productive struggle there needs to be an acute focus on the student participants. Hence, we recommend more student focused studies across different grade spans and mathematics subject-matter are necessary to move this line of inquiry forward.

There were serious inconsistencies in the examination and reporting of mathematics content and teaching processes across the observed studies. Moreover, there was a lack of clear explanation as to how the productive struggle was operationalized and measured. For advancing consistency, we propose furthering the empirical study of the characteristics, tasks, and effects of productive struggle implementation on mathematics learning outcomes. Providing substantial evidence to establish a consensus is critical to the considerations and expectations guiding the implementation of the productive struggle to facilitate progress, develop efficacy, and alleviate implementation challenges for students and teachers. Additionally, advances in consistency would support implementation efficacy and help to develop consistent operational definitions for the implementation and measurement of the productive struggle across mathematics content and grade levels. Consistent theoretically sound operational definitions for the productive struggle and more explicit reporting of the mathematics content focus across studies will help to improve the quality and efficacy of the measurement of the productive struggle within the mathematics educational context.

## Conclusion

The present scoping review summarizes extant literature on the productive struggle in mathematics education using scoping review methods. The results of this study have empirical and theoretical implications that facilitate synergy between research and practice. Specifically, as we characterize study demographics and interventions, we are promoting evidence evaluation by examining representative trends and identifying possible gaps in the field. Moreover, as we examine the methodological paradigms and approaches used across studies, we are facilitating future research as we are inherently identifying questions that have not yet been answered. Finally, as we unpack study outcomes, we will begin to understand better the true significance of the productive struggle as a mechanism for transforming mathematics teaching and learning.

In conclusion, we contend that there are opportunities for rigorous research designs reporting on observable trends in the implementation of the productive struggle explicitly targeting underexamined NCTM content and process standards and focusing on how the productive struggle can support equity in mathematics. We believe that these opportunities should be investigated, to provide a broader evidence-base for developing meaningful experiences in mathematics for school-aged children that capitalize on the interconnectedness of attribution, meta-cognition, and controlled frustration in mathematics learning. Ultimately, when asked “How Productive is the Productive Struggle?”, we assert that the productive struggle though in its infancy is burgeoning line of inquiry that simply needs more rigorous operationalization to bring it into the forefront of mathematics education research.

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