Exploring Students' Work in Solving Mathematics Problem through Problem-Solving Phases

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Abstract: This study explores students' use of mathematical objects in each problem-solving phase based on an onto-semiotic perspective. The subjects of this research are students who solve problems in different ways but all with the correct result. The first student uses organizational data by applying the concept of permutations. In contrast, the second student uses visual representation (making pictures) by applying the concept of filling in slots or multiplication rules. The results showed variations in the formation of mathematical objects in each problem-solving phase and indicators of activity using mathematical objects. The detail of students' work will be discussed comprehensively.

Keywords: mathematical object; onto-semiotic approach; problem-solving; combinatorics.

INTRODUCTION

One of the essential activities in learning mathematics is problem-solving. The main goal of learning mathematics is to develop the ability to solve various complex mathematical problems (Baykul & Antalya, 2011; Csachová, 2021; Prayitno et al., 2020). Godino and Batanero (2020) state that problem-solving activities are central to constructing mathematical knowledge. Problem-solving has always been part of the mathematics curriculum (Bien et al., 2020). Furthermore, Piñeiro et al. (2019) suggest that problem-solving is the primary indicator in demonstrating one's mathematical competence, evaluating the quality of the education system, and being an essential aspect of teacher learning and knowledge. However, although problem-solving is very essential, there is no formal activity for using mathematical objects explicitly in the problem-solving process. For example, indicators that are commonly used in the understanding phase of the problem are writing down what is known and what is asked without being explained explicitly about the use of mathematical objects in the activity. Docktor and Heller (2009) explain that despite many efforts to improve problem-solving across the education system, there is no standard way to evaluate written problem-solving that is valid, reliable, and easy to use. It underlies the need to study problem-solving processes using a theoretical perspective emphasizing adherence to mathematical objects. Compliance with the explicit use of mathematical objects in each problem-solving phase is expected to make students successful problem-solvers. In his article, Starikova (2010) states that the choice of representation of abstract objects can lead to breakthroughs and significant concept

development. Furthermore, Godino et al. (2011) say that getting a better analysis of mathematical activities requires the introduction of mathematical objects.

Several theoretical perspectives can be used in mathematics education research, including APOS (Action, Process, Object, and Scheme), semiotics, and onto-semiotics. Each theoretical perspective has advantages over other theoretical perspectives. The onto-semiotic approach (OSA) was conceived to complement the APOS theory (Font et al., 2010). The onto-semiotic approach is a lens that can provide more complex descriptions and extensions of observations that focus on conceptual and procedural understanding. It is as the description that an onto-semiotic approach is a configuration tool that facilitates a detailed description of the mathematical practice involved in solving a problem (Godino et al., 2021). Based on the description of the advantages and several studies that have used the onto-semiotic approach, no research results have been found that specifically integrate problem-solving theory with the use of mathematical objects in the onto-semiotic approach. The onto-semiotic approach has been used in analyzing combinatoric problems (Godino et al., 2005) but has not been specifically linked to theories related to the stages of problem-solving. In addition, in this study, the subjects observed were students in high school. Furthermore, several studies in Indonesia have used the onto-semiotic approach to analyze students' understanding of algebra (Amin et al., 2018). However, no research has been found that analyzes secondary entities or cognitive dualities in the onto-semiotic approach.

The object of mathematics in the onto-semiotic approach is anything that can be used, recommended, or directed when doing, communicating, or learning mathematics (Montiel et al., 2012). The results of studies related to the use of mathematical objects in each phase of problem-solving can provide an overview of the direction of completion that allows students to arrive at the result or the correct solution to the problem. Teachers need to know the use of a conceptual framework or methodological approach to plan, implement, and assess the mathematics learning process (Burgos et al., 2019; Giacomone, Godino, & Beltrán-Pellicer, 2018).

This research is to answer the question: How does the appearance of mathematical objects in the onto-semiotic approach on each problem-solving phase by students? The study of the use of mathematical objects can show the process that can be done to arrive at the correct solution to a mathematical problem and can show the difficulties experienced in solving a problem. It is like the statement that one of the dilemmas that most often arise in learning is regarding the introduction of mathematical objects (Nachlieli & Tabach, 2012). In line with that, previous research revealed that difficulties in understanding the meaning of mathematical objects are related to semiotic representations occurring at every level of education (Font et al., 2015).

Empirical evidence showcased that the students' habit of giving short answers has an impact on the lack of use of mathematical objects in the structure of the answer to a problem. Then the lack of use of mathematical objects causes students to produce wrong solutions. Furthermore, the habit of overly believing in the completeness and correctness of answers without carrying out activities to check the suitability of the result with the problem situation can also impact the wrong solution. It underlies the need for research to obtain information about using mathematical objects based on the onto-semiotic approach in each problem-solving phase carried out by students to produce

solutions. These findings, regarding the formation of the use of mathematical objects in each phase of problem-solving, become an essential framework or guide to be applied in the mathematics learning process.

LITERATURE REVIEW

Onto-Semiotic Approach

The development of the onto-semiotic approach is based on several theories. Godino (2014) states that four groups of models underlie the development of the onto-semiotic approach, namely: (1) mathematical epistemology; (2) semiotic-based mathematical cognition; (3) an instructional model on a socio-constructivist basis; and (4) a systemic ecological model. The onto-semiotic approach includes an explicit typology of mathematical objects, which facilitates the description and analysis of mathematical activity (Giacomone, Díaz-Levicoy, et al., 2018). The onto-semiotic approach introduces the notion of the observable as an entity or object that can be identified by the subject or observer using a particular theory by reference (Bencomo et al., 2004). This approach is named onto-semiotic because of its essential role in language and the categorization of various objects that appear in mathematical activity (Wilhelmi et al., 2021). The onto-semiotic approach can be used as an analytical tool for the processes and objects involved in mathematical activities or practices, a tool for analyzing the learning process in the classroom, as well as for meta-didactic reflection and reflection on the suitability of learning (Godino, 2017). Thus, it can be stated that the onto-semiotic approach is a theoretical perspective or lens that can be used as an analytical tool to use mathematical objects and the position of each part of the object in mathematical activities.

There are two primary components of the onto-semiotic approach, known as entities. The primary entity consists of six aspects, namely: language, situations, concepts, procedures, propositions, and arguments (Burgos et al., 2019; Burgos & Godino, 2020; D'Amore & Godino, 2006; Font et al., 2007; Giacomone, Godino, & Beltrán-Pellicer, 2018; Godino, 2002; Godino et al., 2005, 2006, 2007; Godino, 2019). Furthermore, primary entities can be viewed according to five pairs of points of view known as secondary entities or cognitive dualities. A complete description of the components of the onto-semiotic approach, as developed by Godino et al. (2007), can be seen in Figure 1.

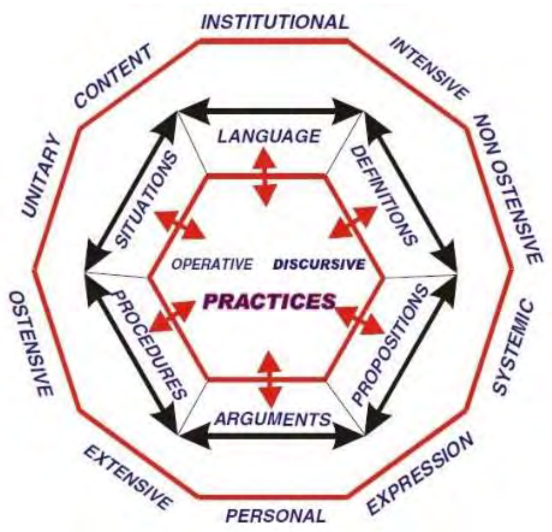


Figure 1: The components of the onto-semiotic approach developed by Godino et al. (2007)

Problem Solving

Several experts have given opinions on the definition of the problem. Avcu and Avcu (2010) concluded that a problem is a situation faced by a person with several obstacles. Mathematical problems are an instrument for developing thinking and problem-solving skills, including those related to everyday life (Arfiana & Wijaya, 2018; Pimta et al., 2009). Polya (1957) stated that problems are broadly divided into two types, namely: (1) routine problems and (2) non-routine problems.

According to Polya (1957) a problem is routine if a problem can be solved by substituting specific data into the problem or by following steps from similar problems that have been solved. Meanwhile, non-routine problems arise when a person faces a particular situation and intends to achieve the critical situation but needs to know how to achieve the goal (Arfiana & Wijaya, 2018; Elia et al., 2009). Nonetheless, Avcu and Avcu (2010) explain that routine problem-solving is essential in developing computational skills.

There are eight classifications of problems that Johnstone developed in 1993, where problems are divided into eight types based on three components, namely the data provided, the methods that can be used, and the objectives (see Table 14.1 page 306 in the work of Bien et al. (2020)). In this study, the first task is a question that is a problem with complete data, non-routine methods, and closed objectives (type 2). At the same time, the second task is a question that is a problem with complete data, routine methods, and closed objectives (type 1).

There are four phases of problem-solving, according to Polya (1957). Several experts have proposed modifications related to the problem-solving phase developed by Polya, such as Zelaso's team, as cited by Kotsopoulos and Lee (2012). Carson (2007) develops problem-solving phases into five steps based on Krulik and Rudnick's modification work for these phases. According to

John Dewey, George Polya, Stephen Krulik, and Jesse Rudnick, the different problem-solving phases can be seen in Carson's article (2007). However, several theories about the problem-solving phase are used in this study, such as Polya's four phases of problem-solving.

Furthermore, Kılıç (2017) and Bien et al. (2020) have described each problem-solving phase. The indicators of problem-solving activities used to classify student activities in this study can be seen in Table 1.

Phase of Problem Solving	The Indicators of Activity
Understanding the Problem	• write or state what is known • write or state what develops
Devising a Plan	• state the method or concept that will be used to solve the problem • state or write down certain rules that will be used to solve the problem • state the reason for using a concept
Carrying out the Plan	• carry out procedures by applying the planned concept
Looking Back	• re-check the procedures that have been carried out • check the final result with the problem situation

Table 1: The Activity Indicators in Each Problem-Solving Phase

METHOD

This qualitative research describes the use of mathematical objects used in the problem-solving process by students based on an onto-semiotic theoretical perspective.

Instrument

The main instrument in this research is the researcher himself. Qualitative researchers, as human instruments, determine the research focus, select informants, analyze and interpret and make conclusions based on research findings. The supporting instruments used are assignments in two number description questions and semi-structured interview guidelines. However, the first question to apply the concept of combination or addition needs to be answered correctly by all prospective research subjects. So, the answer analyzed is the answer to the second question, which is a question that can be solved by applying the concept of permutation or multiplication. Prior to use, the supporting instruments were validated. Mathematical tasks that can be answered correctly by prospective subjects are shown in Figure 2.

Ali wants to change his email account password. To create a password that is difficult for others to guess, Ali created a password that is 8 characters long, consisting of 4 different letters taken from the letters a, b, c, d, and e and followed by 4 different numbers taken from the number 0, 1, 2, 3, 4, and 5. Specify as many possible new passwords for Ali's email account!

Figure 2: Math Assignment

Research Subject

The subjects of this study were twelve-grade students who had obtained the material on the rules of enumeration. Of the 28 students, there were 24 students whose answers got the correct result. By paying attention to the order of completion of the 24 students' answers, there are two groups of correct answers. The first group comprises 17 students who make two subsections: letter and number permutations. The second group consists of 7 students who use multiplication rules to count the number of possible numbers and letters. The selected subject is one person from each group who has similar answers. In administering the test, the researcher does not suggest that students use permutations or filling slots. Students are left to determine how to solve according to their compassionate nature. Students are given this test precisely one week after the teacher provides the material about permutations. It was first asked of the teacher. Students selected from both groups have complete, correct answers among friends who use permutations or filling slots.

Data Collection

The data collected were students' written answers in the form of photos of student answer sheets sent via a google form, student explanations regarding answers in the form of videos which are also sent via a google form, and interview result data. Data collection begins with giving math assignments to students. Furthermore, semi-structured interviews were conducted based on the results of student work. The interview emphasized confirming the use of mathematical objects in problem-solving based on data from answer sheets and explanation videos.

Data Analysis

The steps taken in the data analysis of each subject were: (1) transcribing the explanation video data related to the answers; (2) transcribing the recorded interview data; (3) reviewing the results of written answers, explanatory video transcripts, and interview recordings; (4) perform data reduction; (5) tabulating the use of mathematical objects; (6) describes the position of mathematical objects; (7) interpreting and meaning of the data; and (8) draw conclusions. Furthermore, the units and categories are arranged for making drawings to interpret and describe the data. The units and categories used in this study were adapted from the onto-semiotic perspective developed by Godino, et al. (2007). It is described in detail as follows.

1. Arrange Units

The arrangement of the units in this study is based on the component approach onto-semiotic. The use of color is intended to facilitate differentiating each component students raise. Units' use of mathematical objects can be seen in Table 2.

Category	Descriptor	Code
Language	Written statements (words, symbols, signs, and pictures) and verbal statements used in solving problems	Red trapezoid image
Situations	The information contained in the given problem	Orange trapezoid image
Definitions/ Concepts	Statements relating to certain concepts	Yellow trapezoid image
Procedures	The steps taken in implementing a concept or strategy	Green trapezoid image
Propositions	A statement about the principle or formula used in solving the problem	Blue trapezoid image
Arguments	A statement used to justify a proposition or procedure	Indigo trapezoid image
Personal	Student point of view which can be in the form of student answer (personal side)	Black line
Institutional	The institutional point of view is a reference for understanding and evaluating the teaching and learning process (in this case, including problem-solving)	Red line
Ostensive	Objects that appear to be used explicitly or can be observed directly because they are written on the answer sheet such as symbols and pictures	Brown line
Non-ostensive	The object is only stated verbally without being accompanied by a written form that can be observed on the answer sheet (for example, a multiplication sign that is not written but there is a multiplication process)	Yellow line
Extensive	Object is used as a specific case, e.g., $P_4^6 = \frac{6!}{(6-4)!}$	Dark green line
Intensive	Object used as a more general form, e.g., $P_r^n = \frac{n!}{(n-r)!}$	Light green line
Unitary	Mathematical object used as a unified entity (object that should have been known beforehand)	Dark blue line
Systemic	The system being studied. For example, in teaching the general formula for permutations, factorial, division, and subtraction are considered as something known (unitary). While the same object (factorial, division, and subtraction) in a particular class or on a particular occasion, must be treated as a systemic and complex object to be studied	Light blue line
Expression	A symbol that represents a certain meaning	Purple line
Content	The meaning represented by a symbol	Light purple line

Table 2: Units of Use of Mathematical Objects

2. Organize Categories

Subanji, quoted by Sukoriyanto (2017), states that the arrangement of categories is carried out to facilitate data interpretation, simplify problems, and facilitate the thought analysis process of

research subjects. Categorization or coding is distinguished by color coding and image coding, presented in Table 3.

Object Name	**Color Code**			**Image Code**
	Red	***Green***	***Blue***	
Language	255	115	115	Trapezoid
Situations	255	185	115	
Definitions/ Concepts	255	255	125	
Procedures	125	255	125	
Propositions	145	255	255	
Arguments	255	115	255	
Personal	0	0	0	Line
Institutional	255	0	0	
Ostensive	128	64	0	
Non-ostensive	255	255	0	
Extensive	128	128	64	
Intensive	128	255	0	
Unitary	0	0	255	
Systemic	0	255	255	
Expression	128	0	255	
Content	255	0	255	

Table 3: Coding of Primary Entity and Secondary Entities Components

The subsections of language components and definitions/concepts are coded using letters followed by an index number based on their occurrence. The description and code of each language component subsection and concept are provided in Table 4.

Category	**Descriptor**	**Code**
Language	Write words or sentences (in the form of text), for example known, asked, numbers, letters, possibilities, etc.	T
	Sate a word or sentence (in the form of an oral statement)	L
	Using symbols, for example: 1, 2, 3, 4, etc.; *a*, *b*, *c*, *d*, and *e*; P (permutation symbol)	S
	Using signs, for example: Operation signs +, -, x, or.,: atau /, (), ^, and √; Relation signs (=, <, >, ≤, ≥, ≠, etc.); and Factorial sign (!)	Ta
	Creating images, for example: lines, flat shapes, building spaces, etc.	G
Concepts	Permutation	C1
	Factorial	C2
	Distribution	C3
	Subtraction	C4
	Multiplication	C5

Table 4: Coding Subsection of Language and Concept Components

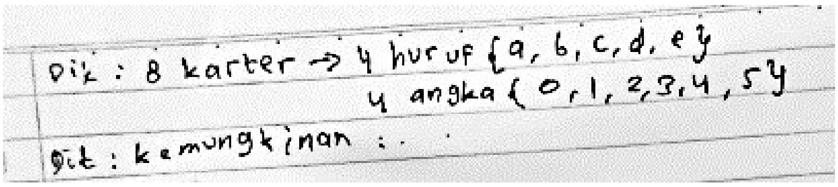
RESULTS AND DISCUSSION

This section contains exposure and analysis of data and research findings. The problem-solving process carried out by each subject is described in four phases, namely: understanding the problem, devising a plan, carrying out the plan, and looking back. Furthermore, in each phase of problem-solving, all mathematical objects raised by the subject are presented, which can be categorized as primary entities in an onto-semiotic theoretical perspective: language, situation, definitions/concepts, procedures, propositions, and arguments. Furthermore, the position of the primary entity used by the subject when viewed from a second entity or cognitive duality is presented, including personal-institutional, ostensive-non-ostensive, extensive-intensive, unitary-systemic, and expression-content.

Data Exposure of Subject 1 (S1)

Understanding the Problem

After reading about the given problem, S1 writes down the known elements and the asked elements of the problem. The snippet from the S1 answer sheet, which can be interpreted as part of the phase of understanding the problem, can be seen in Figure 3.


Dik : 8 karter → 4 huruf {a, b, c, d, e}
4 angka {0, 1, 2, 3, 4, 5}
Dit : kemungkinan : . .

Translating in English:
Given: 8 characters→ 4 letters {a, b, c, d, e}
4 numbers {0, 1, 2, 3, 4, 5}
Asked: possibly: …….

Figure 3: S1's Part Answer

Before specifying all the mathematical objects used by S1, it is also observed in the section that shows the Phase of understanding the problem in the explanatory video transcript. The explanatory video fragment that corresponds to the written answer by S1 is:

> *It is known that Ali wants to create an email password that consists of eight characters. It consists of four letters colon (in this case it means "of") a, b, c, d, and e. It is followed by four different numbers, namely-from 0, 1, 2, 3, 4, and 5. Four different letters and four different numbers. Keep asking for possibilities. I asked if it was possible, with many possible passwords.*

From Figure 3. and the transcript of the explanatory video, S1 uses two primary entity components: language and situation. The object implied in the practice of mathematics by S1 is the concept of a set. The concept of this set was confirmed through the interview session as one of the objects used by S1. The transcript of the interview with S1 related to the object is as follows:

P: What do the curly braces mean here? (While showing the first line written in curly braces)
S1: The set, the set of letters.

From the analysis results related to the use of primary entities and the position of each sub-section of each primary entity when viewed from the five pairs of cognitive dualities by S1 at the Phase of understanding the problem, it can be seen in Figure 4.

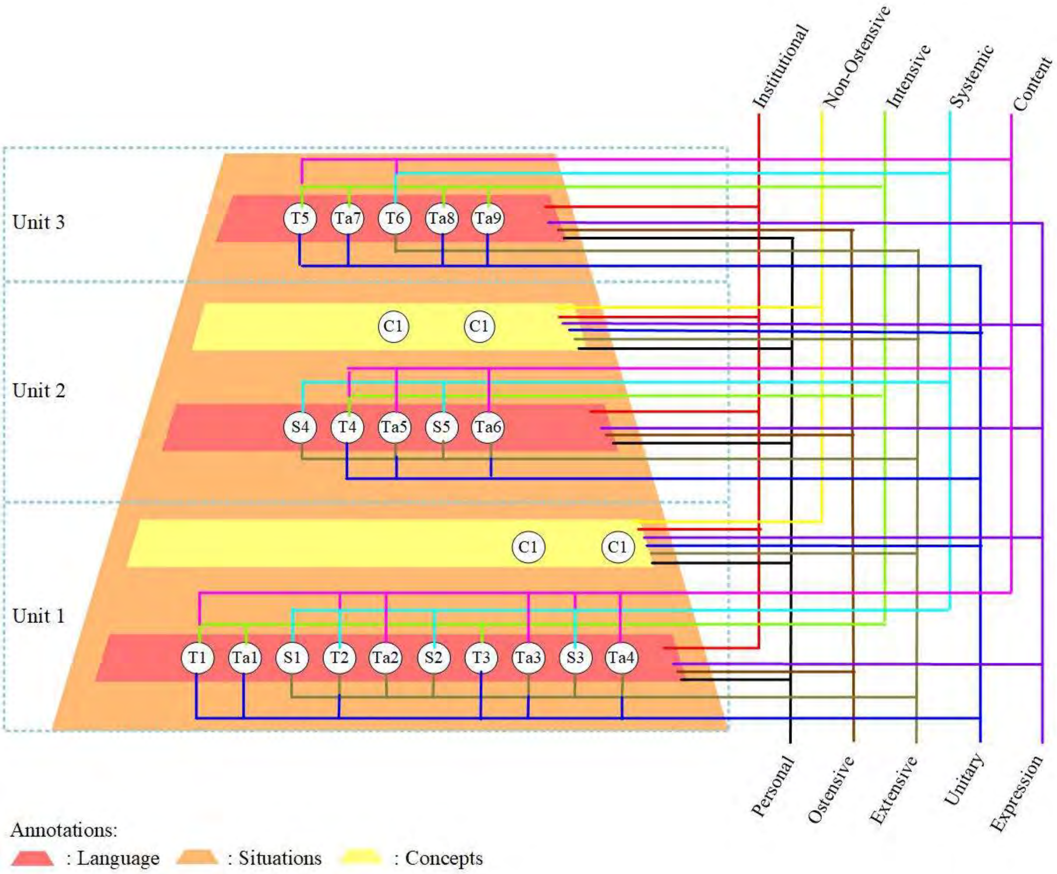


Figure 4: The Use of Mathematical Objects by S1 at the Understanding the Problem Phase

Devising a Plan

After writing down the information that is known and asked from the problem, S1 determines a problem-solving strategy using permutations. However, no specific section on the answer sheet shows this section. The part of the problem-solving process that shows the phase of planning a solution is contained in the explanatory video recording. This work is as in the S1 statement fragment in the following video transcript:

Continue to solve it using permutations because he says different, four different letters and four different numbers. This is a permutation.

This was also revealed in the interview session. Excerpts of interview transcripts that show S1 is planning a solution by stating the use of the permutation concept, namely:

P: Is this your answer? (While displaying the answer sheet S1)
S1 : Yes
P: Please explain why it is answered like this!
S1 : That's a permutation of four out of five because there are five letters and four are taken. So, I use permutations.

From this statement, it can be stated that the mathematical objects used by S1 in the planning phase of completion are situations, language, concepts, and arguments. The use of mathematical objects by S1 in planning solutions can be seen in Figure 5.

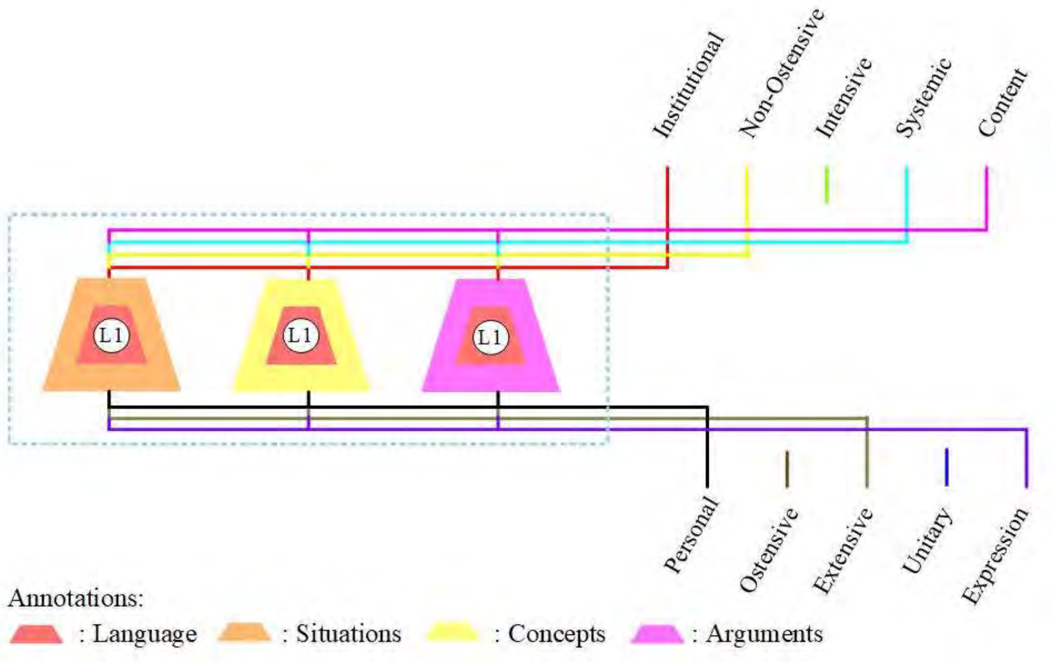
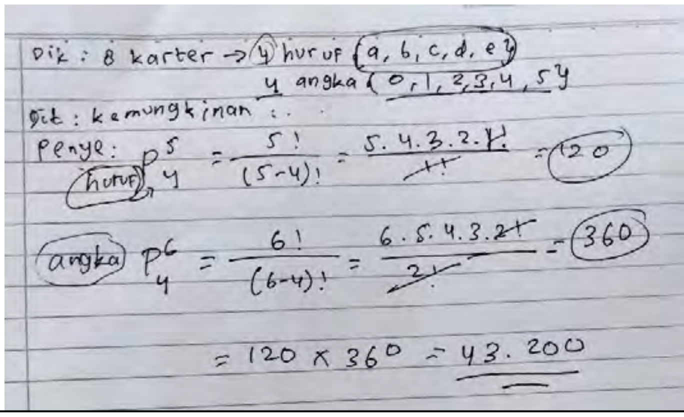


Figure 5: The Use of Mathematics Objects by S1 at Devising a Plan Phase

Carrying out the Plan

After determining the strategy that can be used, S1 calculates the permutation of letters, calculates the permutations of numbers, then multiplies the results of the permutations of letters and numbers. A piece of the answer sheet that shows the process of carrying out the completion by S1 can be seen in Figure 6.



Translating in English:
Given: 8 characters→ 4 letters {a, b, c, d, e}
4 numbers {0, 1, 2, 3, 4, 5}
Asked: possibly: …….
Completion:
Letter
Number

Figure 6: S1's Part Answer

Furthermore, it is observed simultaneously with the explanatory video fragment by S1, who corresponds to the written answer. The video snippet that shows the phases of carrying out the completion by S1 are:

First permutation four out of five. Because four are chosen, and there are five letters (then circle the number 4 and the letter in curly brackets in the first line of the answer part about the information that is known from the question). So, five factorials per five minus four factorials equal five times four times three times two factorials per one factorial equal one twenty. So, this is a letter. It is a letter (circling the letter word that has been written and making an arrow pointing to the form of four out of five permutations).

Continue to number (while writing the number of words and circling the number word). Also, use permutations because they do not repeat. So, this is four out of six because four numbers are chosen from six numbers (then make a line under the number 4 and the numbers 0, 1, 2, 3, 4, and 5 in the second line of the answer section about the information that is known from the question). Six factorials by six minus four factorials. By two factorials. Six five four three two factorials. This crossed-out equals the result three sixty.

Continue this (while circling the number 120) and this (while circling the number 360) multiplied. So, one twenty times three sixty equals four thousand, forty-three thousand two hundred. This way (while drawing a line twice).

From Figure 6. and the video transcript, there are several mathematical objects used by undergraduate students: language, situation, concept, procedure, and argument. The statement regarding the concept of permutations and the formula for calculating permutations was confirmed in the interview, and the results showed that S1 used propositions, but it was not stated in the video. The part of the interview that shows the use of propositions is:

P: Why don't you write down the general form of the permutation first?
S1: to keep it short.
P: Please re-explain your answer!
S1 : P(5,4) because 4 letters will be chosen from 5 letters (a, b, c, d, and e)
P(6,4) because 4 numbers will be selected from 6 numbers (0, 1, 2, 3, 4, and 5)
P: Have you ever received a similar question?
S1 : ever
P: Is there any other way to solve this problem?
S1 : that is my way, ma'am.

The interview excerpt found that before starting the procedure in completing, S1 gave arguments and stated the propositions related to the formula for calculating permutations. Mathematics objects by S1 can be seen in Figure 7.

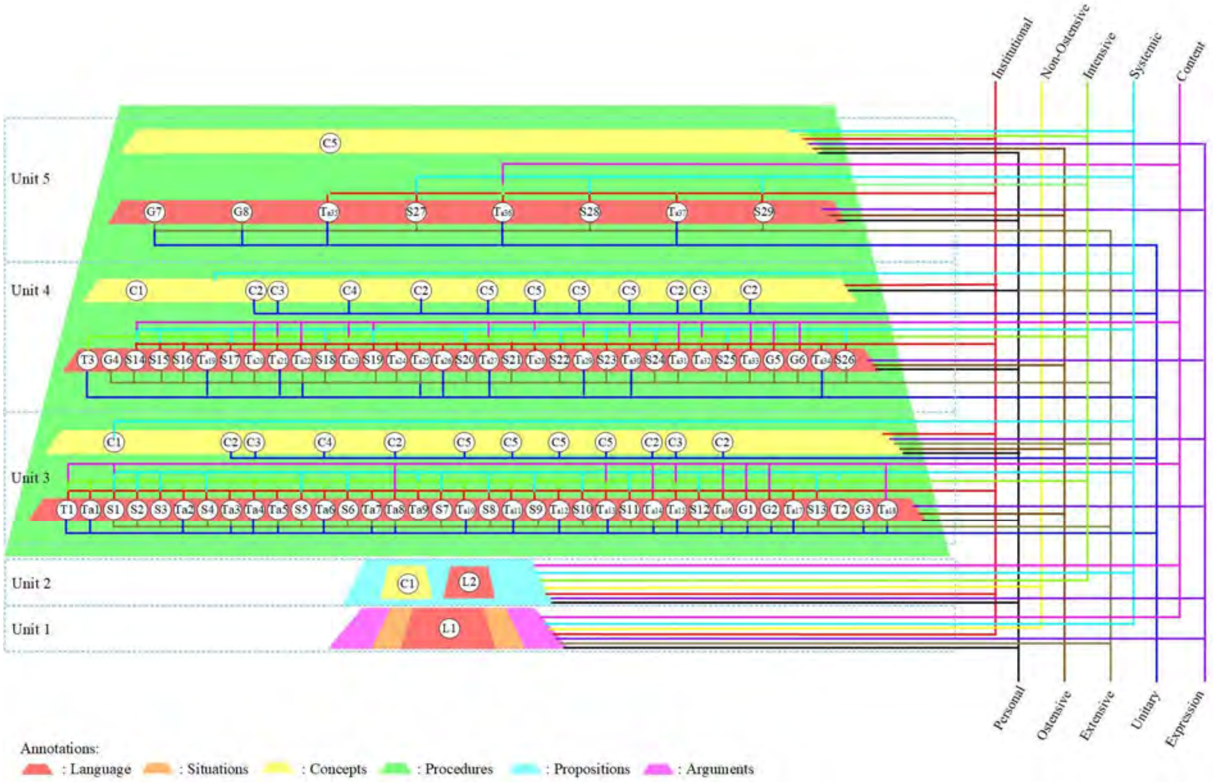


Figure 7: The Use of Mathematical Objects by S1 at the Phase of Carrying out the Plan

Looking Back

Part of the problem-solving process carried out by S1, which can be considered part of the re-examination phase, occurs when S1 carries out the completion phase. The activity of checking procedures carried out previously by circling and underlining certain sections indicates that the checking activity is carried out by S1. The part which is the looking back phase is marked in Figure 8.

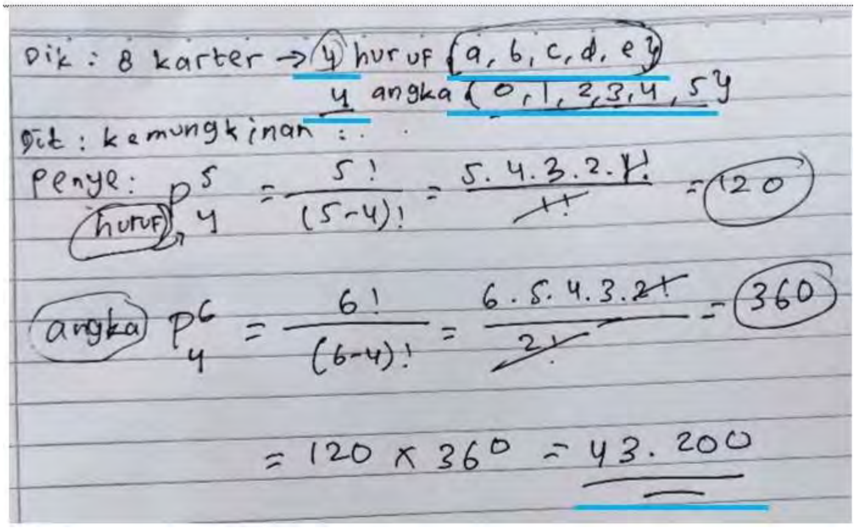


Figure 8: S1's Part Answer

The use of objects by S1 at the looking back phase cannot be observed directly from the answer sheet. An explanatory video transcript was showing the activity at the re-examination phase co-occurred at the phase of completing (in the image, it is marked with a blue underline). S1 performs the check-back phase back and forth. After performing specific procedures, S1 checks again by verifying the parts that support the procedures' justification. This is done repeatedly every time the procedure has been carried out until the result is obtained as an answer to the existing problems. The mathematical objects used by S1 are language, situations, procedures, and arguments. The use of mathematical objects by S1 can be seen in Figure 9.

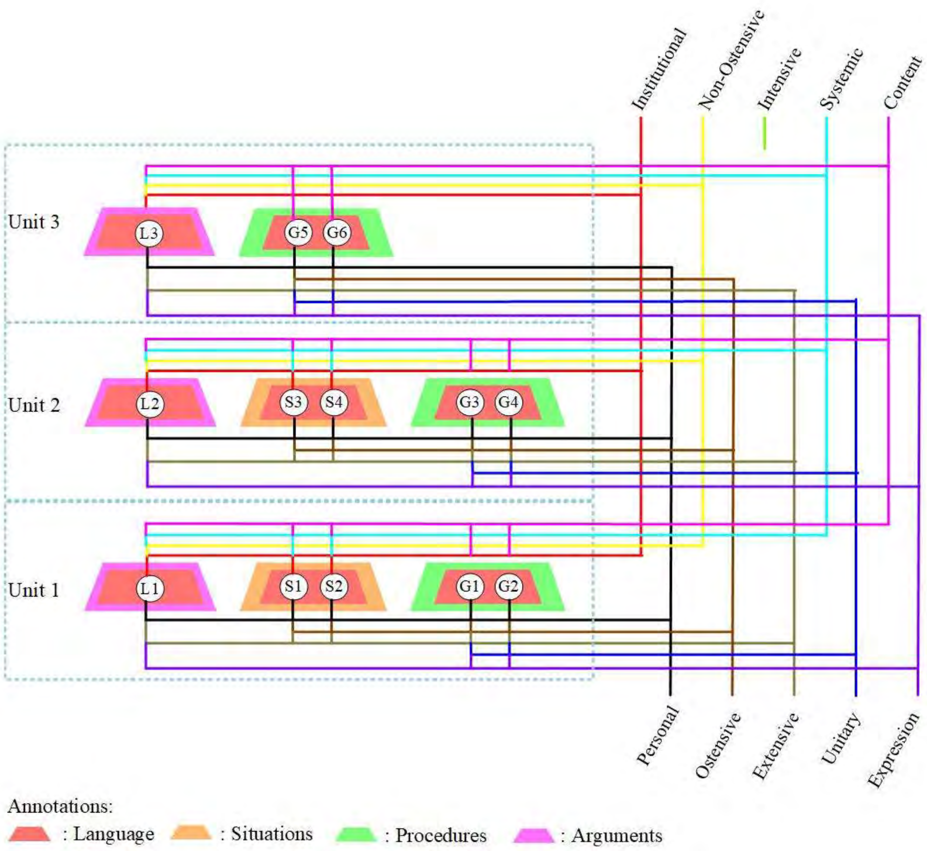


Figure 9: The Use of Mathematics Objects by S1 at the Looking Back Phase

Data Exposure of Subject 2 (S2)

Understanding the Problem

No part of S2's answer sheet can be determined directly as an activity to understand the problem. Explanation fragments from S2 that show the phase of understanding the problem are:

> *So, Ali made a password that was eight characters long, consisting of four different letters taken from the letters a, b, c, d, e and followed by four different numbers zero one two three four five. Determine the number of possible passwords?*

The S2 statement confirming the given task's information shows the use of mathematical objects. The mathematical objects used by S2 at the phase of understanding the problem are language and situations. An illustration of the use of mathematical objects by S2 can be seen in Figure 10.

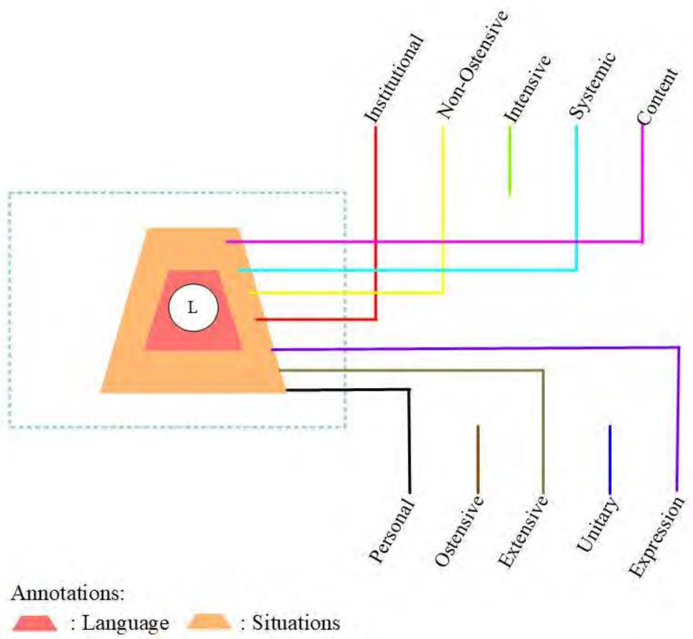


Figure 10: Use of Mathematical Objects by S2 at the Understanding the Problem Phase

Devising a Plan

What S2 does as part of the planning phase for completion is to draw eight lines. It is adapted to the existing problem situation. The pieces of S2 answers that show the activities in the planning phase of completion can be seen in Figure 11.

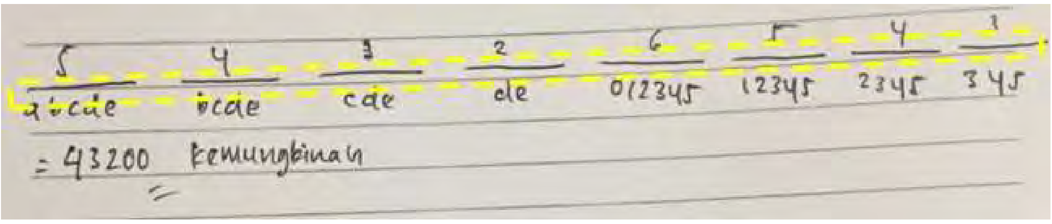


Figure 11: S2's Part Answer

Oral statements showing part of the planning phase for completion can also be obtained from the explanation video by S2. The fragment that contains the part planning for completion by S2 is as follows:

> *So, a, b, c, d, e are five letters so five per abcde, then it can't be the same so bcde has four letters, then it can't be the same anymore so cde becomes three letters and de is two.*

Based on data from written answers and explanation videos, it can be stated that S2 uses two mathematical objects. The mathematical objects used by S2 at this phase are language, concepts, procedures, and arguments. From the personal side, S2 is planning a solution using language

objects and arguments. In addition, during the interview session, it was also clarified that the phases of planning the completion were carried out. The excerpts of the interview related to the Phase of planning this settlement are as follows:

> *P: Here's the answer. Tell me about the process!*
> *S2 : Starting from the line, because it's eight characters, what I learned at the tutoring center was first to draw the line.*

From the results of interviews related to the process carried out by S2, there are statements indicating the use of the concept. It shows the use of mathematical objects in the form of concepts in devising a planning phase by S2. The use of objects by S2 can be seen in Figure 12.

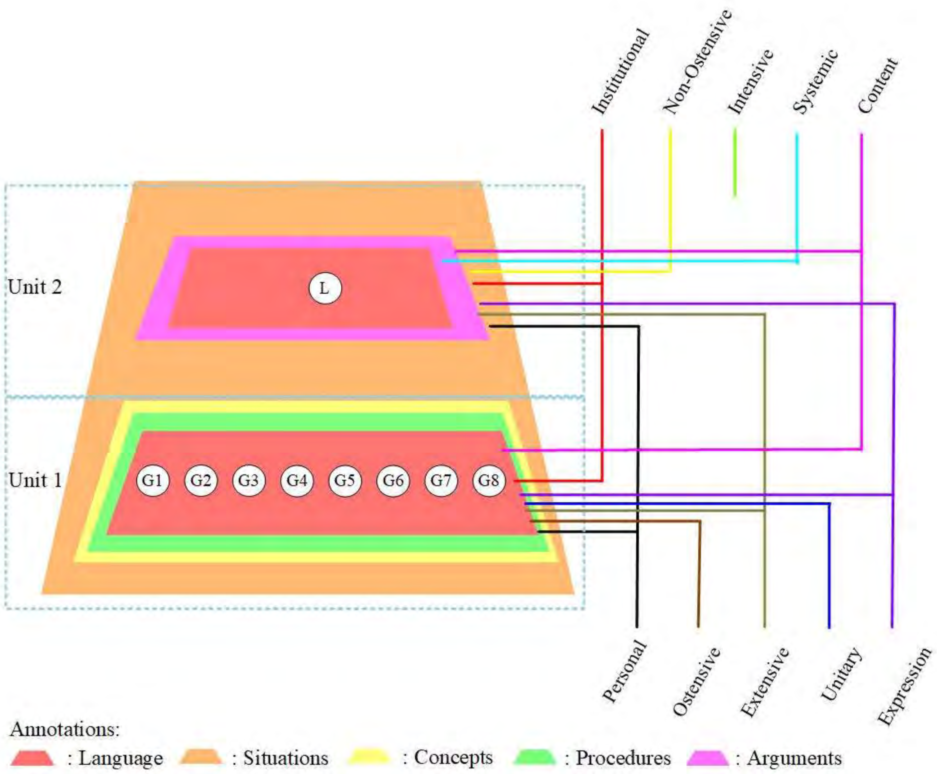
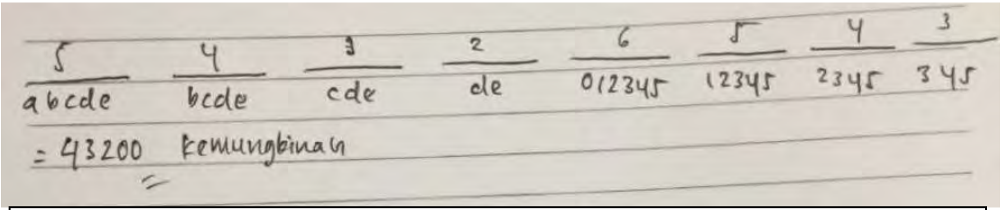


Figure 12: Use of Mathematics Objects by S2 at the Devising a Plan Phase

Carrying out the Plan

After planning the solution, S2 applies the rules for filling in places to count the number of letters and numbers that can occupy certain positions according to the problem. The answer sheet that shows the use of mathematical objects in the process of carrying out the completion by S2 can be seen in Figure 13.



Translating in English:
= 43200 possibilities

Figure 13: S2 Answer Pieces

From the video explanation of the answers by S2, it can also be seen about the activities in the phase of carrying out the completion. The transcript showing the phases of carrying out the completion by S2 are:

> *So, a, b, c, d, e are five letters so five per abcde, then it can't be the same so bcde has four letters, then it can't be the same anymore so cde becomes three letters and de is two. Then the numbers can't be the same, so zero one two three four five is six letters, one two three four five is also five, two three four five four letters, three four five three letters to four. All times.*

From Figure 13 and the transcript of the explanatory video, the mathematical objects used by S2 in the completion phase are language, situations, concepts, procedures, propositions, and arguments. In the content definition/concept object, the concept of subtraction and the concept of multiplication are used as objects from the non-ostensive side.

> *P: So here it was reduced?*
> *S2: Yes.*
> *P : O. So, after finishing this one, did you start writing again six five four three?*
> *S2: Yes*
> *P: Only six, it keeps decreasing again?*
> *S2: It's reduced.*
> *P: So how do you get this? (While circling the result on the answer sheet.*
> *S2 : Multiplied by the top number, the top number is five four three two six five four three.*

The mathematical objects used by S2 can be seen in Figure 14.

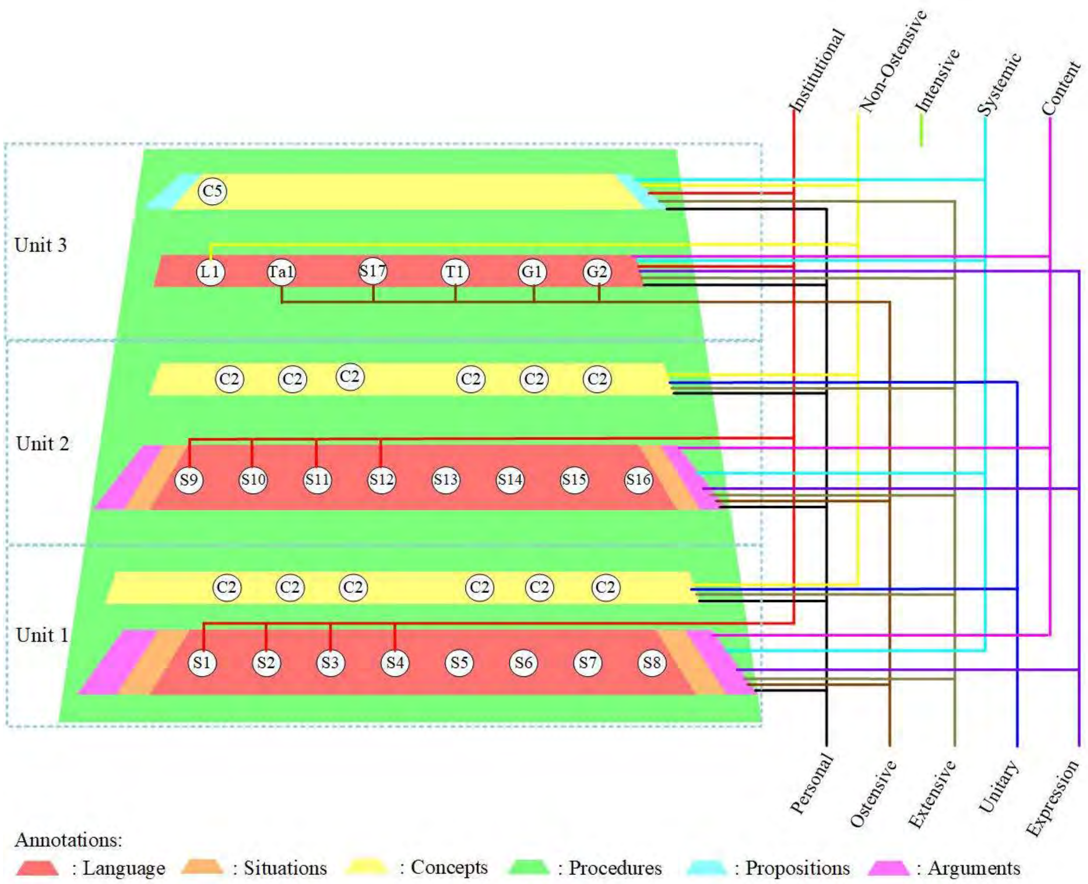


Figure 14: Use of Mathematical Objects by S2 at the Phase of Carrying out the Plan

Looking Back

On the S2 answer sheet, it cannot be directly determined which activities are part of the looking back phase. Data regarding the looking back activity was obtained while simultaneously observing the answer sheet and the video along with the explanation video transcript (not separate from the completion activity). From the video transcript, as in the implementing section, it can be stated that the looking back phase of S2 uses language objects and arguments. The use of mathematical objects by S2 can be seen in Figure 15.

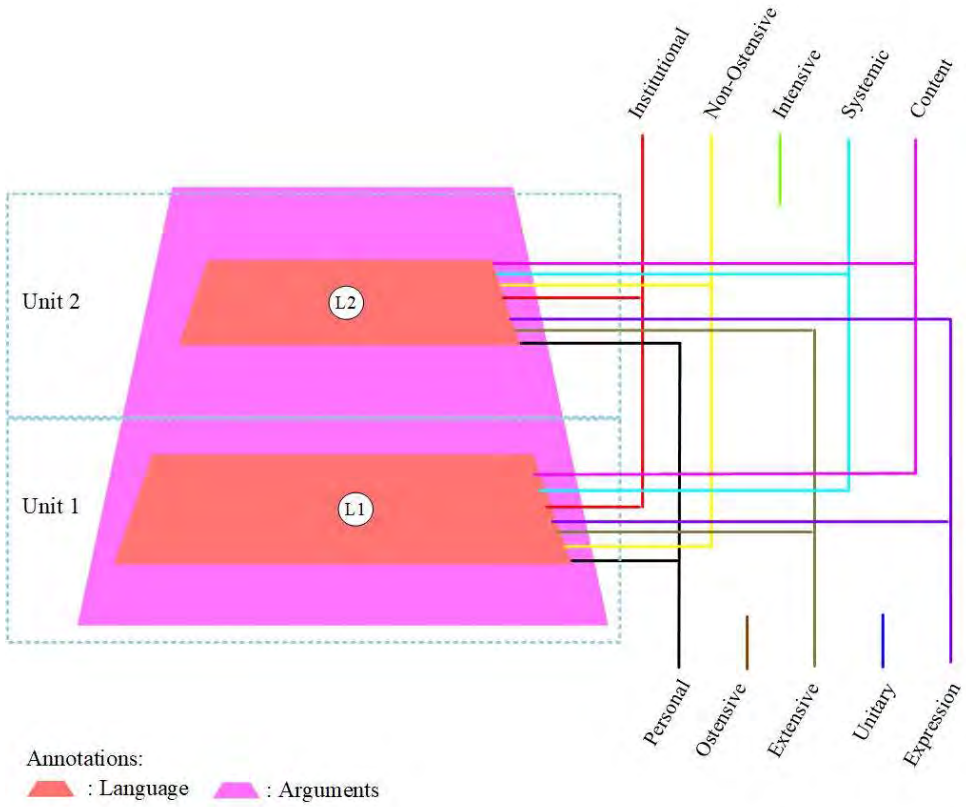


Figure 15: Use of Mathematics Objects by S2 in the Looking Back Phase

Based on the previous data exposure, a summary of the use of mathematical objects from each subject is made to make it easier to see the slices or combinations of the use of mathematical objects between subjects. The summary of the use of mathematical objects by Subject 1 is presented in Figure 16.

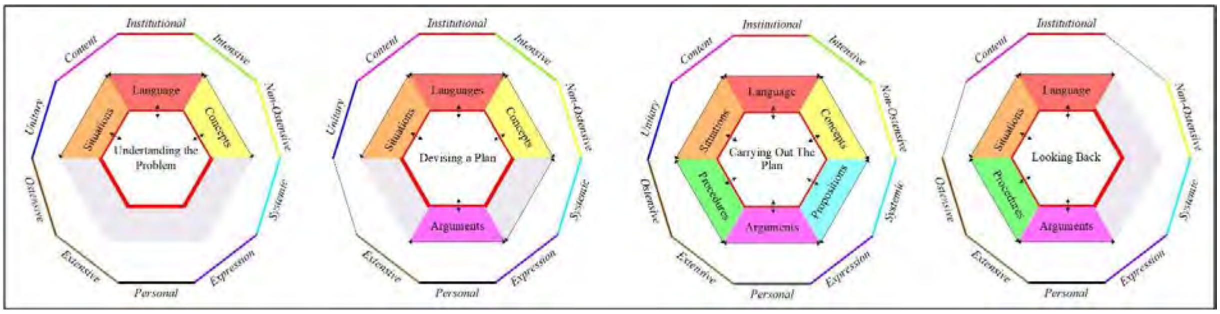


Figure 16: Summary of the Use of Mathematical Objects of Subject 1

Furthermore, a summary of the use of mathematical objects in each phase of problem-solving by Subject 2 can be seen in Figure 17.

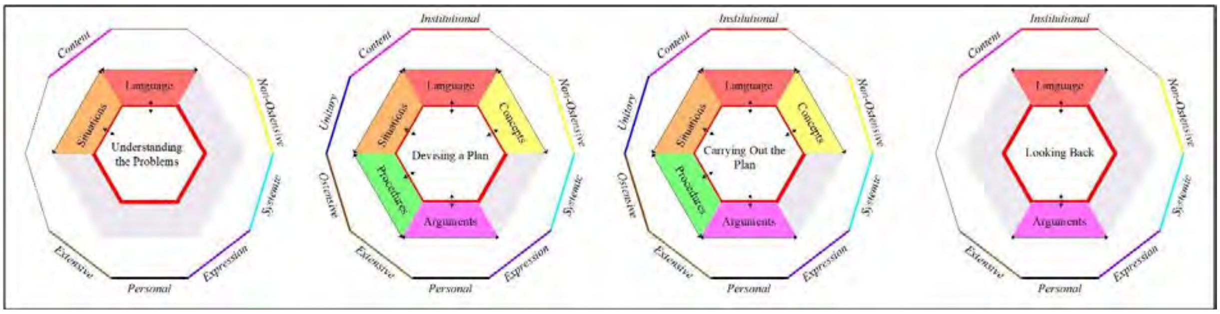


Figure 17: Summary of the Use of Mathematical Objects of Subject 2

Finding

Based on the analysis results and the use of objects between subjects, it can be stated that there are variations in the formation of the use of mathematical objects between the phases of problem-solving. The use of objects in each phase can be seen in Figure 18.

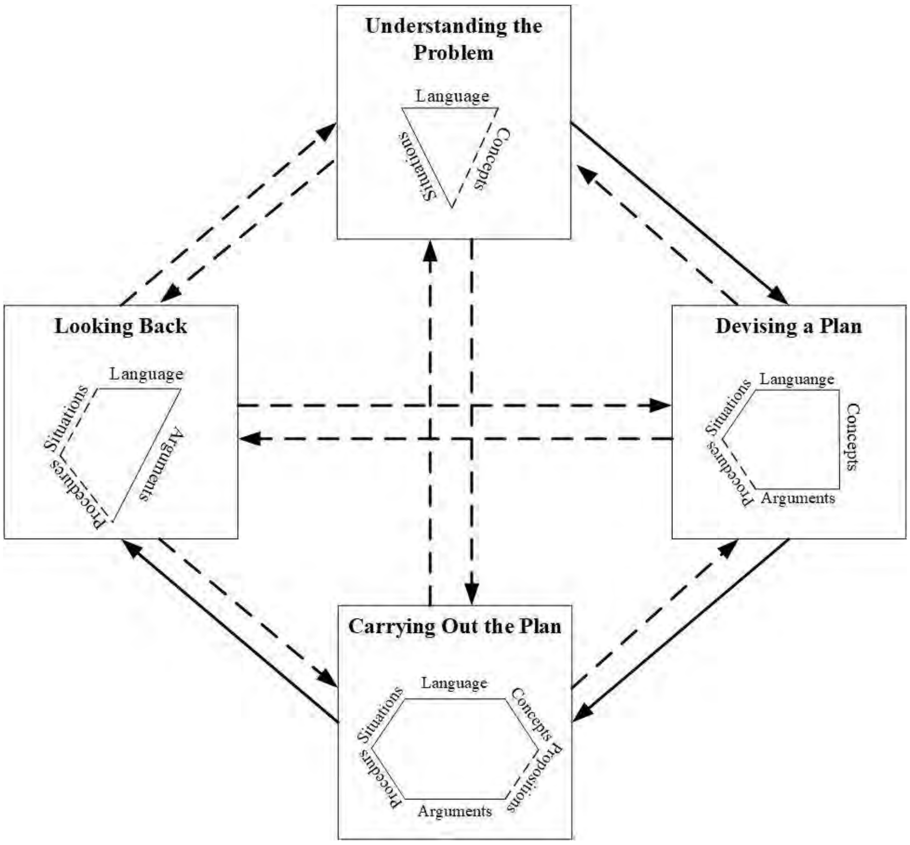


Figure 18: The Use of Mathematical Objects in Each Problem-Solving Phase

Figure 18 shows activity indicators formulated in each problem-solving phase because of integrating mathematical objects with an onto-semiotic approach. In detail, the indicators referred to can be seen in Table 5.

Phase of Problem Solving	Indicators of Mathematical Objects
Understanding the Problem	Using written language objects (words, symbols, signs, and picture) or spoken language in presenting problem information (problem situations)
Devising a Plan	• State the concept that will be used to solve the problem • Provide arguments against selected concepts
Carrying out the Plan	• State a proposition or write a general formula. • Perform procedures involving language objects, situations, and concepts.
Looking Back	• Examine procedures using language objects, other procedures, and arguments. • Check the suitability of the result with the problem situation (can use language objects, situations, procedures, and arguments)

Table 5*:* Indicators of Using Mathematical Objects in the Problem-Solving Phase

By integrating the theory of problem-solving and the onto-semiotic approach, the minimum indicators of activity that can be used in each problem-solving phase can be seen.

DISCUSSION

This section discusses the research findings, namely the variations in the formation of the use of mathematical objects in each problem-solving phase. Furthermore, the mathematical objects that appear in each phase of problem-solving need references in their use. The reference in question is given as indicators for using mathematical objects. The use of mathematical objects in each problem-solving phase is described as follows.

Understanding the Problem

The research findings show that the mathematical objects used by students in the problem-understanding phase are language, situations, and concepts. The results of this study indicate something new to complement the existing theory about understanding the problem, where it is obtained to produce the correct solution. It is enough to use two or three of the six primary entity components in the onto-semiotic approach. This result is in line with Kılıç (2017) research, which states that this phase is significant for the right solution and involves understanding the problem situation and determining and deciding facts and goals. Every student needs to understand the problem to have a chance to come up with a solution. Understanding the problem is a phase that involves determining what is needed regarding problems related to mathematical concepts and

procedures, accessing prior knowledge, and isolating relevant information from irrelevant contexts (Kotsopoulos & Lee, 2012).

The results showed that, in general, students understanding the problem used mathematical objects as part of understanding on the personal side. Students use mathematical objects that are not by the institutional side. Godino (2018) states that students are expected to adjust progressively to institutional meaning by participating in relevant practices to achieve a combination of initial personal and institutional meaning in the learning process.

The results showed a variation between the use of objects that could be observed directly on the answer sheet (ostensive) and using objects that were only used in students' minds (no ostensive) in the phase of understanding the problem. The use of this ostensive object is very influential in assessing the competence results of students, especially if the assessment is only in the form of a written test. Students who are more dominant in using non-ostensive objects need confirmation to get used to writing parts that have the potential to be elements that the teacher assesses. As Hough and Gluck (2019) state, performance-based measurement is the most popular technique for assessing human understanding. However, the measurement process needs to be modified to assess understanding. It is necessary that more complex cognitive models can be broken down into components for appropriate empirical testing.

In understanding the problem, students are more dominant in using objects specifically (extensive) than objects in general form (intensive). In this phase, students use the object being studied (systemic) more than objects that have been understood previously (unitary). Some expressions have linguistic meanings, situations, and concepts in this phase. The use of mathematical objects marks the students' understanding of the information on the given task. Students classify the information in the test given as one indication of an understanding of the test passed. Fuentes (1998) describes that part of the expected results in the learning process is that students can understand text math problems because this is the right way to solve problems. Reading comprehension is one of the cognitive factors that can play an essential role in solving high school students' issues (Öztürk et al., 2020).The basic understanding possessed by students plays an essential role in the problem-solving process (Lee, 2011). It is difficult for a person to have a good idea about problem-solving. It is even impossible to have an idea if he does not have knowledge and ideas related to problem-solving based on previously acquired knowledge or experience (Polya, 1986).

Devising a plan

The research findings indicate that in the planning phase for completion, all students use mathematical objects in the form of language, situations, concepts, and arguments. In addition, there are mathematical objects in the form of procedures by students. These results provide a specific answer compared to the previous theory about planning completion. In planning completion, students can use four or five of the six primary entity components in the onto-semiotic approach. Kotsopoulos & Lee (2012) stated that designing a settlement plan involves selecting the mathematical processes and operations appropriate to the problem and establishing the procedures to be applied to solve the problem.

The results showed that in the planning phase of completion, students used mathematical objects as part of their understanding but institutional ones. In planning the completion of mathematical objects, non-ostensive objects are dominated by ostensive objects, students are more dominant in using extensive objects than intensive objects, and students use systemic objects more than unitary objects. Furthermore, some expressions mean situations, concepts, and arguments. Students' use of mathematical objects in the planning phase shows an understanding of concepts that can be applied based on problem situations. The problem given is a problem with the context of the email password with the consideration that high school-age children in their daily lives have a lot to do with the use of passwords. The choice of the context of the problem was also chosen to make it easier to understand and more accessible to plan the steps for solving it. As stated by Batanero et al. (2021), context dramatically influences children's strategies, so it is necessary to provide different contexts, including situations in children's lives and different media such as dice, chips, and coins. Vásquez et al. (2021) also describe that problem-solving is one of the keys to demonstrating mathematical competence, including managing knowledge, skills, and emotions to achieve goals that are more towards practical situations and in the context of everyday life.

Carrying out the plan

The research findings show that in the phase of implementing the completion, students use all components of the primary entity in an onto-semiotic approach. Students use mathematical objects in language, situations, concepts, procedures, propositions, and arguments. This result adds to the details of the previous theory that carrying out the solution is a part that requires a mathematical process, including operations, to produce a solution (Kotsopoulos & Lee, 2012). This study shows that the mathematical process of carrying out the solution involves all components of the primary entity from the onto-semiotic approach, not only procedures that involve mathematical operations.

The results show that students generally use mathematical objects corresponding to institutional objects in the completion phase. In completing mathematical objects, ostensive objects are dominated by non-ostensive objects. Students first use intensive and extensive objects and use systemic objects more than unitary objects. Some expressions mean situations, concepts, propositions, and arguments. Differences in the use of mathematical objects also occur in this phase, even though they both lead to the same correct result due to differences in the formulas or procedures used. Giacomone et al. (2018) state that each procedure used to solve problems can mobilize different mathematical objects and lead to necessary consequences in mathematical activities. Under certain conditions, the aspects of planning and implementing a settlement must be separated. In line with this, preparing and implementing plans are two aspects that become an integrated whole (Nurkaeti, 2018).

Looking back

From the research findings, the mathematical objects students use in looking back are dominated by language and arguments. However, there is also the use of situation objects and procedures. These results provide more operational information about the mathematical practice carried out in the review phase to complement the previous theory that in reviewing and evaluating, the key idea is to explore the problem solution by evaluating whether the results are reasonable and the

reliability and validity of the results (Kotsopoulos & Lee, 2012). The practice of mathematics in the looking back phase is carried out by using two or four of the six primary entity components in the onto-semiotic approach, namely language and arguments or language, situations, procedures, and arguments.

The results show that in the looking back phase, the mathematical objects used as personal understanding still need to be per the mathematical objects on the institutional side. The same thing has previously been part of the conclusion of the study by Kazemi et al. (2010) that the most common student difficulty in solving combinatoric problems is the inability to ensure the correctness of the answers they find. Students need help looking back at the truth of the answers to questions caused by incorrect understanding, planning, and implementation of problem-solving (Nurkaeti, 2018).

In looking back phase, mathematical objects are dominated by non-ostensive objects rather than ostensive objects. These results, as in the study by Giacomone et al. (2018), found that justifications, arguments, or explanations do not appear explicitly in test books in general, nor specifically in the tasks given. These results are in line with some of the conclusions from the research of Moguel et al. (2020), which stated that at secondary-level learning in Mexico, mathematics usually does not have official evidence, which implies that teachers are not accustomed to providing evidence to justify the solution of a problem. The same thing was also concluded by Arfiana and Wijaya (2018) that the lowest result for middle and high school was the phase of re-checking the answers.

In this phase, all objects used by students are extensive objects; intensive objects are not used. As in the previous phase, all objects used by students are systemic objects; there is no use of unitary objects. Mathematical objects used by students are more dominant in expressions that have the meaning of arguments. However, some students use situational objects and procedures. The lack of meaningful expressions of situations and procedures is evidence that, in general, students need to check the results concerning the problem situation to verify the solutions obtained. Similar results were obtained in the study of Moreno et al. (2021), where the study showed that none of the students interpreted the solutions found about the actual situation.

CONCLUSIONS

The results of the research and discussion show that students' onto-semiotics in solving combinatoric problems provide insight into the variations in the formation of the use of mathematical objects in each problem-solving phase. Integrating problem-solving theory with an onto-semiotic perspective provides the basis for using mathematical objects in each problem-solving phase. The variations in the use of mathematical objects in the problem-solving process as a result of integration with the onto-semiotic approach, namely

1. In the phase of understanding the problem, students use written language objects (words, symbols, signs, and pictures) or spoken language in presenting problem situations or problem information.

2. In the devising a planning phase of completion, students state the concept that will be used to solve the problem and provide arguments against the chosen concept.
3. In the carrying out the planning phase, students state propositions or write general formulas and perform procedures involving language objects, situations, and concepts; and
4. In the looking back phase, students examine the procedure using language objects, other procedures, and arguments and check the suitability of the final result with the problem situation using language objects, situations, procedures, and arguments.

The process suggests several things, results, and discussion: (1) in each problem-solving phase, there is the use of non-ostensive mathematical objects. Hence, teachers need to conduct an authentic assessment by assessing the mathematical objects that are not explicitly written on student answer sheets; (2) language objects and arguments dominate the use of mathematical objects in the looking back phase by students. Thus practically, the checking activities need to be familiarized teachers and students with the mathematics learning process; (3) in order to improve students' problem-solving skills, it is recommended for teachers to emphasize the components of mathematical objects that must be raised in each phase of problem-solving as indicators of the activity of using mathematical objects as found in this study; (4) further researchers are advised to further explore the differences in the proportions of the area of each trapezoid (use of mathematical objects) by using specific indices such as how many words/ sentences, time students use/ spend in the area of situations, language, concepts, etc. and (5) further research can be done to see variations in the use of mathematical objects specifically in terms of differences in problem-solving strategies. In this case, the instrument in the form of a mathematical task is confirmed to be a task that can be solved in several different strategies.

References

[1] Amin, M. E. I. A., Juniati, D., & Sulaiman, R. (2018). Onto semiotic approach to analyze students' understanding of algebra based on math ability. AIP Conference Proceedings. https://doi.org/10.1063/1.5054481

[2] Arfiana, A., & Wijaya, A. (2018). Problem solving skill of students of senior high schools and islamic high schools in Tegal Regency in solving the problem of PISA based on Polya 's stage. Jurnal Riset Pendidikan Matematika, 5(2), 211–222. https://doi.org/10.21831/jrpm.v5i2.15783

[3] Avcu, S., & Avcu, R. (2010). Pre-service elementary mathematics teacher's use of strategies in mathematical problem solving. Procedia - Social and Behavioral Sciences, 9, 1282–1286. https://doi.org/10.1016/j.sbspro.2010.12.321

[4] Batanero, C., Álvarez-Arroyo, R., Hernández-Solís, L. A., & Gea, M. M. (2021). El inicio del razonamiento probabilístico en educación infantil. PNA, 15(4), 267–288. https://doi.org/10.30827/pna.v15i4.22349

[5] Baykul, Y., & Antalya. (2011). Problem solving in elementary mathematics curriculum. 2nd International Conference on New Trends in Education and Their Implications, 1149–1157. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.1065.6457&rep=rep1&type=pdf

[6] Bencomo, D., Godino, J. D., & Wilhelmi, M. R. (2004). Elaboración de redes ontosemióticas de configuraciones didácticas con atlas/ti. Concept Maps: Theory, Methodology, Technology, 1–4. https://cmc.ihmc.us/Papers/cmc2004-135.pdf

[7] Bien, N. Van, Krause, E., & Tho, C. C. (2020). Problem solving. In Comparison of Mathematics and Physics Education I (pp. 345–368). Springer Spektrum. https://doi.org/10.1007/978-3-658-29880-7_14

[8] Burgos, M., Beltrán-Pellicer, P., Giacomone, B., & Godino, J. D. (2019). Onto-Semiotic Analysis of a Lesson on Percentages. INTED2019 Proceedings, 1, 1524–1533. https://doi.org/10.21125/inted.2019.0462

[9] Burgos, M., & Godino, J. D. (2020). Modelo ontosemiótico de referencia de la proporcionalidad: Implicaciones para la planificación curricular en primaria y secundaria. Avances de Investigación En Educación Matemática, 18, 1–20. https://doi.org/10.35763/aiem.v0i18.255

[10] Carson, J. (2007). A problem with problem solving: teaching thinking without teaching knowledge. The Mathematics Educator, 17(2), 7–14. https://files.eric.ed.gov/fulltext/EJ841561.pdf

[11] Csachová, L. (2021). Looking back in problem solving with future primary school teachers. International Symposium Elementary Mathematics Teaching, 157–166. https://www.semt.cz/proceedings/semt-21.pdf#page=157

[12] D'Amore, B., & Godino, J. D. (2006). Punti di vista antropologico ed ontosemiotico in didattica della matematica. La Matematica e La Sua Didattica, 1, 9–38. http://www.dm.unibo.it/rsddm/it/articoli/damore/548 Punto di vista.pdf

[13] Docktor, J., & Heller, K. (2009). Robust assessment instrument for student problem solving. Proceedings of the NARST 2009. https://groups.physics.umn.edu/physed/Talks/Docktor_NARST09_paper.pdf

[14] Elia, I., van den Heuvel-Panhuizen, M., & Kolovou, A. (2009). Exploring strategy use and strategy flexibility in non-routine problem solving by primary school high achievers in mathematics. ZDM - International Journal on Mathematics Education, 41(5), 605–618. https://doi.org/10.1007/s11858-009-0184-6

[15] Font, Godino, J. D., & D'Amore, B. (2015). Representations in mathematics education: an onto-semiotic approach. Journal Internacional de Estudos Em Educação Matemática, 2(1). https://doi.org/10.17921/2176-5634.2010v2n1p%25p

[16] Font, V., Godino, J. D., & D'Amore, B. (2007). An onto-semiotic approach to representations in mathematics education. For the Learning of Mathematics, 27(2), 2–7. https://flm-journal.org/Articles/7856D4C00321D131A92B3A9C9B7580.pdf

[17] Font, V., Godino, J. D., & D'Amore, B. (2010). Representations in matematics education: an onto-semiotic approach. Jornal Internacional de Estudos Em Educação Matemática, 2, 58–86. https://doi.org/10.17921/2176-5634.2010v2n1p%25p

[18] Fuentes, P. (1998). Reading Comprehension in Mathematics. The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 72(2), 81–88. https://doi.org/10.1080/00098659809599602

[19] Giacomone, B., Díaz-Levicoy, D., & Godino, J. D. (2018). Análisis ontosemiótico de tareas que involucran gráficos estadísticos en educación primaria. Revista Digital: Matemática, Educación e Internet, 18(1), 1–13. https://doi.org/10.18845/rdmei.v18i1.3256

[20] Giacomone, B., Godino, J. D., & Beltrán-Pellicer, P. (2018). Developing the prospective mathematics teachers' didactical suitability analysis competence. Educacao e Pesquisa, 44, 1–19. https://doi.org/10.1590/S1678-4634201844172011

[21] Giacomone, B., Godino, J. D., Wilhelmi, M. R., & Blanco, T. F. (2018). Desarrollo de la competencia de análisis ontosemiótico de futuros profesores de matemáticas. Revista Complutense de Educacion, 29(4), 1109–1131. https://doi.org/10.5209/RCED.54880

[22] Godino, J. D. (2002). Un enfoque ontólogico y semiótico de la cognicion matematica. Recherches En Didactique Des Mathématiques, 22(2.3), 237–284. https://www.ugr.es/~jgodino/funciones-semioticas/04_enfoque_ontosemiotico.pdf

[23] Godino, J. D. (2014). Síntesis del enfoque ontosemiótico del conocimiento y la instrucción matemática: motivación, supuestos y herramientas teóricas. Universidad de Granada *cita En El Doc, 1–60. http://www.ugr.es/local/jgodino/eos/sintesis_EOS_24agosto14.pdf%5CnJuan

[24] Godino, J. D. (2018). Bases epistémolgicas e instruccionales del enfoque ontosemiótico en educación matemática. Universidade de Granada, 1–29. http://enfoqueontosemiotico.ugr.es/JDGodino_bases_epins_EOS.pdf

[25] Godino, J. D. (2019). Perspectiva de las investigaciones sobre educación estadística realizadas en el marco del enfoque ontosemiótico. https://digibug.ugr.es/handle/10481/55031

[26] Godino, J. D. (2017). Articulación de teorías socio-culturales en educación matemática desde la perspectiva del enfoque ontosemiótico. Conferencia Plenaria. RELME 31, Lima, Perú, September 2017. https://www.researchgate.net/profile/Juan-Godino/publication/335571756_Articulacion_de_teorias_socio-culturales_en_educacion_matematica_desde_la_perspectiva_del_enfoque_ontosemiotico_Networking_socio-cultural_theories_in_mathematics_education_from_the_ont

[27] Godino, J. D., & Batanero, C. (2020). El enfoque ontosemiótico: implicaciones sobre el carácter prescriptivo de la didáctica. Revista Chilena de Educación Matemática, 47–59. https://doi.org/10.46219/rechiem.v12i2.25

[28] Godino, J. D., Batanero, C., & Font, V. (2007). The Onto-semiotic Approach to Research in Mathematics Education. ZDM - International Journal on Mathematics Education, 39, 127–135. https://doi.org/10.1007/s11858-006-0004-1

[29] Godino, J. D., Batanero, C., & Roa, R. (2005). An onto-semiotic analysis of combinatorial problems and the solving processes by university students. Educational Studies in Mathematics, 60(1), 3–36. https://doi.org/10.1007/s10649-005-5893-3

[30] Godino, J. D., Burgos, M., & Gea, M. M. (2021). Analysing theories of meaning in mathematics education from the onto-semiotic approach. International Journal of Mathematical Education in Science and Technology, 1–26. https://doi.org/10.1080/0020739X.2021.1896042

[31] Godino, J. D., Font, V., & Wilhelmi, M. R. (2006). Análisis ontosemiótico de una lección sobre la suma y la resta. Relime Número, Especial, 131–155. https://dialnet.unirioja.es/descarga/articulo/2161569.pdf

[32] Godino, J. D., Font, V., Wilhelmi, M. R., & Lurduy, O. (2011). Why is the learning of elementary arithmetic concepts difficult? Semiotic tools for understanding the nature of mathematical objects. Educational Studies in Mathematics, 77(2–3), 247–265. https://doi.org/10.1007/s10649-010-9278-x

[33] Hough, A. R., & Gluck, K. A. (2019). The understanding problem in cognitive science. Advances in Cognitive Systems, 8, 13–32. http://www.cogsys.org/journal/volume8/article-8-3.pdf

[34] Kazemi, F., Fadaee, M. R., & Bayat, S. (2010). A subtle view to metacognitive aspect of mathematical problems solving. Procedia Social and Behavioral Sciences, 8(5), 420–426. https://doi.org/10.1016/j.sbspro.2010.12.058

[35] Kılıç, Ç. (2017). A new problem-posing approach based on problem-solving strategy: Analyzing pre-service primary school teachers' performance. Educational Sciences: Theory & Practice, 17(3), 771–789. https://doi.org/10.12738/estp.2017.3.0017

[36] Kotsopoulos, D., & Lee, J. (2012). A aaturalistic study of executive function and mathematical problem-solving. The Journal of Mathematical Behavior, 31, 196–208. https://doi.org/10.1016/j.jmathb.2011.12.005

[37] Lee, S. Y. (2011). The effect of alternative solutions on problem solving performance. International Journal for Mathematics Teaching & Learning, 1–17. https://www.cimt.org.uk/journal/lee.pdf

[38] Moguel, L. E. S., Landa, E. A., & Cabañas-Sánchez, G. (2020). Fases del razonamiento inductivo que presentan profesores de matemáticas al resolver un problema de generalización. PNA, 14(2), 118–140. https://doi.org/10.30827/pna.v14i2.9118

[39] Montiel, M., Wilhelmi, M. R., Vidakovic, D., & Elstak, I. (2012). Vectors, Change of Basis and Matrix Representation: Onto-semiotic Approach in the Analysis of Creating Meaning. International Journal of Mathematical Education in Science and Technology, 43(1), 11–32. https://doi.org/10.1080/0020739X.2011.582173

[40] Moreno, A., Marin, M., & Ramirez-Ucles, R. (2021). Errores de profesores de matemáticas en formación inicial al resolver una tarea de modelización. PNA, 15(2), 109–136. https://doi.org/10.30827/pna.v15i2.20746

[41] Nachlieli, T., & Tabach, M. (2012). Growing mathematical objects in the classroom-the case of function. International Journal of Educational Research, 51, 10–27. https://doi.org/10.1016/j.ijer.2011.12.007

[42] Nurkaeti, N. (2018). Polya's strategy: an analysis of mathematical problem solving difficulty in 5th grade elementary school. EduHumaniora: Jurnal Pendidikan Dasar |, 10(2), 140–147. https://doi.org/10.17509/eh.v10i2.10868

[43] Öztürk, M., Akkan, Y., & Kaplan, A. (2020). Reading comprehension, Mathematics self-efficacy perception, and Mathematics attitude as correlates of students' non-routine Mathematics problem-solving skills in Turkey. International Journal of Mathematical Education in Science and Technology, 51(7), 1042–1058. https://doi.org/10.1080/0020739X.2019.1648893

[44] Pimta, S., Tayruakham, S., & Nuangchale, P. (2009). Factors influencing mathematic problem-solving ability of sixth grade students. Journal of Social Sciences, 5(4), 381–385. https://doi.org/10.3844/jssp.2009.381.385

[45] Piñeiro, J. L., Castro-Rodríguez, E., & Castro, E. (2019). Componentes de conocimiento del profesor para la enseñanza de la resolución de problemas en educación primaria. PNA, 13(2), 104–129. https://doi.org/10.30827/pna.v13i2.7876

[46] Polya, G. (1957). How to Solve It: A New Aspect of Mathematical Method. In Princeton University Press. https://doi.org/10.2307/2306109

[47] Polya, G. (1986). How to solve it: mathematical problem solving. In Science (Vol. 232, Issue 4753).

[48] Prayitno, L. L., Subanji, Susiswo, & As'ari, A. A. (2020). Exploring Student's Representation Process in Solving Ill-Structured Problems Geometry. Participatory Educational Research (PER), 7(2), 183–202. https://doi.org/10.17275/per.20.28.7.2

[49] Starikova, I. (2010). Why do mathematicians need different ways of presenting mathematical objects? The case of Cayley graphs. Topoi, 29(1), 41–51. https://doi.org/10.1007/s11245-009-9065-4

[50] Sukoriyanto, S. (2017). Karakterisasi interferensi berpikir mahasiswa dalam menyelesaikan masalah permutasi dan kombinasi. Universitas Negeri Malang.

[51] Vásquez, C., Coronata, C., & Rivas, H. (2021). Enseñanza de la estadística y la probabilidad de los 4 a los 8 años de edad: una aproximación desde los procesos matemáticos en libros de texto chilenos. PNA, 15(4), 339–365. https://doi.org/10.30827/pna.v15i4.22512

[52] Wilhelmi, M. R., Godino, J. D., & Lacasta, E. (2021). Didactic effectiveness of mathematical definitions: the case of the absolute value. International Electronic Journal of Mathematics Education, 2(2), 72–90. https://doi.org/10.29333/iejme/176