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Problem-based learning for improving problem-solving and critical thinking skills: A case on probability theory course

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ABSTRACT

Problem-based learning (PBL) has been widely applied as an alternative to improve learning outcomes, but it is still little studied in the context of the probability theory course. This study described how implementing the PBL model improves students' problem-solving and critical thinking skills in probability theory course and evaluates its quality. This design research involved 58 undergraduate students and two probability theory course lecturers from two universities in Indonesia as participants. Data collection used observation to describe PBL implementation, then questionnaire and pretest-posttest to evaluate the quality of the model. This study produced a PBL model for the probability theory course which is implemented through five steps: i) Orienting students on problems; ii) Organizing students to study; iii) Assisting individual and group investigations; iv) Developing and presenting work or solutions; and v) Analyzing and evaluating problem solving processes. Lecturers and students consider that the implementation of PBL is practical. PBL implementation can also improve students' problem-solving skills and critical thinking in the probability theory course. Thus, implementing PBL can be used as a solution to optimize learning outcomes in the probability theory course.

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1. INTRODUCTION

Critical thinking and problem-solving are two essential skills in learning mathematics. Critical thinking skills in learning mathematics are needed to solve mathematical problems rationally and make the right decisions [1]–[3]. Research results have also reported that critical thinking skills are not only needed in solving mathematical problems but also needed in various other knowledge disciplines [4], [5], including one of them in detecting false information or hoax [6] which is a significant issue in today's rapid development of technology and communication media. As has been stated by many experts [7]–[12], critical thinking has now been the primary goal of education. Not only in the field of education, but critical thinking is also a necessary component in the work place [7], [9], [11], communication [9], even in terms of technology utilization [9]. Critical thinking allows individuals to gain a more complex understanding of the information they encounter and make good decisions and problem solving in real-world situations [7], [11]. Critical thinking also enables a person to make better decisions and judgments in complex situations, reduce cognitive biases and involve heuristic thinking, and become an informed and actively participating citizen [7].

Apart from critical thinking, problem-solving skills are also required in the context of education and everyday life. Problem solving is vital for each learner because these skills have become one of the focuses in learning mathematics [13]–[18]. From a global view, problem-solving skills can help individuals to overcome problems in everyday life [15]. In addition, Özreçberoğlu and Çağanağa [15] added that individuals with problem-solving skills have self-confidence, creativity, and become independent thinkers. In mathematics education, problem solving in mathematics helps students have experience in solving problems in everyday life by applying their mathematical knowledge and skills [14]. Several reasons why problem-solving skills are important, as stated by Osman *et al*. [14], include: i) Problem-solving skills make students have many choices to make decisions; ii) Problem solving helps students to reason in various aspects of life practice; and iii) Through the problem solving process, students will use their skills to analyze, think of solutions, determine causes, evaluate possible strategies or solutions to solve problems and determine the most effective solutions. In addition, problem-solving skills, besides emphasizing cognitive aspects, also emphasize affective aspects of the learning process [19]. Problem solving is also seen as the first step for students in developing ideas to construct new knowledge and other mathematical skills [20].

Efforts to develop students' critical thinking skills through the learning process are inseparable from developing problem-solving skills. Compared to creative thinking, problem-solving skills have a closer relationship with critical thinking skills [2]–[5]. Ling and Loh [21] report that the ability to think critically has a very close relationship and is a significant predictor of the ability to recognize patterns (pattern recognition). Recognizing patterns is one of the important skills needed in solving problems, especially mathematics problems [22]. That is, if educators successfully carry out learning that develops students' critical thinking, then educators also have the potential to be successful in developing problem-solving skills, as has been reported by previous research results [23], [24]. One learning model that can be used to develop students' critical thinking skills and problem solving is problem-based learning (PBL) [25]. Some of the advantages obtained through PBL such as students being actively involved in exploring their learning experiences [25]–[27], instilling students the ability to think flexibly and become successful problem solvers [27]–[29], and involves collaboration [25], [27]. Thus, PBL represents learner-centered learning through meaningful activities for developing thinking skills and collaboration.

Yew dan Goh [25] state that PBL has been widely adapted to various educational fields and contexts to promote critical thinking and problem solving in authentic learning situations. Furthermore, Gijbels *et al*. [29] underlined that PBL has implications for educational practice in higher education level that involve inter-subject and inter-disciplinary influences. This indicates that research on PBL is not only relevant for the elementary education level [30]–[32] and secondary [33]–[36], but has also been widely applied at the higher education level [37]–[42]. This study focuses on the implementation of PBL at the higher education level. The implementation of PBL at the higher education level which focuses on developing critical thinking and problem solving has also been extensively researched. Cahyono *et al*. [38] investigated the effect of PBL supported by scaffolding on critical thinking in prospective teacher. The implementation of PBL in this study was carried out in an algebraic structure course. Evendi *et al*. [37] investigated the effect of PBL on students' critical thinking in terms of cognitive style. In this study, PBL was implemented online in general mathematics courses for undergraduate students. Several studies focus on problem solving, for example, Klegeris and Hurren [43] investigated the impact of PBL in a large classroom setting on the problem-solving skills of undergraduate biochemistry students at a university in Canada. Kadir *et al*. [44] investigated the effect of implementing PBL on the problem-solving skills of undergraduate business students at a university in Malaysia. PBL in this study is implemented in practical communication courses for fourteen weeks. Most recently, Kök and Duman [45] investigated the effect of PBL implementation on problem-solving skills in learning English. The implementation of PBL in this study involved 46 undergraduate students from the school of foreign languages at one of the state universities in Turkey. Existing studies show that PBL has become one of the strategies for developing critical thinking and problem solving in various disciplines for higher education.

Although PBL has been widely used to develop students' critical thinking and problem-solving skills at the higher education level, it is still rare to find studies that reveal how PBL is implemented in probability theory course. This study therefore seeks to fill that gap. In addition, studies that attempt to investigate efforts to improve problem-solving and critical thinking skills in probability theory courses still need to be found. The findings of this study are expected to contribute to improving the quality of learning and student learning outcomes in probability theory courses. Thus, this study aims to describe the implementation of PBL to improve students' problem-solving and critical thinking skills in probability theory courses and evaluate its practicality and effectiveness.

2. RESEARCH METHOD

This study was conducted as design research [46], [47]. The focus of this study is to describe the excellent practice of PBL in probability theory courses as an effort to improve students' problem-solving and critical thinking skills. The implementation of PBL in probability theory courses refers to the model that has been developed as shown in Figure 1. In addition, this study also evaluates the practicality and impact of the model in probability theory courses. The practicality of implementing the PBL model in probability theory courses is evaluated based on the assessments of user lecturers and students. The impact of implementing the PBL model is seen in the improvement of students' problem-solving skills and critical thinking in probability theory courses.

2.1. Participant

The implementation of PBL in this study involved two lecturers and 58 students from two undergraduate mathematics education study programs in Indonesia. In this study, the two lecturers acted as teachers. The two lecturers taught probability theory courses at their respective universities. One lecturer (lecturer A) comes from a state university and the other (lecturer B) from a private university. Students are divided into three classes, which are two classes at a state university (class A, *n*=24; and class B, *n*=18.) and one class at a private university (class C, *n*=16). When this research was conducted, all students as participants were contracting for probability theory courses at their respective universities. In this study, lecturer A implemented PBL in classes A and B while lecturer B in class C. Even though there were differences in the lecturers who taught, all classes received the same treatment.

2.2. Research procedure

2.2.1. Designing a model

This research begins by designing a PBL model for probability theory course. The PBL model in probability theory course is developed based on the constructivism paradigm. This model was developed to improve students' problem-solving and critical thinking skills in probability theory course. The components contained in the model include syntax or lecture steps, social system, principles of reaction, support system, and instruction and nurturant effects as presented in Figure 1.

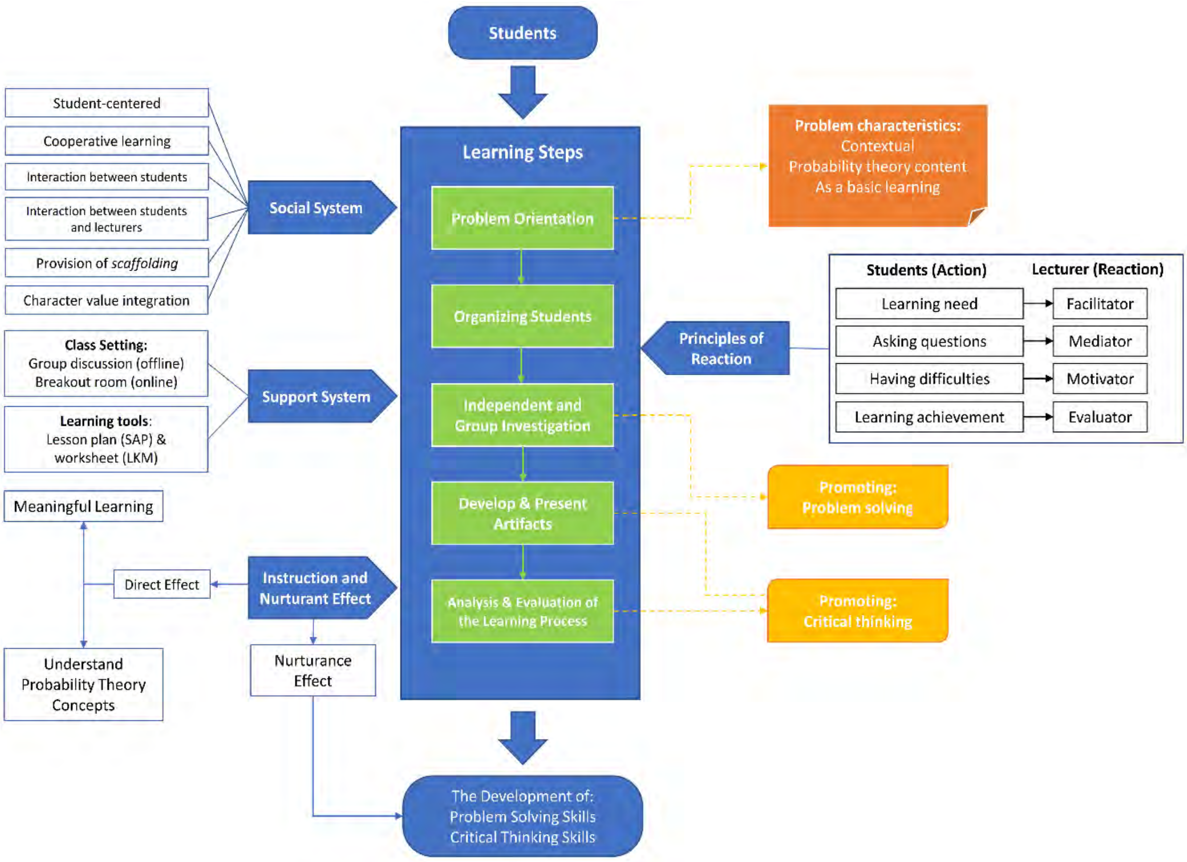

Figure 1. The model of problem-based probability theory course

In the PBL model in probability theory course, the lectures' success or failure is determined by the problems that will be given to students. Thus, the problem is the core of the model. The design of the problem in the probability theory course model is carried out by noticing the characteristics, context, environment, and learning resources. The problems selected in the developed model consider several matters: i) The suitability of the problem with the probability theory subject; ii) The suitability of the problem with real world situations; iii) Variations in complexity level of the problem; iv) The relationship between the problem and other disciplines; v) Possible alternative solutions to the problem; and vi) The relationship between the problem and the concepts that have been studied. Furthermore, the problem context proposed in the model considers the following: i) The problem context is ill-structured; ii) The context of the problem motivates and raises students' curiosity to find solutions to problems; and iii) The problem context has novelty. Besides that, in designing the problems in the model also consider matters related to the environment and learning resources, including: i) The problems presented must stimulate students to collaborate, discover, and group discussions and ii) The problems presented require a variety of relevant learning resources to find solutions.

2.2.2. Model validation

Six experts then validated the design of the model: one expert in mathematics, three experts in mathematics education, and two experts in mathematics learning assessment. The model validation stage is carried out in three stages. First, the researcher asked the validator to provide qualitative suggestions according to his expertise to improve the developed model. Second, revise the model according to suggestions from all validators. After revision, the draft model is then submitted back to the validator. Third, asking the validator to provide a quantitative assessment based on aspects of the model component assessment. At this stage the expert can still provide input if it is found that there are still parts in the model that need to be revised. The results of the validation of the PBL model in probability theory course by six experts are presented in Table 1. Based on the expert's assessment the developed model was declared valid and practical to be implemented in probability theory course.

Table 1. Validation results of the PBL model in probability theory course by experts

Model components assessed	Validator assessment results						Mean
	1	2	3	4	5	6	
Supporting theory	24	4.90	25	25	25	25	4.90
Syntax	5	4.83	5	5	5	4	4.83
Social system	29	4.81	30	30	30	29	4.81
Principles of reaction	12	4.56	14	15	15	14	4.56
Support system	16	4.63	19	20	20	19	4.63
Instruction and nurturant effect	14	4.72	15	15	15	14	4.72
Implementation guidelines	15	4.83	15	15	15	15	4.83
Total	115	106	123	125	125	120	
Mean	119						
Category	Very good						

2.2.3. Model implementation

Implementing the model in probability theory course is guided by a mathematics education lecturer with experience teaching probability theory courses. In this study, two lecturers became instructors in probability theory course. Because this study involved students from three different classes, one of the lecturers became an instructor in two different classes. So that the lecturers have the same perception, the researchers conducted informal discussions with them to provide technical explanations regarding the implementation of the model in class. Researchers have also prepared supporting documents for model implementation, such as lecture material summaries, learning scenarios, student worksheets, pretest, and posttest. Lecturers are allowed to study these documents. Researchers also provide opportunities for lecturers to ask questions that are considered unclear from these documents.

Due to the pandemic, model implementation is done online using the Zoom meeting platform. Researchers use the breakout rooms facility on the Zoom application for group discussion activities. Each group discusses in their respective rooms, and the lecturer is assisted by researchers and observers who can monitor the discussion activities of each group in turn. The implementation of the PBL model for probability theory course was carried out in six meetings with different topics, they are combinatorics; probability, probability properties, conditional probability, total probability and Bayes' theorem; random variables and discrete random variables distribution; random variables and continuous random variables distribution; as well as the expected value and its properties. However, before implementation, each class was given a pretest and after six implementations, all classes were given a posttest. During implementation, three observers assisted the researcher in monitoring the learning activities. All learning activities for six meetings are recorded live via Zoom meeting.

2.3. Data collection

Data collection in this study was using observation, questionnaires, and tests. Observations were done to monitor the implementation of the learning stages and activities carried out by instructors and students during lectures. Observations were carried out by researchers and assisted by three observers, namely graduates of the mathematics education master's degree. The researcher trained the three observers to make observations and record important and unique points that occurred during the course. Field notes from all observers will be confirmed with the results of lecture recordings so that comprehensive descriptive data is obtained.

In this study, a questionnaire was used to capture the perceptions of user lecturers and students regarding the practicality of implementing the PBL model in probability theory course. This study used a 5-point Likert scale: 1=very poor, 2=poor, 3=acceptable, 4=good, and 5=very good. The 10 indicators are measured through the lecturer's questionnaire: i) Ease of implementing learning syntax; ii) Smooth running implementation of learning; iii) The accuracy of time allocation; iv) Achievement of lecture objectives; v) Student interest in attending lectures; vi) Student motivation to learn; vii) Student activity in asking questions; viii) Student activity in group discussions; ix) Student activity in completing worksheet; and x) Students' self-regulation in constructing knowledge. For student questionnaires, the indicators used to measure the practicality of model implementation include: i) Enthusiasm for learning; ii) Ease of participating in learning activities; iii) Ease of understanding concepts/materials; iv) The usefulness of each learning step; v) Development of problem-solving skills; and vi) Development of critical thinking skills. Before being used, both questionnaires had been validated by four experts (one mathematician, two mathematics education experts, and one mathematics education assessment expert). Feedback from experts is used to revise the questionnaire before it is used in data collection.

The test evaluates the effectiveness of PBL implementation in probability theory course. In this study, the test consisted of a pretest and a posttest. Pretest and posttest measure the same indicators related to students' problem-solving and critical thinking skills. The pretest and posttest each consist of 9-item descriptions. Four items measure problem-solving skills, and five measure critical thinking skills. The pretest and posttest instrument grids for measuring problem-solving skills are presented in Table 2, and critical thinking is presented in Table 3. Before being used, the pretest and posttest instruments were validated by four experts (one mathematician, two mathematics education experts, and one mathematics education assessment expert). Feedback from experts is used to improve the test before it is used in data collection.

Table 2. The blueprint test of problem-solving skills

Learning content	Indicator
Probability and its properties	Given problems in everyday contexts related to probability properties, students can solve them correctly
Conditional probability	Given problems in everyday contexts related to conditional probability properties, students can solve them correctly
Continuous random variables	Given a function for the domain of a continuous random variable, students can correctly prove that the function is a probability density function
Expected value	Given problems in everyday contexts related to expected values (calculating the mean and variance), students can solve them correctly

Table 3. The blueprint test of critical thinking skills

Learning content	Indicator
Combinatorics	Ask alternative questions
Probability and its properties	Planning an action or experiment
Discrete random variables	Formulate hypotheses and inferences
Continuous random variables	Evaluate the truth of an information
Expected value	Decide what to believe or do

2.4. Data analysis

Data analysis to describe the steps of PBL implementation in probability theory course was carried out retrospectively with reference to the phases in design research [47]. In the retrospective analysis, the researcher used the learning syntax of the model as a guide for the analysis. First, the analysis is focused on examining whether implementing the PBL model in probability theory course is in accordance with the learning design. This analysis utilizes the results of the recording of each meeting. Second, the results of field observations are compared with each learning activity's syntax and objectives. This analysis is carried out to determine how effective the activities are and how students gain an understanding of the concepts being studied. Furthermore, the actual learning process is compared with the predicted learning process in the developed model. The results of the interaction analysis between learning design and empirical observations serve as a basis for developing instructional theory. After retrospective analysis, the initial learning design was adjusted to get a new learning design.

This study conducted a descriptive analysis to evaluate the practicality and effectiveness of PBL implementation in probability theory course. Descriptive statistics are used to analyze quantitative data from lecturer and student questionnaires. The results of this data analysis are used to describe the level of practicality of PBL implementation in probability theory course. In this study, the implementation of PBL in probability theory course was declared practical if the results of the lecturer's assessment were in the minimum good category ($M \geq 98.58$) and the results of student assessments were in the minimal good category ($M \geq 50$). Descriptive statistics are also used to analyze pretest and posttest data. After obtaining the data description (such as M and SD) of students' problem-solving and critical thinking skills, researchers will analyze the increase in the average score of problem-solving skills and critical thinking after students attend the course. In this study, the implementation of PBL in probability theory course was declared effective if there was an increase in students' problem solving and critical thinking skills.

3. RESULTS AND DISCUSSION

This study focuses on describing how implementing the PBL model improves problem-solving and critical thinking skills in the probability theory course and evaluates the practicality and effectiveness of the model. First, authors describe how best practice PBL implementation is in probability theory course based on the model syntax that has been designed. In this study, lectures were conducted online using Zoom platform. Next, authors focus on presenting the practicality and effectiveness assessment results from implementing the PBL model in probability theory course.

3.1. PBL implementation in probability theory course

In this study, the implementation of PBL in probability theory course was carried out through five steps: i) Orienting students to problems; ii) Organizing students to study; iii) Assisting individual and group investigations; iv) Developing and presenting works or solutions; and v) Analyze and evaluate the problem-solving process. In general, each stage in the probability theory course with PBL model is designed to facilitate students constructing their knowledge while simultaneously stimulating their problem-solving and critical thinking skills improvement. An explanation regarding the steps for implementing PBL in probability theory course is presented:

3.1.1. Step 1: Orient students to the problem

At this stage, the lecturer gives an apperception by conducting questions session to students to explore and recall the concepts students have learned regarding the material to be studied later. The lecturer will also convey the goals to be achieved in this course. At this stage, the lecturer will also briefly explain the planned lecture activities to be carried out. If there are things that need to be discussed related to the objectives and plans for lecture activities, the lecturer allows students to express their opinions. Furthermore, the lecturer invites students to look at the problems presented on the worksheet. In this activity, students are expected to be motivated and challenged to find solutions to these problems. An example of a problem in the worksheet is presented in Figure 2.

> Problem-1 (Observing Electronic Components)
>
> An experiment observing three electronic components was done to determine whether the electronic components still function properly or do not function properly.
>
> Draw a function that states many fail electronic components in the experiment above with an arrow diagram. Name the function x.

Figure 2. An example of a problem given in worksheet

3.1.2. Step 2: Organize students for study

Learning activities at this stage started by creating small groups of 3-4 students. After the group is formed, the lecturer conveys the learning scenario that will be carried out at this meeting. An example of a learning scenario delivered by the lecturer is presented in Figure 3.

Learning Scenario

1. The learning started with problems.
2. The problems in students' worksheet are discussed and done within groups.
3. The representative presents the result, other groups are invited to give responses or asking questions.
4. In the end of learning, it will be summary, homework, and a follow-up action.

Figure 3. Learning scenario

Next, the lecturer explains the activities that students will carry out during group discussions as shown in Figure 4. Because lectures are conducted online through the Zoom platform, group discussions are conducted through breakout room feature.

Enter your Group.

1. Please click "breakout rooms".
2. Go to your group.
3. Discuss and solve the problem on your worksheet, by:
   a. Directly write on the available paper, or
   b. Write on the paper, scan, and then paste into the available space.
4. The discussion will run in 30 minutes.
5. The representative presents the result.

Figure 4. The instruction for group discussion

After joining their respective groups, each student is allowed to re-examine the problems found in the worksheet. In this activity, students are allowed to ask questions, either to the lecturer or fellow group members, if there is information that has not been understood from the given problems. In this case, it needs to be underlined that the questions posed by students are not related to solutions to problems but focus on things that will be used as initial information for students to find solutions to problems. Thus, the lecturer will only respond to student questions regarding what matters are known and ask about the problems, while the lecturer does not explicitly respond to questions that lead directly to the final solution of the problems. The following is an example of lecturer activity in responding to students' questions. One of the results of students' work on a problem asked of the lecturer is shown in Figure 5.

"*Is there any difficulties?*" (Lecture)

"*Can we write this problem (referring to Problem-1, Figure 2) like this, ma'am? (Referring to Figure 5)*" (Student 1)

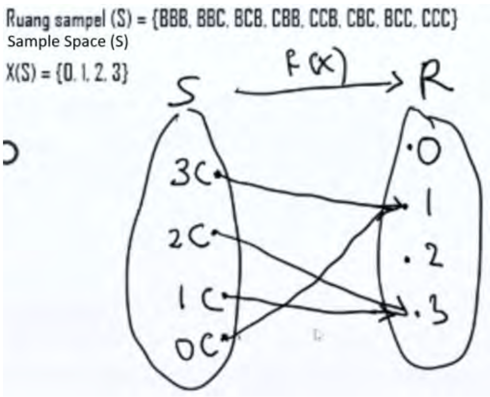

Figure 5. The result from the students' initial discussion

"*OK, you were already writing the S, that is good, very detailed. Now try to pay attention again to what is X defined as?*" (Lecturer)

"*X is the number of probabilities, ma'am.*" (Student 1)

"*Try reading on it again, in the problems presented above.* "*Draw a function that states many fail electronic components in the experiment above with an arrow diagram. Name the function X*". *So, x defined as?*" (Lecturer)

"*The function?*" (Student 1)

"*What kind of function?*" (Lecturer)

"*The number of electronic components that failed.*" (Student 1)

"*OK, the number of electronic components that failed. So, you have determined all the possible outcomes of the experiment, right? You have written down the sample space there. My question is, for the first sample point 'BBB', what is the value of x?*" (Lecturer)

"*The value of x for 'BBB' is 0, ma'am.*" (Student 1)

"*What is the value of x? (Confirming the answer of student 1)*" (Lecturer)

"*0 (Answered confidently)*" (Student 1)

"*Why (is the value of x equal to) 0?*" (Lecturer)

"*Because no one is failed, ma'am.*" (Student 1)

"*OK, so because there is no fail... in the arrow diagram, in the S set, you should write down the outcomes of S, BBB, then where do you map it to? Because the R here denotes the number of components which?*" (Lecturer)

"*Fail.*" (Student 1)

"*Then, BBB is mapped to which number? 0, 1, 2, or 3?*" (Lecturer)

"*Should be 0, ma'am.*" (Student 1)

"*OK, so it is understandable right?*" (Lecturer)

"*Yes, ma'am.*" (Student 1)

3.1.3. Step 3: Assist independent and group investigation

In this step, the lecturer instructed students to discuss in their respective groups. Group discussions focused on finding strategies or devising plans to solve problems in the worksheet. During the discussion, students were given the opportunity to gather various relevant learning resources in order to help find ideas to find solutions to problems. In this activity, the lecturer acted as a facilitator where when group experienced problems, the lecturer gave the group the opportunity to ask questions. Questions expected to arise from students were not questions related to the final solution to the problem, but questions that led students to find ideas that they would use to arrive at a solution to the problem. The following is an example of scaffolding that the lecturer has given to help students find ideas to solve problems which in this case the problems are presented in Figure 6. Meanwhile, Figure 7 shows an illustration that the lecture use to help students find ideas to solve these problems.

"*Ma'am, I want to ask. What does this problem mean? (Referring to Problem-2, Figure 6)*" (Student 1)

Problem-2 (The Probability of Motorcycle Racer to Win the Race)
Look at the following illustration.
One day, a motorcycle racer took part in a motorcycle racing competition in two rounds. The probability of him winning in the first round is 0.7. The probability of him winning in the second round is 0.6. The probability of him winning on both rounds is 0.5. Determine the probability that
a. He won at least one round
b. He won on one round
c. He did not win in both rounds

Figure 6. One of the problems in the worksheet that students need to solve through group discussions

"*In the second problem (referring to Figure 6), it is known that there are events A (he won in the first round) and B (he won in the second round), the probability of event A to occur and the probability of event B to occur are also known, as well as the probability of intersection of two events A and B is known. These probabilities can be determined by considering regions of Venn diagram so that it can help us in figuring out how to determine the probability that he won at least one round. He won at least one round means that he won in the first round only (A), he won in the second round only (B), or he won in both rounds (A and B). Does that mean it's OK if he wins in both rounds?*" (Lecturer)

"*Yes.*" (Student 1)

"*That is OK ma'am.*" (Student 2)

"*So, try to present it (the existing problem) in a Venn diagram so that it will be clearer which event is meant, including later the probability can be determined, that is all.*" (Lecturer)

"*Yes ma'am. Next, is the probability that he did not win (in both rounds) also be considered?*" (Student 1)

"*Does your question refer to the third question, "he did not win in both rounds"?*" (Lecturer)

"*Yes ma'am.*" (Student 1)

"*OK. If he did not win in both rounds it means he did not win in round A, did not win in round B, or did not win in rounds A and B.*" (Lecturer)

"*If so, does that count as a sample space?*" (Student 1)

"*Yes, that of course goes into the sample space as well. OK, now let us look at the following figure.*" (Lecturer)

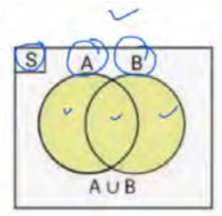

Figure 7. Illustrations that lecturers use to explain to students

"*Take a look at the figure presented (referring to Figure 7). There are the probability of event A to occur, the probability of event B to occur, the probability of at least one event to occur, and the probability of event A and B to occur represented by regions of the Venn diagram. Please attempt to associate them in terms of their probabilities.*" (Lecturer)

"*All right, ma'am, we will try it first.*" (Student 1)

At this step, the guidance that the lecturer provides can be directed at individuals or groups, depending on the discussion situation. If there are group members who seem passive, the lecturer needs to approach the students and ask what the obstacles are. In this regard, the lecturer needs to identify the obstacles and difficulties experienced by these students and help them to overcome existing obstacles and difficulties as soon as possible through the right stimulus. The same thing can also be done by lecturers for groups that experience problems and difficulties in the discussion process. The stimulus given by the lecturer can be in the form of trigger questions that lead to what the group should do to find a solution to the problem being discussed. However, the stimulus provided by the lecturer during the discussion process is dynamic, meaning that it adapts to the situations and conditions that occur during the discussion process. Through activities carried out at the step of assisting individual and group investigation, students are encouraged to develop their problem-solving skills, especially those related to the application of the concepts they have learned.

3.1.4. Step 4: Develop and present artifacts or solutions to the problems

The main activity at this step was presenting the results of the discussion of each group. In this regard, each group representative was asked to present the results of their group discussion. In this presentation, students not only reported the solutions to the problems obtained, but they also needed to report important information contained in the problem, the formulation of the problem to be solved, the chosen plan or strategy to obtain a solution to the problem, as well as the results of their work and conclusions obtained. At this step, after the representatives of one of the groups have finished presenting the results of their group discussions, the other groups were allowed to ask questions, provide feedback or objections to the results of the presentation. During the question-and-answer activity, each member of the presenter group could respond to questions or objections from other groups. When the presenter group had difficulty in responding to questions from other groups, the lecturer provided stimulus through trigger questions to direct the presenter group to be able to answer the questions. The activities carried out by students at the step can encourage them to develop their critical thinking skills. The following is an example of a presentation of the results of one group's work in solving Problem-1 on worksheet 3A as presented in Figure 8 and the provision of stimulus by the lecturer during class discussion.

Problem-1 (Give Away Door Prizes)
At a company's end-of-year event, door prizes are provided for employees who attend the event. The prizes provided are in the form of cellphones with three brands and two conditions as shown in the following table.

	Brand		
	A	B	C
ORI (Original)	8	7	6
KW (Counterfeit)	4	3	2
	12	10	8

If an employee receives a brand B cellphone, what is the probability that the cellphone received is an original cellphone? Explain your answer!

Figure 8. One of the problems that students need to solve in worksheet 3A

The solution to Problem-1 as shown in Figure 8 presented by group 4 which is presented in Figures 9-11. The worksheet also contains critical thinking activities, in which students are asked to find other ways to find solutions to problems. An example of the results of critical thinking activities presented by group 4 is presented in Figure 10. Figure 11 presents an example of the conclusions a group came to after a problem-solving activity.

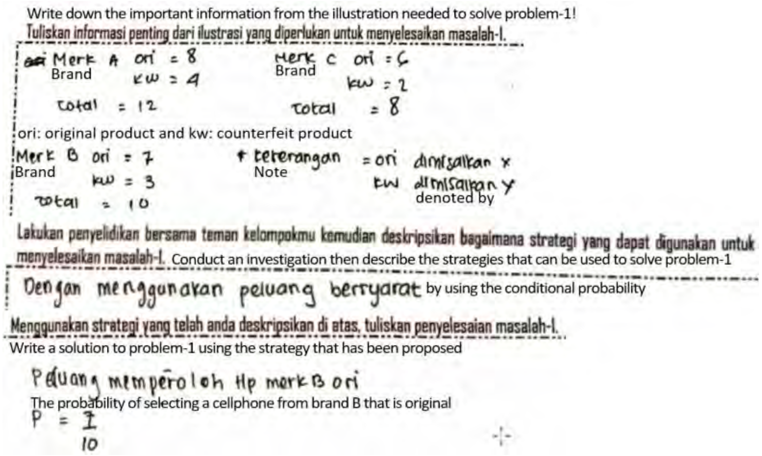

Figure 9. Example of group discussion results

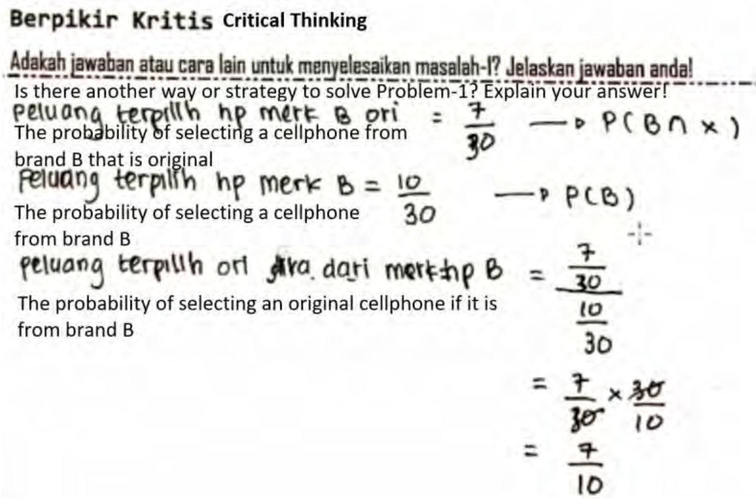

Figure 10. An example of a result of a critical thinking activity: Finding other ways to determine the solution to a problem

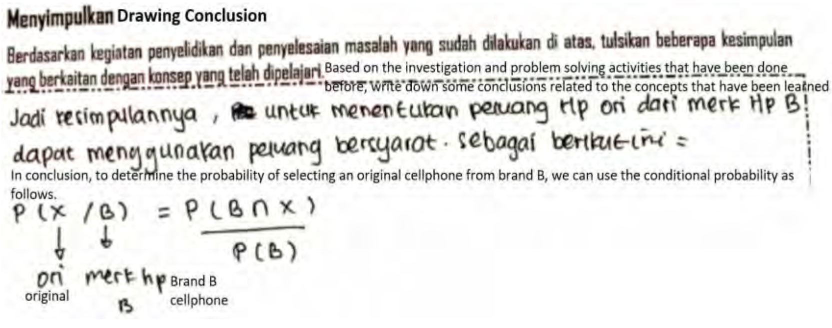

Figure 11. An example of the conclusion a group made after working on the activity in worksheet 3A

After group 4 finished their presentation, the lecturer invited other groups to provide feedback. The following is a transcript of feedback by a member of another group (group 5).

"*We welcome anyone who wants to comment (on what group 4 has presented).*" (Lecturer)

"*Actually, there is no difference in the end result, but to find a solution in the critical thinking part (looking for another way to solve the problem) we use a different way.*" (Student 1)

"*Oh... OK, that means we get the same answer, namely 7/10, but the method used is different. Where is the difference?*" (Lecturer)

"*During group discussions, for another method, we use the probability method with combinations, because there is only one event, so we use combinations like that, ma'am.*" (Student 1)

"*OK, present your group's work, please.*" (Lecturer)

"*OK ma'am.*" (Student 1)

While waiting for group 5 to present the results of their presentation, the lecturer provided feedback for group 4's presentation.

"*OK, starting from the supposition (referring to Figure 7), here the original is supposed to be x. This supposition can be more complete, i.e., suppose that X is the event that the original cellphone was selected and Y is the event that the counterfeit cellphone was selected. Like that. Please for group 4 (and all students), the supposition can be made even more detailed.*" (Lecturer)

Furthermore, group 5 presented their work as shown in Figure 12.

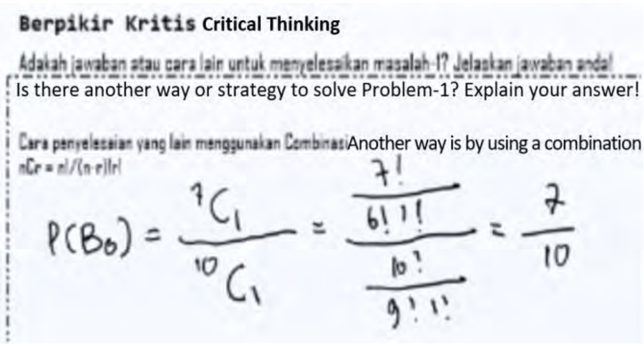

Figure 12. Another way proposed by group 5

The feedback that the lecturer gave to the method proposed by group 5 is:

"*OK. It says that the probability of choosing an original cellphone from brand B is represented by the combination of seven choose one. Which cellphone is chosen for this case?*" (Lecturer)

"*The original cellphone from brand B.*" (Student 1)

"*Then, what brand of cellphone is chosen which is indicated by the combination of ten choose one? The one chosen is brand B, isn't it? Is this actually the same as the previous activity? (Referring to previous problem solving).*" (Lecturer)

"*Hmm. That seems to be the same, ma'am.*" (Student 1)

"*All right. So, actually 7/10 can also be found in that way (referring to the method proposed by group 5). Do you get it?*" (Lecturer)

"*Yes, I get it, ma'am.*" (Student 1)

"*OK. Now I return to what was presented by group 4. What way or method did group 4 use to solve the existing problems?*" (Lecturer)

"*They used the method of conditional probability, ma'am.*" (Student 4)

"*That is right. They used the method of conditional probability and the result is the same, right?*" (Lecturer)

"*Yes ma'am.*" (Student 4)

3.1.5. Step 5: Analyze and evaluate problem solving processes

After the question-and-answer activity between the presenter group and other groups was over, the lecturer gave appreciation for what had been presented by the presenter group. Furthermore, the lecturer provides reinforcement and clarification to what the presenter group has explained and questions/responses from other groups. The reinforcement and clarification from the lecturer were used as material for improvement or correction if what the presenter group has explained was not optimal. If the explanation of the presenter group has been optimal, it could be used as the final solution to the problem being discussed.

Afterwards, the lecturer gave the opportunity for students to ask questions if there was material or things that they did not understand. If there are no more questions from students, the lecturer provides reinforcement of the results of solving the problems that have been carried out by each group. In providing reinforcement, the lecturer needs to focus on the main concepts that students should master after involving in probability theory learning facilitated by applying PBL model. Furthermore, lecturers and students conclude the material or concepts that have been studied. The process of drawing conclusions could also be used to check the understanding of some students, especially for students who seem less active throughout the learning process, of the material they just learned. Next, the lecturer provides follow-up in the form of giving homework. The homework aimed to strengthen students' understanding of the concepts they have learned. Lastly, the lecturer informed students regarding the topics to be studied at the next meeting. Overall, the activities carried out by students at the step of analyzing and evaluating the problem-solving process can train and develop students' critical thinking skills.

3.2. Practicality of PBL implementation in the probability theory course

After carrying out face-to-face learning in online mode for six meetings using the PBL model, two user lecturers gave their assessment to the PBL model developed in this study. It was found that, in general, they considered that the learning syntax of the learning model was easy (practical) to implement. The assessment results of the two user lecturers are presented in Table 4. The table reveals that one aspect that still needs to be optimized in the implementation of PBL in the probability theory course is the management of time allocation.

Table 4. The results of user lecturers' assessment on the practicality of PBL implementation in the probability theory course

Aspects assessed	Assessment results	
	Lecturer 1	Lecturer 2
Ease of implementing learning syntax	Very easy	Easy
Smoothness of learning process	Very good	Good
Accuracy of time allocation	Very good	Acceptable
Attainment of learning objectives	Very good	Good
Interest of students to be involved in learning	Very good	Very good
Motivation of students to learn	Very good	Good
Activeness of students to ask questions	Very good	Good
Student activeness during group discussion	Very good	Very good
Student activeness in working on worksheets	Very good	Very good
Student independence in constructing knowledge	Good	Good

In addition to the assessment by user lecturers, the practicality of implementing PBL in the probability theory course was also carried out by students. The results of the practicality assessment of PBL implementation by students are presented in Table 5. Table 5 shows that in general students gave a positive response to the implementation of PBL in the probability theory course. The average results of the practicality assessment of PBL implementation by students are in the practical category. This indicates that students are able to follow each step of learning properly. In addition, students gave the highest rating on the indicator of ease of participating in learning activities. This indicates that the learning steps of PBL in the probability theory are easy for students to follow. However, students gave the lowest rating on the indicator of ease of understanding the material or concepts being studied. This is understandable because students are not too familiar with learning procedures that require them to be actively involved in constructing their knowledge. In terms of the potential of PBL to support the development of students' problem solving and critical thinking skills in the probability theory course, students gave positive responses to these two aspects. This shows that students believe that the implementation of PBL in the probability theory course can foster their problem-solving and critical thinking skills.

Table 5. The results of practicality assessment of PBL implementation by students (*n*=58)

Assessment indicators	Mean	Min.	Max.	Indicator attainment *
Enthusiasm in learning	12.90	6	18	71.67%
Ease of following learning activities	7.48	3	9	83.12%
Ease of understanding concepts/learning materials	3.79	2	6	63.17%
Usefulness of each learning step	6.92	3	9	76.89%
Development of problem-solving skills	11.87	5	15	79.14%
Development of critical thinking skills	8.40	4	12	70.00%
Overall	51.79	23	69	75.06%
Interpretation	Practical			

Note. *: The attainment of each indicator is the percentage of the mean relative to the maximum score of each indicator

3.3. Effectiveness of PBL implementation in the probability theory course

The effectiveness of PBL implementation in the probability theory course can be examined from the increased score in problem-solving and critical thinking skills after the implementation of PBL. In this regard, we describe the development of students' problem-solving and critical thinking skills by comparing the conditions before and after they engaged in the probability theory course. Descriptions of students' problem solving and critical thinking skills are presented in Table 6.

Table 6. Descriptive statistics of students' problem-solving and critical thinking skills before and after PBL implementation (*n*=58)

	Problem solving					Critical thinking				
	M	*SD*	Min.	Max.	Ideal	*M*	*SD*	Min.	Max.	Ideal
Pretest	2.90	1.66	0	7	12	1.10	1.36	0	6	18
Posttest	5.74	2.52	1	11	12	4.10	2.18	0	10	18
Gain	+2.84					+3.00				

Table 6 demonstrates an increase in the average score of students' problem-solving and critical thinking skills after implementing PBL in the probability theory course. Even though the average posttest score is below the ideal maximum score, an increase in score from pretest to posttest indicates a positive impact on the development of students' problem-solving skills and critical thinking skills from the implementation of PBL in the probability theory course. Thus, the implementation of PBL in the probability theory course is quite effective in improving students' problem-solving and critical thinking skills.

3.4. Discussion

This study focuses on describing the implementation of PBL in the probability theory courses and evaluating its practicality and effectiveness in terms of promoting problem-solving and critical thinking skills. Our study reveals that PBL can be implemented well in the probability theory course. Similar to the findings of previous studies [31], [41], [45], [48]–[50], in the probability theory lecture setting, the strength of PBL is that it provides opportunities for students to construct knowledge related to the concepts being studied. Knowledge that is constructed directly by students through problem solving procedures indirectly encourages them to think critically [25], [45]. In addition, the knowledge that is constructed through student involvement makes learning more meaningful for them [42], [48], [50]. The meaningfulness of this learning contributes to increasing students' conceptual understanding of the topics studied.

One of the keys to the successful implementation of PBL in opportunity theory lectures is the provision of scaffolding. Scaffolding is needed when students experience obstacles in figuring out ideas to solve problems. Several studies [32], [33], [51] have confirmed that scaffolding is required when implementing PBL. The focus of scaffolding is to provide a stimulus – for example through trigger questions and providing feedback – for students to generate ideas to solve problems, not get solutions to problems directly. Scaffolding is situational, meaning that it is given only when there are groups which show no progress during the discussion process. On the other hand, when there are other groups that are able to independently generate ideas to solve problems, scaffolding is not needed.

Although previous studies have succeeded in developing and examining PBL-based instructional designs [34], [37], [43], [48]–[50], [52], [53], the focus of these studies is not on the probability theory course. In addition, although this study also develops a PBL-based instructional design, the novelty of the findings of this study lies in the target of utilizing the results of the development, in which the resulting instructional design is specifically designed to be implemented in the probability theory course. Best practices related to PBL implementation in this study can be used as a reference for educators and researchers to design varied and meaningful instructional designs for students in higher education. The findings of this study can also be used as a guide for educators to improve students' problem-solving and critical thinking skills.

One aspect that needs to be evaluated in the development of instructional designs is practicality [54], [55]. The findings of this study reveal that even though the implementation of PBL is considered as a new thing in the probability theory course, educators and lecturers claim to be able to follow learning activities easily. This indicates that PBL is practically applied to the probability theory course. The findings of this study thus corroborate previous studies which show that PBL can be implemented well in various disciplines and levels of education [34], [37], [48]–[50]. The fulfillment of the practical aspects of the PBL model in the probability theory course demonstrates several things. First, learning steps can be easily understood and implemented properly by lecturers and students. Second, the support system in the form of worksheets is useful for facilitating student learning activities and can be used easily. Third, the instructional design developed is relevant to the course objectives. Fourth, students were enthusiastic and actively involved in the learning process showed by actively asking questions, discussing, working on worksheets, and even seeking to construct knowledge independently.

This study also revealed that there was an increase in students' problem-solving and critical thinking skills after being actively involved in probability theory lectures that apply PBL. Even though there has been an increase, it did not satisfy our expectations. This result thus fall short of our predictions and inconsistent with the findings of previous studies which have succeeded in uncovering the effectiveness of PBL [35], [38], [48]–[50]. However, this finding is consistent with the results of other studies reporting that PBL is not sufficiently effective [56]–[58]. Several things are considered as possible causes that hinder the optimal improvement of problem-solving and critical thinking skills in the probability theory course applying PBL. First, students were not used to PBL. Jailani *et al*. [48] reported that one of the challenges in implementing PBL is related to learning habits, where students who were accustomed to traditional learning tended to need time to adapt to PBL. Dolmans *et al*. [57] suggest that when students are not used to PBL, it causes the group discussion cannot work as it should. Given that group discussion is a crucial aspect in PBL, when it cannot work properly, learning activities also cannot take place optimally. Thus, in order for PBL to be implemented optimally, students' habituation with PBL is needed.

The second thing is related to time allocation constraints, where students sometimes spent too much time during group discussions. Constraints related to time allocation are one of the main obstacles in implementing PBL [48], [56]. Thus, a sufficiently long duration of time is required so that all steps of learning can be carried out. When one learning step takes too long, it can cause disruption to the next learning step. It can even cause some of the learning activities that have been designed to be not carried out. However, PBL is also not recommended to be carried out for too long because this can reduce students' positive attitudes towards learning [59].

Third, students' low mastery of the prerequisites to probability theory course such as differential and integral calculus is thought to prevent them from achieving an optimal improvement in problem-solving and critical thinking skills. In this study, it turned out that at one university the Integral Calculus course was offered in the same semester as the probability theory course. Students should ideally have taken and passed the integral calculus course before enrolling in the probability theory course. Hung [60] argues that mastery of prerequisites is crucial in PBL. Prerequisites can be used to develop new knowledge related to the interpretation and representation of problems [61]. Thus, to optimize the implementation of PBL in the probability theory course, students need to be equipped with adequate mastery of the prerequisites to probability theory course as well.

Fourth, the field trial of the implementation of the PBL model conducted online through the Zoom platform contributed to the not optimal learning process in the probability theory course. Even though all steps of learning could be carried out well, when conducting group discussions students needed quite a long time to align perceptions and obtain solutions to the problems. Abdelkarim *et al*. [62] reported that one of the weaknesses of PBL when implemented online is that students become more individualistic. This also happened in this study, where when group discussions were online, students with high academic abilities dominated, while other students tended to be passive without attempting to contribute ideas or thoughts in solving problems. This results in a gap in understanding between students with high and low abilities.

3.4.1. Study limitations and implications

This study has some limitations. First, because the field trial was carried out in the midst of the COVID-19 pandemic, the field trial was carried out online. Even though efforts have been made to use a blended learning system in one of the universities, the number of students who were willing to take face-to-face in the classroom is very small – less than five students. Although the implementation of PBL in online mode could still be carried out, some learning activities have become less than optimal, especially in terms of monitoring student learning progress. Second, because the trial process was carried out online, there were several technical problems experienced by students, such as internet network problems and computer or laptop devices that experienced technical problems. Internet network constraints experienced by students were generally caused by weather factors that cannot be predicted and anticipated by us. The technical constraints that arose had an impact on the field trials which were not optimal, especially in terms of time allocation.

The limitations encountered in this study indicate that PBL implementation in online mode is challenging. Further studies are still needed to investigate and find the best solution to optimize PBL implementation when it is carried out in online mode. In addition, considering that the implementation of PBL in the probability theory course is something new, other studies are needed to investigate how to implement PBL in offline mode. Future studies can also compare the effectiveness of PBL in online and offline mode. Although this study was successful in providing initial indications that the implementation of PBL can improve students' problem-solving and critical thinking skills in the probability theory course, the effectiveness of the instructional design developed has not been tested in (quasi) experimental design. Thus, we encourage researchers to conduct (quasi) experiments to investigate the effectiveness of PBL on students' problem-solving and critical thinking skills, especially in the probability theory course. Future experiments are also expected to consider other aspects such as gender differences, learning modes (online vs. offline), and the level of student interest in learning. It is hoped that the results of these studies will further strengthen the findings of this study so that PBL is truly tested as a solution to improve student learning outcomes in the Probability theory course.

4. CONCLUSION

The implementation of PBL in the probability theory can be done through several steps of learning: i) Familiarize students with problems, organize students to solve problems; ii) Guide individual and group investigations; iii) Develop and present the results of problem solving; and iv) Analyze and evaluate problem solving procedures and solutions. PBL implementation encourages students to be actively involved in constructing knowledge and concepts on topics studied in the probability theory course. Lecturer skills in providing scaffolding in the problem-solving process are an important key to successful PBL implementation in the probability theory course. Apart from training problem-solving and critical thinking skills, the implementation of PBL also encourages students to develop their collaboration and communication skills. Both lecturers and students gave positive responses to the implementation of PBL in the probability theory course. They feel that the designed learning steps are easy for lecturers to implement and for students to follow. The findings of this study also reveal that the implementation of PBL can improve students' problem-solving and critical thinking skills in the probability theory course. However, the implementation of PBL in online mode is very challenging and requires optimal preparation in terms of supporting devices and learning contents. Based on best practices and the findings of this study, we recommend PBL to be adapted as a variation of learning model to facilitate the probability theory course.

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