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FIFTH-GRADE STUDENTS' ARGUMENTATION STRUCTURES IN THE POOL OF GEOMETRIC SHAPES[1]

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ABSTRACT

In this study, it was aimed to examine the argumentation structures of 5th-grade middle school students, while they were structuring geometric concepts in argumentation-based activities of geometric shapes. To this end, 7 fifth-grade students studying in a public middle school were included in the study. In the study, five geometry tasks involving activities based on materials of geometric shapes were carried out. The geometry tasks were structured based on argumentation, and the argumentation structures of students in the processes of structuring geometric concepts were analyzed. In the study, the argumentation structures of source-structure, spiral-structure, and reservoir-structure emerged from the student discussions and communications. According to these results, teaching activities structured based on geometric shapes and materials were observed to be effective in revealing students' different argumentation structures.
Keywords: argumentation structures, quadrangles, middle school students.

GEOMETRİK ŞEKİL DENİZİNDE ÖĞRENCİLERİN ARGÜMANTASYON YAPILARI

ÖZ

Bu çalışmada, ortaokul 5. sınıf öğrencilerinin argümantasyon temelli geometrik şekil etkinliklerinde geometri kavramlarını yapılandırırken ortaya çıkan argümantasyon yapılarını incelemek hedeflenmiştir. Bu amaçla, bir devlet ortaokulunda öğrenim görmekte olan 7 beşinci sınıf öğrencisi çalışmaya dahil edilmiştir. Çalışmada, geometrik şekil oluşturma materyallerine dayalı etkinlikleri kapsayan beş geometri görevi gerçekleştirilmiştir. Geometri görevleri argümantasyon temelli olarak yapılandırılmış ve öğrencilerin geometri kavramlarını yapılandırma süreçlerinde argümantasyon yapıları ortaya konulmuştur. Çalışmada, öğrencilerin karşılıklı konuşma ve tartışmaları sistematik olarak incelenmiş ve bu inceleme sonucunda kaynak-yapı, spiral-yapı ve rezervuar-yapı argümantasyon yapıları ortaya çıkmıştır. Bu sonuçlara göre, geometrik şekil oluşturma materyallerine dayalı olarak yapılandırılan öğretim etkinliklerinin öğrencilerin farklı argümantasyon yapılarını ortaya çıkarmada etkili olduğu görülmüştür.
Anahtar kelimeler: argümantasyon yapıları, dörtgenler, ortaokul öğrencileri.

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INTRODUCTION

With the discovery of the importance of social learning environments in mathematics education, the orientation toward mathematical reasoning and communication activities has increased in teaching environments. Since the mathematics teaching environment is a social context, communicative elements appear in the teaching process (Lerman, 2000). In mathematics teaching, students share their opinions in written and verbal forms and structure an in-depth understanding process by evaluating the shared opinions with their own reflections (Lampert & Cobb, 2003). In this process based on argumentation, mathematical inquiries are realized in the form of reasoning, supporting, or refuting each other's opinions in various ways (Hunter & Anthony, 2011). In these inquiry processes, students challenge each other's opinions, make mathematical explanations to defend their mathematical views, and if someone else questions their opinions, they strengthen their claims with mathematical data. Accordingly, mathematical discourse, reasoning, and arguments appear as parts of mathematics teaching (e.g., National Council of Teachers of Mathematics [NCTM], 2000, 2014).

Argumentation is considered as a tool for students' participation in mathematical discourse when they "criticize their peers' reasoning using examples and counter-examples to refute arguments" (NCTM, 2014, p. 35). The purpose of argumentation is defined as statements comprised of rhetorical tools to convince individuals of the accuracy or inaccuracy of a statement (Antonini & Martignone, 2011). Arguments are the last statements structured by participants in the argumentation process and eventually accepted or refuted by all (Krummheuer, 1995). From these definitions, argumentation is a logically linked mathematical discourse process (Mason, 1996; Vincent, 2002), and arguments are the end product of argumentation. During the argumentation process, students express their opinions by directly participating in social learning, discuss and defend them by presenting evidence to convince others (Stein et al., 2008). In the mathematics teaching process, students' evaluation of each other's opinions, questioning of opinions, and structuring of mathematical concepts in direct interaction with each other are also quite effective in terms of conceptual learning (Yackel & Cobb, 1996). Many researchers have found the contribution of the argumentation process to students' mathematical knowledge (Kosko et al., 2014; Walter & Barros, 2011; Yackel & Cobb, 1996) and skills (Driver et al., 2000; Heinze & Reiss, 2007). In this regard, the argumentation process is extremely effective in students' learning processes and contributes positively to their high-level thinking skills.

This study aimed to examine the argumentation structures of fifth-grade middle school students while forming geometric concepts with geometric shapes materials. To this end, the study aimed to (i) carry out argumentation-based geometric shapes activities with fifth-grade middle school students and (ii) examine the argumentation structures that emerged from the students' discussions while they were structuring geometric concepts. For this purpose, argumentation activities were carried out and presented in this study. By examining the effectiveness of the argumentation activities in revealing the argumentation structures of the students, it was ensured that exemplary argumentation activities that middle school mathematics teachers could use in teaching geometry were developed. Thus, activities that will make middle school geometry teaching effective and support students' argumentation process will be created and presented to the middle school mathematics teachers.

Toulmin's Argumentation Model

Argumentation is a form of mathematical discourse (Mason, 1996). Students' production of arguments, backing of each other's arguments or their efforts to refute each other's arguments using counter-examples, and their criticism of their peers' reasoning are indicators of student participation in mathematical discourse (NCTM, 2014). Toulmin's argumentation model defines such an argumentation process (Toulmin, 1958). In this model, Toulmin (1958) associated the components/elements of an argument - claim, data, and warrant - with each other. Moreover, the model has three auxiliary components: qualifier, backing, and rebuttal. Although these auxiliary elements are not the main components of the argumentation process, they can be present in arguments (Rumsey, 2012). *The claim/conclusion* [C] is the main component that each argument should have. The statement in which the argument provides a warrant is the claim. The statement that forms the basis of the claim is *data* [D]. To back the claim or conclusion of any argument, there must be some facts, information, or other statements referring to the data (Yackel, 2002). There should be data in the argument since a claim without data remains unbacked, and therefore there is no argument (Toulmin, 1958). The statements presented to enable applicability are *warrants* [W]. Backing [B] is another factual statement that justifies the warrant (Hitchcock & Verheij, 2005). Moreover, *qualifiers* [Q] are required to express the degree of reliability. Finally, exceptional cases where the claim is invalid, if any, can be added to the argument as *rebuttal* [R]. Toulmin (1958) reveals the relationship between these components in a specific order or with a scheme in Figure 1.

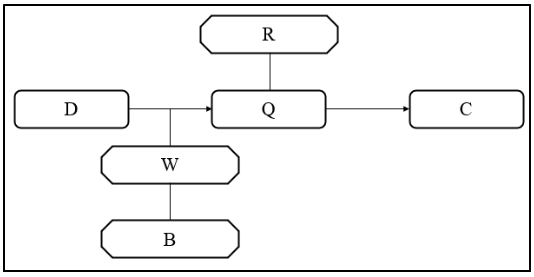
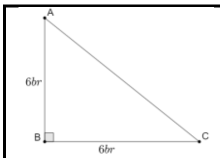


Figure 1. Toulmin's Argumentation Steps

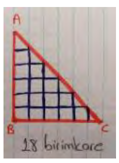
In a sample argumentation activity, the teacher asked the students, "*How can you find the area of this triangle?*" Then, the students were asked to make claims, justify these claims, and present their data on what they base their claims. In the classroom discussion process, the students have the aim of convincing each other and the teacher. For this reason, an argumentation process is carried out by evaluating each other's claims and justifications (see Figure 2).



$|AB| = |BC| = 6br$

$A(ABC) = ?$

Student 1: First I drew the triangle on the squared notebook, then I found the area of the triangle by counting the squares in the triangle using my notebook.



Student 2: Using my knowledge, I completed the triangle into a square and calculated the area of the square.

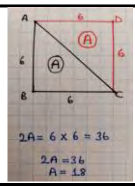


Figure 2. Sample Argumentation Process

During the argumentation carried out, different argumentation structures emerge as students make claims, and their justifications and rebuttals are put forward. In Figure 3, the argumentation process that emerged in the sample activity is schematized.

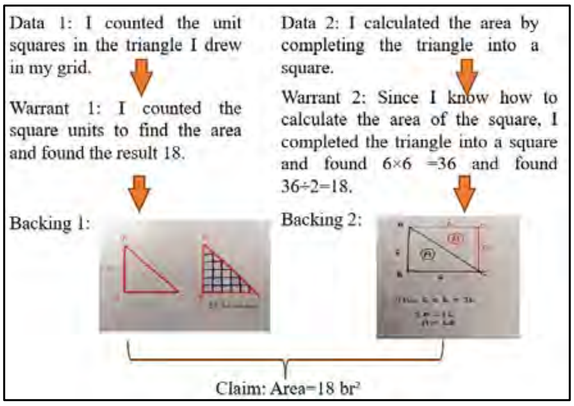


Figure 3. Sample Argumentation Process

Argumentation Structures

The components in Toulmin's argumentation model are described as argumentation steps or *local arguments* (Knipping & Reid, 2010). Many proofs include sub-proofs of a larger proof structure. Argumentation steps do not generally occur within linear chains; since the results of some steps are recycled as data for others, these steps are combined in argumentation streams (AS) (Knipping & Reid, 2010). Argumentation streams are interconnected in more complex ways and form the argumentation structure together. The argumentation process progresses from the fine structure in individual steps toward the structure of the entire argumentation. According to Reid and Knipping (2010), these argumentation streams do not usually progress linearly, and thus, argumentation structures become more complex and difficult to analyze. Based on this situation, Knipping (2008) proposed an analysis method (Global Argumentation Analysis) that would make it easier for researchers to analyze complex argument relations and recommended further studies to examine argumentation structures.

Four types of argumentation structures, which emerged in the proof processes in mathematics classes, were defined: source-structure, spiral structure, reservoir-structure, and gathering-structure (Knipping, 2008; Reid & Knipping, 2010). These argumentation structures are presented in Figure 4 and explained in detail.

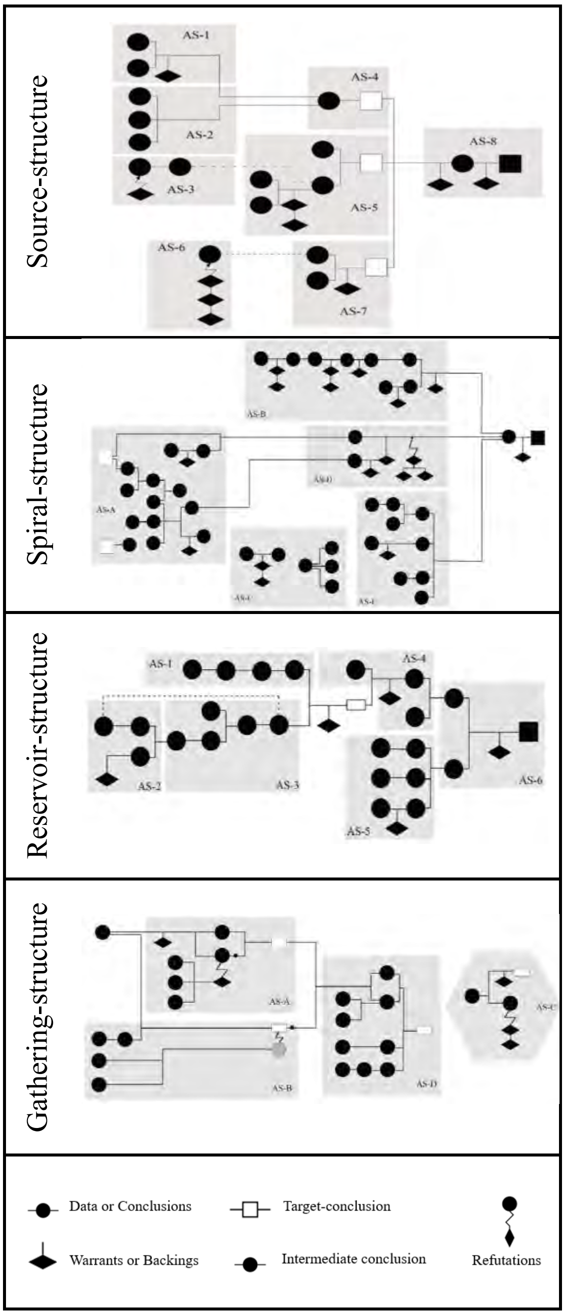


Figure 4. Argumentation Structures (Reid & Knipping, 2010)

In *source-structure*, different arguments from different data are presented. Reid and Knipping (2010) explained this structure with the metaphor stating that brooks originating from different streams merge to form rivers. The distinctive characteristics of the structure are as follows: the presence of parallel arguments defending the same claim, argumentation steps with multiple data, and the presence of refutations (Reid & Knipping, 2010).

In spiral-structure, parallel arguments reach a single main argument. In addition to parallel arguments, there are also arguments (AS-C) that are disconnected from the structure and do not impair the spiral structure in general. The major difference between spiral-structure and source-structure is that the claim/data is the conclusion in source-structure while the target is the conclusion in spiral-structure.

The *reservoir-structure* defined by Knipping (2008) includes intermediate claims. These intermediate claims form the ways of transition in reaching the main claim. The distinctive characteristic of this structure is that there is occasional backward and then forward continuity between the arguments. Making backward inferences allows this structure to involve more in-depth discussion. Thus, arguments are re-evaluated, and additional explanations are made.

Gathering-structure is the structure in which all data are not mentioned in the beginning and the data that emerge during discussions over time are included. This structure has no parallel arguments and backward/forward orientation.

METHOD

In this study, a case study design, one of the qualitative research methods, was used. The case study is a research approach that enables the in-depth and holistic examination of individuals, phenomena, and events (Fraenkel et al., 2012). In the present study, the case study design was used since it was aimed to examine the argumentation structures of a group of students in a holistic way as they engaged in geometry activities.

Participants

The study participants consisted of fifth-grade students attending a public middle school. Seven students were selected among the fifth-grade students and activities were carried out-of-classroom. Purposive sampling was used in this study. Since this study focused on the argumentation process, students who could express their opinions clearly and were active and willing to participate in the discussions were included in the study to carry out the process efficiently. The characteristics of the study participants are presented in Table 1.

Table 1. Characteristics of the Participants

Participant	Gender	Age	Grade
S1	Female	11	5
S2	Male	11	5
S3	Female	12	5
S4	Male	12	5
S5	Male	12	5
S6	Male	12	5
S7	Male	12	5

Procedure

The students were supported to structure their argumentation processes using geometric shape materials in their development of an understanding of geometric concepts. To this end, a geometric shapes set was used. The geometric shapes set helped students create their 2D shapes and thus activate their creativity. The parts in the set (Figure 5) and their properties are presented below.

Figure 5. Geometric Shapes Materials

Set Content:

- 16 orange connectors (10 holes)
- 18 red connectors (8 holes)
- 16 long purple sticks
- 24 turquoise medium sticks
- 20 small green sticks
- 20 blue curved sticks

The set consisted of 80 sticks and 34 connectors of different lengths. The students could create various 2D geometric concepts using these materials. Orange-colored connectors had holes with gaps of 45^0, and red connectors had holes with gaps of 60^0. Hence, the set content was suitable for forming shapes with different angle values. In the study, four sets were used for each student's use so that each student could individually structure geometric shapes.

Learning Objectives

It was aimed to help students to structure the argumentation process regarding the concepts of polygon and quadrangle using the geometric shapes set in this study. The content was formed according to the Ministry of National Education (MoNE) curriculum (MoNE, 2018). The topics of the activities are polygons and quadrilaterals. In this context, the concepts of the trapezoid, parallelogram, rectangle, and square were discussed. The topic episodes of the geometric concepts are given in Appendix 1.

In the topic episodes where the relevant concepts were discussed, the students were requested to create these concepts using geometric shapes materials. Afterward, the students were asked to think out loud about the properties of the structured shapes and discuss how they created the shapes. In this process, the students were encouraged to evaluate each other's opinions. The image of a teaching activity example from the study is presented in Photograph 1.

Photograph 1. Image of the Activity Setting

The researcher guided the students so that they could produce productive discourses and make inquiries during the implementation. To enable the students to justify their claims in their discourses, they were asked questions such as "Why did you say that? What makes you say that?" In the classroom discourses structured in this way, focus group interviews were held with seven students. Accordingly, the argumentation processes structured by the students were reached. Argumentation processes were conducted for about 20-40 minutes for each topic episode.

Data Analysis

In the study, a three-step process was followed in structuring students' arguments with geometric shapes materials. In the first step, the students' process of creating geometric concepts and reasoning was divided into sections. Thus, the general topics that emerged in classroom discourses were determined, and the order of geometric concepts was structured. Revealing the different sections of the process allowed making the analysis of arguments in these sections more accessible. After the stream and order of the topics were revealed, the construction and analysis of arguments started. After arguments were constructed, the argumentation structures of the participants were created using the argumentation structures proposed by Knipping (2008) and Reid and Knipping (2010). The analysis of the obtained data in line with the previously determined themes is defined as descriptive analysis (Merriam, 2009). For this purpose, a multi-stage process was followed in the descriptive analysis carried out in the study. For data analysis, first, the data in five geometry tasks were independently read and coded by the researchers. After the coding process done by the researchers separately, the researchers presented the codes they created to each other and discussed them. They mutually explained with which label and the reason the coding was made. As a result of the code evaluations made jointly by the researchers, a consensus was reached, and the data analysis was completed.

FINDINGS

In this section, the argumentation structures that emerged from the students' discussions during the geometry activities will be presented. The students' argumentation structures throughout the five tasks are summarized in Table 2.

Table 2. Distribution of Argumentation Structures in Geometry Tasks

	Argumentation structures
Task-1	2 Source-structure 1 Spiral-structure
Task-2	1 Source-structure 3 Spiral-structure
Task-3	2 Spiral-structure
Task-4	2 Source-structure 1 Spiral-structure
Task-5	1 Reservoir-structure 2 Source-structure

When the students' argumentation structures that emerged while structuring geometric concepts are examined, it is seen that the structures differed according to tasks. Findings related to each structure observed are presented below.

Source-Structure

In the study, the source-structure argumentation structures of students were observed in Task-1, Task-2, and Task-4. In this subsection, the activity process performed in Task-1 and Task-2 and the sample argumentation structure that emerged in Task-2 are presented.

Task-1: Trapezoid

Firstly, it was aimed to reveal students' preliminary knowledge about regular polygons. For this purpose, cards with different geometric shapes were distributed to the students (Figure 6) and they were asked which ones were polygons, regular polygons, and trapezoids:

> S2: When we were describing triangles, we used to say that the sides should be straight, so we cannot call the shapes curvature polygons.
> Researcher (R): So, what can we call the curved shapes?
> S4: They are also shapes, but we cannot say polygons, regular polygons.

Figure 6. Geometric Shapes Card

When the explanations of the students about the concept of polygons were examined, it was seen that they have the understanding that they should not be curved and that shapes containing this feature are not polygons. After revealing the students' prior knowledge of polygon concepts, the concept of a "regular polygon" was questioned:

> R: What properties must a shape have for it to be smooth?
> S4: Its sides must be equal.
> S1: Angles may need to be equal. But I am not sure.

R: Which of the shapes in the picture do you think are regular polygons?
S6: Regular quadrilaterals; trapezoidal, rectangular, square, etc. This is how we learn things.
R: You said the sides and angles must be equal though. Is this what you're talking about edges and equality?
S4: So, the sides and angles don't have to be equal.

The focus was on the concept of the trapezoid, which the students expressed after the inferences they made for a polygon to be regular. The question "What is a trapezoid?" was asked to the students. The students were asked to create trapezoids using the geometric shape materials given to them.

Task-2: Parallelogram

The spiral-structure that emerged in Task-2, in which the concept of parallelogram was structured, and its properties were addressed, is presented in Figure 7. And then the steps of the argumentation are detailed.

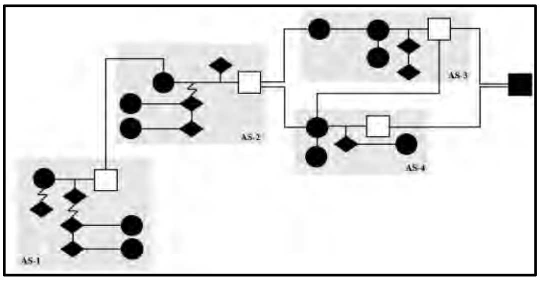
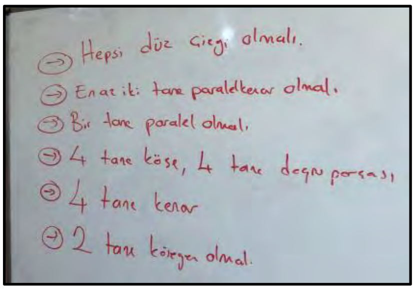


Figure 7. Source-Structure in Task-2

In Task-2, the concept of parallelogram was presented to the students. Students were asked to create properties related to parallelograms using materials (see Photograph 2). Questions were asked by the researcher about the shapes created by the students using the material:

- Why do you think the parallelogram is like this?
- Can you make parallelograms in different shapes? Why didn't you accept the other shapes as parallelograms?
- What properties must a shape have in order to be a "parallelogram"? How do you say these features?
- How can you convince your friends that a shape is a parallelogram?
- Does everyone agree? Anyone have a different opinion than your friend?
- Has anyone created a parallelogram using these sticks differently?
- Why do you think this is a parallelogram?



Photograph 2. Parallelogram Properties Listed

By asking the relevant questions, an argumentation process was structured. It was aimed to reveal the claim, data, warrant, qualifier, backing, or rebuttal elements of the students.

AS-1. Student comments falling under this argumentation type are presented below.

S4: [By forming a parallelogram (D) (Photograph 3)] The measurements of all angles of the parallelogram are equal (T-C) since their sides are also equal (D/T-C).
S2: [By showing the parallelogram in his hand] But the sides are not equal (W).
S6: Yes, only the opposite ones are equal (W). You did the shape wrong; you should not have used the same sticks (R).

Photograph 3. Example of Forming a Parallelogram

AS-2. Student comments falling under this argumentation are presented below.

S6: [By summing the dimensions of the angles at the connection points (D)] If this is the case, then the sum of the values of the internal angles exceeds 360^0 (W), so their angles should not be equal (R).

S2: [By adjusting the sum of the dimensions of the angles at the connection points to 360^0 (D)] All of them must be 90^0 for the dimensions of the angles to be equal (W).
S1: [By forming a square (D)] If their angles are equal, it becomes either a rectangle or a parallelogram (R).
S5: But the rectangle and square are also parallelograms (B).
S6: If all the angles of the parallelogram are equal (W), we will say, "This shape is a rectangle or a square" (D/T-C).

AS-3. Student comments falling under this argumentation type are presented below.
S2: [By connecting the adjacent angles at the tips in the parallelogram (D) (Photograph 4)] Because they are correct (W), the sum of these angles is 180^0 (T-C).
S4: Yes, it is like an angle on a straight line (D), they became supplementary angles (B).

Photograph 4. Example of Constructing a Straight Angle

AS-4. Student comments falling under this argumentation type are presented below.
S2: Then, these two angles (opposite angles) are also equal (T-C).
S7: [By forming a parallelogram and determining the degrees of the connection angles (D)] Yes, they become equal (B).
S7: Because if we use the parallels (D), they become equal (B).
S3: They should be equal because, by calculating the measurements of the adjacent angles (W), the sum of these two angles is 180^0 (D), then the other angle should be equal to it (B).

When the argumentation process was examined, four argument streams about the properties of the angles of the parallelogram were seen. When the structure is reviewed, it is observed that it is a funnel shape, and the claim/data and target claim are formed based on the data from different sources. Therefore, it is concluded that the argumentation structure of the students in this task is source-structure.

Spiral-Structure

Spiral-structure was observed in Task-1, Task-2, Task-3, and Task-4. The spiral-structure that emerged in Task-3, in which the concept of the rectangle was structured, and the area properties were discussed, is presented in Figure 8. Then, the argumentation steps are detailed.

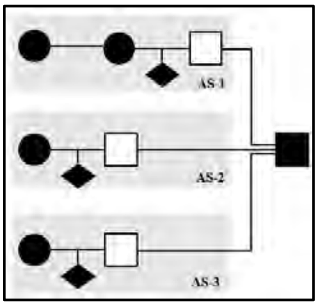
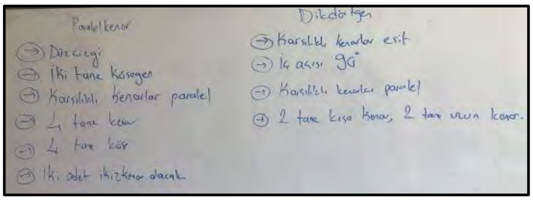


Figure 8. Spiral-Structure in Task-3

Task-3: Rectangle

In Task-3, the concept of the rectangle is discussed. Students were asked to describe the rectangle and explain its properties (Photograph 5). It was observed that the students formed their arguments by using the parallelogram, triangle, and trapezoid properties in the area calculation of the rectangle. The explanations of the students and the argumentation structures that emerged in this direction are presented below.



Photograph 5. Comparative List of Parallelogram and Rectangle Properties

AS-1. Student comments falling under this argumentation type are presented below.
S6: If I make the angle between these parts 90^0, it becomes a rectangle (W) (see Photograph 6). We calculate the area by base×height in a parallelogram (D). So, we can calculate the area in the same way in this (C).

Photograph 6. Example of Creating a Rectangle from a Parallelogram

AS-2. Student comments falling under this argumentation type are presented below.

S3: If I add a diagonal to this shape, it turns into two triangles (W) (see Photograph 7). We find the area of a triangle with $\frac{a.h_a}{2}$ (D) because there are two triangles in a rectangle; when we multiply this operation by 2, the area is found as $a.h_a$. a is the base, and h_a is the short side of the rectangle (C).

S7: Here, we can also create four triangles with diagonals and find their areas one by one (D).

Photograph 7. Finding the Area of a Rectangle by Creating a Triangle

AS-3. Student comments falling under this argumentation type are presented below.

S5: [By creating a trapezoid with materials (D) (see Photograph 8)] When I carry the part on the side, a rectangle is also formed with the trapezoid (W). The area of a trapezoid is found with $\frac{(\text{lowerbase+upper base}).\text{h}}{2}$ (D). When I moved the piece here, the long side of the rectangle became equal to the bottom of the trapezoid, and the short side became equal to the height. Therefore, $\frac{2.\text{long side.short side}}{2}$. In other words, the area of the rectangle is calculated with the 'short side×long side' (C).

The students asserted the same claim that the area of the rectangle could be calculated with the long side×short side and proved their claim in different ways. Accordingly, it was observed that parallel argument structures were structured for the same argument at the end of the process, and therefore the argumentation process had a spiral-structure.

Photograph 8. Example of Creating a Trapezoid

Reservoir-Structure

In this study, reservoir-structure only emerged in Task-5. The findings are presented below.

Task-5: Association

This task aimed to allow students to analyze and associate the properties of the geometric concepts discussed. For this purpose, the researcher asked the following questions to the students: "Can you explain the relationship between square-rectangle-parallelogram-trapezoid using the sticks you have?"

AS-1. Student comments falling under this argumentation type are presented below.

S5: [Creating a rectangle from a trapezoid using materials (D)] The rectangle is a trapezoid (C), because its opposite sides are equal (W).

S2: [Creating a rectangle from a trapezoid using materials (D)] Yes, a rectangle is a trapezoid (C) the sum of the interior angles of both is 360^0 (W).

AS-2. Student comments falling under this argumentation type are presented below.

S1: When you make the parallelogram like this [by making the angles of the parallelogram formed with the materials 900], it will be trapezoidal (D).

S3: If we bend it like this [by examining the square he created with the materials], it becomes a parallelogram (D).

S5: In the book, it says that a parallelogram is also a trapezoid (D).

During the argumentation process regarding the properties of the relevant concepts, S5 claimed, "The diagonal lengths of the parallelogram are equal." with respect to the properties of diagonals. The students could not be sure about the claim asserted by S5 regarding this property, and they had questions in their minds. In the continuation of the process, diagonal properties were discussed again in examining the rectangle and square, and the claim asserted by S5 was re-visited. After the related claim was handled, another discussion started. Accordingly, the resulting reservoir-structure is presented in Figure 9.

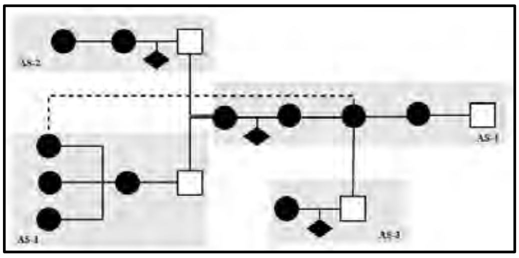


Figure 9. Reservoir-Structure in Task-5

During the process of Task-5, a retrospective study was carried out once, and discussions were held about the relevant data source. After these discussions, a prospective study was conducted, and the final conclusion was reached. Therefore, it was revealed that the process was of a reservoir-structure.

CONCLUSION AND DISCUSSION

In the study, it was aimed to examine the argumentation structures of fifth-grade students in the processes of structuring geometric concepts using materials. It was observed that the students used the properties of the concepts they created with geometric shapes materials in structuring their arguments and creating their data, claims, and warrants. Based on this, it was concluded that the geometric shapes materials used in this study were effective in various components of students' argument steps. In this way, the students could directly justify the geometric concepts they created via materials. Furthermore, it was seen that backing or rebuttal took place through geometric shapes created with materials. Especially rebuttals increase the awareness of the validity of arguments and enable students to identify errors in others' arguments (Solar & Deulofeu, 2016). Thus, students are given the opportunity to improve their understanding of mathematical concepts (Cervantes-Barraza et al., 2019). Hence, it is concluded that the activities of this study supported the students in the conceptual interpretation of geometric concepts and their properties.

It was observed that different argumentation structures emerged during the activities performed in the study. The resulting argumentation structures are source-structure, spiral-structure, and reservoir-structure. When the literature is reviewed, it is reported that structures are not superior to each other, but some structures are more complex (Erkek & Bostan, 2019). The emergence of complex argumentation structures is possible with the high-level thinking of students (Knipping, 2008). Thus, when the argumentation structures of students are reviewed in this study, it is concluded that high-level thinking skills emerged. As a result of examining the argumentation structures in this study, it can be claimed that the teaching activities created based on geometric shapes materials were effective in supporting students' argumentation processes.

This study has elucidated that the teaching activities based on geometric shapes materials were effective in scaffolding students' argumentation structures. However, elementary and middle school students are not adequately supported in proving, reasoning, and exploring mathematical relations throughout the teaching process in school mathematics and therefore do not have sufficient experience in high-level thinking (NCTM, 2000). Hence, it is important to provide students with effective reasoning tools. Therefore, it is suggested that similar materials can be used in teaching geometry to fifth-grade students.

This study shares geometry activities with middle school mathematics teachers to support their students' argumentation processes. The activities are appropriate for fifth-grade students. In the activities carried out based on argumentation, the emergence of different argumentation structures of the students was supported. For this reason, it is thought that the activities developed in this study can contribute to teaching geometry content in middle grades.

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Appendix-1

The Topic Episodes of the Geometric Concepts

	Topic	Objectives
Task-1	Trapezoid	• Creates a trapezoid, isosceles trapezoid, and perpendicular trapezoid using geometric shapes materials. • Defines the shape and properties of the trapezoid by justifying the conditions of its formation. • Asserts claims about the properties of the trapezoid. • Verifies/falsifies the claims.
Task-2	Parallelogram	• Creates parallelograms using materials. • Defines the shape and properties of the parallelogram by justifying the conditions of its formation. • Asserts claims about the properties of the parallelogram. • Verifies/falsifies the claims.
Task-3	Rectangle	• Creates rectangles using materials. • Defines the shape and properties of the rectangle by justifying the conditions of its formation. • Makes inferences regarding the definition of the rectangle based on the definition of the parallelogram.
Task-4	Square	• Creates squares using materials. • Defines the shape and properties of the square by justifying the conditions of its formation. • Makes inferences about the definition of the square based on the definition of the rectangle.
Task-5	Association	• Asserts claims about the properties of trapezoid, parallelogram, rectangle, and square. • Justifies these inferences using material. • Verifies/falsifies associations.