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Research Article

Investigating mathematical modeling competencies of primary school students: Reflections from a model eliciting activity

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Educators have long emphasized the importance of relating mathematics to everyday life situations. During the primary school years, mathematical modeling activities play an important role in this regard. Through a mathematical modeling activity prepared in accordance with the length and area measurement learning outcomes in the primary school 4th-grade mathematics curriculum, the purpose of this study was to determine students' mathematical modeling competencies. The study adopted a qualitative approach using a teaching experiment model. The study involved three students selected from 33 in a state primary school through criterion sampling. *Farmer Uncle Hüseyin* model eliciting activity (MEA) was given to the focus group to work on, and the whole process was videotaped. Each video recording was transcribed in detail. Transcriptions, student handouts, and researcher field notes were analyzed using the mathematical modeling competencies cycle developed by Blum and Kaiser and adapted by Maaß (2006). Modeling competencies of students were found to be at different levels. Based on the study, the students' warm-up activities, the context of the MEA, and their previous modeling experiences all contributed to the students' representation of different levels of modeling competency. Due to the frequent use of multiple-choice questions in their classrooms, students also had difficulty interpreting qualitative data and understanding the mathematical task they were given.

Keywords: Mathematical modeling; Model eliciting activities; Primary school; Teaching experiment

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1. Introduction

The educational approach of the last quarter of the 20th century focused on raising individuals who used and constructed knowledge, and as the information age has progressed, the idea of developing problem-solving skills has become increasingly popular. In the era we live in, our understanding of education has also changed, leading to some changes in the curriculum as well. Among these changes is the inclusion of the term, mathematical modeling, in the 2015 primary school Turkish mathematics curriculum (Ministry of National Education [MoNE], 2015b, p. 10). Additionally, the updated curriculum in 2015 and 2018 also incorporated mathematical modeling applications to teaching as "points to consider in implementing the program" (MoNE, 2015b, p. 15;

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2018b, p. 15). Among the main purposes of mathematics instruction is to prepare students to solve real-life problems and to associate mathematics with real-life fields such as engineering, architecture, sports, economics, medicine, and statistics.

Positive attitudes toward mathematics have an important impact on mathematics achievement (MoNE, 2018b, p. 15). The teaching of mathematics at schools with abstract concepts and memorized rules would negatively affect students' interest and performance in mathematics (Baki, 2014). During primary school, teachers should implement methods that promote students' positive attitudes toward succeeding in mathematics. As a result, students are able to master mathematics, enjoy it, and develop positive attitudes toward it. Positive attitudes towards mathematics will enable students to use it in their daily lives.

Various opportunities can be provided through model eliciting activities to enable students to use mathematics effectively in their daily lives, to associate mathematics with real life situations, to produce different solutions to problems they encounter, and to think analytically (Blum & Niss, 1991; English & Watters, 2004; Lesh & Doerr, 2003). Turkish mathematics curriculum includes the expression "*It is aimed at solving length problems involving addition and subtraction using models or modeling*" (MoNE, 2018b, p. 11) regarding modeling activities at primary school level, and modeling activities are discussed in a limited way for numbers and geometry learning areas. However, Model Eliciting Activities [MEAs] include different cognitive processes and relate mathematics to everyday life. Turkish mathematics curriculum emphasizes problem-solving, communication, reasoning, and associating as essential skills for primary school students. Mathematical modeling is included in the curriculum for the first time in 2015, but no explanation is provided regarding how these skills should be acquired and developed. Accordingly, the findings of this study can contribute to the field in terms of explaining how MEAs should be implemented.

Students' mathematical competencies and mathematical literacy are assessed as part of the Program for International Student Assessment [PISA] and the Trends in International Mathematics and Science Study [TIMSS], in addition to assessing reading comprehension skills and various science skills. In a report released by Organization for Economic Co-operation and Development [OECD] (2019), the role of mathematical literacy was emphasized. Recent PISA and TIMSS assessment projects have considered mathematics applications based on reading comprehension, especially mathematical modeling applications. The performance of Turkish students was below the OECD averages (MoNE, 2011, 2015a, 2018[Y1] [Y2] a; OECD, 2019). Using mathematics effectively is not only important for PISA and TIMSS tests, but also for everyday life. As a contribution to the field, having students work with mathematical modeling activities from the primary school years is considered important.

At primary school level, MEAs are shown to differentiate and develop students' metacognition and critical thinking skills as well as contribute to their conceptual knowledge (English & Watters, 2004). Moreover, it was concluded that students had significantly improved their ability to use mathematical expressions, adjust to group work, read tables, and interpret graphics (Watters et al., 2004). In addition to expressing the structures they work on to provide a solution, it also allowed students to evaluate their thinking from a critical perspective and to learn different approaches to problem-solving (English, 2015). In terms of both mathematical modeling ability and mathematical reasoning ability, heterogeneous groups performed better than homogeneous groups (An & Oh, 2018). In comparison to traditional textbook-centered courses, MEAs were statistically more likely to develop mathematical aptitude (Ko & Oh, 2015). In addition, MEAs offer teachers and students an opportunity to generate knowledge through the iterative process of knowledge construction (Suh et al., 2017). Most of the studies on mathematical modeling in Turkish literature are carried out by secondary school, high school, undergraduate, and mathematics teachers (Canbazoğlu & Tarım, 2021; Çavuş Erdem & Gürbüz, 2018; Deniz, 2020;Deniz & Akgün, 2016; Doğan et al., 2018; Hıdıroğlu, 2022; Hıdıroğlu & Bukova Güzel, 2016; İncikabı, 2020; Özaltun Çelik & Bukova Güzel, 2018; Özer & Bukova Güzel, 2016; Şahin et al., 2017; Tekin Dede & Yılmaz, 2016) while the studies on MEA at primary school level are limited (Canbazoğlu & Tarım, 2021; Işık, 2016; Şahin, 2014, 2019; Şahin & Eraslan, 2017, 2018; Ulu, 2017). In a study by Işık (2016), the impact of mathematical modeling activities on perceptions of difficulty and success in the subjects that primary school 4th-grade students perceive to be difficult were examined. Based on the results, mathematical modeling activities were found to be more effective than traditional problem-solving activities in terms of computing knowledge and concept-computing relationship. In conclusion, students acquired a positive attitude toward mathematics and developed metacognitive skills necessary to establish a concept-computing relationship. In another study by Şahin (2019), in which 4th grade primary school students examined in terms of model-eliciting activities and model-eliciting processes, students were able to produce and develop mathematical ideas, choose and try factors related to the problem, and test and revise the model they created. A further conclusion was that some students were able to offer realistic solutions, while others were unable to do so because they could not visualize the situation.

1.1. Theoretical Framework

To better understand mathematical modeling, it is important to first explain what the terms model and modeling mean. Despite their common etymological origins, modeling and model have different meanings based on their etymologies. While the model is a product that will help us solve a problem by going through different mental processes in the problem solving process of a real-life situation (Lesh & Doerr, 2003), modeling entails creating a model of a situation in order to respond effectively (Sriraman, 2005). Modeling involves interpreting the problem situation, making arrangements for solving it, and evaluating the solution while models include equations, diagrams, and embodied representations. Considering both definitions, a model and modeling are defined as the creation of mathematical expressions that represent complex systems numerically (Lesh & Doerr, 2003). Mathematical modeling has been defined by many researchers as a method of analyzing real-life problems by transferring them to the mathematical world through mathematical methods (Borromeo-Ferri, 2006; Bukova Guzel, 2016; Maaß, 2006). Mathematical modeling has two prominent elements. First, we consider mathematical modeling as a process, and second, we examine the relationship between the mathematical world and the real world. In comparison to open-ended, routine word problems, mathematical modeling refers to problem-solving processes associated with real life with no single correct answer. During the mathematical modeling process, Bukova Guzel (2016) indicates that the real world situation is expressed through mathematizing, the problem and the factors that affect its solution are specified, and based on assumptions, answers about the problem are attempted to be reached. According to Lesh and Doerr (2003), mathematical modeling is a phase of MEAs. In this context, MEAs are non-routine problems in which students are asked to produce solutions that are based on more than one assumption in order to generalize the model they have created, and where different possible solutions are re-evaluated, by working in groups, students propose different solutions to real-life problems.

According to Maaß (2006), there are some competences and sub-competencies associated with understanding a problem, translating it into mathematical language, solving it mathematically, interpreting and verifying the solution found in the implementation process (see Table 1).

MEAs develop students' problem-solving and communication skills in a group discussion setting. During these discussion settings, students learn to respect each other's ideas and have a common opinion with their friends. In MEAs that students perform in groups, students develop various competencies, such as understanding the problem situation, simplifying, mathematizing, working mathematically, interpreting and verifying the solution proposal, and sub-competences based on these competencies. In the competence of understanding the problem, students work on interpreting real-life problems based on their past experiences, using their reading, understanding, and interpreting skills, as well as creating graphics and tables. In simplification, students who

Table 1
Mathematical modeling competencies and sub-competences (Maaß, 2006, p. 116-117)

1. Competencies to understand real life problem and construct a model based on real situation
1 a. Making assumptions for the problem and simplifying the situation
1 b. Identifying, naming, and identifying important variables that affect the problem situation
1 c. Identifying relationships between variables
1d. Finding appropriate information and distinguishing necessary/unnecessary information to solve the problem
2. Competences in constructing a mathematical model with the aid of a real model
2a. Mathematically specifying relevant quantities and the relationships between them
2b. Simplifying the relevant quantities and the relationships between them, as well as reducing uncertainty with the quantity and number of relationships
2c. Choosing appropriate mathematical representations and displaying situations graphically
3. Competences in solving mathematical problems with the created mathematical model
3a. Using appropriate problem-solving strategies
3b. Using mathematical knowledge to solve the problem
3c. Dividing the problem into smaller parts, relating the problem to other problems and expressing it differently
3d. Finding solutions to the problem with different strategies by diversifying the quantities and appropriate data
4. Competences in interpreting mathematical results in real situations
4a. Interpreting mathematical results in non-mathematical contexts
4b. Generalizing solutions developed for a specific situation
4c. Interpreting the solution using the appropriate mathematical language associated with the solution of the problem
5. Competencies to validate the solution
5a. Checking the result with a critical approach by making assumptions
5b. If the solutions are not consistent with the situation, examining different solutions, revising some parts of the model and revising the mathematical modeling process again if necessary
5c. Considering whether the result can be produced differently by solving the problem
5d. Querying the obtained model in general

make assumptions regarding the problem situation determine the relationship between the variables and distinguish the necessary and unnecessary information. The competencies expected from the students in the mathematization phase are to express the data mathematically, simplify the relevant quantities, and express them with figures, tables and graphics by selecting the appropriate mathematical representations. Mathematical work requires the assimilation of mathematical knowledge for the solution of a problem situation, on the other hand. Students demonstrate their ability to interpret mathematical solutions by relating results to real life, generalizing solutions developed for specific situations, and communicating about solutions mathematically. Some parts of the model are reviewed during the validation phase of the interpreted solution, if the solutions are not consistent with the real-life situation. An important aspect of this competency is to discuss how the model could be improved by considering possible solutions and evaluating the model in general.

During the implementation phase of MEAs, students exchange ideas among themselves about the problem situation and engage in peer learning, one of the most effective learning methods. Lesh and Doerr (2003) described model eliciting activities as a process for students to better explain and define mathematical concepts and to explain why they exist. By applying model eliciting activities, students can demonstrate a variety of competencies. These competencies can appear sequentially or in different cycles. Table 2 presents a cycle developed by Maaß (2006) that was used to analyze the data in this study.

Table 2
Mathematical modeling cycle and competencies of Borromeo Ferri and mathematical modeling competencies and sub-competencies developed by Maaß

Modeling cycle (Borromeo Ferri)	Modeling competencies (Borromeo Ferri)	Modeling competencies (Blum & Kaiser cited in Maaß, 2006)	Modeling sub-competencies (Blum & Kaiser cited in Maaß, 2006)
RS→RLS	Understanding	----	1. Expressing what is given and requested in the problem(A1) 2. Drawing a representation of the problem situation(A2) 3. Associating the problem with background knowledge(A3) 4. Associating the problem with real life(A4)
RLS → RM	Simplification	Competence to understand the real situation and build a model based on reality	1. Making assumptions for the problem situation, simplifying the situation (B1) 2. Identifying the quantities that affect the situation, naming them, and identifying key variables (B2) 3. Determining the relationships between the variables(B3) 4. Accessing the existing information by distinguishing the necessary/unnecessary information (B4)
RM→ MM	Mathematization	Competence in constructing a mathematical model from a real model created	1. Expressing the data for the solution of the problem situation mathematically. (C1) 2. Making simplifications among relevant quantities. (C2) 3. Expressing with figures, tables and graphs by choosing appropriate mathematical representations. (C3)
MM → MS	Mathematical Study	Competence in solving mathematical questions within the mathematical model	1. Using heuristic strategies for the solution by making the necessary assimilation for the solution of the problem. (D1) 2. Using mathematical knowledge for the solution of the problem. (D2)
MS → CS	Interpretation	Competence in interpreting mathematical results to real situation	1. Interpreting mathematical results by relating them to real life. (E1) 2. Generalizing the solutions developed for a particular situation. (E2) 3. Discussing the solution of the problem and/or communicating about solutions using appropriate mathematical language. (E3)
CS→ RLS	Verification	Ability to validate the solution	1. Checking the solutions through critical paradigm. (F1) 2. Review parts of the model or go through the mathematical modeling process again if the solutions are not consistent with the situation. (F2) 3. Thinking on possible solutions or reflecting on the result that did/did not develop in a different way. (F3) 4. Evaluate/question the model in general(F4)

Note. RS: Real Situation, RLS: Real Life Situation, RM: Real Model, MM: Mathematical Model, MS: Mathematical Solution, CS: Commented Solution)

Using modeling activities in a learning environment, this study examines 4th-grade students' mathematical modeling competencies and the factors that affect them. In line with this general purpose, the following research questions were addressed:

RQ) What are the primary school 4th-grade students' a) understanding, b)simplifying, c) mathematization, d) mathematical study, e) interpretation, and f) verification competencies when working with mathematical modeling activities?

2. Method

Using MEAs, this study determines the modeling competencies of primary school 4th grade students. The study aimed to identify the competencies that students exhibit during the mathematical modeling process. Because the teaching activities in this research can be improved and revised based on student learning throughout the study, it is a teaching experiment and conducted in consecutive teaching sessions, and the researcher himself serves as the teacher (Steffe, 1991; Steffe & Thompson, 2000). The primary purpose of teaching experiments is to explore and understand students' mathematical activities (Steffe & Thompson, 2000). In ıther words, the purpose of teaching experiments is to examine the development of students in a certain process, and based on the results of the examinations, to make changes to related aspects of instruction in a way that will contribute to student learning.

The role of researcher teacher had some limitations. The key to qualitative research is to be able to be a "marginal native" in terms of belonging to a group. Before working with the group as a researcher, students have explored how mathematics can be used in daily life by attending classes with members of the group. Data collection and student presentations are carried out together with the classroom teacher in this study. Although the classroom teacher was not very skilled in applying mathematical modeling in an academic context, his command of the class allowed the study to be more effective. Throughout the research, it was important to apply procedural knowledge to mathematical modeling activities.

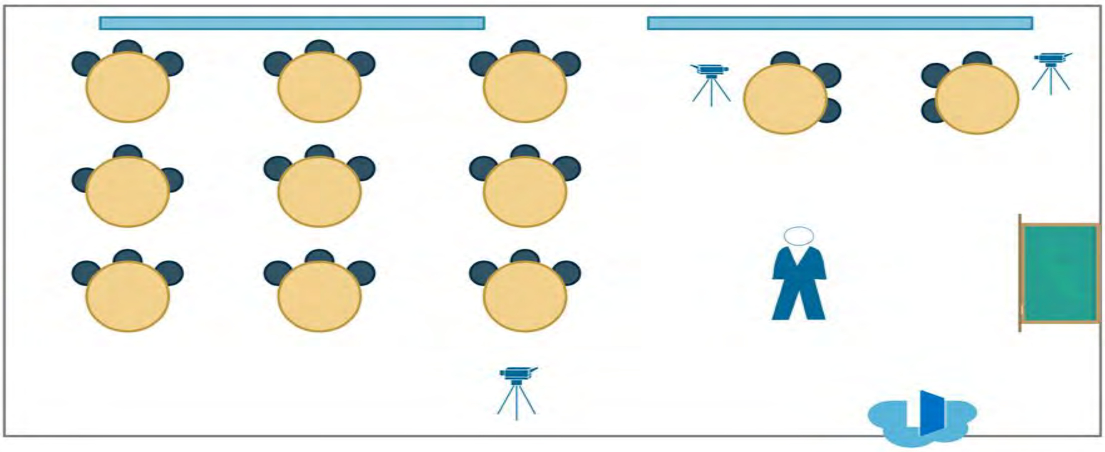
2.1. Participants

Participants in the study were 4th-grade students in a public primary school in Konya city for the 2019-2020 academic year. A convenience and criterion sampling method (Patton, 2015) was used to determine the participants. Three focus students were selected from thirty-three/six students in eleven groups of three each. In order to determine which students to include in the focus, the following criteria were taken into consideration; a) Obtaining a score of 70 and above on the Success Evaluation Form (SEF) presented in Appendix 1 before conducting research.
b) Assuring active participation in the "use of mathematics in daily life" session following the SEF application. As a result of evaluations made with the classroom teacher, it was identified which students get along well with their friends and are good at group work.

2.2. Setting

The application was conducted in the *cognitive education classroom* of the school, as illustrated in Figure 1. Choosing this class was due to the fact that it had tables and chairs suitable for the seating arrangement that allowed for student interaction, and because it had more space than the others. In contrast to traditional classrooms, the cognitive education classroom has desks where students can communicate face-to-face. As an example for the future steps to be conducted, the classroom activities have a table seating arrangement, emphasizing the importance of doing so in an environment where students can interact and develop social skills. There were 33-36 students in the class, divided into 11 groups of three. Three cameras recorded the entire class during the process, which prohibited communication between groups, but allowed communication within groups.

Figure 1
Seating arrangement of cognitive education classroom



2.3. Instruments

First, the Success Evaluation Form (SEF) presented in Appendix 1 was used to determine the focus students to be studied during the research. The teachers interviewed students who achieved sufficient scores on the SEF, which was prepared based on the curriculum acquisitions. A key aspect of the research was the use of procedural knowledge in mathematical modeling activities. In this context, students are required to receive 70 points from SEF in order to carry out the activities. Results from the SEF showed that the students in the class received scores between 45 and 95. A total of four students were unable to pass this threshold, including two mainstreaming students. Six students scored 70-79, 12 students scored 80-89, and 14 students scored 90-100. A modeling activity, *Farmer Uncle Hüseyin*, developed by the researcher and presented in Appendix 2, was applied to the students. MEA was prepared taking into account the objectives in the curriculum, as well as the content of the subjects that students had previously encountered or might have encountered in their vicinity and that were thought to be of interest to them. Moreover, mathematical modeling activities were designed to incorporate the features that mathematical modeling activities should possess (English, 2009; Lesh et al., 2000; Lesh & Caylor, 2007). In preparation of the MEA, the teacher of the class where the application would be proceeded was consulted, and the activity was evaluated according to its suitability and difficulty level. The activity was also reviewed by two field experts for their opinions and sent to two Turkish language teachers for linguistic compatibility checks. Throughout the research, video camera recordings, researcher field notes, student worksheets, recordings of student presentations, and researcher diaries were used.

2.4. Data Collection

As part of the geometry learning domain, the *Farmer Uncle Hüseyin* modeling activity relates to the side lengths of the square and rectangle. The students were expected to solve the problem as in real life situations by associating the elongation of the roots of the olive trees with the elongation of the branches of the olive trees in the modeling activity that was developed in the context of how many olive trees could be planted in a rectangular area 2 times its width. Students were asked how many meters apart the trees should be and how many trees should be planted in total. They were encouraged to use their past life experiences by adding the condition of passing a tractor through the olive trees. In preparation for the application, students had to research the olive tree and its growing conditions, and they had to understand and simplify the information given in the event of a problem, explain how the solutions relate to the real-life situation, as well as interpret and evaluate the finalized solutions. During the preparation of this MEA, students were expected to create a generalizable model that could be applied under different assumptions and in different situations. Students were also expected to achieve the relevant outcomes of the mathematics curriculum, to present their solutions orally and in writing, and to perform effective group work.

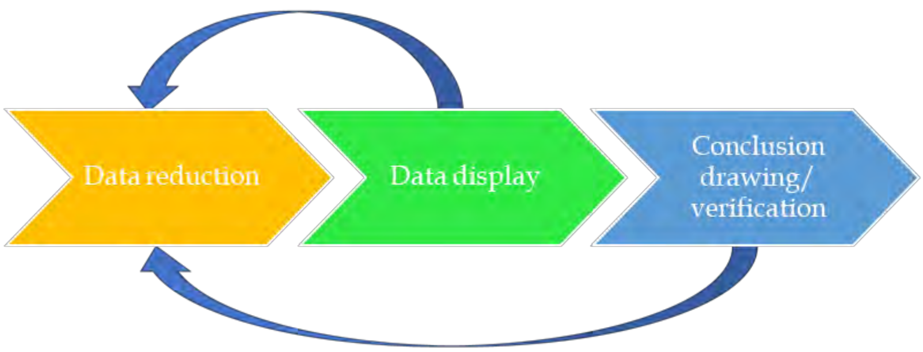
Prior to the implementation of the MEA, students were given research tasks appropriate to the context of the problem situation. In order to prepare students for the modeling process, warm-up questions were posed. A video camera was used to record students' performances during the implementation of the MEA. Additionally, field notes and diaries kept by the researcher supported the data obtained through the video camera recordings. In approximately 40+40 minutes, the students were able to solve the MEA and share their solutions. Students presented their solutions in the classroom, and had an opportunity to associate and verify their solutions with real-life situations.

Post-primary mathematics activities include an "introductory article" application in the process. In primary schools, it is difficult for students to be online, so warm-up exercises were given two days before the activities. These warm-up exercises have been used for the purpose of an introductory article.

2.4. Data Analysis

This study examined the mathematical modeling competencies of primary school 4th-grade students in designed learning environments. Data were collected from observations, student handouts produced by the students during the study, field notes kept by the researcher, and video recordings during the research process. A spiral analysis was used to compile and analyze video recordings, student worksheets, the researcher's diary, and field notes during the data analysis. A content analysis technique was used to analyze all data obtained through different tools. The basic principle of content analysis is to collect similar data based on certain concepts and themes and to interpret them in a way that the reader can understand (Yıldırım & Şimşek, 2013, p. 260). Coding the data set obtained from the research is the first step in content analysis. Researchers independently analyzed the data and coded how students' mathematical modeling competencies and sub-competencies related to these competencies were developed. In addition, the coding included the process of mathematical modeling for those competencies. For the creation of the codes, mathematical concepts from the mathematical modeling cycle were primarily considered. The MAXQDA package program was used for analyzing research data. The three-step analysis process developed by Miles and Huberman (1994, p. 23) was applied to the analysis of mathematical modeling competencies of students and their sub-competencies. Figure 2 illustrates the three-step analysis process.

Figure 2
Three-Step Analysis Process (Miles and Huberman, 1994, p. 23)



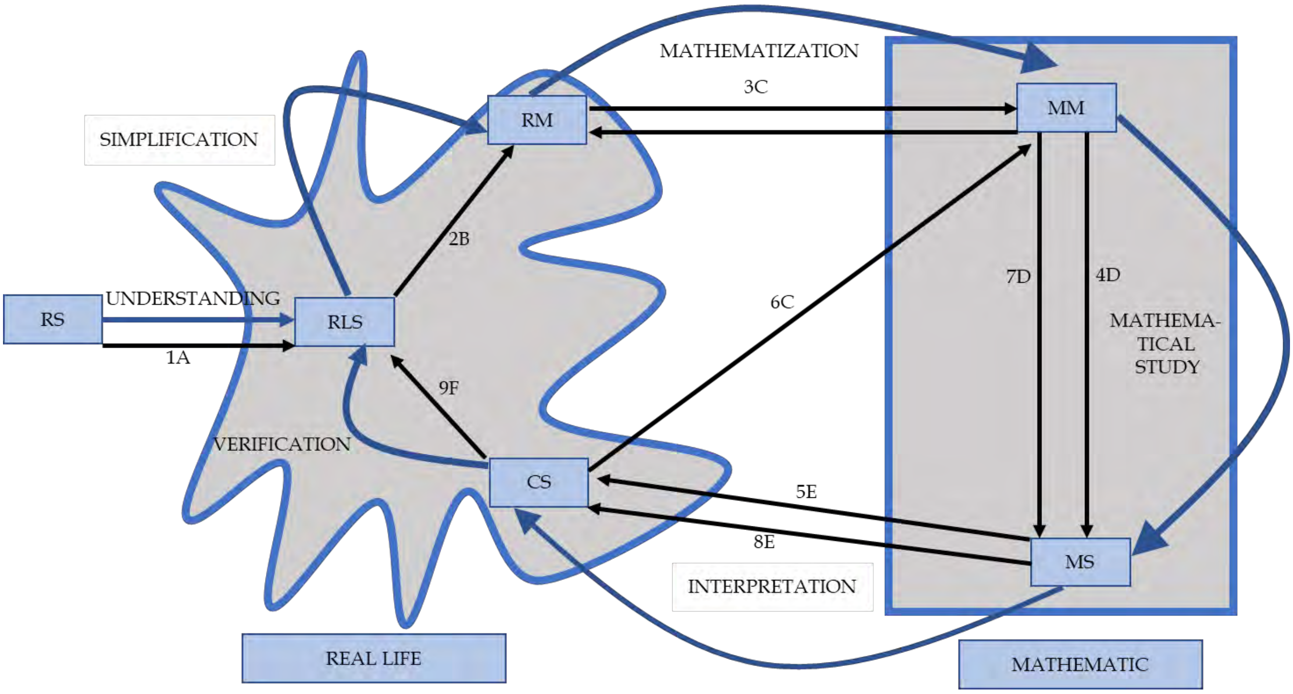
Data reduction is the first step in the analysis process obtained using qualitative data collection tools. As a result of organizing, processing, and reducing the collected data, the researcher organizes the data outside the themes that are created according to the context of the research (Baltacı, 2017, p. 7). Using processing to reduce data prevents possible coding errors. Based on Miles and Huberman's (1994) process, the data obtained after the group activities with students were reduced. Students' dialogues not related to the context of the question posed in group work were not transcribed. Moreover, the situations that students repeated to their friends several times during the process were taken into account.

Data display is the second step in Miles and Huberman's (1994) analysis process. Information about the reduced data is organized, correlated, and combined at this stage. In order to analyze non-reduced qualitative data, the data should be transformed into a systematic format using various presentation methods (Baltacı, 2017, p. 9). The data is presented visually using tables, figures, graphs or diagrams. Through the analysis process, the data obtained through the activities performed with the students were arranged as tabulations of modeling competencies and sub-competencies.

The third step of Miles and Huberman's (1994) analysis process is conclusion *drawing and verification*. The main purpose of reducing and visualizing the data is to make it easier to draw conclusions. This step involves comparing the results obtained during the analysis and making implications for the possible results of the study. The propositional states are verified with the formatting of the data obtained from the research. The validity of every proposition that is confirmed will become evident (Miles & Huberman, 1994). Data obtained from the group studies were compared in order to ensure completeness and verify the data. Those data were evaluated for conceptual consistency, and the results were validated regarding modeling competency.

Figure 3 shows the process of mathematical modeling competencies of the focus group participants.

Figure 3
A diagram of the process of mathematical modeling competencies of the students



Note. (RS: Real Situation, RLS: Real Life Situation, RM: Real Model, MM: Mathematical Model, MS: Mathematical Solution, CS: Commented Solution) A: The process between RS and RLS; B: The process between RLS and RM; C: The process between RM and M. The numbers at the beginning of the letters show the order of operations performed by the students.

3. Findings

Before the implementation, students were asked to make an investigation regarding the characteristics of the olive plant prior to the *Farmer Uncle Hüseyin* modeling activity. During the implementation phase of the activity, students utilized their pre-learning they gained from their research. Figure 4 presents the images of the students' investigation about the olive tree they conducted before the activity.

Figure 4
Deren's investigation about the characteristics of the olive plant

[Translation: When I came home today, I had a research assignment about olives given to us by teacher İsmail. I researched olives on the internet, and discovered that they can grow up to 15 meters tall. It can live for a thousand years. My most astonished reaction to this information was the following: provided that its leaves never change, it stays green year-round like a pine tree. There are four types of olives, as far as I know. Green olives, yellow olives, black olives, and pitted olives. A number of regions in the world grow olives, including South Australia, the Eastern Mediterranean, Central Asia, Crimea and California. During the months of May to Autumn, olives are collected and dried under shade. Imagine the cycle of a tiny olive we eat each morning.]

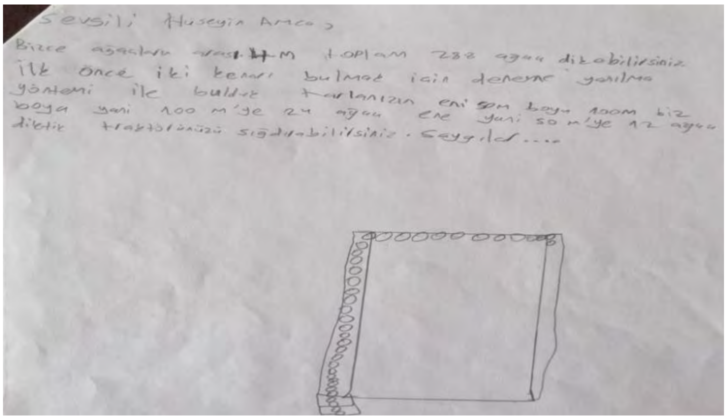
Including the preparation phase, the *Farmer Uncle Hüseyin* modeling activity lasted 55-70 minutes. The data set was not provided to the students ready, and they were expected to create it themselves. The students found the width and length of the field where olives will be planted with different results in the group studies. The problem was attributed to the fact that students were unlikely to understand the real-life situation. Based on the field shape they have reached, the students determined how many meters apart the olive trees should be planted. Students created a model for the solution, but it was observed that they were unable to create a model suitable for the real-life situation. Furthermore, they were able to create figures, tables, and graphics for the solution to the problem. Based on their previous experience, they estimated the tractor's width. Mathematical operations were performed based on the width of the tractor and the length of tree leaves. Using intuitive strategies, students were able to plant trees of different lengths. However, the condition of planting the highest number of trees in the question caused the students to reconsider their answers. Qualitative expressions such as "logical, appropriate" were used by the students to justify their solutions. During the text part of the activity and the modeling activity "Farmer Uncle Hüseyin", the teacher asked the class the questions in Table 3 aloud. Table 3 presents how the applied MEA was carried out.

Table 3
The Syllabus of Teaching Section of Farmer Uncle Hüseyin Activity

Items	*Syllabus of teaching section*
Student difficulties and mistakes they made	They could not predict how long the tree's petals could grow. They could not know for what purpose the tractor would enter the field, and this situation affected the solution of the question. They thought that trees should be planted only on the edges of the field and the middle should be left empty. They made mistakes in arranging distance between the trees as less than it should be.
Action that caused misunderstanding	They could not explain the relationship between root lengths and petals of trees. Quantities affecting the problem situation could not be named and no contribution could be made to the solution of the problem situation by using key quantities. They could not make assumptions related to real life.
Findings, concepts/contexts of focus group	They tried to find the width and length of the field with the "trial and error" method. They explained that a question might have more than one answer. They explained that planting the highest number of trees per unit area was possible by minimizing the distances between the trees.
Findings concepts/contexts of the whole group	They stated that there might be different ways to solve the problem. It was seen that they developed different hypotheses for the solution by evaluating the relevant quantities and the relations among them.
Learnings in student presentations	They stated that predictions could be made for the solution of the problem by using heuristic strategies. They realized that in order for an olive tree to be planted, the distance between two trees had to be at least four meters.
Role of the teacher	During the activities, the teacher asked students the following questions to have them understand what the modeling competencies and its sub-competencies related to those competencies were. • Do you think this could happen in real life? Have you ever planted a tree? • Could you simplify the question in your own words? What do you think "high yield" means? • What are the important and unnecessary information for solving the problem? • Can you relate the problem situation with mathematical expressions and express the situation with figures, tables and graphs?
General assessment of the process	The students were observed to have difficulty in finding the length and width of the field during the preparation phase and the presentation of the activity, which lasted 55-70 minutes. Students who tried to relate the problem situation to real life made different suggestions for the solution of the problem situation. Although the research task given two days in advance helped the students in the solution process, some students could not come up with suggestions suitable for the real-life situation about why the tractor should enter the field and how far the distance among the trees should be. It was an important step towards solving the problem that the students in the focus group reached the data set based on the "trial and error" method. Especially the students who offered different solutions for the problem situation were eager for the presentation. In this activity, which was applied in the seventh week, it was observed that the students realized an understanding of how modeling activities should be solved and demonstrated their solution-oriented modeling competencies. However, they achieved the competence of interpretation and validation at a partially sufficient level. The competence of students could be explained by the fact that the study was conducted in further weeks, on the other hand, the partial level of interpretation and validation competencies could be explained by the cognitive status of the students.

In Figure 5, a student worksheet for the *Farmer Uncle Hüseyin* modeling activity is presented.

Figure 5
Student worksheet for modeling activity



First, the group members used intuitive strategies to determine the width and length of the field where olive trees should be planted. The group that made the presentation stated that the values they gave were accurate by drawing a figure in response to the objections made by the class. Using the text part of the question, the students stated that it was important knowledge that the tree roots would grow two to five meters long. The group members drew a bird's-eye view of the field in order to choose appropriate mathematical representations and to show the situations graphically, however, they could not give a real-life contextual explanation for the appropriateness of the distance between the trees to the questions from the class. Using the width and height of the field as indexes, they determined the number of trees to be planted in the whole field by dividing the problem situation into smaller solvable parts. The following dialogues relate to the presentation of the "Problemciler" (Deren, Ali, and Eren) group after the modeling activity was completed.

> Deren: There was a method suggested by Ali. We firstly decided to try it.
> Researcher: What was this method?
> Deren: It was a trial and error method. (Group member Eren draws a rectangular field on the board.) Whatever we multiplied with would give us 5000. At first, we multiplied 500 by 100. As we understood it didn't happen, we multiplied 50 by 100, according to what Ali said.
> Researcher: How did you determine these numbers?
> Eren: By trying.
> Researcher: I understand, but why didn't you give larger or smaller numbers??
> Eren: We considered the size of the field.

The group members used intuitive strategies to determine the width and length of the field where olive trees should be planted. Ali stated that the group called this method the "error method", but Deren corrected it as the "trial and error method.". Eren explained the heuristic strategy by stating that the values for the width and height were based upon the area of the field.

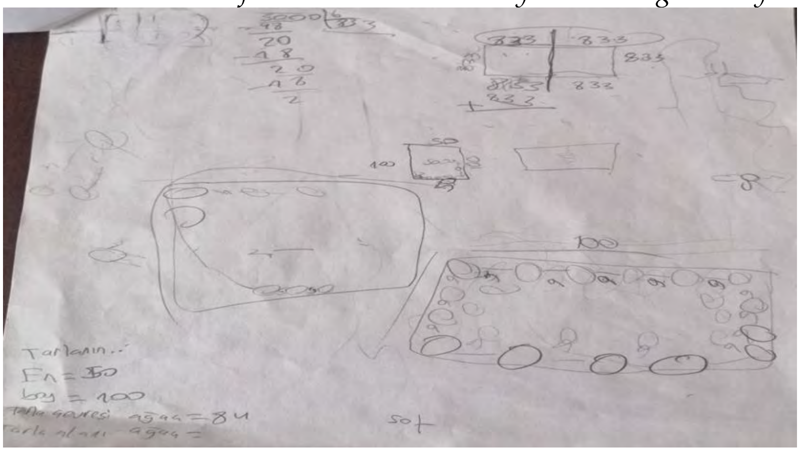
> Deren: Since its length is twice its width, this was suitable for us as well.
> Researcher: What did you do next?
> Deren: This question seemed a little long to us; we read the first part first and then we did it part by part. Then we moved on to the second paragraph. The most important part for us here was that the roots of olive tree were up to two and five meters. This was important information because how many trees we would plant depended on it.
> Eren: We started from two meters, from the least.
> Deren: Because we were told to plant too many trees in the question. If the roots of a tree are 2 and the roots of the other tree are 2 meters, it is very suitable, but it also told us about the tractor here.

The students learned how to simplify the relevant quantities and their relationships after reading the problem and considering the information in the first paragraph of the modeling activity. Their statement that the roots of trees would grow two to five meters long was important. Associating the problem situation with real life helped the group members solve the problem when they were able to divide the problem into smaller pieces.

> Researcher: Can you explain what those rectangles are??
> Deren: We drew a bird's-eye view of the field, we drew a lot.
> Researcher: OK, you can go on.
> Deren: The tractor was an important piece of information for us here, the front of the tractor would be one and a half meters and two and a half meters behind, we thought that if we add these, we could catch something.
> Researcher: Eren, have you ever seen a tractor in real life?
> Eren: Yes, I have.
> Researcher: Have you ever got on a tractor?
> Eren: No, I haven't.
> Researcher: How will adding the length of front and back of the tractor contribute to you?
> Deren: As we understood that it did not help, we gave up on this path and believed that the tractor would pass from four meters. We thought the roots of a tree were four meters, but it seemed unreasonable to us.
> Researcher: Why did it seem unreasonable for a tree to have roots of 4 meters?
> Ali: A tree of four meters and another tree of four meters would be eight meters and it would be too much. This was too much for us. And the tractor could pass from eight meters very easily.

In Figure 6, the students worksheet for Farmer Uncle Hüseyin modeling activity, is presented.

Figure 6
Students worksheet for Farmer Uncle Hüseyin modeling activity



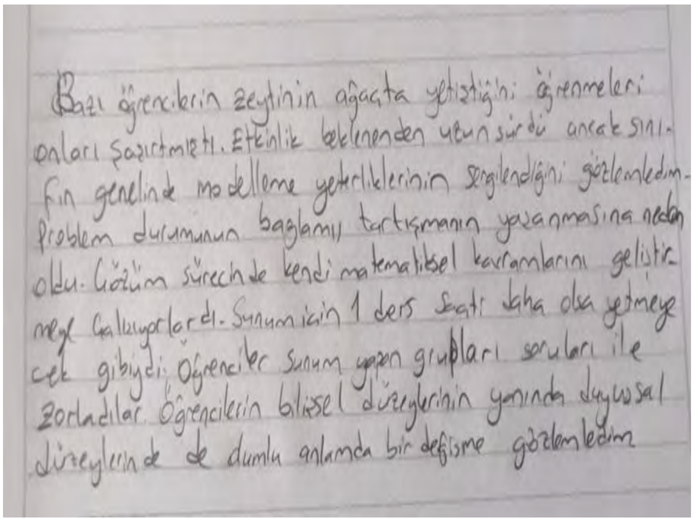
To choose the appropriate mathematical representations and to illustrate the situations graphically, group members drew too many bird's-eye views of the field. Group members made assumptions about the problem and attempted to simplify it by stating that the tractor would be four meters wide and the distance between trees should be determined accordingly for it to pass through them. Additionally, the students interpreted the mathematical results in non-mathematical contexts when they stated that a tree root of four meters would not be appropriate for planting a high tree.

> Deren: Then we divided 100 by four to find out how many trees we would be able to plant in the length of the field and we found 25 trees.
> Researcher: Can 25 trees be planted?
> Deren: No, since we calculated the gaps, there will be gaps at the edges. We will plant 24 trees for 100. (She means that they need to plant 24 trees across the field.)
> Researcher: How many trees are planted in the width of the field?

Eren: We divided 24 into two because its length is twice its width, so 24 trees are planted in length, and 12 trees are planted in width.
Deren: Not only that, trees will also be planted inside; we multiplied 12 by 20 to find them all, and we found that 288 trees should be planted. (The members of the group finished their presentations by reading the letter they wrote to Farmer Uncle Hüseyin.)
Researcher: Does the class have any questions?
Eren: You made two plus two, four meters, but won't this tree ever grow?
Deren: I don't know if it will get taller, but we made it according to its current form.
Eren: What if the trees get taller?
Deren: We did it according to this question, if it gets longer, we could have an answer accordingly.
Kaan: It said the perimeter of the field was 5000; wouldn't it be better if you made it two meters wide by 2500 meters long
Deren: No, it wouldn't. It said its length twice its width.

During the process of evaluating their solution, the group members applied the solution found in real life to achieve "the capability to think of how the result could be created in a different way". From the statements "we made it according to the current state of the trees, and if they grow taller, we can make it accordingly", it appears that they did not sufficiently relate the problem situation to the real-life situation. In answering their classmates' questions, the group members explained the question about the growth status of the trees in the following years in a way that did not reflect reality. Moreover, the researcher kept a diary on the day of the MEAs in order to document the process and confirm its validity and reliability. Figure 7 shows what the researcher wrote in his diary about the activity.

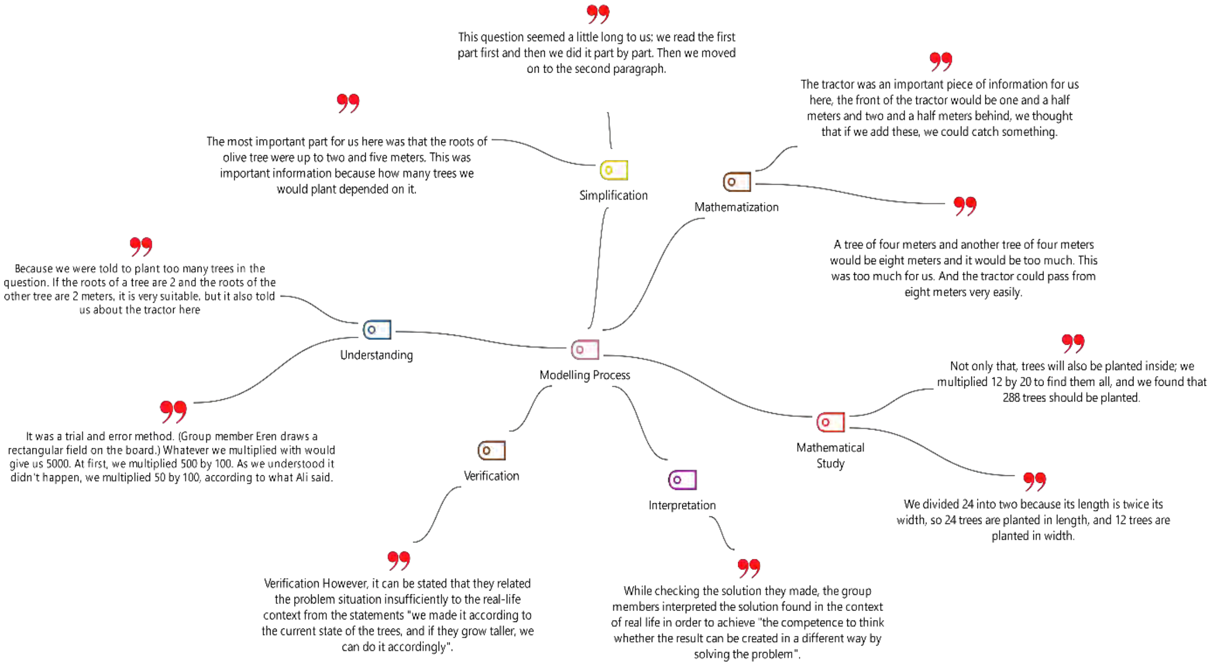
Figure 7
Researcher Diary of the Activity


Bazı öğrencilerin zeytinin ağaçta yetiştiğini öğrenmeleri onları şaşırtmıştı. Etkinlik beklenenden uzun sürdü ancak sınıfın genelinde modelleme yeterliklerinin sergilendiğini gözlemledim. Problem durumunun bağlamı tartışmanın yaşanmasına neden oldu. Çözüm sürecinde kendi matematiksel kavramlarını geliştirmeye çalışıyorlardı. Sunum için 1 ders saati daha olsa yetmeyecek gibiydi. Öğrenciler sunum yapan grupları soruları ile zorladılar. Öğrencilerin bilişsel düzeylerinin yanında duyuşsal düzeylerinde de olumlu anlamda bir değişme gözlemledim

[Translation: Students were surprised to learn olives grow on trees. Despite taking longer than expected, I observed a high level of modeling proficiency throughout the class. The context of the problem situation gave rise to debate. During the solution process, they developed their own mathematical concepts. It appeared that one more lesson would not be enough. Students challenged the presenting groups with their questions. I observed a positive change in the students' cognitive and affective levels.]

An example of how the analysis-related coding is done in the Maxqda package program is presented in Figure 7.

Figure 4
Modeling process of students



4. Results

The results of this study are presented in the context of mathematical modeling competencies and sub-competences related to them. During the MEA implementation, students demonstrated mathematical modeling competencies by applying nonlinear solution stages in a variety of ways. During the solution process, the students used different strategies to understand the problem situation and made assumptions in order to simplify the problem situation and made associations among the quantities affecting the situation. In Farmer Uncle Huseyin, students expressed themselves and discussed their mathematical ideas in settings that allowed them to express themselves. The students created a suitable model by performing the necessary mathematical operations, choosing appropriate mathematical representations, and illustrating them with figures, tables, and graphics. In their drawings, they represented what was given in the problem situation. Although students were partially able to interpret mathematical results by associating them with real life, they were unable to perform at the expected level when it came to verification. The students, however, were unable to relate the problem situation to real life situations because they disagreed on how to apply their previous knowledge.

In the *understanding* competence part of MEA, the students attempted to create the data set that wasn't provided to them ready-made. Different ideas emerged at this stage, and the students tried to persuade one another. In particular, different opinions were expressed about the width and length of the field. It was difficult for students to determine how many meters should be between the two trees when trying to quantify the qualitative data. In the preparatory work they were given before the activity, the students attempted to understand the question based on the relationship between the petal and the root. Another point that caught the students' attention was their desire to move on to the solution stage before fully understanding the problem. Because students were used to solving routine problems in natural learning environments, there was only one correct answer to these problems. When the students were in the process of understanding the problem, they constantly exchanged ideas among themselves and tried to convince one another their ideas were correct. In order to gain a better understanding of the problem situation, the students asked questions to the researcher to confirm their own strategies.

During the process of *simplifying* the problem situation, students identified key variables by making associations among the relevant quantities in order to develop a valid model. As a result of their discussions, students simplified the important information about the solution to the problem by stating that the width of the tractor passing through the trees was an important variable. The *mathematization* competence involves the students developing a valid model by performing four operations on concepts such as width, height, order, minimum, and maximum by expressing the data mathematically. They stated that the trees to be planted in the width of the field would be calculated in the length of the field, while the trees to be planted at each corner would be calculated in the length of the field. In accordance with their own conceptual systems, students demonstrated this competence by constructing a bird's-eye view of the field. In addition to their competence in understanding and simplification, the students also achieved their competence in *mathematical study* by assimilating the problem solution and using heuristic strategies. When students used heuristic strategies to find the width and length of a field, they called their method the error method. The students used their mathematical knowledge for solving the problem by using their advanced mathematical operation skills.

In comparison to the desired level of modeling competence, students had difficulty interpreting the solutions they arrived at in the context of real life until they reached the *interpretation* competency. When generalizing solutions developed for a special case, they made a superficial transfer by saying that the model they developed could also be applicable to different kinds of trees. Despite their attempts to communicate the solution to their friends using appropriate mathematical language and responding to criticisms within the framework of logic, the students could not communicate about their solutions.

During the *verification* competence, students reported that they were confident of the correctness of the solution they reached, but couldn't critically evaluate it. However, they explained with an understanding of causality that the distance of 4 meters between the trees was appropriate and that they reached the most appropriate solution after considering possible solutions. The students explained the problem situation in the letter they wrote to the addressees and drew the figures. They also supported their presentations verbally and responded to criticisms from their friends stating that the solution was not appropriate at the end of the presentation.

5. Discussion and Conclusion

During the analysis of the student works, it was found that the comprehension competence, which is the beginning and first competence of the modeling process, had been achieved to a sufficient level. The students expressed what was given in the problem and what was asked, associated the problem with their prior knowledge, related the problem situation to real life, but had difficulty drawing the representation of the problem. Several studies in the literature show that implementing such activities, which were unfamiliar to students at first, develops with experience (Aydın-Güç, 2015; Bukova-Guzel, 2011; Leiss et al. 2010). Although the discussion within the group was not at a high level, the competence among the members was evident, and studies conducted with groups indicated positive results. This situation coincides with the claim of Maaß (2007) that in-group discussions are effective in demonstrating the comprehension competence of MEA.

Several studies have shown that the ability to make assumptions and simplification is associated with the ability to understand, and participants in different studies have not shown these abilities (Blum, 2015; Chan et al., 2012; Maaß, 2007; Sekerak, 2010). It is evident from the competencies demonstrated by the students in identifying and naming the quantities that affect the situation and determining the key variables that the modeling activity played an important role in their development. The poor interpretation of the activity context was cited by Aydın-Güç (2015) as the reason that this sub-competence was partially demonstrated adequately. Students' previous experiences and the context of the activity are considered in the present study as influencing the model development process. This could be seen as an attempt to develop mathematical models using heuristic strategies or non-systematic methods. In her study, Tekin-Dede (2015) found that participants developed models that did not fit the problem context by making random guesses that were not suitable for the modeling process of quantities. According to Zubi et al. (2018), students learned different knowledge from their friends during classroom presentations. A high level of interaction between students during group presentations in the classroom is considered to be an important aspect of a student's development.

Students in this study were not provided with a ready-made data set, but were asked to create their own. Metacognitive thinking processes such as estimation and evaluation were found to be used by students during this process. The creation and interpretation of the data set is an important factor in the modeling process and in the mathematization of the model (Doerr & English, 2003; English, 2006a, 2006b; English & Watters, 2004). Further, many studies have indicated that it takes longer to complete modeling activities than to solve other problems, since modeling activities involve real-life situations and are more complex than routine mathematical problems (Blomhaj & Jensen, 2003; Biccard & Wessels, 2011; Chan et al, 2012; Haines & Crouch, 2007; Maaß, 2007). During this study, students expressed concerns about the lack of time, especially after the first 30 minutes. Therefore, it was concluded that the students performed partially or insufficiently towards solving the problem situation and that it was unnecessary to draw figures, tables and graphs to represent the situation.

During the examination of the subcompetence of using heuristic strategies for solving the problem, one of two subcompetences of the mathematical work competence, the student makes the assimilations to solve the problem. Through the model-elicitation process, the students suggested heuristic strategies for solving the problem. In addition to the difficulty of constructing the mathematical model, students also experienced difficulties during the activity. In this difficult situation, students demonstrated procedural skills such as trial and error, valuing, and using heuristic strategies. English and Watters (2004) noted that students could compare data intuitively, however, Doerr and English (2003) found that students could create models based on the methods they arranged according to their cognitive abilities when they had difficulty working through mathematical operations. In using mathematical knowledge for the solution of the problem, another sub-competence of the mathematical work competence, the students attempted to solve the complex structure of the modeling activities by doing simple mathematical operations - not routine mathematical problems. Although students had enough prior knowledge to solve the entire modeling activity, they used simple mathematical knowledge to continue their solutions. In the literature, there are several studies with similar findings (Aydın-Güç, 2015; Biccard, 2010; Biccard & Wessels, 2011; Blum, 2015; Chan et al., 2012; Çiltaş, 2011; Şen-Zeytun, 2013; Tekin-Dede, 2015). The students were able to verbally express mathematical concepts in their presentations and group work during the application, but they struggled with the mathematical tasks. Based on this situation, it was evident that the students were unable to assimilate the concepts, since they only had superficial knowledge of the content.

Students performed partially adequately regarding their ability to interpret mathematical results. In this case, it is believed that the situation is influenced by the students' previous experiences, the context of the problem situation, and the discussion environment within the group. Similar to this study in the literature, the studies of Aydın-Güç (2015) and Çakmak (2018) show that although this competence was not at the desired level, it developed as a result of experience in mathematical modeling, and that the designed learning environment was effective in facilitating the development of the interpretation sub-competence by integrating the results into real-life situations. One of the findings of this study was that the participants did not display the sub-competence of generalizing the solutions developed for a specific situation exhibited by the study authors Çakmak-Gürel (2018) and Şahin (2019). Several factors can explain this situation, including the context of the modeling activity and the students' modeling experiences. Additionally, in their natural learning environments, students often assume the answer to multiple choice tests and routine mathematical problems is complete when they find it. The solution, however, must be interpreted and verified through mathematical modeling.

Verification competence was one of the stages in which students were inadequate. Similarly, Ji (2012) and Tekin Dede and Yılmaz (2013) found that students with or without modeling experience were incapable of verifying the validity of the model. In the study by Şen-Zeytun (2013), it is stated that the sub-competence of reviewing some parts of the model was not displayed at a high level and that this was considered due to the students' belief that the teacher was responsible for verifying the solutions. This is similar to the desire of the students to have the researcher verify the solutions they reached during the application. Moreover, the students' insistence on the correctness of their solution prevented them from exploring other possibilities. In his study, Şahin (2019) also found similar results and mentioned that students generally attempted to reach the result quickly after determining a single solution. Aydın-Güç (2015) also concluded that because mathematical modeling activities took place over prolonged periods of time, pre-service teachers were convinced that one solution would suffice to explain the situation. The students' inability to manage time effectively and the long questions of the MEA caused them to start a solution in prejudices in this study. Similar to this result, Tekin-Dede (2015) evaluated the students' demonstration of modeling competencies in two course hours and concluded that the time allocated was insufficient. The appearance of this situation may have been caused by students' performance of routine tasks and multiple-choice questions within the specified time frames in their natural learning environments.

The modeling competencies of the students in this study were examined, and it was observed that they performed at the desired level in terms of comprehension, simplification, mathematization, and mathematical study, as well as related subcompetencies. However, they performed inadequately or partially adequately in the areas of interpretation and verification, as well as related sub-competencies. To achieve the solution of the problem situation as quickly as possible, the students attempted to reach the result without performing processes in mathematical modeling competencies. In particular, the presentations they made at the end of the activity are considered to have contributed to their awareness of different options. It was also very important to discuss the best model that helped students develop their modeling competencies. The researcher and all of the students in the class worked together after the presentations, and all of the groups contributed to the solution. Bringing suggestions for solutions and feeling that their own thoughts matter motivate students to participate in model-eliciting studies because they feel their own ideas are important. It is believed, however, that students do not display their modeling skills due to the desire to reach the result as soon as possible, as well as the time limit. Observations showed that the students did not have any difficulties in working on the mathematical model they obtained and in expressing the real-life situation with mathematical operations. However, their performance was found to be either insufficient or partially adequate in terms of interpreting the results in real life and verifying the interpretations.

6. Recommendations

For students to perform mathematical modeling activities in primary schools, teachers should possess these competencies. The pre-service training of primary school teachers in this context is considered significant. Teachers who intend to use MEAs in their classrooms must be trained in the effective teaching of mathematics models so that they can practice in their classrooms. In the first grade, students begin to learn mathematical concepts through multiple-choice tests and activities containing a single correct answer. To solve these types of questions, it may be sufficient to teach students problem solving methods, concepts, and thinking skills without explaining different problem solving techniques. Contrary to these types of questions, applications that include different solutions, such as MEAs, which are conducted in groups and reveal the cognitive and metacognitive structures of the participants, should be conducted in appropriate learning environments from the early stages of primary school and even in pre-school, because they are performed in groups. The physical conditions of the school are also considered important in deciding where the application will be carried out. For students to develop their modeling skills, schools should provide them with opportunities to apply MEAs. Studies carried out with mathematical modeling activities at primary school level are not at the desired level in Turkey. In this context, it is recommended that researchers involved in mathematical modeling activities be supported in improving the physical facilities of their programs, projects, and schools. It is important to encourage new researchers to work on this topic.

Upon reviewing the results of this study and some studies in the literature, it was concluded that students were not able to implement the interpretation and verification competencies found in MEAs. In order to assist students in developing these two competences, future studies should find out why they have difficulty in these areas, and students should be educated using MEAs in learning environments that will support the development of these two competences. Efforts should be made by researchers to develop and implement training on interpretation and validation competencies that are not adequately demonstrated. This study examined the modeling competencies of three focus students who received 70 or more points on the SEF developed by the researcher. In future studies, it is recommended to examine how competencies of students in heterogeneous groups will be modeled. Due to the fact that MEAs increase the achievement of students in mathematics, it is possible to conduct pre-test and post-test experimental studies to assess the effect of the intervention. Finally, teachers should be encouraged to apply well-structured MEAs in their classrooms.

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Appendix 1. Success Evaluation Form

1) Write the spelling and pronunciation of the numbers below.
a) Forty eight thousand one hundred nine:..................
b) 8008:..

2) 1078 is given. Analyze this number and round to the nearest tens and hundreds.
analysis:... ....
nearest tens:.....................
nearest hundreds:....................

3) 780201 is given. What will be the new number when the tens digit replaces the tens thousands digit?
New number: ....

4) Do the following operations and find the missing ones.

25467	59634	9A3B
+74895	+ [ ]	+C8D6
	76013	14302

5) The median of five consecutive natural numbers is 47. What is the sum of these numbers?

6) Add the 57 to the number that is when substracting 505 we find 242?

7) There is a 5-year age difference between Bade and her sister Cemre. Considering Bade was born in 2009, what will be the sum of their ages in the 100th year of Turkish independence?

8) Pose a problem using the following expressions: 2965 walnuts, 5784 walnuts, basket, and collect.
My problem: ……………………………………………………………………………………………………
:……...…
………?

9) Do the following operations.

960284	65420	7A43B
-782317	- ☐	-39CD8
	8347	

10) 78638 – 54947 > ....". What is the largest natural number that can be written in the blank?

11) How do we get the total of 15723 by adding the greatest four-digit even number?

12) Do the following operations.

54	769	825× ☐ =13925
×38	× 37	
		47×65×29=65×29× ☐

13) Pose a multiplication problem using these expressions: 24, 35, 5 friends, weight, and sugar.
My problem:……………………………………………………………………………………………….
……….

14) There are 24 rows with 37 seats and 127 rows with 48 seats for Fazil Say's concert in Konya. 6400 tickets were sold, how many people will watch the concert standing?

15) Do the following division operations. Find the missing ones.

84 | 4   378 | 16

6A6÷38=17   4128÷ ☐ =96

16) Estimate the number of digits of the divison results in the following operations without making any calculations.

75 | 6   87 | 9   576 | 6

has...digit   has… digit   has…digit

17) Ali reads the 597 page book in nine days. He read 64 pages on the first day and 78 pages on the second day. In the remaining days, he read an equal number of pages. How many pages did Ali read in the remaining days?

18) Pose a problem that requires division with using the following expressions: 450, 18, student, and bus.

My problem:……………………………………………………………………………………………….
……….

Note. Questions 13 and 18 are 10 points each. Other questions are 5 points.

GOOD LUCK ☺ ☺

Appendix 2. Farmer Uncle Hüseyin Task

Farmer Uncle Hüseyin has a field where he plans to plant olive trees. The area of this field, which size of width is twice as long as its lenght, is 5000 square meters. Uncle Hüseyin wants to plant the largest number of olives in his field. However considering development and yield of olive trees, he must plant with certain intervals this trees. When he goes to minstory of town agriculture, took this informations.

-Olive trees must have certain intervals with each other for getting high performance.

- The roots of an adult olive are not in the form of piles, but in the form of fringes. These roots go up to 2-5 meters.

- The length of the crown (body length of the leaf) of the trees should be as much as the fringe root.

- If we want to have productive trees; field is should been plowed with a tractor twice a year.

With the direction of these informations help to the uncle hüseyin about suitabilityof how many olive trees are should planted on this field and with how much intervals using panoramic sketch.

Explain your help to the Uncle Hüseyin with writing a letter.