Journal on Mathematics Education
Volume 13, No. 2, 2022, pp. 239-256

Future early childhood teachers designing problem-solving activities

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Received: 4 November 2021 | Revised: 2 June 2022 | Accepted: 10 June 2022 | Published Online: 15 June 2022

Abstract

This work aims to identify the criteria to design activities based on problem-solving tasks that emerge when future early childhood education teachers jointly plan their activities and reflect on them. The participants were 76 students from the Didactics of mathematics subject that was carried out in the 2nd year of the Early Childhood Education Degree of a Catalan public university. This is qualitative research in which the phases of the thematic analysis have been adapted: familiarizing with the data; systematically applying the categories to identify the student criteria emerged; triangulating the analysis with experts; reviewing and discussing the results. The Didactic Suitability Criteria (DSC), from the Ontosemiotic approach (OSA) framework to design tasks and their indicators, were used to categorise and analyse the tasks performed by future teachers. As a result, it was identified that when the future teachers adopt consensually design their activities, they are implicitly based on the Didactic Suitability Criteria (DSC). Still, not all their indicators emerge since their reflection is spontaneous and is not guided by an explicit guideline that serves them to show their didactic analysis in detail. The study concludes that it would be convenient to offer future teachers a tool, such as DSC, to have explicit criteria to guide the designs of their mathematical tasks. In this sense, a future line of research opens, much needed, to adapt the DSC to the singularities of this educational stage.

Keywords: Didactic Suitability Criteria, Early Childhood Education, Ontosemiotic Approach, Problem-Solving, Task Design, Teacher Training

How to Cite: Sala-Sebastià, G., Breda, A., & Farsani, D. (2022). Future early childhood teachers designing problem-solving activities. *Journal on Mathematics Education, 13*(2), 239-256. http://doi.org/10.22342/jme.v13i2.pp239-256

Problem-solving is considered, in the field of Mathematics Education, one of the optimal scenarios for teaching and learning skills in this discipline (Sapti et al., 2019); once this approach encourages the development of mathematical processes, such as reasoning and proof, the establishment of connections and representation, and communication, among others (NCTM, 2003; Niss, 2002). In this sense, teachers must develop the competence to design and pose problems in their instructional processes (Singer et al., 2015) since problem-solving and problem-posing are complementary approaches (Halmos, 1980; NCTM, 2003; Rafiepour & Farsani, 2021).

Developing the task resolution competence from an early age is crucial since it is a stage where the curriculum presents mathematical contents and processes and the relationships established between them in an integrated manner (Alsina, 2016). In this sense, the tasks posed by the teachers –problem, research, exercise, etc.– to the students are the starting point of their activity, which, in turn, produces their learning as a result (Pochulu et al., 2016).

According to several authors (Ball et al., 2008; Blömeke et al., 2014; Kaiser et al., 2017; König et al., 2014; Pino-Fan et al., 2015; Putman & Borko, 2000; Shulman, 1986; Silverman & Thompson, 2008), the didactic and mathematical training of teachers is a relevant field of research for the international community of researchers in mathematics education and of interest to educational administrations worldwide. The main reason, according to Hill et al. (2005), is that the development of students' thinking and mathematical skills depend fundamentally on the teaching received and, consequently, on the training of their respective teachers. Once teachers' teaching practice determines student learning, designing, executing, and evaluating tasks is a crucial aspect and one of the skills that future teachers must develop in their training.

Among the studies related to the development of teachers' professional skills (Silverman & Thompson, 2008; Even & Ball, 2009), there is the teacher's didactic-mathematical knowledge and competences model -DMKC model- (Godino et al., 2018) within the framework of the Ontosemiotic Approach (OSA) (Godino et al., 2007). This model assumes that the key competencies of the teacher who teaches mathematics are two: mathematical competence and analysis and didactic intervention competence; in the second one competence the design, application, and assessment of learning sequences through didactic analysis techniques are crucial. So, the use of Didactic Suitability Criteria (DSC) became essential to establish cycles of planning, implementation, assessment and approach to proposals for improvement (Breda, Pino-Fan, & Font, 2017). It is a competence long studied by the OSA with mathematics teachers where the results showed that the Didactic Suitability Criteria (DSC) were a valuable tool to organize their reflections and evaluations and improve their task designs (Breda, Pino-Fan, Font, 2017; Sánchez et al., 2021). However, this type of study has not already been carried out with future early childhood education teachers.

In this work, we focus on the DSCs that emerge from the future early childhood education teachers when asked to reflect together to justify the design of their tasks, in this case, problem-solving. The objectives are 1) To study what aspects future early childhood education teachers consider essential when planning and designing problem tasks for the teaching and learning of numbering; 2) Which of these aspects can be identified as DSC; 3) To identify which elements of the DSC are not considered by the students.

Design of Problem-Solving Tasks on Numbering for the First Ages

Within the teaching-learning process of mathematics, tasks occupy a central place in student learning and are defined as the work proposal that a teacher performs for a student, intentionally and carefully planned by the teacher to achieve a specific learning objective (Ponte, 2014).

For Gusmão and Font (2020), the tasks can be classified as exercise, problem, or research project. Exercise-type tasks require training, and excessive use of routine calculations and algorithms, thus falling within the reproduction level. Due to its characteristics, levels of requirements and practicalities, it ends up taking up most of the time dedicated to the study and learning of mathematics. Problem-type and research project tasks require students to look for unknown elements, interpret information, identify relevant elements, and connect mathematical concepts and ideas (intra-mathematical connections) with other curricular components and everyday situations (extra-mathematical connections). Students must use different strategies to solve the same situation, which helps promote the development of their autonomy and communicative competence.

Although there is no consensus definition, in the literature, various authors have presented what they understand by "problem" (Arcavi & Friedlander, 2007; Borasi, 1986; Kilpatrick, 1985; Polya, 1986; Schoenfeld, 1985). Malaspina (2017) suggests that to specify what is understood by the problem and by creating mathematical problems, it is essential to take into account that the problems must bring: a) Information: quantitative or relational data that are given in the problem; b) Requirement: what is requested to be found, examined or concluded; c) Context: it can be intra-mathematical or extra-mathematical; d) Mathematical environment: the global mathematical framework in which the mathematical concepts that intervene, or may intervene, to solve the problem are located.

On the one hand, concerning context, Ponte (2005) considers three possible contexts for working with problems: real-life/reality, pure mathematics, and semi-reality. On the other hand, mathematical problems can be classified as open or closed. On the one hand, open problems admit several correct answers, vary in duration between medium and long, offer spaces for arguments, and justifications and have a high degree of challenge (Gusmão, 2019). On the other hand, problems of a closed nature admit a single correct answer once the statement usually gives clues or specifies what is given and what is requested (Ponte, 2005).

According to the NCTM (2003), the aspects that must be considered when working on problem-solving in the classroom are:

1. Build new mathematical knowledge through problem-solving.
2. Solve problems that arise from mathematics and in other contexts.
3. Apply and adapt a variety of appropriate problem-solving strategies.
4. Control and reflect on the process of solving mathematical problems.

Along the same lines, there are some key aspects regarding the use and meaning of problems in the classroom and, therefore, should be considered when designing the problems. There is an agreement among researchers that a problem situation must be a new situation for which the resolution method is not known in advance. There is also agreement on how one learns to solve problems by manipulating, simulating, discussing, sharing, imagining, observing, visualizing, etc. Moreover, in this sense, in the resolution process, each child should be allowed to use the strategy that best suits his or her possibilities: a drawing, a diagram, mental calculation, the manipulation of a particular material, etc. (Alsina, 2016).

Focusing on the mathematical activity that students develop when solving tasks, there are different teaching approaches but, in the case of early childhood education, citing Baroody and Coslick (1998), the following can be distinguished: 1) the skills approach; 2) the conceptual approach; 3) the problem-solving approach; and 4) the investigative approach, which is a combination of conceptual and problem-solving approaches, the primary purpose of which is for students, with the mediation of teachers, to reach their conclusions through reflection, reasoning, representation, problem solving and research. In this study, students were asked to work assuming the problem-solving approach.

Didactic Suitability Criteria in the Design and Management of Tasks

“To what extent, how, under which conditions, can (or must) didactics set value judgments and normative prescriptions in order to provide criteria about how to organize and manage study processes?” (Gascón & Nicolás, 2017, p. 9). In response to this question, Godino et al. (2019) suggest that the OSA considers the didactics of mathematics as a scientific and technological discipline, since it must address descriptive, explanatory, and predictive questions, which are characteristic of scientific knowledge, and also prescriptive and evaluative questions, which are specific to technological knowledge. In the OSA, the prescriptive and evaluative view of the didactics of mathematics leads to the generation of theoretical constructs such as the Didactic Suitability Criteria (DSC). The didactic suitability is the degree to which the mathematical teaching process has certain characteristics considered as optimal or adequate to succeed on the adaptation between the personal meanings attained by the students (learning) and the intended or implemented institutional meanings (teaching), considering the circumstances and available resources (environment) (Godino et al., 2007; 2019). The OSA considers the following DSC (Breda, Pino-Fan, & Font, 2017; Godino, 2013;): 1. Epistemic Suitability, to assess whether the mathematics being taught is “good mathematics”. 2. Cognitive Suitability, to assess, before starting the instructional process, if what is to be taught is at a reasonable distance from what the students know, and after the process, if the acquired learning is close to what was intended to teach. 3. Interactional Suitability, to assess whether the interactions resolve doubts and difficulties of the students. 4. Mediational Suitability, to assess the adequacy of the material and temporal resources used in the instructional process. 5. Affective Suitability, to assess the involvement (interests and motivations) of students during the instructional process. 6. Ecological Suitability, to assess the adequacy of the instructional process to the educational project of the centre, the curricular guidelines, and the conditions of the social and professional environment.

To make the Didactic Suitability Criteria operational requires a set of observable indicators, which allow assessing the degree of suitability of each of these criteria. For example, there is consensus that it is necessary to implement "good" mathematics, but it is possible to understand very different things from it. For some DSC, the indicators are relatively easy to agree on (for example, for the Mediational Suitability criteria), for other DSC like in the case of Epistemic Suitability, it is more difficult. When the six Didactic Suitability Criteria have been determined, each one of them is split into components and indicators, which make them operational (Breda, Pino-Fan, & Font, 2017).

Based on the Didactic Suitability Criteria, Gusmão and Font (2020) defined a set of observable indicators for the design and analysis of mathematical tasks. In particular, the indicators serve to assess, in an operational way, the degree of suitability of each of these criteria in the design and analysis of tasks. In the case of task design, the indicators are those included in Table 1.

For instance, the indicator “Is it based on the prior knowledge of the students?” (CoS1) is specified in two aspects: a) the students have the prior knowledge necessary for the study of the subject (either they have been studied previously or the teacher plans their study), b) the intended meanings can be achieved (they have a manageable difficulty) in its various components (they are in the student's zone of proximal development).

As mentioned, the indicators of the DSC (Table 1) have been made considering the according to the perspective of the fact, that is, the trends (presentation of contextualized mathematics, giving importance to teaching mathematical processes -problem-solving, mathematical modelling, etc.-, teaching and learning of an active (constructivist) type, principle of equity in obligatory mathematical education and incorporation of new technologies of information and communication, principles and standards of NCTM (2000), and results of research in the area of didactics of mathematics (Bikner-Ahsbahs & Prediger, 2010; Breda et al., 2018). Particularly, for Epistemic Suitability, this principle has been considered: mathematical objects emerge from practices, which embody their complexity (Font et al., 2013; Rondero & Font, 2015). From this principle, a component is derived (representativeness of complexity) whose objective is to consider the mathematical complexity in the design and redesign of didactic sequences (Calle et al., 2021; Monje et al., 2018; Pino-Fan et al., 2013).

Table 1. Task design categories and indicators according to Didactic Suitability Criteria (Gusmão & Font, 2020)

Epistemic Suitability (EpS)
(EpS1) Is the task's description in a clear, correct, and appropriate language for the level of education? (EpS2) Are different languages and forms of mathematical expression used (verbal, graphic, symbolic, pictorial, etc.)? (EpS3) Is the selection of tasks representative and varied, and whether it includes tasks of a closed and open nature? (EpS4) Are the tasks of different types? (EpS5) Does the generation of hypotheses promote open thinking (reversible, flexible, decentralized thinking) and encourage the use of argumentation and justification processes?
Cognitive Suitability (CoS)
(CoS1) Is it based on the prior knowledge of the students? (CoS2) Is knowledge expanded, reinforced, and systematized? (CoS3) Is the level of cognitive development of the students respected? (CoS4) Is the use of different, creative, and original resolution strategies encouraged? (CoS5) Are different learning objectives met, and are students developing different cognitive and metacognitive skills?
Interactional Suitability (InS)
(InS1) Are there moments of dialogue and discussion between students or between teacher and students? (InS2) Is the resolution of tasks individually, in pairs, or groups encouraged? (InS3) Does it allow the generation of cognitive conflict (in the Piagetian sense) and the negotiation of meanings? (InS4) Do they promote responsibility for the study (exploration, formulation, and validation)?
Affective Suitability (AfS)
(AfS1) Does it promote interactivity, attraction, fun, and inclusion, raising self-esteem, the feeling of inclusion, and a taste for mathematics? (AfS2) Are the different types of reasoning and responses valued? (AfS3) Is participation encouraged and interest generated? (AfS4) Do they favor the perception of the usefulness of mathematics in life and at work? (AfS5) Is student involvement promoted in solving tasks (return of learning in Brousseau's sense)? (AfS6) Are there possible challenges to be achieved, triggering levels of thought, each one more complex? (AfS7) Do they present the application and beauty of mathematics?
Mediational Suitability (MeS)
(MeS1) Are manipulative and/or technological materials provided, or is their use recommended? (MeS2) Is sufficient time allowed for its completion and the maintenance of concentration and interest? (MeS3) Are the times appropriate for each of the different types of tasks? (MeS4) Are adequate spaces provided for its realization? (MeS5) Are moments of hands-on experimentation provided to aid understanding of concepts and their applicability?
Ecological Suitability (EcS)
(EcS1) Are official curricular documents (national and local) taken into account? (EcS2) Is the articulation between different contents of Mathematics and between different areas of knowledge sought? (EcS3) Are the tasks contextualized with the social and cultural environment? (EcS4) Are the contents of the tasks useful for social and work life?

METHODS

Context of the Study

The participants were 76 students from two groups of the Didactics of Mathematics subject that was carried out in the 2nd year of the Early Childhood Education Degree of a Catalan public university, during the first semester of the 2020-2021 academic year. The course began in September with mixed teaching and, given the evolution of the health situation due to Covid19, became a virtual course in November. At this point, the teacher had to follow the subject's content block called "Numerical Thinking". The teacher of these groups redesigned the sessions to adapt their teaching methodology to the new virtual teaching modality. Before the Covid19, when the sessions were face-to-face, students could experiment in the classroom with the material resources when they resolved the problems proposed. They could ask the teacher for their feedback immediately. In the same way, the teacher could ask students and observe them in action to assess their learning. Thinking of virtual teaching modality, the teacher wanted to collect other pieces of evidence to assess their students' achievements'. For this, he prepared various tasks that are part of the context of a broader study than the one presented in this work (some parts of which we are still analysing), with the general objective of studying how future early childhood education teachers build their didactical and mathematical knowledge around solving problems at an early age.

It should be noted that both individual and group work within the classroom and that carried out in groups independently and outside the classroom were carried out virtually through videoconferencing platforms and online applications. When the interaction was within the course schedule, the Black Board Collaborate application was used, which has made it possible to configure rooms for work in small groups. When students worked independently outside of class sessions, they could choose the platform that best suited their preferences and needs.

One of the tasks before the one analysed in this work carried out by the participants - we will call it Task A - had the purpose that the students knew a didactical and mathematical source, as a practical example, on which to base the development of their next task we will call it Task B. This work only focuses on Task B because it is part of the analysed data and responds to the specific objectives indicated in the introduction section.

Table 2. Description of steps followed by the future teachers doing the Task A

Step	Description
1	Individually, carefully read the article De Castro and Escorial (2007)
2	In your workgroup, share doubts and reflections and listen actively
3	In group, choose 3 of the 7 problems of the article and describe them: type of problem and strategies (according to the table in the article) and materials resources used to solve them
4	In group, solve the problems selected with different resources than children in the article and check if your strategy is different too, or not.
5	In group, explain steps 3 and 4 in a document and hand over it to the Moodle platform
6	In group, share and discuss your conclusions with other groups in a specific session

To carry out Task A, the students organized into heterogeneous workgroups of 3, 4, or 5 people (in total, 19 workgroups), and carried out an analysis task (Task A) autonomously (outside class hours) from De Castro and Escorial (2007) reading, "Solving verbal arithmetic problems in early childhood education: an experience with an investigative approach." This article explains, among other aspects, how a teacher proposes seven problems to her 5-year-old students and the development of the sessions where they solve them. The problems and the resolution strategies used by the children were classified based on a table prepared by the authors of the reading article and based on the typology of Carpenter et al. (1993). Table 2 summarized task A.

Description of Task B Analysed

This task, which was presented to the students in a virtual joint session, consisted of developing a group work (with the same groups as those in Task A) autonomously, outside the hours of the virtual joint sessions, which consisted of designing a minimum of three problem-solving tasks for a classroom of 20 5-year-old boys and girls. For this, the consultation of the early childhood education curriculum, the article by De Castro and Escorial (2007) analysed in Task A, and the rest of the consultation documents of the content block, available on the subject campus, was recommended. It was also part of the work to present the recording of a videoconference where the group members discussed the aspects of the design and the implementation methodology that they considered essential to incorporate and reach a consensus and/or seek solutions.

Before the final delivery of the works, a 2-hour class session (which was recorded) was dedicated to presenting their designs and obtaining feedback from their classmates and the teacher when deemed necessary. Thus, if they saw fit, they were given time to incorporate improvements in the design of their tasks. Finally, all the 19 working groups handed over a file in pdf format with the design of the problem tasks activities and a file in mp4 format with the recording of their decision-making session (videoconference). Some groups also sent the supporting document to their presentations.

The data analysed came from the following documents: 19 recordings of the videoconferences (one per group), 19 documents of the design of the problem tasks (one per group) and the audio teacher recording' of the presentation session. The unit of analysis was the 19 written documents of the working groups' design of the problem tasks and their recordings of the videoconferences. The audio recording of the presentation session and the working groups' supporting presentation documents were helpful to the authors to have a first exposition about the tasks designed by the groups and complementary notes.

Analysis Methods

This is a study of qualitative characteristics on the aspects that the participants consider essential to consider in their designs of a sequence of problems for the teaching and learning of numbering for 5-year-old students and analyse which of these aspects can be identified with DSC and which elements of the DSC are not considered. The phases of the thematic analysis proposed by Braun and Clarke (2012) have been adapted in this study.

In the first phase, to familiarize ourselves with the data, the working groups' supporting presentation documents were revised by the authors, complemented by listening to the audio recording of the presentation session. After that, all the group's videoconference recordings were viewed concerning the planning of the work to be carried out, and aspects of the design and management of the problem-solving tasks addressed by the participants were identified. Then, it was compared with the written works submitted as a final document to verify that these aspects were reflected in their final works and/or complement consider other aspects that emerged. After viewing each group videoconference', the first author read each group's written work submitted and identified all the criteria they had considered to plan the problem tasks. The first author classified the evidence extracted from the written work in an Excel table according to the criteria. For instance, if the group considered starting the task with the teacher asking questions to activate the children's previous knowledge, this evidence went to the row of cognitive criteria or Cognitive Suitability (CoS); if the tasks designed were 3 different types of problem task (representative of the different kind of problems studied in the paper of the Task A), this evidence was intended for the row of Epistemic Suitability (EpS); if a certain children's organization (in pairs or cooperative groups, for example) to solve the tasks is considered, this evidence was wrote down to the row of Interactional Suitability (InS); if they planned to offer manipulative resources for modelling and problem solving, this evidence went to the row of Mediational Suitability (MeS); if they showed were worried to design tasks engaging enough to involve children, this evidence was included in the row of Affective Suitability (AfS); or if they justify the design quoting some part of the curricula, the evidence belongs to the Ecological Suitability (EcS), etc. When this first classification was completed, the three authors individually saw all the videoconferences to compare each group's discussions and agreements with the classification in Excel. Then, the authors refined it together.

In the second phase, the authors systematically applied the indicators in Table 1 to evidence the aspects that the students considered necessary for the design and management of their resolution problems tasks. This evidence had been yet classified in Excel, above mentioned, based on the type of criteria (DSC). The evidence of each row (or DSC) was categorized with the corresponding indicator according to the questions in Table 1. For instance, the evidence that responds to the question: "Are manipulative and/or technological materials provided, or is their use recommended?", which could be the list of the manipulative resources the future teachers include in their tasks design, was categorized with the indicator (MeS1). After the categorization, the authors analysed in a descriptive and interpretive way which criteria could be identified with the DSC and which aspects of these were not contemplated.

The third phase was focused on the categorization revision. Also, in this phase, the identification, analysis, and interpretation of all the evidence were triangulated with a recognized researcher who is an expert in using the DSC. The expert revised and guaranteed that all the evidence in Excel had the right indicator (when necessary, the works submitted by the students' groups, or their videoconferences were revised too) and then participated in discussion with the three authors to describe and interpret the results. In the last phase, the three authors discussed the results.

RESULTS AND DISCUSSION

Although the participants were not given any training on DSCs, all groups implicitly used DSCs to justify decisions about the design of their tasks, but no group evidence is observed for all indicators. The indicators assigned were synthesized in Table 3.

Designs have high Affective Suitability (AfS), so all the indicators emerged. For example, in videoconferences, the 19 participating groups express the intention that the activities motivate students to participate in them (AfS3) and find them fun (AfS1) and allow them to express their results in their different ways (AfS2). To do this, seeking to activate empathy, characters such as the class mascot are incorporated, who supposedly live-in problematic situations and ask for help (AfS4), games are created, and stories, in their own words, "a story that calls them the attention" (AfS3), etc. For example, in one problem proposed the teacher receives a letter from the school cook' (Figure 1) in which she asks students for help to count the meatballs that she prepared for lunch.

Table 3. Overview of indicators assigned to the future teachers' designs

Indicators	Future Teachers' Working Groups																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
EpS1		*															*		
EpS2																			
EpS3																			
EpS4	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
EpS5	*					*	*	*		*	*				*	*	*		*
CoS1																	*		
CoS2	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
CoS3	*	*	*	*	*	*	*	*	*	*	*	*	*			*	*	*	*
CoS4	*		*	*	*	*	*	*	*	*	*	*		*	*	*	*	*	*
CoS5	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
InS1											*						*		
InS2	*	*	*		*	*	*	*	*	*	*	*		*	*	*	*		*
InS3											*					*	*		
InS4																	*		*
AfS1													*			*	*	*	*
AfS2	*		*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
AfS3									*	*	*	*	*	*	*	*	*		*
AfS4													*			*	*		*
AfS5	*		*	*	*				*	*	*	*	*	*	*	*	*		*
AfS6	*	*	*	*	*	*	*		*	*	*	*	*	*	*		*		*
AfS7	*	*	*	*	*				*		*								
MeS1	*		*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
MeS2		*	*	*	*	*	*		*	*	*	*	*	*	*	*	*		*
Me3S		*	*	*	*	*	*		*	*	*	*	*	*	*	*	*		*
MeS4																*			
MeS5	*	*	*	*	*	*	*	*	*	*	*	*				*	*	*	*
EcS1	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
EcS2																			
EcS3											*	*				*	*		*
EcS4		*	*	*	*	*					*					*	*		

The symbol * means the indicator has been assigned to the design.

Regarding Mediational Suitability (MeS), all groups organized the implementation time of their designed task in some parts. For example, one group said: "To work on numerical thinking, three sessions have been designed to solve numerical problems, with three phases of execution". Moreover, each session was organized in parts indicating the time the teacher should invest; for instance, Session 1: "To start, 15 minutes", "Teaching-learning, 20-25 minutes" and "To finish, 5-10 minutes. In addition, they plan the development of the session in phases. Firstly, the "Introduction", in which the teacher introduces the scenario of the problem. Secondly, the phase they called "Institutionalization", that they describe as following: "After presenting the scenario the teacher will present the problem and write it on the board so that the children can understand it. Children were given a few minutes to find a solution to the problem using different materials". Thirdly, the phase they called "Validation" to share, compare and discuss the solutions each other. And, finally, the “Devolution” to decide the better strategy and the correct solution of the problem, and when the teacher could intervene if realize something wrong. These pieces of evidence had the indicator (MeS2, MeS3 and MeS5).

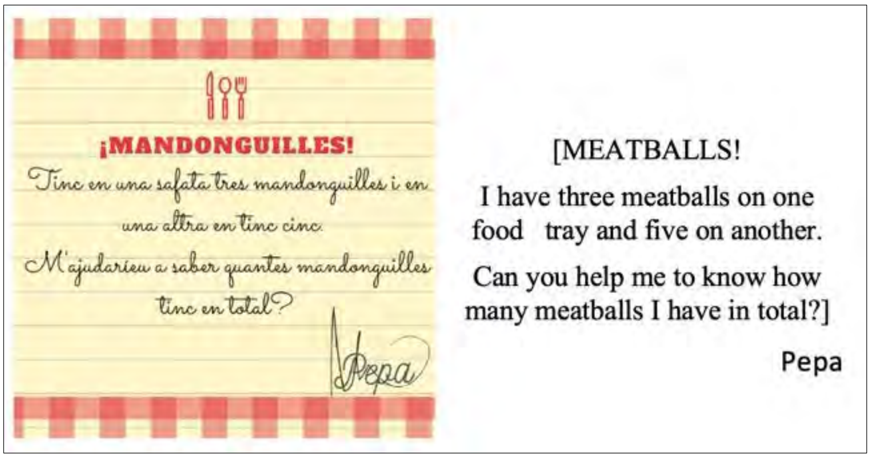

Figure 1. Message from Ms Pepa, the school cook, sharing her problem with the students

All the designs are considered to offer material resources to students to solve their problems. For example, the previous group mentioned was planning to offer the resources in the phase of implementation they called “Institutionalization” in all the sessions designed. Depending on the problem, they offer the resources (Pythagorean table, pencil and paper, Cuisenaire’s rods, dried beans, Parcheesi chips, multilink blocks, etc.). This evidence had the indicator (MeS1), although the future teachers did not justify their resources selection.

However, all the groups decided to include some material resources for children could experiment with strategies to solve the problems, all adhere to the resources seen in the training classes and in the paper of Task A and add non-specific materials (pencils, pasta, stones, chickpeas, etc.) and others related to the context of the problem (reproductions of fish, drawings of trees, and apples, sports medals, etc.). Only one of the guideline designs offers five specific material resources and a program that rotates through the small groups of students formed for each task (MeS1, MeS3 and MeS5). However, it is not contemplated that each student can choose and experiment with the material that he or she sees fit, and it is not considered that some of the resources are better than others depending on the problem to be solved. In addition, in all the designs except one, the activities are planned to be carried out entirely in the classroom environment, they did not consider it important to think about which could be the best space to implement the task. Only one design, which was the only categorized with the indicator (MeS4), schedules tasks on the playground and in the psychomotor room as well as in the classroom.

Regarding Epistemic Suitability (EpS), a large part of the designs, 14 of the 19, are concerned that the organization of the students and of the time includes a space for the explanation and argumentation of the process through which the answer has been found (EpS5). However, only two designs specifically explain how to manage it. The only one provides possible dialogues with good questions with the students to promote the generation of hypotheses, argumentation, and justification. Although future teachers have studied problem-solving from an investigative approach, and have analyzed an example, none of the designs considers it. All the groups work from verbal statements, with clear and straightforward questions (EpS1), and freedom is left for different proposals for resolution considering different ways to express and represent them to arise (EpS2) that will be put together (only 5 of the 19 designs do not foresee a pooling to explain the possible diverse resolutions of the students). Although the participants proposed different types of problems (EpS4), all of them are closed, with a single valid result (indicator EpS3 didn't emerge).

There is less evidence that students care about the cognitive adequacy (CoS) of their tasks (there was no evidence of indicators CoS3 and CosS4), but all designs consider introducing problems with a gradual increase in their difficulty (CoS2). In their dialogues, they start from what theoretically 5-year-old students should know according to the curriculum (EcS1) (for example, in their words: "they have to know how to count to 20", "we have to take into account the 5-year level"), but only one of the designs incorporates prior knowledge exploration activities. This only design, in the first activity, planned a phase before presenting the problem where it can be read: "To start the session, the first step will be to remind the children of their knowledge, so they will be asked to count out loud up to the number that is indicated, in this case, up to 10" (CoS1). In their dialogues, future teachers do not debate about which concepts the teacher is going to reinforce or systematize; they only say that "she will have a guiding role," "she will interact with the students," and "she will go around the groups to review the processes and results," "at the end the students will stand in a circle so that the teacher can clarify doubts." On the other hand, we have observed aspects related to the evaluation of students that, although implicit in cognitive (and ecological) suitability, are not explicitly contemplated with indicators in the DSC. The 19 works analysed are concerned with evaluating the students' achievement related to the learning objectives of the tasks (CoS5), with various proposals such as rubrics, lists of actions to observe, or lists of questions for students to answer. For example, one group proposed the teacher will have to ask children: "What does it mean when I say "add"? and to assess the comprehension of the concept of adding she "will have to evaluate the children's interest to answer the question".

Regarding the Interactional Suitability (InS), all the students' designs organize the activity to solve the problem in small groups, arguing: "so that they help each other" (InS1 and InS2) but they didn't plan how to manage these moments of dialogue and discussion. When they want to increase the difficulty, they propose that they be individually resolved. However, no design reflects how to get 5-year-old boys and girls to collaborate and, therefore, no working group considered that the activity implies that they must negotiate meanings and promote it. None of the analysed designs is the designed activities self-validating; that is, the validation of the process and the results must always go through the teacher's approval. So, we didn't find evidence of the indicators InS3 and InS4.

Evidence of Ecological Suitability (EcS) is obtained since curricular objectives and competencies are incorporated for the second cycle of early childhood education, specifying them for each activity (Ec1). However, none of the groups attempts to articulate different mathematical contents than numbering since, apart from the typology of problems studied, no other concepts are incorporated (such as, for example, logic, which is a block of knowledge dealt with in the course in the months before this work), or other knowledge areas (no evidence of EcS2). The statements of the problems contain elements those future teachers think are every day for children but are based on imaginary stories with children's characters, more in the sense of activating empathy and obtaining a high emotional fitness. For example, the problem in Figure 2 pretends to be a situation close to students' day to day, but the character is not a person, is fair, and the image of the nest associate doesn't correspond to a right nest of the mentioned bird species in the statement.

El niu de l'oreneta tenia 3 ous, però la fada Flora s'ha llevat aquest matí, i n'ha vist 4 més, quants ous hi ha en total ara?
[The swallow's nest had 3 eggs, but when the fairy Flora woke up this morning, she saw 4 more, how many eggs are there in total now?]

Figure 2. The problem proposed in a work analysed and their associated image

Only 3 of the 19 designs are based on real and semi-real school contexts (EcS3 and EcS4): athletics games at school, activities in the school playground, and a trip to a lake. However, they are only motivating contextual excuses to put elements in the statements of the problems, and in no case are the activities proposed in an interdisciplinary way. For this reason, the indicator (EcS2) was not assigned, as you can see in Table 3.

CONCLUSIONS

It can be affirmed that, in the training of early childhood education teachers, it is essential to promote dialogue to make consensual decisions in the design of tasks for the teaching and learning of suitable mathematics, since group reflection enables the generation of proposals, opinions, and decisions taken as a group.

Regarding the first and second objectives of the study, the aspects that future preschool teachers consider essential when planning and designing activities with problem tasks and which of these are identified with the DSC, future teachers are implicitly based on criteria identified with the DSC. Thereupon, all the designs presented high Affective Suitability (AfS), for which all the indicators emerged. This result corroborates one of the most relevant research results around the last 45 years: the strong tendency to consider affective and emotional aspects of students' mathematical learning (Gutiérrez & Boero, 2006).

Concerning the third objective, not all its indicators emerge since their reflection is spontaneous and is not guided by specific criteria because they do not have an explicit guideline to guide their didactic analysis. On the one hand, from the epistemic point of view, the result that the future teachers did not contemplate in their proposals for the resolution of problems from an investigative approach can be explained by the difficulty that the teachers or future teachers present, which agrees with the international results that the teachers they present difficulties in interpreting epistemic aspects of the tasks and identifying their educational potential (Stahnke et al., 2016). On the other hand, Ecological Suitability indicators concerning interdisciplinarity tasks and the promotion of making connections between different mathematical contents did not emerge either.

This result agrees with the conclusion reached in recent works such as Pino-Fan et al. (2020), Breda (2020), Breda, Pino-Fan, Font, et al. (2017) and Sánchez et al. (2021). In this sense, it is observed that DSC appears as a regularity when teachers or future teachers want to justify the criteria on which their decisions are based without being taught the use of this construct. The reason could be related to the fact that DSCs reflect consensus on what good mathematics teaching should be, widely assumed by the educational community. From a didactic point of view, this study offers indications that it would be convenient to offer future early childhood education teachers a tool such as DSC so that they have explicit criteria to guide the designs of their mathematical tasks. In this sense, a future line of research opens, much needed, to adapt the DSC to the singularities of this educational stage.

Acknowledgements

This work was supported by the MCIU/AEI/FEDER UE, under the Research Project PGC2018-098603-B-I00 and ANID PAI77200008.

Declarations

Author Contribution	:	GSS: Conceptualization, Methodology, Writing - Original Draft, Formal Analysis, Writing - Review, and Editing Revised Version. AB: Conceptualization, Data Gathering, Formal Analysis, Review, Validation, and Supervision. DF: Methodology, Review, Editing, and Formal Analysis.
Funding Statement	:	This work was supported by the MCIU/AEI/FEDER UE, under the Research Project PGC2018-098603-B-I00 and ANID PAI77200008.
Conflict of Interest	:	The authors declare no conflict of interest.

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