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Math is beautifully intimidating: Analyzing the conflict between teacher affective disposition and observed positioning-by-others

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ABSTRACT

Teacher productive disposition is considered as one of the key strands of mathematical proficiency. Teacher disposition and positioning (disposition in action) toward mathematics influence student learning. However, teachers' productive disposition does not always translate into productive positioning in the mathematics classroom, and vice versa. In this study, we selected teacher dis/position as the unit of analysis to explore the phenomenon of two middle school mathematics teachers' self-reported affective disposition and observed positioning-by-others. Grounded in positioning theory the relationship between teacher disposition and positioning-by-others was examined utilizing a cross-case analysis. Results of the study indicate that dispositional characteristics such as attitude, self-concept, and nature of mathematics were significantly different between the cases. The study findings also suggest that interconnectedness between teacher core disposition and positional situatedness could potentially contribute to understanding and addressing the complexity of teaching and learning in the mathematics classroom.

Keywords: complexity of education, teacher affective disposition, positioning-by-others, self-reported disposition

INTRODUCTION

What teachers say does not necessarily correspond to what they do in the classroom. As depicted by Davis et al. (2020), the literature suggests that there is a "growing gap between what we thought we were hearing and what we were confident we were seeing" (p. 410). The question of why productive disposition does not always translate into productive positioning remains unanswered. We attempt to answer that question (a gap in the research literature) by using multiple sources of data to report noticeable gaps between teachers' beliefs and what is occurring within the classroom (situated teacher). Davis et al.'s (2020) findings support our notion that self-reporting by teachers and what is observed in classrooms is widely diverse resulting in gaps between reported and observed data; Davis et al. (2020) suggest "inviting others into the discussion of the complex entanglements of these matters" (p. 411). Responding to this invitation, we extend the discussion to unpacking the relationship between teacher self-reported affective disposition and positioning-by-others as a critical element in understanding the why of the complexity of mathematics teaching and learning. The purpose for this study was to explore teacher reflection in action and on action, i.e., affective disposition and positioning toward mathematics. In an educational context, positioning is a habitual inclination formed by teacher and/or students to navigate through academic content, settings, and interactions (Bass & Mosvold, 2019). Conversely, disposition includes beliefs, but do not necessarily reflect actions.

In this study, dichotomous self-reported disposition and positioning-by-others were observed and analyzed through the criteria-based, purposefully selected case studies of two middle school mathematics teachers (Sage and Thyme, pseudonyms) This study sought to explore the interconnectedness of disposition and positioning and associations (or lack thereof) of disposition and positioning constructs on teaching and learning, recognizing the significance of unique contextual variables such as, languages, ethnicities, and economics in a US southwest border region. The aim of the study was twofold:

1. To document self-reported and observed middle school teachers' affective dis/positional characteristics within the context of mathematics, and its teaching and learning practices.
2. To analyze connections between teacher self-reported disposition and positioning-by-others.

Recognition of self-reported affective disposition and positioning-by-others is a critical element in understanding the complexity of the mathematics classroom that is conducive to the promotion of mathematics learning. Cai et al. (2017) have emphasized the importance of research that "impact practice by studying the specification of learning goals and productively aligned learning opportunities" (p. 342). Adding significant practical value to knowledge gained from research on teacher disposition and positioning may serve as a key component in the creation of productive learning opportunities and environments. Utilizing a cross-case analysis approach, the purpose of the study was to answer the following research questions:

1. What are the participating middle school teachers' self-reported affective dispositions toward mathematics, its learning, and teaching?
2. What are the participating middle school teachers' observed positioning-by-others?
3. How does self-reported teacher disposition compare to observed positioning-by-others?

This paper includes multiple sections. First, we provide an extended literature review of complexity theory and dis/position construct. Then we discuss the methodology of the study consisting of the research design, participants, procedure, data collection, and analysis. Finally, we will present the results of the study followed by a discussion and conclusion.

THEORETICAL BACKGROUND

The main unit of analysis of this study is teacher dis/position. Dis/position has two states: static disposition (e.g., I believe I am a good teacher) and dynamic positioning (e.g., I am practicing good teaching). The fluidity of positioning is rarely intentional and originates as a product of responsiveness to moments in time and varying environments. Fels (2004) stated,

> "Becoming a teacher is a curricular adventure, a generative framework of possibility that invites stops, interruptions, hesitations, elated moments of recognition, loss, and recovery; a unique journey shared by educator and students across an emerging landscape that unfolds with each footstep" (p. 88).

Kilpatrick et al. (2001) identified productive disposition as one of five strands for mathematical proficiency. They defined this construct as "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (p. 131). This study exemplified the presence of productive and nonproductive dispositions as influenced by positioning and positioning-by-others. This dynamic creates storylines based in discourse and context; and on the connections between the knower, knowledge, and learner, accounted for by the intersection of dis/positioning with the phenomenon of complexity (Davis & Phelps, 2005). Rather than considering contrasting aspects of theory and practice, consideration of positioning and conflictive storylines, specifically the feeling of intimidation, provides an understanding of the intricacy of affective disposition.

Researching the phenomenon of teachers' self-reported disposition challenged by positioning-by-others is a critical component in studying dynamics of a teacher's role in mathematics classrooms. The dynamics and multifaceted nature of teacher disposition contribute to resultant emergent and shifting student mathematical disposition. Identification of patterns in teacher self-reported affective disposition, as well as the nature of positioning-by-others affords the necessity of consideration of the "who" and "why," in addition to the "what" and "how," of the instructional process. Without the inclusion of the "who" and "why", navigation in complex educational settings is limited.

Positioning Theory

Positioning theory (van Langenhove & Harré, 1999) provides a framework for evaluating the affective disposition characteristics and its manifestation in teacher self-positioning and positioning-by-others. As pertinent to the study, analysis of teacher dis/position focused on the social realm of positioning toward mathematics, mathematics teaching, and learning as informed by affective disposition characteristics. Utilizing the components of the positioning triad (e.g., position, social force, and storyline), positioning theory "pictures a dynamic stability between actors' positions, the social force of what they say and do, and the storylines that are instantiated in the sayings and doings of each episode" (van Langenhove & Harré, 1999, p. 10). Encompassing diversity of purpose, positioning theory allows for framing research focus and concepts within social discourse analysis of affective teacher disposition and positioning. Disposition is not equivalent to positioning. The conceptual difference between disposition and positioning is that disposition is more stable and habitual (Beyers, 2011) whereas positioning is dynamic and situational (van Langenhove & Harré, 1999).

Disposition and positioning occur at multiple levels of perspective and productivity, accounting for, and as an interaction/reaction with extraneous variables. A lack of awareness of self-reported disposition and positioning-by-others contributes to the challenges of navigation in the complex realm of educational context (Davis, 2008).

Teacher Affective Disposition

In a review of prior studies, evidence documents a relationship between teacher and student affective disposition, but to what extent and encompassing the characteristics and factors had not been sufficiently substantiated. According to Burton et al. (2022), key to researching affective disposition is identifying and being aware of the connections between dispositions and educational practice. Beyers' (2011) synthesis of the literature identified two key impacts of disposition on learning: (1) "(…) teachers play an essential role in shaping students' dispositions -affect student learning by means of opportunities to learn" (p. 70). To collect the data needed for identification and analysis of characteristics of affective disposition, the phenomenon of affective disposition was thematically categorized as nature of mathematics, self-concept, attitude, worthwhileness, usefulness, sensibility, and anxiety (adapted from Beyers, 2011).

Table 1. Operationalized definitions of affective disposition characteristics

Characteristic	Definition	Examples of Semantic Expressions
Nature of mathematics	A belief about mathematics being procedural/conceptual, logical/irrational, precise/chaotic, beautiful/dull, intellectually challenging/boring, creative/mundane, concrete/abstract, etc. It also could include the acknowledgment that mathematics plays/does not play a central role in modern culture with its broad/narrow range of applications.	Procedural/conceptual, logical/irrational, precise/chaotic, beautiful/dull, intellectually challenging/boring, creative/mundane, concrete/abstract, computational, algorithmic, rule-based/intimidating, etc.
Worthwhileness	A value judgment that the time and/or effort spent engaging in mathematics has an intrinsic/extrinsic payoff/penalty leading to increased/decreased interest in mathematics.	Worthwhile, payoff, worthless, etc.
Usefulness	A belief about the contribution/detraction of mathematics for meeting current or future need, performance, success, etc. It also could include the acknowledgment that mathematics plays/does not play a central role in modern culture with its broad/narrow range of practical/impractical applications.	Utility, useful, advantage, serviceable, practical, purposeful, etc.
Sensibleness	A belief that mathematics is (un)reasonable, understandable/confusing, meaningful/meaningless, (dis)connected, etc.	Un/reasonable, ir/rational, un/wise, im/prudent, in/coherent, un/realistic, un/sound, il/logical, etc.
Self-concept	One's subjective feelings, ideas, and/or self-perception as a confident/insecure learner/user/knower of mathematics, as well as the ease/difficulty or (dis)comfort that one experiences with mathematics.	Confidence, insecurity, intimidation, comfort, discomfort, easy, hard, difficult, etc.
Attitude	A (un)favorable state of mind/view and/or a positive/negative feeling influencing an emotional (re)action toward a (dis)investment in mathematics.	Like, hate, do not like, love, favorite, least favorite, dis/tasteful, etc.
Anxiety	Experiencing an unpleasant/threatening/stressful/apprehensive psychological/physiological reaction resulting from engagement in mathematics.	Intimidation, fear, threat, danger, conflict, nervous, apprehension, stress, distress, scared, lack of safety, frustration, worry, angst, suffering, agony, misery, grief, anguish, etc.

Table 1 provides operationalized definitions of affective characteristics along with examples of semantic expressions used in the study. The National Council of Teachers of Mathematics (NCTM) identified characteristics of productive dispositions to include usefulness and worthwhileness, sensibleness, persistence, self-efficacy, flexibility, interest, curiosity and inventiveness, and reflection (Gerson et al., 2011). Further refinement of criteria designating productive and nonproductive teachers was necessary to address the key construct (**Table 1**). A teacher with productive disposition analyzes, designs, and constructs outcome-based teaching to provide effective learner-centered environments and experiences for student success, whereas a nonproductive teacher establishes a teacher-centered environment, which limits learning opportunities (Lynch-Arroyo, 2013). The importance of distinction between a productive and a nonproductive teacher is supported by the research of Schukaljow et al. (2012), asserting "the data analyses show that the student-oriented, 'operative-strategic' form of teaching tended to have stronger effects on students' enjoyment, value, interest, and self- efficacy, as compared with 'directive' teaching" (p. 231). Within the complex nature of education, practitioner awareness of disposition and positioning-by-others is acutely necessary to achieve effective teaching and learning.

METHODOLOGY

Research Design

The cross-case study provided a venue to observe critical variances in self-reported disposition and positioning-by-others, as well as the lack of teacher self-awareness of the variances. The selection of the case study method was appropriate because our study, as suggested by Eisenhard (1989) and Yin (1994), was intended to investigate a complex social phenomenon (self-dis/position in mathematics education) in real-life contexts (middle school math classrooms). Moreover, the case study method allows researchers to observe formal as well as informal processes (e.g., social interactions) and collect a wide array of data (Hartley, 1994). The case study approach in this investigation allowed multifaceted and authentic data collection and analyses of affective disposition characteristics, the conflict of self-reported disposition and positioning-by-others, and the resulting complexity involved in the positional conflict.

Embedding case study within the framework of positioning theory allowed for the identification of "dynamic stability between actors' positions, the social force of what they say and do, and the storylines that are instantiated in the sayings and doings of each episode" (van Langenhove & Harré, 1999, p. 10). Within our thesis, encompassing the diversity of purpose, positioning theory allowed for framing the research focus and concepts within social discourse/meaning-making analysis utilizing constructs of affective disposition to ascertain self-reported disposition and positioning-by-others (**Figure 1**).

From a larger sample of an initial study of middle school teachers' disposition toward mathematics, two teachers were identified as cases to be analyzed in depth due to their self-reported dispositions and positioning that were opposites of each other. Through purposeful selection of cases that represented both sides of the positioning continuum, interconnectedness between teacher core disposition and positional situatedness could be cross analyzed.

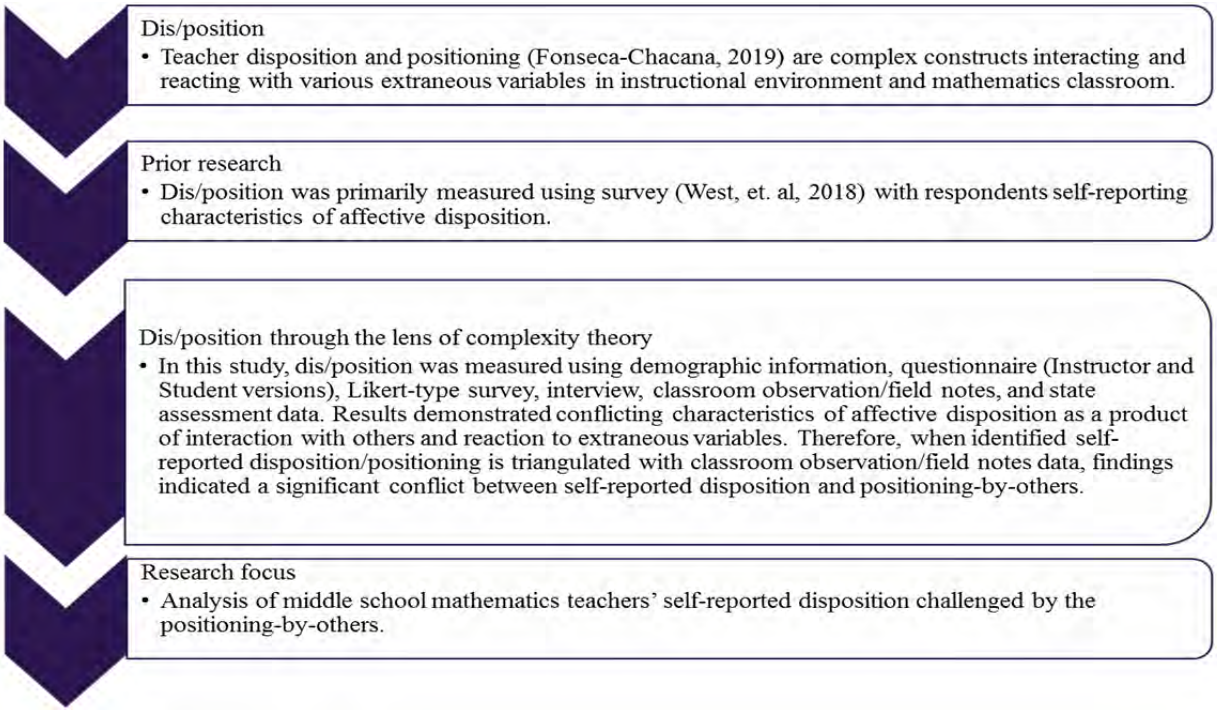

Figure 1. The thesis of the study (Source: Lynch-Arroyo, 2013)

Setting and Participants

Utilizing the researcher-developed written questionnaire using metaphors to ascertain productive and nonproductive positioning/disposition and thematic coding of narrative responses, two independent cases were selected based on dichotomous variability in participants' responses. The case studies of Sage and Thyme provided an observational window into two unique middle school mathematics classroom experiences.

Sage and Thyme taught in the same large urban school district (50,000+ students) and middle school campus (2,000+ students) in a border region of Mexico and the United States i.e., a homogeneous sample in terms of demographic and environmental criteria. Both Sage and Thyme were middle grade, female mathematics teachers and possessed middle school mathematics teaching certifications. Each teacher had accumulated over twenty years of teaching experience. Sage had been teaching for thirty-five years and Thyme had been teaching for twenty-six years at the time of the study. Sage is a native of the region and Hispanic. Thyme is a native of the upper Midwest United States and white. Both teachers have spent their entire teaching careers in the same district at a variety of middle school and high school campuses.

Data Collection

Data sources consisted of demographic information, questionnaire (Instructor and Student versions), Likert-type survey, interview, classroom observation/field notes, and state assessment data. Implemented in the study were two primary data sources: researcher-developed questionnaire and Likert-type survey. The use of a researcher-developed questionnaire (**Appendix A**) allowed for a conceptualization of reality utilizing open-ended metaphor prompts and Likert-type survey for data collection encompassing the context and meaning of participant responses/storylines. Stages of data collection, coding, and analysis built a scaffold conducive to valid data interpretation leading to key conclusions for each stage. The collection, analysis, and synthesis of the data required a holistic approach reflective of the authentic responses/storylines of the participants. The second data source was a Likert-type survey (**Appendix B**) generated from the analysis of questionnaire results.

Considering the complexity of the main construct of the study, as well as differences between disposition and positioning discussed previously, the data were collected from multiple qualitative and quantitative sources. The researcher-developed Likert-type survey, the open-ended questionnaire using metaphorical prompts, teacher interview, classroom observations/field notes conducted by a nonparticipant researcher served as sources of identifying and analyzing teacher dis/positioning.

Table 2 captures and describes data sources for measuring teacher self-reported disposition and positioning-by others.

Questionnaire: Open-ended metaphor

Bullough Jr (2014) suggests that in each practitioner action there is an implicit 'root metaphor,' which works as a private theory explaining and shaping the assumptions made within the boundaries of everyone's worldviews. Bullough Jr (2014) explained:

> "Behind every educational decision reside 'root metaphors' ... tacitly shaping thought and action. Metaphors, particularly root metaphors, operate as implicit theories, loose schemas, that shape how the world of people and things is understood and establish boundaries for meaning making" (p. 161).

Table 2. Data sources for measuring self-reported disposition and positioning-by-others

Teacher self-positioning	Teacher positioning-by-others (students)
Questionnaire–Teacher version - Four open-ended opinion questions and nine metaphorical prompts.	Questionnaire–Student version - Three open-ended opinion questions and nine metaphorical prompts.
Likert-type survey - Three Likert-type ratings assessing positioning toward mathematics, mathematics learning and mathematics teaching	Likert-type survey -Three Likert-type ratings assessing positioning toward mathematics, mathematics learning and mathematics teaching.
Interviews. - Series of three paired questionnaires and Likert-type survey rating individual-specific responses shown to the interviewee. - Interviewees asked to identify any differences or similarities in the responses and describe what they thought the differences or similarities meant. - Based on responses, clarifying interview questions posed.	Interviews - Series of three paired questionnaires and Likert-type survey rating individual-specific responses shown to the interviewee. - Interviewees asked to identify any differences or similarities in the responses and describe what they thought the differences or similarities meant. - Based on responses, clarifying interview questions posed.

In this study, participants' root metaphors were grounded in personal experiences tacitly shaping their disposition. Examples (from the supporting data) driving the research impetus include the following self-reported teacher excerpts:

> "If mathematics were the weather, it would be a tornado. Although necessary, it seems to leave a trail of destruction."
>
> "If mathematics were a plant, it would be poison ivy. There are people who do not ever want to get near it."
>
> "If mathematics were a plant, it would be a Venus Fly Trap. It has been known to swallow children whole!"

The researcher-developed questionnaire had two versions: one for the teacher and a second one for the students (instructor version is included in **Appendix A**). Each questionnaire consisted of three sections: demographic information, open-ended questions, and metaphor prompts. The questionnaire provided data to evaluate and measure self-reported affective disposition. Although some scholars have questioned the use of metaphors (Handal, 2008), others consider metaphors as a powerful means for identifying and interpreting teacher and student learning, disposition, and reasoning (Abrahamson, 2010; Cai & Merlino, 2011; Karolia, 2022; Presmeg, 1992, 1998). The application of open-ended metaphorical prompts afforded an opportunity for the authenticity of responses. Abrahamson (2012) argues, for teachers and students, metaphors are tools to convey to others what and how they see things. In this sense, the use of metaphors gave teachers voice and provided the researcher with a vehicle to gather and analyze data from the perspective of the responder i.e., conscious, and unconscious positioning.

Likert-type survey

The purpose of the Likert-type survey developed by the researchers (**Appendix B**) was to confirm or deny productive or nonproductive positioning, as identified in the researcher-developed questionnaire, specifically the identification of positioning toward target social forces–mathematics, mathematics teaching and mathematics learning. The scale utilized consisted of a continuum from one to five with one being the least productive and five representing the most productive and included a request for narrative of ratings chosen. The Likert-type responses were a basis for the development of the follow up semi-structured interview prompts. In clarification, although Likert-type data were quantitative by nature, responses did include descriptions and explanations of responses requiring reflective analysis.

Semi-structured interview

To clarify differences and similarities in the Likert-type survey and researcher-developed questionnaire responses, Sage and Thyme were interviewed. The design of the interviews provided venues for in-depth study, to collect additional qualitative data, and to confirm researcher initial coding, rating, interpretation, and analysis (Carey, 1994; Miller & Glassner, 2004). In the interviews, responses to linked researcher-developed questionnaires and Likert-type survey responses were shown to interviewees. Interviewees subsequently identified any differences or similarities in the responses and described what they thought the differences or similarities meant (**Appendix C**).

Classroom observation/field notes

Classroom observations/field notes provided the venue to identify teachers who make those connections and teachers who do not make connections (content, real-world, prior experience) with students. Dragon et al. (2008) state, "Classroom observations are a useful exploratory strategy because human observers can intuitively discern high-level behaviors and make appropriate judgments on limited information" (p. 30). Classroom observations/field notes conducted by a nonparticipant researcher were used to identify any self-positioning movement over time, but more importantly, positioning-by-others that contributed to a phenomenological conflict between teacher self-positioning and positioning-by-others.

State assessment data

For purposes of triangulation of multiple instrument findings, student state assessment data were reviewed for each case study. State assessment data were considered outcome data reflective of self-positioning and positioning-by-others.

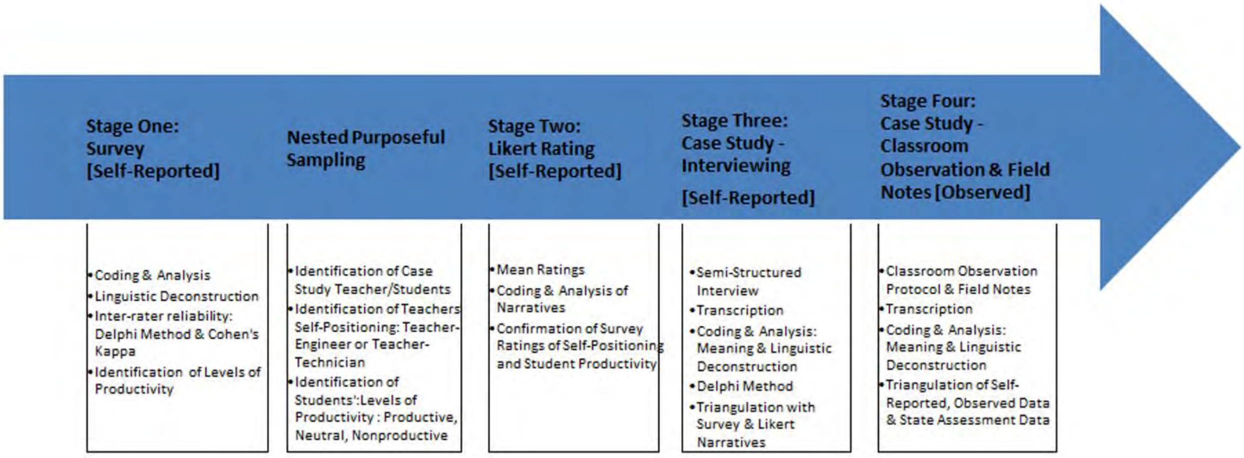

Figure 2. Research procedures (Source: Lynch-Arroyo, 2013)

Table 3. Teacher affective disposition characteristics and its descriptors

Characteristic	Productive dis/positioning	Nonproductive dis/positioning
Nature of mathematics	Focuses on concepts Learning by doing mathematics Learner-centered approach	Focuses on procedures Learning by rote memorization Teacher-directed approach
Worthwhileness, usefulness, & sensibleness [W.U.S.]	Creates learning opportunities Sense making and understanding Uses real-world applications	Limits learning opportunities Focuses on teaching to the test Lacks contextual relevance
Self-concept	Facilitates and shares knowledge Encourages collaboration Possesses content mastery Becomes a resource for other teachers	Regurgitates information Limits collaboration Possesses weak content knowledge Not a resource for other teachers
Attitude	Utilizes constructive and effective criticism Accepts challenge Enthusiastic Believes all students can learn mathematics	Utilizes destructive and ineffective criticism Avoids challenge Non-enthusiastic Believes mathematics ability is innate
Anxiety	Possesses productive anxiety Open-minded Inspired	Possesses non-productive anxiety Has fixed mindset Uninspired

Data Analysis

Consistency in definitions was present throughout the study when identifying and coding affective disposition characteristics for both teachers and students. Affirming the importance of student affective disposition, "(…) the National Research Council recognized the affective component of learning mathematics saying, '[s]tudents' disposition toward mathematics is a major factor in determining their educational success" (Bishop, 2012, p. 35). Grounded in this perspective, affective dispositional characteristics, within the context of the complexity of education, are reflective of conflictive self-reported disposition and positioning-by-others.

Stages of data collection, coding, and analysis are depicted in **Figure 2**.

Analysis of the Likert-type survey and researcher-developed questionnaire used a quantitative scale of disposition (from 1 to 5) along with a qualitative analysis of teacher open-ended responses. Interview narrative, classroom observation/field notes data were used for confirming data analysis. Linguistic deconstruction, meaning making, and discourse analysis (Kvale & Brinkmann, 2009) were the main tools of coding the qualitative data. The analyses of data substantiated the cases of Sage and Thyme via manifestation of affective self-reported disposition and observed positioning-by-others.

For uniformity of responses, narratives from the questionnaire and Likert-type survey were linguistically deconstructed and compared. The targets as social forces (mathematics, mathematics learning, and mathematics teaching) were used to establish parameters for analysis (Weston et al., 2001) leading to a thematic technique of coding. "A complex system is characterized by emergent behavior resulting from the interaction among its parts (…)" or for purposes of the study being reported, emergent teacher positioning rose from "(…) dynamics of interaction" (Ferreira, 2001, p. 1).

Each episode of the storyline was deconstructed using linguistic analysis and meaning coding (Kvale & Brinkmann, 2009), recognizing internal and external cues. Independent expert raters determined levels and intensity of productive and nonproductive dispositional inclinations from researcher-developed questionnaire responses. As described in **Table 3**, thematic coding was considered not as a precursor to analysis, but as a unit of analysis and the "best representation of (…) thinking about the phenomenon at a particular time" (Weston et al., 2001, p. 391)

Table 4. Samples of teacher responses on non-productive/ productive continuum

Non-productive disposition	→	Productive disposition
If mathematics were a plant, it would be … poison ivy. There are people who do not ever want to get near it. *Target: Mathematics*		If mathematics were an animal, it would be … a Panda Bear. Big and Cuddly. *Target: Mathematics*
If mathematics were a plant, it would be … Venus Fly Trap. It has been known to swallow children whole! *Target: Mathematics learning*	If mathematics were a plant it would be … Venus Fly Trap. The students are like the flies hovering around the plant (math). When they get a concept, they are secure in their learning (the plant closes on the fly). *Target: Mathematics learning*	If mathematics were a plant, it would be…rose. A rose starts out as a bud. As lighter and water is given to the rose, it starts to bloom into a beautiful plant. *Target: Mathematics teaching*
If mathematics were the weather, it would be…tornado. Although necessary it seems to leave a trail of destruction. *Target: Mathematics*		If mathematics were the weather, it would be…a combination of bright and glowing days. *Target: Mathematics*

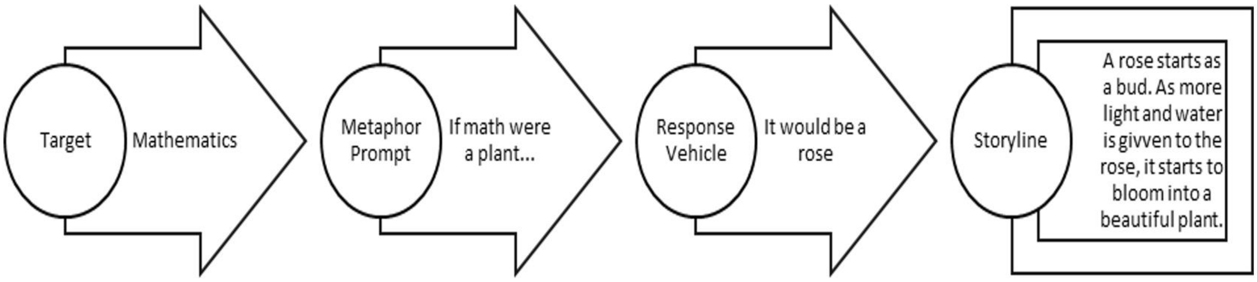

Figure 3. Example of metaphor structure: linguistic deconstruction (Source: Lynch-Arroyo, 2013)

Therefore, the focus of data analysis was on the meaning coding of the characteristics of affective self-reported disposition and positioning-by-others using descriptors presented in **Table 3** as depicted in **Table 4**.

As part of the interview process, the Likert-type survey was utilized as a source to cross-analyze coding and rating of the researcher-developed questionnaire using metaphor responses. Interview prompts grew from individual responses to the researcher-developed questionnaire and Likert-type survey, identifying convergent and divergent response elements of discourse. Researcher-developed questionnaire and metaphorical prompt responses/storylines were coded according to levels of defined affective dispositional constructs; additionally, expert raters using descriptors ranked responses as presented in Table 3. Likert-type surveys and storylines were numerically ranked, examined through linguistic deconstruction, and comparatively analyzed with the researcher-developed questionnaire results.

Interview transcripts, classroom observations/field notes were analyzed using descriptors of affective dispositional characteristics. Meaning coding and frequency of response/observation measurements were utilized to identify themes of affective disposition reflective of productive and nonproductive positioning. Student level state assessment data were collected to provide additional quantitative data as a source of indirect positioning-by-others. Through the analysis, findings supported that an open-ended researcher-developed questionnaire using metaphor, as opposed to a closed-ended Likert-type survey, allowed participants to express themselves authentically and not locate their response on a predetermined scale (Lynch Arroyo et al., 2012).

The structure of metaphor consists of a target, a prompt, a vehicle, and a storyline (Cameron & Maslen, 2010). **Figure 3** presents an example of the application of the metaphor structure using linguistic deconstruction of one of the teacher responses.

Linguistically deconstructed open-ended storylines facilitated two phases of coding and rating:

(1) rating of affective dispositional inclinations on a continuum of one to five within operationally defined categories of productive to nonproductive disposition, and

(2) meaning coding of participant responses as a storyline.

Inter-Rater Reliability

The resulting questionnaire data were comparatively analyzed for reflections of Likert-type survey ratings. The intent was to confirm and elaborate productive or nonproductive positioning. The design of the rating scale was applied consistently to meet a condition of using Cohen's kappa. For education research, an acceptable kappa value is 0.70. Differing from other inter-rater statistical tests, Cohen's kappa takes into consideration agreement that occurs by chance. The basic structure of calculating kappa involved establishing a proportion of actual agreement among raters adjusted by the proportion of chance agreement (**Figure 4**).

Three expert raters participated in a consensus process, using the Delphi standard as compared to questionnaire ratings, with no more than a plus or minus one-point deviation in divergent Likert-type survey overall ratings of productivity. Key to the Delphi process was independent ratings without consultation or influence (Helmer, 1967; Hsu & Sandford, 2007; Rowe & Wright, 1999; Skulmoski et al., 2007). The three expert raters were all within the field of mathematics education. For purposes of confirmation, self-reported Likert-type survey ratings were specifically compared to questionnaire ratings for prompts, as depicted in **Table 5**.

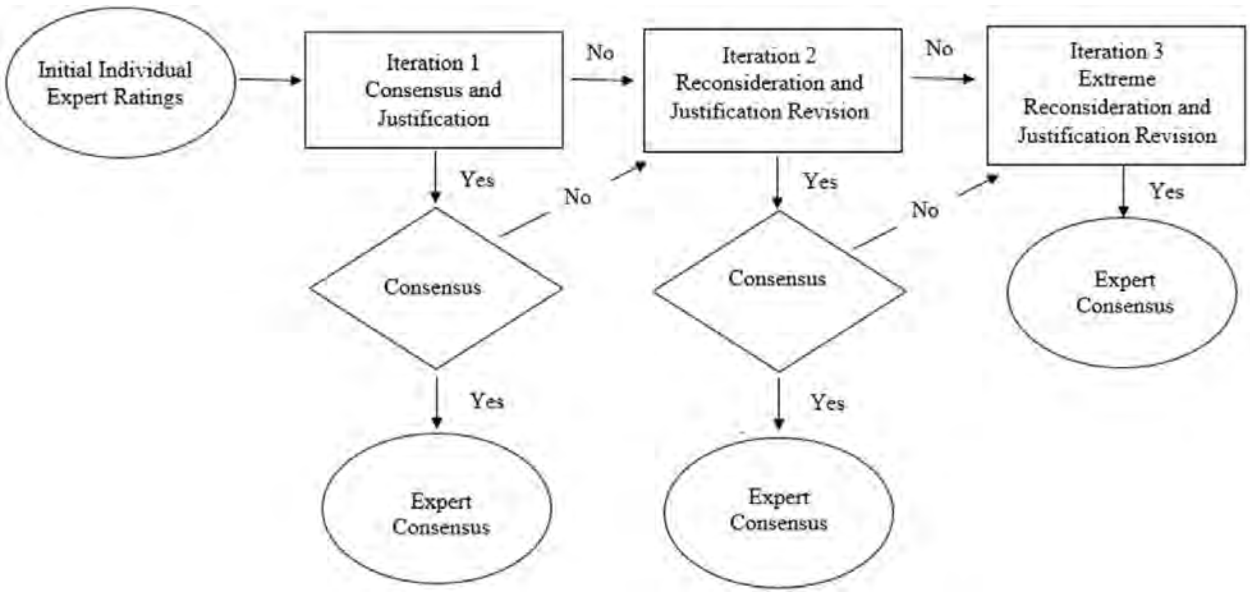

Figure 4. Delphi process (Source: Lynch-Arroyo, 2013)

Table 5. Comparative open-ended questionnaire and Likert-type survey data

Participants	Questionnaire (open-ended)	Survey (Likert-type)
Sage	3.67	4.00
Thyme	2.67	3.67

Table 6. Overall experts' mean ratings of linked teacher and student disposition using metaphor questionnaire

Participants	Teacher disposition mean rating	Student disposition mean rating
Sage	2.67	2.58
Thyme	3.33	3.19

Open-ended questionnaire items and metaphorical responses were linguistically deconstructed to facilitate rating of affective dispositional inclinations on a continuum of one to five within operationally defined categories of productive to nonproductive disposition. Three independent expert raters determined levels and intensity of dispositional inclinations from questionnaire responses. Of the three expert raters, one holds a Ph.D. in mathematics education and the other two raters were doctoral students at the time of the study with concentrations in mathematics education. According to operationally defined affective characteristics, open coding of self-reported and observed data included frequency and meaning coding. Inter-rater reliability was achieved through use of the Delphi Method of consensus and Cohen's Kappa accounting for three raters. Cohen's Kappa was noted at k=.875 (within the range of 'almost perfect agreement'). Utilizing operationally defined affective characteristics, open coding of self-reported and observed data included frequency and meaning coding.

RESULTS

Affective dispositional characteristics, within the context of the complexity of education, were critical factors in the identification of conflictive self-reported disposition and positioning-by-others. According to the data, the disposition rates ranged from 2.67 (non-productive) to 3.33 (productive). As shown in **Table 6** and to illustrate the nature of the dichotomous positioning and complexity, we refer to the two case studies of Sage and Thyme.

The two case studies rendered not only the presence of productive and nonproductive dispositions but also the fluidity of positioning by teachers between dispositional stances that influence the learning opportunities. Sage is nonproductively positioned-by-others (students) as a 'sage on the stage' and Thyme, in response to the classroom discourse, integrates a student-centered approach utilizing instructional 'time' productively (hence the pseudonyms). Emerging from the case study analysis, Sage is not cognizant that she self-positions herself productively while simultaneously was positioned by others as nonproductively utilizing teacher-centered, traditional pedagogical approaches. Conversely, Thyme described herself using nonproductive characteristics based on her insecurities about mathematics content mastery ("Math is beautifully intimidating"). Yet, simultaneously, Thyme was positioned by others as productive in facilitating learning through the creation of a student-centered learning environments. This conflict in positioning is rooted in the complexity of education itself (Davis, 2005), fluid teacher positioning or disposition in action (Harré, 2011, 2012; van Langenhove & Harré, 1999), and the dynamics of affective disposition (Beyers, 2011; Kilpatrick et al., 2001).

Table 7. Examples of Sage's influencing experiences

Self-reported experiential influence (data source–teacher interview)	Self-reported productive positioning (data source–teacher interview)	Observed teacher nonproductive positioning (data source–field notes)
Most memorable math experience: "Because my 3rd-grade math teacher just inspired me I guess on that particular day … just by her doing manipulatives."	"I use the manipulatives, I … especially in higher algebra problems. Ah, I do not have them out because we're done through with them. But I do have lots of manipulatives …"	Manipulatives are not a primary component of the teaching process, even though her most memorable positive math experience was a 3rd-grade teacher who used manipulatives. She 'guesses' that she was 'inspired' by this 3rd-grade teacher–doubt exists and is possibly in conflict with her traditional direct instruction teaching style.
"I'm anti-computer." "I tell the students the computer only does what you tell it to do. So, if you do not have the knowledge up here, the computers are not going to do anything for you."	"Here's something else I feel about teachers and the computer. The computer can be unplugged. I cannot be … until I die. [laughter] That's the theory that I say to them all the time."	Technology not integrated other than worksheets projected. Integration of technology seems to be in conflict with participant's self-concept and attitude, as well as how she positions herself as 'Sage-on-the stage'–computers cannot replace her knowledge and experience.

Table 8. Examples of Thyme 's influencing experiences

Self-reported experiential influence (data source–teacher interview)	Self-reported productive positioning (data source–teacher interview)	Observed teacher nonproductive positioning (data source–field notes)
"I think I had poor experiences growing up I was a student in the 70's and early 80's where it was more drill and kill. They give you the same problem to solve over and over again. And I was not a very good math student because I did not … I was very bored."	"I think I'm a struggling learner, so I do not take things for granted… Because I'm going through the process of learning and so are the students. So, if I question something I know the students are questioning it."	Participant indicates that learning is facilitated by a project-based, kinesthetic, hands-on approach–"projects, building, hands-on", "hands-on experiences of mathematics", "kinesthetic." They need to be hearing it, seeing it, feeling it and then it becomes internal and "visualize in their mind"
"… one of my favorite classes was methods of mathematics, was a required course, and I enjoyed that class because it studied the science of mathematics."	"I did not understand the problems. But it was beautifully intimidating … I love math. I think I just love the concept and I would feel good about myself when I solved the problems."	When given enough time, the participant indicates satisfaction in problem-solving –"I really enjoy problem-solving" and "I liked math because they would give me a problem and I would try to solve it."

Table 9. Examples of metaphorical responses

Metaphorical prompt/vehicle	Sage's self-reported productive positioning (Data source–open-ended survey)	Thyme's self-reported productive positioning (Data source–open-ended survey)
"Mathematics is like …"	"Mathematics is like a balloon. When you 1st start school, you do not have much knowledge about math. As the years go by and you learn math more, and the balloon starts to inflate with the knowledge. Eventually, it will stop inflating like your brain."	"Mathematics is like a wolf–beautiful yet intimidating."
"If mathematics were the weather, it would be …"	"If mathematics were the weather, it would be a calm and cool day. Because math is calm and cool when you know how to do it.	"If mathematics were the weather, it would be a hurricane. Powerful–frequently overwhelming for students to grasp knowledge and apply over several situations."
"If mathematics were a plant, it would be …"	"If mathematics were a plant, it would be a rose. A rose starts as a bud. As lighter and water is given to the rose, it starts to bloom into a beautiful plant."	"If mathematics were a plant, it would be a cactus. You have to be careful and strategic when solving problems."
"If mathematics were an animal, it would be …"	"If mathematics were an animal, it would be a lion. Lion is the king of the land. Just like math–without it, you would not be able to succeed."	"If mathematics were an animal, it would be a cheetah: Direct-sleek-scientific."

Teacher Self-Reported Disposition

In self-rating, Sage reported a mean rating of 4.0 (productive) in all areas on a 5-point scale, emphasizing the following: "I share my knowledge with students. I leave it mostly to them to learn from my experience." This self-rating was consistent with the researchers' expert productive rating of 3.67 (productive). Thyme self-rated as nonproductive (2.67), emphasizing: "I am not afraid to learn alongside with my students and this encourages and facilitates a positive learning environment."

Teachers' experiential backgrounds influenced the formation of teacher self-reported disposition, both negatively and positively. Manifestations of conflict were portrayed for each teacher case study participant, Sage (**Table 7**) and Thyme (**Table 8**). Sage was predisposed to self-report as productive generally but was positioned-by-others as nonproductive.

Thyme self-reported positive and negative mathematical learning experiences, which contributed to a self-reported conflicted lack of confidence in her mathematical knowledge.

Influenced by a mathematics methods class, Thyme based her teaching practice on positive and negative experiences varying from self-perception as nonproductive to a demonstration of productive competencies "necessary to integrate content and teaching strategies" (Lynch-Arroyo, 2013).

Presentation of metaphorical prompts was the nucleus of the questionnaire designed to elicit authentic narratives and external representations of storylines, assisting us in unpacking teachers' positioning (**Table 9**).

Table 10. Cross-case analysis of self-reported disposition using frequency of coded references

Affective characteristics	Sage	Thyme	χ^2	p-value
Self-concept: Nonproductive dis/positioning	1	24	2.3683	.123819
Productive dis/ positioning	1	3		
Attitude: Nonproductive dis/positioning	15	22	16.2397	5.6E-05
Productive dis/positioning	39	8		
Nature of mathematics: Nonproductive dis/positioning	12	14	1.0736	.300132
Productive dis/positioning	18	12		

Table 11. Cross-case analysis of positioning-by-others using frequency of coded references

Affective characteristics	Sage	Thyme	χ^2	p-value
Self-concept: Nonproductive dis/positioning	12	3	4.9665	.0258
Productive dis/ positioning	9	12		
Attitude: Nonproductive dis/positioning	15	4	9.5372	.0020
Productive dis/positioning	8	17		
Nature of mathematics: Nonproductive dis/positioning	49	7	28.5580	.0000
Productive dis/positioning	4	14		

Results of the questionnaire and Likert-type survey ratings for Sage and Thyme revealed inflated and deflated self-reporting of disposition toward mathematics, mathematics teaching, and mathematics learning. Sage self-reported inflated productive characteristics while Thyme self-reported nonproductive characteristics (**Table 6**, **Table 7**, and **Table 8**). Identification of affective characteristics was not considered in terms of linear delineation of characteristics contributing to affective disposition, but as identification of primary contributors to positioning that gave rise to conflict and complexity within the construct of affective disposition. The identified characteristics and measurement classifications included in **Table 10** provided a lens to view affective dispositions as significant contributors to overall self-reported affective mathematical dispositions. Identification of affective characteristics contributing to positioning that gave rise to conflict and complexity in the construct of self-reported affective disposition primarily manifested in the characteristics of self-concept, attitude, and nature of mathematics.

Cross-case analysis of self-reported disposition using Chi-square statistics was applied to measure the difference in the distribution of coded narrative responses (frequency). The data did not present evidence of significance for affective characteristics of self-concept and nature of mathematics. However, the dispositional characteristic of attitude (p=5.6E-05) was significantly different in favor of Sage. Attitude, for the purpose of this study, was defined as "an (un)favorable state of mind/view and/or a positive/negative feeling influencing an emotional (re)action toward a (dis)investment in mathematics" (Lynch-Arroyo, 2013). Based on the level of significance of this characteristic presented in the data, it was surmised that attitude plays a central role in determining one's disposition as a teacher. Awareness of one's attitude toward mathematics, mathematics learning, and mathematics teaching is critical to the formation of disposition and fluidity of positioning movement. Selected anecdotal storylines reflected characteristics of affective disposition while at the same time linguistic deconstruction provided for coding as either productive or nonproductive. For example, demonstrating the affective characteristic of self-concept, Sage self-reported her productive disposition as following: "I know I'm not average (…). There are always different ways to solve problems." Conversely, Thyme self-reported her nonproductive disposition as following: "I'm a struggling learner (…) I was not a very good math student."

Observed Positioning-by-Others

Classroom observation/field notes provided a venue to identify characteristics of self-reported affective disposition present in teaching practices, the interactive nature of positioning and the presence of conflict. In an analysis of classroom observation/field notes protocol (**Appendix A**) for teachers and students, the phenomenon of student reflection of teacher practice in their positioning demonstrated converse positioning of both teachers–Sage as non-productive and Thyme as productive. Like Davis' (2008) claim, "(…) the teacher-participants did not seem to be aware that they were jumping among different levels of phenomena (…)" (p. 8), or specific to this study, Sage and Thyme were not aware of simultaneous positioning-by-others (**Table 11**).

When positioned-by-others, levels of significance using Chi-square statistics illustrated van Langenhove and Harré's (1999) assertion, "clearly, persons are constantly engaged in positioning themselves and others. The concrete forms such positioning will take differ according to the situations in which they occur" (p. 30). Affective characteristics, such as attitude (p=.0020), self-concept (p = .0258), and nature of mathematics (p=0), were found to be the dominant categories of disposition significantly impacted by positioning-by-others as a response to the complexity of classroom interactions (**Table 11**). These findings are important in respect of narrowing the focus of research to identify contributing elements of those characteristics and to bring more awareness of fluidity of self-positioning and positioning-by-others (productively and non-productively) in teaching practices.

Affective characteristics of self-concept and attitude presented in the data contributed to a conflictive state of self-positioning and positioning-by-others (level-hopping). As the data support (**Table 10** and **Table 11**), individual's self-concepts were defined and refined by the people around them (Fein & Spencer, 1997; Rodríguez, 2005). Additionally, Douglas (2009), citing a study conducted by Palardy and Rumberger (2008), stated, "(…) teacher attitude and practices combined account for more variance in student learning than do teacher background qualifications" (p. 518). The importance of these data is supporting prior findings that self-concept and attitude are key components of affective disposition and contribute to the conflictive/complex nature of mathematics education–productive or nonproductive approaches to teaching.

Table 12. State assessment passing rate

Case	State assessment passing rate	χ^2	p-value
Thyme	66.50	5.2475	.021978
Sage	52.18		

Comparison of Teacher Self-Reported Disposition and Observed Positioning-by-Others

Sage and Thyme demonstrated a lack of awareness of their multiple conflicting positions as productive and nonproductive. This was not to say that their self-reported dispositional characteristics were unrealistic, but rather inflated/deflated due to "the multitude of diverse and often conflicting 'voices' that are speaking in a text" (Davis, 2008, p. 14). Linguistic deconstruction of self-reporting allowed for the identification of shifting positioning not in terms of linear delineation, yet with consideration of complexity of the construct and the influence of extraneous factors.

Sage positioned herself as productive. However, Sage was observed demonstrating nonproductive characteristics as evidenced in 97% of all semantic references identified in the thematic coding of narrative responses to the questionnaire, interview, and in classroom observations/field notes. Classroom observation/field notes quoted Sage: "Turn to page 23 and copy what I have here. Turn to page 12 of your STAAR interactive notebooks. Page 17, my bad." Goes through true/false answers. Continues reading answers "now lesson 15, 1-10". The description of basic, systematic procedures (task propensity) is representative of Sage's nonproductive positioning-by-others. Sage's positioning as nonproductive was reflected in one of her positioning statements, "I would have more fun in the education, because I know how it feels to be bored to death by work."

In contrast, Thyme positioned herself as nonproductive, yet was positioned-by-others as productive. Thyme was thoughtful, and more individualized in her teaching practice exhibiting productive characteristics, including "persistently check[ing] for student understanding and supporting student learning through a variety of tasks" (Shepard, 2005, p. 66). Classroom observation/field notes stated, "Students interacted around the topic. Learning was enriched by conjecture, investigation, and analysis. The instruction was related to something relevant to the students. She asked for explanations and justifications". Moreover, Thyme was positioned-by-others as productive despite of her self-reported nonproductive self-concept. For example, "I did not understand the problems. But it was beautifully intimidating (…) I love math" was representative of Thyme's overall positioning toward mathematics as well as its teaching and learning practice in general. Most importantly, Thyme's positioning as productive was reflected in one of her students' positioning statements: "Math is fun (…) my teacher explains it well to me". Sage self-reported positioning as productive was challenged by positioning-by-others (student). One of her students' interview statements reflected the extent of positioning-by-others: "(…) because I know how it feels to be bored to death" and conversely, "I might be like more interested in what to actually do it and what to understand it more than being bored and not understanding it." Anecdotal examples from Sage's classroom observation/field notes established her positioning-by-others as nonproductive.

Data Confirmation

Standardized state assessment data were collected as an indirect positioning-by-others data source. For the purposes of this study, we considered the overall percentage passing rate achieved by students linked to the case studies during the academic year as a means of conflict resolution between teacher self-reported disposition and observed positioning-by-others (**Table 11**).

As indicated in **Table 12**, the difference in participating teachers' linked student passing rates on state assessment was significant at $p<.05$ using Chi-square statistic. Considering limitations of the case study, we are hesitant to claim a distinct relationship between state assessment scores and teacher disposition; however, the study findings suggest that longitudinal research may provide greater insight into implications of this source of data.

Distinct divergences in self-reported disposition and positioning-by-others, and fluidity, demonstrated in passive (nonproductive) and active (productive) teaching behaviors. Based on Thyme's expert disposition rating at a non-productively inclined level (2.67); the concept of conflict was demonstrated by reported student passing rate (66.50%) in the top quartile of reported scores for all teachers in the larger sample from an initial study of volunteer middle school mathematics teacher participants from the entire school district. The case of Thyme personified the dynamics involved in self-reported disposition and resulting conflict in positional shifting. Throughout the study, Thyme self-positioned herself as nonproductive. Observationally, and as supported by state assessment data, she was positioned-by-others (observer and students) as productive. "One individual can thus undertake several varieties of positioning" (van Langenhove & Harré, 1999, p. 30). Situational factors were distinct contributors to Thyme's productive positioning-by-others. Conversely, Sage self-reported as productively inclined (3.67), yet students' passing rates on the state assessment (52.18%) were in the lowest quartile for the initial sample of teachers. Meaning coding and interpretation produced multiple anecdotal examples as evidence of self-reported disposition as productive, while positioning-by-others as nonproductive and the existence of complexity and conflict. In the case of Sage, the affective characteristics which contributed to a phenomenological conflict between teachers' self-reported disposition and positioning-by-others, were nature of mathematics, self-concept, and attitude as measured by meaning-coding utilizing thematic analysis. Revealing evidence of the conflict was observed when Sage self-positioned as productive stating in the interview "There are always different ways to solve problems" yet demonstrated in the positioning-by-others example (classroom observation/field notes), "D is the answer" as nonproductive and provided no further extension of learning.

These examples bore out the assertion that simultaneously one can be self-positioned productively yet be positioned-by-others (observed) on the opposite end of the continuum. This pattern suggested the existence of conflict/level-hopping indicating greater scrutiny was needed to establish characteristics of self-reported disposition and positioning-by-others, that is, confirmation of self-reported data was substantiated in an extended exploration of additional observational data collected.

DISCUSSION AND CONCLUSIONS

The aim of the study was to ascertain if there was a pattern of dichotomous positioning, which contributed to the conflict of teacher self-reported disposition and positioning-by-others toward the complexity of mathematics teaching and mathematics learning. This study had a distinct focus on cross-case comparison allowing for in-depth data analysis from multiple sources. Within traditional educational environments, teachers are positioned in the role of transmitting knowledge to their students. We call this type of passive nonproductive positioning "teacher-as-technician" (Tchoshanov, 2013). In learner-centered environments, a teacher assumes active productive positioning called "teacher-as-engineer" (Tchoshanov, 2013). Attributed to Bourdieu's (1984) reflective theory of didactics, Uljens (1997) described teacher positioning as a method of ascertaining "how instructional processes in the institutionalized school may be experienced" (p. v) or the influence of teacher positioning on the "learner's intentional activity" (Uljens, 1997, p. 51). A teacher may function solely as a conduit of information, or a teacher may productively facilitate student learning through the process of transformative constructive pedagogical practices. Tchoshanov (2013) has identified productive facilitation of learning as "a social constructivist learning cycle, helping students to build new understandings and develop ideas from prior experiences" (p. 9). This study examined a conflict between teacher self-reported disposition and positioning-by-others within the productive and nonproductive dichotomy, while unpacking the complex characteristics of affective disposition and resulting positioning toward mathematics, mathematics teaching and learning. Fels (2004), as cited by Davis (2005), contends "Becoming a teacher is realized within messy, chaotic, generative spaces that we call learning, as emergent possible worlds-as-yet-unlived unfold." She further asserts that we must "(…) recognize the complexities and complicity inherent within pedagogical spaces" (p. 6). With the research purpose of understanding the dynamics of teacher and student disposition and positioning toward mathematics, networked structures of ideas/ concepts/ information within the classroom environment are important to acknowledge in identifying affective disposition and positioning (Davis, 2008). Evidence of the existence of fluid teacher dis/positioning contributed to conflict and complexity connected to teaching practices and student performance. This study supports Fels' (2004) assertion that changes within the dynamics of a learning environment, even minute changes, can impact the relationships and positioning resulting in a multiplicity of positioning. These findings further support Beyers' (2011) assertion of relationships between teacher and student disposition with the potential of affecting student-learning outcomes. From the perspective of the psychology of mathematics education, acknowledging and addressing the conflict between self-reported teacher disposition and positioning-by-others is critical to educational reform efforts, as evidenced in Davis et al. (2020) findings that teachers occupied a more conflicted state between reform and traditional stances. Teacher recognition of self-reported disposition and awareness of positioning-by-others may result in teaching practice transformation, or conscious positioning alignment and alteration influencing student affective disposition toward mathematics and mathematics learning. Higher student achievement, when working with mathematical problems, was associated with productive disposition/positioning compared to teacher-centered (non-productive) approaches. Most importantly from an outcome-based perspective, students' state assessment passing rates (**Table 12**) exposed the complexity of positioning

In response to the first research question, what are the participating middle school teachers' self-reported affective dispositions toward mathematics, its learning and teaching? the two participants exhibited a lack of self-awareness of divergent disposition and positioning for each teacher and thus the presence of conflictive positioning to self-reported disposition. Ultimately, dichotomous teacher dis/position may contribute to student outcomes as evidenced in the state assessment data and anecdotal student narratives. Sage, as positioned-by-others in the role of nonproductive positioning, demonstrated dispositional characteristics/interactions resulting in overall students' lower state assessment passing rates than Thyme who was positioned-by-others as productive. Unconscious level-hopping between self-reported disposition and observed positioning was supported in the data as conflictive and contributes to productive and nonproductive dis/position.

In relation to the second research question, what are the participating middle school teachers' observed positioning-by-others? our analysis indicates dramatic shifts between stated disposition and observed positioning-by-others. Within the classroom setting, shifts from productive to nonproductive and vice versa were documented in classroom observation/field notes.

Sage, who self-reported as productive, was observed to utilize a procedural approach or nonproductive approach to teaching. She identified 'wrong answers, but not really explored' i.e., little or no error analysis resulting in the absence of student discovery learning. The teaching approach was not identified as inquiry-based, rather integration of closed ended questions, low-level questions, and more concerned with right or wrong answers; "How many got them all right?"–rather than critical thinking. The classroom setting was traditional with rows rather than groups of students and the focus was predominantly paper-pencil tasks. Knowledge was given/shared by the teacher to students.

Thyme who self-reported as nonproductive was observed to utilize a student-centered approach (productive) requiring higher levels of critical thinking as evidenced in the classroom observation/field notes statement 'proof justification was required from students' and 'individual students provided definitions of each technique', as well as the classroom was arranged in collaborative learning groups of four students per group. Technology was integrated and hands-on materials used such as manipulatives. Knowledge was constructed by the students with Thyme as a facilitator.

Without teacher self-awareness of conflictive level-hopping, the impact on teaching and learning is dramatic, particularly in terms of teaching approaches and pedagogical tools and strategies used in the classroom. The internal conflict between disposition and positioning is exemplified in Thyme's response to the Likert-type survey prompt If mathematics were an animal, it would be (…) "a wolf–beautifully intimidating." Her further interview explanation showed both non-productive and productive positioning:

> "I had math first period and if I did not understand something, I would work on it during the day. So, then I was always ready to go back the next morning and re-teach something if I did not understand it. It was beautifully intimidating. I love math. I think I just love the concept and I would feel good about myself when I solved the problems. It was like oh yeah I knew that!"

Although Thyme self-reported as nonproductive and lacking confidence in her skills to teach mathematics, she unconsciously shifted to productive positioning and made sure she had a clear understanding (concept mastery) to be able to facilitate student learning.

In response to the third research question, How does self-reported teacher disposition compare to observed positioning-by-others i.e., situated teacher positioning in terms of fluidity and movement? education is complex at best, but the addition of conflictive dis/position can result in productive or nonproductive positioning, ultimately impacting teaching and learning. Responding to the third research question, one must consider the dynamic nature of positioning within the context of position, social force, and storyline. Each teacher's story was unique based on their dichotomous positioning and, because of, their belief systems in conflict with the classroom reality when faced with positioning-by-others. The comparison of Sage and Thyme's self-reporting and their situatedness in the classroom presented as significantly diverse in perspective and actual practice. As stated earlier, discourse analysis of affective teacher disposition and positioning requires an awareness of the context (storyline) in which the positioning is occurring; neither teacher seemed to be aware of their own level-hopping and response to extraneous variables (such as positioning-by-others. In both case studies, disposition was not equivalent to positioning.

Implications

Findings of this study provide impetus to those who evaluate and analyze education in terms of reform to recognize the importance of reviewing all components, especially conflict in teacher self-reported disposition and positioning-by-others, prior to making recommendations. Critical to educational reform efforts is the acknowledgment of conflict (level-hopping) integral to self-reported disposition and positioning-by-others. Teacher recognition of self-reported disposition and the fluidity of positioning-by-others may result in teaching practice transformation, or, at a minimum, conscious positioning alignment and alteration influencing student affective disposition toward mathematics and mathematics learning. Acknowledgment must not be in simplistic linear terms which ignore the nature of self-reported disposition and positioning-by-others. Education seen through the viewpoint of complexity and positioning theories expands "(…) the space of human possibility by exploring the space of the existing possible" (Davis, 2008, p. 22) and increases the understanding of the affective domain of disposition and the effects in mathematical learning and teaching.

Future Research

We submit that the examination of these two cases did not render compelling evidence that can be generalized to all teachers of mathematics, therefore additional exploration is needed to substantiate the impact this recognition may have on mathematics teaching, mathematics learning, and positioning toward mathematics. It is possible that in future research utilizing a generative research framework, where questions generated by this study can be further explored. Specifically, positioning-by-others, as evidenced by the affective characteristics of self-concept, attitude, and nature of mathematics, was significantly influenced by positioning interplay. These results correspond to findings of a prior study that investigated student-oriented (productive) and teacher-oriented (non-productive) forms of teaching and learning student-centered methods (Bieg et al., 2017).

Teacher attitude presented as a critical dispositional characteristic to be considered in self-reported disposition and positioning-by-others. From a pedagogical perspective, attitude is rarely addressed in teacher preparation programs, yet may be the single-most important dispositional characteristic to bring to the forefront of consideration and future research. Communication of content mastery may be severely inhibited by resultant positioning impacted by one's attitude toward mathematics teaching and learning.

A distinct relationship between state assessment scores and positioning cannot be established, but the study findings suggest that longitudinal research may provide greater insight into the implications of this data.

It is conjectured that self-reported disposition and positioning-by-others (disposition in action) are significantly impacted by conflict occurring in the overlapping of cognitive, conative, and affective disposition constructs and in points of construct intersection. Further investigation of all three domains of disposition interacting with the dynamics of positioning is called for, based on the findings and results of this exploratory study. It is surmised that further evidence of emerging and evolving themes of complexity and conflict in teacher disposition and as reflected in student disposition will be substantiated. Additional research and analyses are needed to confirm associations between disposition and positioning variables, within the framework of complexity, disposition, and positioning, as well as other potential variables including socio-economic-political-policy driven influencers on disposition and positioning.

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APPENDIX A

Instructor Questionnaire

Do *not* put your name on these forms.

Demographic information

Check one of the following or write response in the space provided:

Gender: Male _____ Female _____

Ethnicity: ____

Grade level(s) taught: _____ Languages spoken: ____

What do you consider your primary language? ____

How many years did you attend school in the United States? _____________________________

How many years have you been teaching? ________________

Are you ACP certified? ____________

What is the highest degree you have earned? _____ Major: _____ Minor: _____

Section I: Open-ended questions

Please answer the following questions in the space provided:

1. Would you consider yourself a mathematician? Why or why not?
2. Do your fellow math teachers use your ideas in their teaching practice? Explain.
3. In an environment without restrictions, how would you teach math? Explain.
4. Describe a mathematics classroom where students are best able to learn. Explain and give details.

Section II: Metaphor

Please complete the following statements in the space provided:

5. *Mathematics is like* __________. Explain why.
6. *If mathematics were the weather, it would be*__________. Explain why.
7. *If a mathematics classroom were a plant, it would be*__________. Explain why.
8. *If mathematics were a question, it would be*__________. Explain why.
9. *If mathematics were an animal, it would be*__________. Explain why.

Thank you for your participation in this questionnaire.

APPENDIX B

Likert-Type Survey With Open-Ended Questions

Do *not* put your name on these forms.

Demographic information

Check one of the following or write response in the space provided:

Gender: Male _____ Female _____

Ethnicity: ____

Grade level: _____ Languages spoken: ____

What do you consider your primary language? ____

Students: How many years have you attended school in the United States? _____________________________

On a scale of 1-5, with 1 being "one of the worst" and 5 being "one of the best", how would you position yourself toward *mathematics*?

One of the worst	Below average	Average	Above average	One of the best
1	2	3	4	5

Explain why:

On a scale of 1-5, with 1 being "one of the worst" and 5 being "one of the best", how would you position yourself toward *mathematics learning*?

One of the worst	Below average	Average	Above average	One of the best
1	2	3	4	5

Explain why:

On a scale of 1-5, with 1 being "one of the worst" and 5 being "one of the best", how would you position yourself toward *mathematics teaching*?

One of the worst	Below average	Average	Above average	One of the best
1	2	3	4	5

Explain why:

APPENDIX C

Semi-Structured Sample Interview Prompts

Thank you for agreeing to be interviewed as part of the questionnaire instrument and Likert-type survey you completed in the math methods class. I want to reassure you that your responses and identity will be maintained confidentially and will be anonymous to the researchers.

Are you alright with the interview being recorded? Once the tape is transcribed, it will be destroyed.

Interview questions:

1. I'm going to show you your response to a question from the first instrument and your response to a similar question from the second instrument [question 5 and question 1]. Do you notice any differences between the two responses? If yes, what do you think contributed to those differences?
2. Now I'm going to show you another set of questions. Do you notice any differences? If yes, what do you think contributed to those responses? [question 4 and question 2] In your response mentioned that *"read quote"*. Why do you think inquiry and room to make mistakes are important elements for best learning?
3. Now I'm going to show you a last set of questions. Do you notice any differences? If yes, what do you think contributed to those responses? [question 3 and question 3] In your response mentioned that *"read quote"*. Why is incorporating "real-life scenarios" important to your mathematics teaching?
4. With respect toward mathematics teaching, you position yourself as above average. Please explain why.