Journal on Mathematics Education
Volume 13, No. 2, 2022, pp. 323-336

The role of problem context familiarity in modelling first-order ordinary differential equations

Pilar Johnson[1], Felipe Almuna[2*], Marta Silva[3]

[1] Centro de Docencia Superior en Ciencias Básicas Universidad Austral de Chile, Puerto Montt, Chile
[2] Instituto de Especialidades Pedagógicas, Universidad Austral de Chile, Puerto Montt, Chile
[3] Instituto de Ciencias de la Educación, Universidad Austral de Chile, Valdivia, Chile
*Correspondence: felipe.almuna@uach.cl

Received: 9 March 2022 | Revised: 20 August 2022 | Accepted: 22 August 2022 | Published Online: 8 September 2022

Abstract

Based on the unpredictable effect of context familiarity when students solve real-world problems, this work aims to analyse how certain contexts can be used by pre-service mathematics teachers in the representation and real-world verification of a first-order mathematical model in the classroom in the subject of Ordinary Differential Equations. Specifically, this paper reports a classroom experience in which pre-service mathematics teachers compared the solution of a first-order ordinary differential equations (ODE) with a real-world experimental model. Using documentary records (i.e., students´ hand-written solutions and field notes) and a questionnaire on students´ perceptions on this classroom experience, qualitative results indicated that the pre-service mathematics teachers' familiarity with an authentic context was a fundamental factor they chose a real-world model to represent the solution of a first-order ODE. Our analysis of the results highlights the importance of integrating familiar real-world contexts for pre-service mathematics teachers to model a first-order ODE, which is one of the fundamental principles of STEM disciplines.

Keywords: First-Order Ordinary Differential Equations, Pre-Service Teachers' Problem Context Familiarity, Real-World Context

How to Cite: Johnson, P., Almuna, F., & Silva, M. (2022). The role of problem context familiarity in modelling first-order ordinary differential equations. *Journal on Mathematics Education, 13*(2), 323-336. http://doi.org/10.22342/jme.v13i2.pp323-336

The major developments in the world of Ordinary Differential Equations (ODEs) were motivated by the need to comprehend, study, and provide answers to problems in real-world contexts. These sort of equations, moreover, constitute one of the fundamental principles of the STEM (Science, Technology, Engineering, and Mathematics) disciplines. Space limitations preclude a proper report of the STEM principles here (but see Irish Department of Education and Skills, 2017), although one of them—which is relevant to this paper—is connecting the real-world with the abstract world to make the learning relevant and useful for students. For example, ODEs are utilised to understand the mechanisms of a certain ecosystem when studying population growth, or to date fossils and analyse the decay of radioactive material, whether in the fossil or in the stratum in which it was discovered. A continuous challenge for lecturers of this subject is for university students to generate, refine, and extend forms of intuitive reasoning in order to facilitate more abstract forms of reasoning—and heuristics—for solving an ODE (Rasmussen & Blumenfeld, 2007). In this sense, Rowland and Jovanoski (2004) and Rowland (2006), for example, show that university students encounter difficulties when they attempt to find the meanings of ODEs in context.

The incorporation of context in maths problems has been highly recommended by curricular documents and plans of study in mathematics at the school level throughout the world (e.g., Australian Curriculum Assessment and Reporting Authority, 2015; Department for Education UK, 2013; Ministry of Education Singapore, 2006; National Council of Teachers of Mathematics, 2011), and in STEM courses at the university level (e.g., Fernández-Limón et al., 2018; Rasmussen & Blumenfeld, 2007). At the secondary school level, maths problems in context are particularly relevant in certain international educational large-scale assignments (e.g., NAEP[1], NAPLAN[2], and PISA[3]). These tend to examine students' use of mathematical concepts and procedures to solve real-world problems. However, the use of problems in real-world contexts accounts for dissimilar research results. Although these results originate mainly from the maths classroom at different school levels, key implications can be considered for the impact of real-world contexts on the solving of ODEs. On the one hand, studies in the field of cognition reveal that problems in context should tend to facilitate students' performance, since in general: (i) they allow students to be mentally stimulated, improving accessibility and their motivation to solve problems (Glenberg et al., 2004); (ii) they reduce the cognitive load associated with the problems in context (Clark et al., 2011), and (iii) they improve, in some cases, the accessibility of problems by recalling previous solution strategies (Kotovsky et al., 1985). In addition, anecdotal evidence from the maths classroom suggests that the incorporation of problems in context benefits students' non-cognitive factors as well, such as motivation, confidence, engagement, and interest in the solution to a problem (Almuna, 2020).

On the other hand, some research has revealed (i) that certain students are reluctant to consider context in the formulation and solution of a problem (Almuna, 2020; Busse, 2005; van den Heuvel-Panhuizen, 2005), and (ii) that the interpretation of the context of a problem is not obvious for some students (Boaler, 1994), nor is its comprehension distributed evenly among them (Busse, 2005; Helme, 1994). This underscores how the context of a problem can influence how it is solved. Therefore, the objective of this work is to analyse—from the point of view of context familiarity (later referred to as individual familiar context)—how different contexts can be utilised by pre-service mathematics teachers in the classroom representation and real-world validation of first-order mathematical models in the course Ordinary Differential Equations.

Problem Context

There are various definitions in the literature for the context of a mathematics problem. Nevertheless, these definitions relate to two distinct categories associated with the word *context* in research on mathematics education. One of these is related to the social and physical space in which the problem is presented—e.g., the classroom, learning mathematics; see, for example, Lave (1988) and Säljö & Wyndhamn (1993). However, our interest is related to the category in which the context of a maths problem is connected to the real-world. When this category of context is referenced, distinct terms or

[1] *National Assessment of Educational Progress* is a continuous assessment in the US. It is related with what US students know and can do in fundamental areas of their curriculum (including mathematics). For more information, consult www.naep.org.

[2] *National Assessment Program Literacy and Numeracy* is an assessment applied at the national level in Australia to third, fifth-, and seventh-year students, as well as to students in the first year of high school. For more information, consult www.naplan.edu.au.

[3] *Program for International Student Assessment*. For more information on PISA, consult www.pisa.oecd.org.

meanings for the construct are likewise encountered in the literature, such as cover story (Chapman, 2006; Chipman et al., 1991), thematic content (Pollard & Evans, 1987; Ross et al., 1986), situation (OECD, 2013), and setting (Lave, 1988), all of which are used as alternative labels in mathematics education research.

In each case, these terms were highly influenced by the lines of investigation of researchers who attempted to use the construct of *context.* For example, *cover story* is a term that is generally used in mathematics for simple problems (i.e., word problems). On the contrary, the term *thematic content* is frequently employed in studies on cognition related to the context of the problem, while the term *situation* is utilised when maths problems are integrated in real-world contexts. In effect, the international metric PISA-Mathematics uses *situation* and *context* interchangeably (Stacey, 2015). Meanwhile, *setting* is used specifically to refer to physical sites in the real-world (e.g., a factory) where human activities take place (Lave, 1988). In this study, the *context* of a maths problem will be used specifically to connect the real-world (i.e., extra-mathematical) to that of maths (i.e., intra-mathematical). For this reason, and for the purposes of this study, a working definition of the meaning of the *context* of a maths problem is required. Accordingly, for this work, the context of a maths problem is defined as follows:

*All the information about the real-world situation that is contained and can be derived using common knowledge from the statement of a mathematical problem. The contained and derived information might be necessary or unnecessary for the mathematisation and solution of the problem. The context is distinct from the problem's stimulus but accessed through it. (*Almuna, 2020, p.20*)*

Implicitly, an understanding of mathematical model has been established in the above definition of *context*. However, we hypothesise that an explicit definition is necessary in this study since mathematical model has distinct interpretations. In this study, then, mathematical model is defined based on its classical conception as an adequate mathematical representation of the problem statement in a real-world context in which students must recognise and employ a model of a first-order differential equation to solve a real problem in strictly mathematical terms. We emphasise that the definition provided for mathematical model should not be confused with mathematisation, nor with mathematical modelling competence, which while we considered fundamental, are not related with the focus of this work, which is the empirical corroboration of a first-order ordinary differential equation.

Additionally, in this work the individual familiar context will be understood as the previous personal experiences of university students (within or outside of the university context). In this sense, the experience of a student is not necessarily understood as something that occurs day-to-day, but rather as something that the student can experience at a certain point in time (direct or indirect exposure to a context). However, familiarity with individual contexts must be widely shared among students (e.g., eating breakfast), at least as vicarious experiences in the case of not having experienced an individual familiar context directly (e.g.*, I am not a vegan, but I know what being a vegan entails*).

Lastly, in this work the concept of empirical corroboration in the classroom will be used to describe the real-world corroboration of the solution to a first-order ODE representing a problem in an individual familiar context determined by pre-service mathematics teachers (e.g., preparing filtered coffee, frying an empanada [i.e., meat pie], or the growth of yeast) in front of their classmates and teacher in the course Ordinary Differential Equations.

Arguments for Using Maths Problems in Context

Various theoretical arguments for why problems in context should be used in mathematics are presented below. These have been summarised by Almuna (2020) and constitute a summary of the main arguments available in the literature (e.g., Beswick, 2011; Blum & Niss, 1991; Boaler, 1993; OECD, 2013; Pierce & Stacey, 2006; Wedege, 1999).

1. Formative argument: The emphasis is put on the application of mathematics in context as a means for developing general competencies, attitudes, and skills orientated toward fostering creative and problem-solving abilities as well as "open-mindedness, self-reliance, and confidence in [students'] own powers" (Blum & Niss, 1991, p. 42).
2. Critical competence argument: This argument highlights the importance of preparing mathematically literate students to enable them to "see and judge independently, to recognise, understand, analyse, and assess representative examples of the uses of mathematics, including solutions to socially significant problems" (Blum & Niss, 1991, p. 43).
3. Utility argument: Problems in context may enhance the transfer of mathematics to other contexts. They may increase the chance of students applying mathematics that they have learned at school in other areas in later studies, everyday contexts, or future employment. Mathematics is seen under this argument as a service subject or as a subject of instrumental interest (Helme, 1994). This argument relies on the evidence that the ability to use mathematics in context "does not result automatically from the mastering of pure mathematics but requires some degree of preparation and training" (Blum & Niss, 1991, p. 43).
4. Image of mathematics argument: This argument stresses the importance of providing students with a rich and comprehensive picture of mathematics in all its facets, "as a science, as a field of activity in society and culture" (Blum & Niss, 1991, p, 43). That is to say, mathematics in context reflects the nature of mathematics as a human activity (van den Heuvel-Panhuizen, 2005).
5. Promotion of learning maths argument: This argument insists that mathematics in context is well suited to assisting students in "acquiring, learning, and keeping mathematical concepts, notions, methods, and results, by providing motivation for and relevance of mathematical studies" (Blum & Niss, 1991, p. 44), contributing to the education of students who can think mathematically within and outside of mathematics.
6. Use of mathematics in real contexts argument: The use of context may assist in overcoming the common perception of mathematics as a "remote body of knowledge" (Boaler, 1993, p. 13) with no connection to the real-world. Mathematical problems in real-world contexts may allow students to understand the connection between mathematics and the real-world (Felton, 2010, p. 61), highlighting that mathematics has a relevant meaning in the real-world. Moreover, assessing mathematics embedded in real-world contexts allows students to "discover whether [they] have been well prepared mathematically for future challenges in life and work" (Stacey, 2015, p. 7).
7. The halo effect argument: Pierce and Stacey (2006) showed that some teachers use contexts that appeal to students (for example, a problem about a dog) to improve students' attitude towards learning mathematics by associating the subject with pleasant things.

This brief summary—based on social and cognitive theoretical influences—highlights the benefits of using contexts in mathematics. Considering these benefits and the substantial teaching efforts that ODEs entail (e.g., Rasmussen & Blumenfeld, 2007), the influence of real-world context on ODEs is a topic that cannot be ignored, above all in terms of the familiarity of the context. Although the influence of familiarity with real-world contexts in ODEs does not appear to have been researched previously, research has been done on this general area at the school level, and it is clear, as described in the following sub-section, that this factor can affect the performance of students.

The Role of Context Familiarity in the Students´ Performance

Chipman et al. (1991), in a methodologically robust study, analysed how the context of maths problems could affect problem-solving in the performance of students (n = 256) at a US university. Sixty-four algebra problems were classified into four groups of different contexts according to (i) whether they were stereotyped as appropriate for a certain gender (masculine or feminine) or (ii) whether they were familiar or unfamiliar (the underlying mathematical structure of the problems was controlled). One hypothesis of this study was that students' performance might be decreased by contexts stereotyped as appropriate for the opposite sex, for which no statistical support was found. The other hypothesis was that students could be negatively affected by unfamiliar contexts. Students were found to be more likely to "omit problems of neutral but unfamiliar content and less likely to solve such problems correctly" (p. 910). This hypothesis was supported statistically, but the effect was small in magnitude.

Meanwhile, in a seminal study in context familiarity, Carraher et al. (1985) found that five young Brazilian street vendors (n = 5, aged 9 to 15 years old) correctly solved a maths problem presented in their selling context (i.e., real-world), while their results on a school-type mathematical problem involving identical numbers and operations were lower. Based on this study, Carraher et al. (1985) posited that students could benefit from contexts designed to activate their own knowledge of the real-world.

Lastly, Almuna (2020) used context familiarity, context engagement and levels of context use as part of the set of analytical tools to investigate the effect of an alteration of context on students' performance (Almuna, 2010; Almuna, 2016; Almuna & Stacey, 2014). His mixed-method study examined differences in students' performance when they solved both PISA problems and matched problems that were designed with the same mathematical core but with a different context. One hundred and fifty-one students from eight Year 10 classes at one volunteer school in Melbourne, Australia, solved a total of nineteen problems in a paper-and-pencil format. Because of potential inter-problem effects and limited class time for testing, a rotated design of eight booklets was used. Each student solved three problems, one at each level of context use. Problems used were slightly adapted PISA items and specially constructed PISA-like items. At the level of context familiarity, quantitative findings indicated that a change of context did not influence students' performance for problems that involved more context familiarity for students. Data from interviews revealed some of the ways that students did or did not use their context knowledge, and how it sometimes assisted and sometimes blocked a good solution. Although there are technical and methodological aspects that could have impacted in this result—which are discussed in Almuna (2020)—the above results on context familiarity are best explained by the fact that difficult problems in this research required careful consideration of a problem real-world context, and hence more elaboration in their solutions. The latter is related with consideration of real-world contexts when students solve ODEs.

METHODS

The course Ordinary Differential Equations in the Bachelor of Secondary Mathematics Pedagogy degree program—at one university in southern Chile—includes modelling problems using first-order ODEs as part of its syllabus. This part of the course is generally developed theoretically, with pre-service mathematics teachers solving proposed problems that are typically used in textbooks on this subject (e.g., mixing of H20 and NaCl, bacteria growth, and population growth). However, in the Ordinary Differential Equations course in the second semester of 2019, the syllabus included, in addition, the empirical corroboration of first-order ODEs in the classroom. As stated previously, this is in line with the formative and use of Mathematics of real-world context arguments as well as with the STEM principle of connecting the real-world with the abstract world. The hypothesis that guided this decision was based on the above arguments for using problems in context and on the idea that pre-service mathematics teachers would appropriate the concepts of first-order ODEs addressed in the course based on their individual familiar contexts.

The ethical approval for this study was granted from the School of Mathematics Pedagogy committee on the ethics of human research. In this version of the course, the total number of students enrolled in the course (n=15) agreed to participate in this study, prior to their authorisation with informed consent. In this vein, a traditional approach (i.e., solving lists of problems) was used to analyse how certain contexts (e.g., population growth, the law of cooling, and mixture problems) are favourable for the application of first-order ODE problems. Afterward, as part of the first graded assignment of the course, the pre-service mathematics teachers created and empirically corroborated—using an individual familiar context—a problem that entailed modelling a first-order ODE. For this, pre-service mathematics teachers formed three work groups ($n_1= n_2= n_3=5$) to carry out the activity across three class periods. With the lecturer as a guide, the three classes aimed at empirical corroboration were conducted in the following manner: in the first class, pre-service mathematics teachers discussed the feasibility of the contexts that they could use for corroboration. In the second class, pre-service mathematics teachers chose an individual familiar context that they would use to test the real-world problem chosen in the classroom. The contexts chosen by the groups were (i) the filtration of a mix of coffee and water; (ii) the cooling of an empanada (i.e., a meat pie); and (iii) the growth of yeast.

In this sense, it was expected that through the problem at hand, and utilising an individual familiar context, pre-service mathematics teachers would be capable of identifying the variables involved, their relationships, and additional conditions that could exist in order to solve a first-order ODE modelling problem in context. Generally, in terms of empirical corroboration, pre-service mathematics teachers went through the following steps: (i) identify the variables and their relationships; (ii) construct a first-order ODE to model the situation; (iii) determine the general solution of the first-order ODE; and (iv) if additional conditions exist, adjust the constants. In the third class, pre-service mathematics teachers presented on the reason for the context chosen and how it related to a first-order differential equation. Later, the empirical verification was undertaken. For this, each group wrote the first-order ODE representing their problem in context on the whiteboard and solved it in written form. In parallel, the rest of the participants of each group proceeded to carry out the empirical corroboration of the solution to the equation written on the board. For example, for the solution of the first-order ODE that proceeded from the measurement of the temperature of a cooked empanada (in a time period $t_1 = 5$ minutes) starting at an initial temperature t_0 (in minutes), pre-service mathematics teachers measured the ambient temperature in addition to measuring the temperature of the empanada with a laser thermometer (see Figure 1), and later carried out the measurements of temperature for $t = t_1$ to verify whether the solution of the first-order ODE on the board for $t = t_1$ corresponded with the experimentation in the classroom.

Figure 1. Experiment on the cooling of an empanada (meat pie)

During the experiment field notes were taken in which pre-service mathematics teachers' interactions were recorded, including their dialogs and specifically the diverse forms of reasoning and heuristics that came about when solving the problems. These notes were taken as internal records initially in order to later compile a more detailed description of the results. Additionally, the solutions to the problems, hand-written on sheets of paper by the pre-service mathematics teachers, were collected as documentary records.

Following the experiment, an anonymous questionnaire was applied to pre-service mathematics teachers with the aim of understanding their evaluative perceptions of this learning strategy. The questionnaire consisted of four open prompts: (i) explain in your own words the mathematical model and the familiar context that you chose; explain why you chose this context; (ii) do you feel that this activity was useful for understanding the mathematical models studied in this course? (iii) do you think that it is relevant to carry out this type of activity in the mathematics courses in your program? and (iv) what benefits do you think this activity has provided you for your own learning?

The responses to the four open questions were transcribed in their entirety in order to later be read reiteratively to search for emerging categories of analysis through a content analysis. This analysis sought to understand the evaluation that pre-service mathematics teachers gave the experience of solving the mathematical problems at hand. The analysis of the answers to the questionnaire and the presentation of models created by pre-service mathematics teachers are shown in the following section.

RESULTS AND DISCUSSIONS

The three work groups of pre-service mathematics teachers who participated in this activity presented three individual familiar contexts to carry out the comparison between the algebraic solution of the ODE and its real-world verification in the classroom (i.e., theory vs. experiment). The contexts presented by pre-service mathematics teachers included (i) the filtration of a mix of coffee and water to demonstrate a mixture problem; (ii) the cooling of an empanada to demonstrate the law of cooling; and (iii) growth of yeast in an environment conducive to representing bacterial population growth. In this section, the third experience of bacterial population growth will be reported; the context of yeast was of particular interest to the authors of this paper as pre-service mathematics teachers were exposed to a culturally well-known individual familiar context (i.e., the mothers of the pre-service mathematics teachers in this group prepare homemade bread daily for family consumption) to create a problem that consisted of measuring the time necessary to duplicate an initial quantity of yeast, identifying the variables involved and their relationships (i.e., elapsed time, quantity of yeast as a function of time, and a proportionality constant) to then construct and solve the ODE to model the situation (see Figure 2). Insights on the two other contexts are addressed on the conclusion section.

In the classroom validation experience, pre-service mathematics teachers brought an initial quantity of yeast, warm water, and sugar, mixing the water with the sugar in a precipitate container to obtain an environment conducive for the growth of the yeast. After, the pre-service mathematics teachers added yeast to the mix and measured the time it took for the initial quantity to double. While this was occurring, pre-service mathematics teachers were at the board explaining the theoretical solution of the presentation to the lecturer and their classmates, utilising the model of populational growth (see Figure 2). The result of the model showed that in 8.6 minutes the initial mix would double. The classroom verification was close to the solution of the populational growth model of the first-order ODE, given that the error was considered minor (validation error +5 seconds). Under this parameter of validation error, the experiment was considered successful.

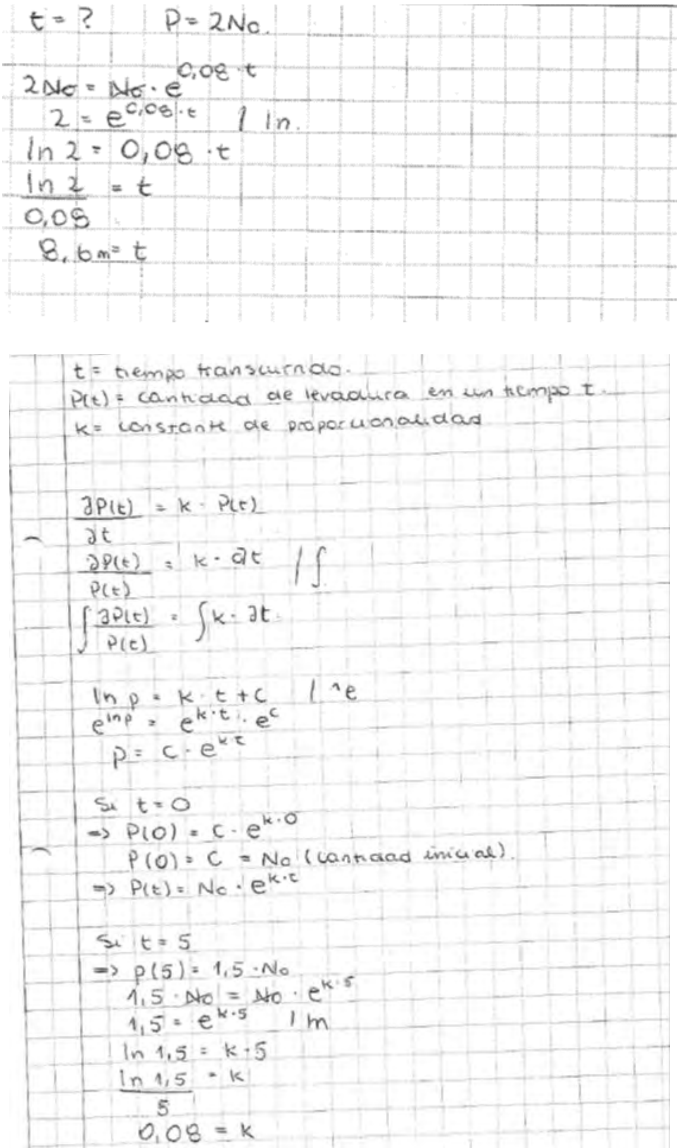

Figure 2. Solution of an ODE in an individual familiar context (yeast)

Regarding the qualitative analysis of the pre-service mathematics teachers' answers to the questionnaire on their perception of this experience, it was observed that the majority valued the transfer of mathematics to reality. This is exemplified in the following answer (translation provided below in footnote [4]) from a pre-service mathematics teacher in the activity (see Figure 3).

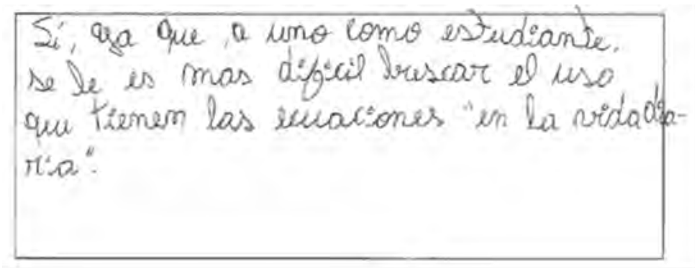

Figure 3. Extract of the answer of a pre-service mathematics teacher regarding transfer

Moreover, pre-service mathematics teachers also placed importance on the use of context to give meaning to the comprehension of the mathematical model used in the activity (see Figure 4, translation provided below in footnote [5]).

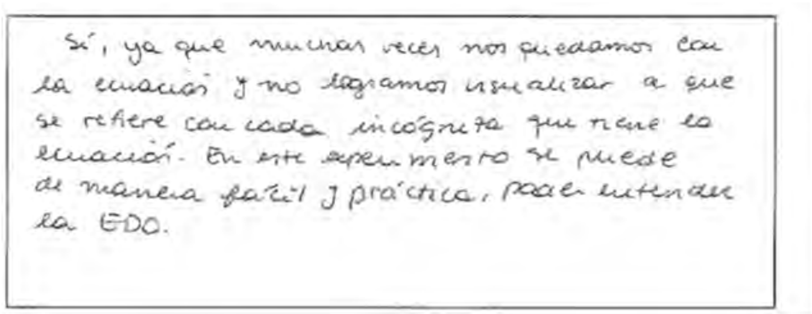

Figure 4. Extract from a pre-service mathematics teacher answer regarding the meaning of the model utilised

This study considered a classroom experience in which pre-service mathematics teachers chose an individual familiar context—different from those habitually used in textbooks on this subject (e.g., mixtures of H_20 and NaCl, populational growth)—to represent and corroborate first-order mathematical models. During this classroom experience, a high level of individual familiarity was utilised, allowing pre-service mathematics teachers to present an individual familiar context for the classroom corroboration of an ordinary differential equation model. This relationship may partly be explained by the individual real-world knowledge of the contexts involved in the experiments. For instance, for the context of growth of yeast in an environment conducive to representing bacterial population growth, students first mixed yeast,

[4] The translated text of the hand-written answer: Yes, since as a student it is more difficult to find the use of equations in daily life. (Original text in Spanish: *Sí, ya que a uno como estudiante se le es mas [sic] dificil [sic] buscar el uso que tienen las ecuaciones en la vida diaria*).

[5] The translated text of the hand-written answer: Yes, since many times we are left with the equation, and we are not able to visualise what each unknown of the equation refers to. In this experiment we could understand the ODE in an easy and practical way. (Original text in Spanish: *Sí, ya que muchas veces nos quedamos con la ecuación y no logramos visualizar a que se refiere con cada incógnita que tiene la ecuación. En este experimento se puede de manera fácil y práctica poder entender la EDO*).

cold water, and sugar in a precipitate container, but no growth was observed. This was because they did not consider real-world assumptions on the yeast grown (i.e., temperature and humidity conditions). With the assistance of the lecturer, pre-service teachers could recall their mothers when making bread. This particular context was performed daily by the mothers of the pre-service teachers; consequently, we hypothesised a higher activation of individual context familiarity for this context than on the two other context. Specifically, from this previous knowledge, pre-service teachers realised they needed a favourable environment for the yeast to growth. Taking this into account, students decided to mix sugar with warm water and then with yeast; this resulted in a favourable environment conducive for the growth of the yeast to double. For the filtration context, students first choice was to mix soil and water, however, from their previous knowledge, they disregard this choice as they knew from their previous knowledge of the local weather (at this Southern Chile location the rainfall is considerable—between 2 000mm to 2 500mm annually—) soil is not soluble in water. Then, again, they used a very familiar knowledge to provide them a choice for the corroboration of an ODE. However, from our field notes, filtering coffee and the frequency of eating/making empanadas at the pre-service teachers´ homes were not done regularly as making bread was. For this particular reason, we reported in this paper only the experience with a higher exposure to an individual context familiarity. This revealed that students were not successful in their first attempts to find a workable method for the corroboration on the contexts less familiar for them. Based on this, in the later version of the course in which this study took place, further directions on real-world context considerations to select context for experimentation are needed. This is because pre-service teachers in this study spent considerable time in finding suitable familiar contexts to carry out the comparison between the solution of the first order ODE and its verification in the classroom (i.e., theory vs. experiment). Despite of the above, it can be hypothesised that the life experience of the pre-service mathematics teachers, understood in this work as their individual familiar context, could influence the activation of real-world knowledge to understand concepts associated with modelling first-order ordinary differential equations.

In this sense, the selected small amount of qualitative evidence presented exhibits the positive evaluation that the pre-service mathematics teachers gave this new experience. On the one hand, not only was a first-order ODE model tested, the understanding of its solution was also promoted; in effect, the pre-service mathematics teachers became aware of the relationship between empirical verification and the comprehension of a solution to an ODE (see Figures 3 and 4). On the other hand, although the researchers of this work did not analyse the use of context on the part of the pre-service mathematics teachers, our previous insights on the contexts considered for this experience, the field notes gathered in the classroom—as well as their handwritten solutions—made it possible to hypothesise that the use of an individual familiar context for the corroboration of an ODE encouraged viewing the context in which the ODE was solved not as a mere addition to the problem, but rather as a principal aspect that allowed pre-service mathematics teachers to test the solution of the equation as a theoretical exercise. Hence, the principal contribution of this research to the Mathematics Education field was contributing to the understanding of the impact of the incorporation of contexts to connect the real-world and the mathematics world when modelling first-order ODE. Therefore, we propose, as Fernández-Limón et al., (2018) also did, that the use of context in the solving of real-world and applied problems should be explored further by lecturers in a manner that fosters transfer between the real-world and the mathematical world. Mathematics education literature has emphasised that this transfer is not immediate and that students must be instructed in transferring from the extra-mathematical to the intra-mathematical (Stillman, 2002).

CONCLUSIONS

This form of classroom work could lead to pre-service mathematics teachers being encouraged in their understanding of the solutions of first-order ordinary differential equations, not from the point of view of solving an algorithm, but rather from a conceptual and real-world perspective as this classroom approach required pre-service teachers to solve real-world familiar problems through the applications of first-order ODE models. In our view, this classroom experience with pre-service mathematics teachers is in line with theoretical arguments on the use of mathematical problems in context (refer to the formative and use of mathematics in real-world context arguments) and the particular STEM principle of connecting the real-world with the abstract world to make the learning relevant and useful for students above stated as—from our field notes— the research experience design allowed them to not only to recognise the relevance between first-order ODEs and the real-world but also apply and communicate the mathematics they know in different real-world and everyday life familiar contexts. In this manner, this connection between the real-world and the mathematical world could also promote their pedagogical abilities since in situ corroborations using mathematics and the real-world could be applied to their future classroom students. In the later version of the course in which this study took place, further directions on real-world context considerations to select context for experimentation are needed. This is because pre-service teachers in this study spent considerable time in finding suitable familiar contexts to carry out the comparison between the solution of the first-order ODE and its verification in the classroom (i.e., theory vs. experiment). In the next version of this course, a quantitative approach will be adopted to test statistically the hypothesis that familiar contexts can be a factor that facilitates the conceptual understanding of first-order ordinary differential equation modelling.

Acknowledgments

The authors would like to express their sincere gratitude to one anonymous reviewer for her thoughtful, helpful, and critical comments on earlier versions of this article and to Mr. Andrew Lee Sigerson who assisted with professional writing and translation services.

Declarations

Author Contribution	:	PJ: Conceptualisation, fund raising, data gathering, and reviewing. FA: Conceptualisation, fund raising, methodology, writing—original and final drafts, reviewing and editing—supervision, validation, and reviewing. MS: Methodology, writing—reviewing and editing—validation.
Funding Statement	:	This work was supported by the Program for Strengthening Initial Teachers' Training-UACh (FID-UACh by its Spanish acronym) under Grant 034-2019.
Conflict of Interest	:	The authors declare no conflict of interest.
Additional Information	:	Additional information is available for this paper.

REFERENCES

Almuna, F. (2010). Investigating the effect of item-context on students performance on mathematics items. *Master Thesis*. Melbourne: The University of Melbourne.

Almuna, F. (2016). Investigating the impact of context on students' performance. In White, B., Chinnappan, M. & Trenholm, S. (Eds.). Opening up mathematics education research. *Proceedings of the 39th annual conference of the Mathematics Education Research Group of Australasia* (pp. 102-109). Adelaide: Mathematics Education Research Group of Australasia. https://files.eric.ed.gov/fulltext/ED572407.pdf

Almuna, F. (2020). Investigating the effect of mathematics problem context on the performance of Year 10 students. [Doctoral dissertation, The University of Melbourne]. University of Melbourne's Institutional Repository. https://minerva-access.unimelb.edu.au/handle/11343/239230

Almuna, F., & Stacey, K. (2014). Item context factors affecting students' performance on mathematics items. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). Curriculum in focus: Research guided practice. *Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia* (pp. 55–62). Sydney: Mathematics Education Research Group of Australasia. https://files.eric.ed.gov/fulltext/ED572560.pdf

Australian Curriculum Assessment and Reporting Authority. (2015). Australian curriculum, mathematics structure. Retrieved June 3, 2020, from https://www.australiancurriculum.edu.au/f-10-curriculum/mathematics/structure/

Beswick, K. (2011). Putting context in context: An examination of the evidence for the benefits of 'contextualised' tasks. *International Journal of Science and Mathematics Education, 9*(2), 367-390. https://doi.org/10.1007/s10763-010-9270-z

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects—State, trends and issues in mathematics instruction. *Educational Studies in Mathematics*, *22*(1), 37-68. https://doi:10.1007/bf00302716

Boaler, J. (1993). Encouraging the transfer of 'school' mathematics to the 'real-world' through the integration of process and content, context and culture. *Educational Studies in Mathematics*, *25*, 341-373. https://doi.org/10.1007/BF01273906

Boaler, J. (1994). When do girls prefer football to fashion? An analysis of female underachievement in relation to 'realistic' mathematic contexts. *British Educational Research Journal, 20*(5), 551-564. https://doi.org/10.1080/0141192940200504

Busse, A. (2005). Individual ways of dealing with the context of realistic tasks—First steps towards a typology. *ZDM*, *37*(5), 354-360. https://doi:10.1007/s11858-005-0023-3

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1985). Mathematics in the streets and in schools. *British Journal of Developmental Psychology*, *3*(1), 21-29. https://doi:10.1111/j.2044-835X.1985.tb00951.x

Chapman, O. (2006). Classroom practices for context of mathematics word problems. *Educational Studies in Mathematics*, *62*(2), 211-230. https://doi:10.1007/s10649-006-7834-1

Chipman, S. F., Marshall, S. P., & Scott, P. A. (1991). Content effects on word problem performance: A possible source of test bias? *American Educational Research Journal*, *28*(4), 897-915. https://doi:10.3102/00028312028004897

Clark, R., Nguyen F., & Sweller, J. (2011). Efficiency in learning: Evidence-based guidelines to manage cognitive load. Pfeiffer.

Department for Education UK. (2013). New maths qualifications to boost numbers studying maths to age 18. Retrieved January 23, 2020, from https://www.gov.uk/government/news/new-maths-qualifications-to-boost-numbers-studying-maths-to-age-18

Department of Education and Skills. Ireland. (2017). STEM Education Policy Statement 2017–2026. Retrieved May 18, 2022, from https://www.gov.ie/en/policy-information/4d40d5-stem-education-policy/

Felton, M. D. (2010). Is math politically neutral?. *Teaching Children Mathematics, 17*(2), 60-63. https://doi.org/10.5951/TCM.17.2.0060

Fernández-Limón, C., Fernández-Cárdenas, J.M., & Gómez Galindo, A. (2018). The role of non-formal contexts in teacher education for STEM: the case of horno3 science and technology interactive centre*. Journal of Education for Teaching*, *44*(1), 71-89, https://doi:10.1080/02607476.2018.1422623

Glenberg, A. M., Gutierrez, T., Levin, J. R., Japuntich, S., & Kaschak, M. P. (2004). Activity and imagined activity can enhance young children's reading comprehension. *Journal of Educational Psychology*, *96*(3), 424-436. https://doi:10.1037/0033-295x.95.2.163

Helme, S. (1994). Mathematics embedded in context: The role of task context in performance, task perceptions and the solution methods of adult women students. [Unpublished Master's thesis]. Australian Catholic University.

Lave, J. (1988). Cognition in practice: Mind, mathematics and culture in everyday life. Cambridge University Press.

Kotovsky, K., Hayes, J. R., & Simon, H. A. (1985). Why are some problems hard? Evidence from Tower of Hanoi. *Cognitive Psychology*, *17*(2), 248-294. https://doi:10.1016/0010-0285(85)90009-x

National Council of Teachers of Mathematics. (2011). Focus in high school mathematics: fostering reasoning and sense making for all students. National Council of Teachers of Mathematics.

Ministry of Education Singapore. (2006). Secondary mathematics syllabuses. Retrieved June 3, 2021, from https://www.moe.gov.sg/education/syllabuses/sciences

Organization for Economic Co-Operation and Development. (2013). PISA 2015 Draft mathematics framework. Retrieved June 23, 2021, from https://www.oecd.org/pisa/pisaproducts/Draft%20PISA%202015%20Mathematics%20Framework%20.pdf

Pierce, R., & Stacey, K. (2006). Enhancing the image of mathematics by association with simple pleasures from real-world contexts. *ZDM, 38*(3), 214-225. https://doi:10.1007/bf02652806

Pollard, P., & Evans, J. (1987). Content and context effects in reasoning. *The American Journal of Psychology*, *100*(1), 41-60. https://doi:10.2307/1422641

Rasmussen, C., Blumenfeld, H. (2007). Reinventing solutions to systems of linear differential equations: A case of emergent models involving analytic expressions. *The Journal of Mathematical Behavior*, *26*(3), 195-210. https://doi.org/10.1016/j.jmathb.2007.09.004

Ross, S. M., McCormick, D., & Krisak, N. (1986). Adapting the thematic context of mathematical problems to student interests: Individualized versus group-based strategies. *Journal of Educational Research*, *79*(4), 245-252. https://doi.org/10.1080/00220671.1986.10885685

Rowland, D. (2006). Student difficulties with units in differential equations in modelling contexts. *International Journal of Mathematical Education in Science and Technology*, *37*(5), 553–558. https://doi.org/10.1080/00207390600597690

Rowland, D., & Jovanoski, Z. (2004). Student interpretations of the terms in first-order ordinary differential equations in modelling contexts. *International Journal of Mathematical Education in Science and Technology*, *35*(4), 503–516. https://doi.org/10.1080/00207390410001686607

Säljö, R., & Wyndhamn, J. (1993). Solving everyday problems in the formal setting: An empirical study of the school as context for thought. In S. Chaiklin & J. Lave (Eds.), *Understanding Practice: Perspectives on Activity and Context (Learning in Doing: Social, Cognitive and Computational Perspectives)*, (pp. 327-342). Cambridge University Press. https://doi.org/10.1017/CBO9780511625510.013

Stacey, K. (2015). The real-world and the mathematical world. In K. Stacey & R. Turner (Eds.), Assessing mathematical literacy. *The PISA experience* (pp. 57-84). Springer International Publishing. https://doi.org/10.1007/978-3-319-10121-7

Stillman, G. (2002). Assessing higher order mathematical thinking through applications. [Unpublished Doctoral dissertation]. The University of Queensland.

van den Heuvel-Panhuizen, M. (2005). The role of contexts in assessment problems in mathematics. *For the Learning of Mathematics*, *25*(2), 2-23.

Wedege, T. (1999). To know or not to know–mathematics, that is a question of context. *Educational Studies in Mathematics, 39*(1), 205-227. https://doi.org/10.1023/A:1003871930181