








We train, but how well? Assessing mathematical knowledge for teaching of future Turkish teachers

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ABSTRACT

This study aims to assess and compare—at the national and regional sizes—mathematical knowledge for teaching of pre-service elementary mathematics teachers (PEMTs). Participants of the study were composed of 1,367 pre-service teachers selected among 21 universities in 12 regions of Turkey. Mathematical knowledge for teaching (MKT) test, which were developed by TEDS-M project were used to collect the data. The instruments were adapted into Turkish by the researchers. According to results, while there wasn't much differentiation among the regions in terms of the MKT, pre-service teachers in the West Marmara, Mediterranean, and Northeast Anatolia regions are more successful than the other regions. PEMTs' mathematics pedagogical content knowledge (MPCK) performance was close but somewhat better than their mathematics content knowledge (MCK) performance, in terms of both regions and universities. In the case of the MPCK test, the regions exhibited a more homogeneous picture, while in the case of MCK, the universities did so. Algebra content domain is where the PEMTs' performance is lowest. When we compared the results obtained from this study with published international averages from TEDS-M study, Turkish PEMTs had appreciably higher performance on both MCK and MPCK components.

Keywords: pre-service mathematics teachers, mathematical knowledge for teaching, content knowledge, pedagogical content knowledge, Turkish regions

INTRODUCTION

In Europe, the 18th century was a critical period of development, setting the stage for substantial intellectual, social, and political revolutions. This rapid change also affected the education picture, and in parallel to rising levels of acculturation, countries began to explore their education system. The study conducted by Marc-Antoine Jullien de Paris (1817) in the first half of the 19th century, investigating the educational implications of European countries' efforts to adapt good examples is accepted as the first comparative education review (cited in Bereday, 1964). There is no doubt that comparative studies on education are crucial and necessary from a number of perspectives. Their leading benefit stems from their use in terms of coming up with successful role models in education, and thereby contributing to the development of policies to enhance the existing education system, armed with an awareness of its strengths and weaknesses, paving the way for the development of multiple education perspectives.

In any discussion of comparative education research, the first studies that gets to be mentioned are usually First International Mathematics Study (FIMS), Second International Mathematics Study (SIMS), Programme for International Student Assessment (PISA), and Trends in International Mathematics and Science Study (TIMSS), characterized by their purported aims of comparing countries in terms of the successes of their students on literacy, mathematics, and science fronts. Such long-term studies with an extended scope and large budgets focus on comparing the achievement levels among the students from the countries taking part in the study. Large scale studies such as TIMSS and PISA regularly review the qualification levels of K-12 students as an indicator of the success of individual countries' education systems. Comparative teacher training studies, on the other hand, is a rather young field of research (Blömeke, 2014). One can safely argue that comparative training studies have seen rising research interest, with a focus on mathematics teacher training in particular (Blömeke, 2014). In this context, some of the leading pieces of research include mathematics teaching in 21st century (MT-21) (Schmidt et al., 2007) presenting an analysis of the pre-service mathematics teachers' teaching knowledge and attitudes towards mathematics, and the teacher education and development study in

mathematics project (TEDS-M) (Tatto et al., 2008) investigating the pre-service teachers' teaching knowledge, attitudes towards mathematics, and learning experiences.

Large-scale international comparative studies offer valuable insights into the learning-teaching practices and qualifications of teachers or pre-service teachers in a number of countries (Blömeke, 2014). On the other hand, such research is also important with reference to the opportunities they provide for comparing the existing state of pre-service teachers enrolled at various universities deemed to exhibit comparable characteristics, in a given country. The knowledge thus gathered would lead to comparative conclusions regarding the level and nature of the knowledge the pre-service teachers get equipped with, and thus, directly (or indirectly) about the characteristics of the institutions training teachers. Therefore, they will bring important fruits in terms of identifying the strengths or weaknesses of individual countries (Blömeke, 2014). Moreover, data such studies produce are also crucial given the opportunities they lead to within the framework of the wider endeavors to identify the similarities and differences between regions, as well as within the wider global perspective (Blömeke, 2012). These characteristics can help in ascribing significance to similarities and differences thus revealed, through developing certain profiles by making connections with the structure of the society/societies, their development levels, and cultural characteristics in again a regional and global perspective. Therefore, these efforts can provide policymakers with important pieces of knowledge and arguments about the policies to be developed, if certain reforms are needed with respect to the education system and teacher training policies. A number of studies presenting comparative results about the achievement levels of the students are useful in this context. Many countries revise their school systems in consideration of these studies' results (Blömeke, 2014; Grek, 2009). For instance, the mediocre achievement levels Germany came up with in PISA 2000 after an extended hiatus in participation in that study produced a minor shock in the country, leading to a revision of the system after extended discussions among the politicians and researchers (Blömeke, 2014; Martens et al., 2014). The results Turkey had in studies such as TIMSS and PISA, which could be labeled as failures, can be noted as inputs for the recent program revisions. The results of PISA also led to renewed debates on the education system, in light of the differences in achievement levels among the schools in Turkey.

A glance at the comparative education research carried out in Turkey reveals that most are limited to the comparison of education programs and teacher training programs of various countries (e.g. Aldemir, 2010; Baki & Bektas-Baki, 2016; Serce & Acar, 2021). In Turkey, there is a dire need for large scale comparative education research focusing on teachers and pre-service teachers, and aiming to compare teacher training programs in terms of future teachers' knowledge. The present study entails a comparative review of mathematics teaching knowledge of pre-service elementary mathematics teachers (PEMTs) enrolled in the final year of the universities located in various parts of Turkey.

Theoretical Framework

Since the last quarter of the 20th century, a great deal of attention has been paid to understanding the role of teacher knowledge on determining student achievement. Studies arguing that teacher knowledge is effective on student achievement (Baumert et al., 2010; Danisman et al., 2019; Guler & Celik, 2021; Hill et al., 2005) point out that teaching knowledge is crucial as an object of study. To be able to carry out such work, however, one should first investigate what teaching knowledge is.

The first studies on the types of knowledge a teacher should have begun to appear in early 80s, and led to the development of a number of distinct models to define teaching knowledge. The study most of these models refer to is that of Shulman (1986). In his critical publication, Shulman (1986) attempts to bridge what he calls the gap between what content (mathematics, physics, etc.) the teachers are required to know, and what knowledge bases they need to possess to make the knowledge meaningful for learners. What makes Shulman (1986) stand out from her predecessors, on the other hand, was pedagogical content knowledge (PCK), which she referred to as the 'missing paradigm' (p. 7). According to Shulman (1986), PCK is "the most useful way of representing and formulating the subject, to make it comprehensible to others", rather than a common pedagogical knowledge such as classroom management (p. 9). PCK is necessary for one to understand students' difficulties and preconceptions that are specific to a topic, and the methods which have the potential to provide effective learning. Therefore, being an effective teacher is not about just being an expert in a field; or solely possessing pedagogical knowledge. That is the main difference between a mathematician and mathematics educator (Baki, 2012).

From the perspective of mathematics education, a number of frameworks were created with the aim of defining the knowledge bases a teacher should possess. While some researchers used the term mathematical knowledge for teaching (MKT) (Baki, 2013; Ball et al., 2008; Hill et al., 2005), mathematics teaching knowledge was also encountered (Chen, 2015; Tatto et al., 2015). It is generally accepted that MKT consists of at least two main components: content knowledge and pedagogical content knowledge. For instance, Ball et al. (2008) proposed a special domain of teacher knowledge and identified domains of mathematical knowledge for teaching. The framework they defined as "mathematical knowledge for teaching" consists of subject matter knowledge (content knowledge) and PCK as two different knowledge bases. The framework constructed mathematics content knowledge (MCK) for teaching as an amalgam of two key elements: "common" knowledge of mathematics, that any well-educated adult should have; and mathematical knowledge that is "specialized" to the work of teaching, and that only teachers need know (cited in Senk et al., 2012, p. 309). On the other hand, PCK in this context consists of knowledge of content and student, knowledge of content and teaching, and knowledge of content and curriculum.

The structure developed by Ball et al. (2008) is crucial in terms of the components to focus on with a view to assessing mathematics teaching knowledge, and to building an understanding of the indicators of each component. On the other hand, large scale studies strive to assess teaching knowledge by developing MCK and mathematics pedagogical content knowledge (MPCK) questions regarding the specific areas of mathematics, and at various cognitive levels concerning such areas (Tatto et al., 2012). One such study, the TEDS-M project made a comparative assessment of the pre-service teachers' MKT in different countries. One of the most important outputs of the project is the provision of validated tools with established reliability to assess

Table 1. Distribution of the sample

Regions	Region code	Number of universities	University code	Number of participants
Istanbul	TR1	1	TR1U1	36
West Marmara	TR2	1	TR2U1	98
Aegean	TR3	3	TR3U1	101
			TR3U2	23
			TR3U3	59
East Marmara	TR4	1	TR4U1	37
West Anatolia	TR5	2	TR5U1	23
			TR5U2	38
Mediterranean	TR6	2	TR6U1	57
			TR6U2	45
Central Anatolia	TR7	3	TR7U1	84
			TR7U2	33
			TR7U3	88
West Black Sea	TR8	1	TR8U1	57
East Black Sea	TR9	2	TR9U1	175
			TR9U2	72
Northeast Anatolia	TRA	2	TRAU1	101
			TRAU2	105
Central East Anatolia	TRB	1	TRBU1	44
Southeast Anatolia	TRC	2	TRCU1	63
			TRCU2	28
Total				1,367

mathematics teaching knowledge within the framework of a given scenario type, using different types of questions (constructed response, multiple choice, etc.). Such tools would allow the assessment of MKT of pre-service teachers enrolled in a number of teacher training institutions at various regions of a given country or in different countries, with a view to coming up with robust deductions regarding the contributions the training provided at these institutions make to the development of teaching knowledge among the pre-service teachers. In the present study, the assessment tools used in TEDS-M study were employed, focusing on the pre-service teachers' MKT in the context of content knowledge and pedagogical content knowledge, with the purpose of making comparisons between regions and universities of Turkey.

Objectives

The main objective of the study is to compare MKT of Turkish PEMTs on a national and regional scale. The researchers have specifically focused on the following research questions:

With reference to the case of Turkey;

1.a. How does the MKT of PEMTs vary between regions?

1.b. How does the MKT of PEMTs vary between universities?

2. How does the MKT achievement of PEMTs vary in terms of content domain of mathematics (algebra, numbers, geometry, and data)?

METHOD

The present study aiming to present a picture of the mathematics teaching knowledge of PEMTs employs the field survey method. The study universe is composed of all senior year pre-service teachers currently enrolled in the elementary mathematics teacher education programs in Turkey. At the time the study was performed, 47 universities in Turkey offered elementary mathematics teacher education programs. The study is carried out with a sample composed of senior year pre-service teachers in 21 universities selected through layered sampling from among the larger set of 47 universities. The study was carried out with the participation of 1367 PEMTs (**Table 1**).

The universities included in the sample were selected with reference to the nomenclature of territorial units for statistics (NUTS) level 1. NUTS is a geocode standard for reference to the subdivisions of countries for statistical purposes. Population, cultural structure, and the development status of the regions are among the criteria taken into account for determining NUTS levels (Tas, 2006; TurkStat, 2012). In this sense, one can argue that, as a classification system NUTS reflects the socio-economic structure of Turkey. Furthermore, TurkStat will reportedly prepare all statistical information and data on the basis of this standard, while the government agencies will also be organized with reference to this scheme (Tas, 2006). In this context, taking into account the important role statistical data presented with reference to this regionalization scheme could play in future decisions to shape education policies, the present study opted for NUTS.

NUTS level 1 stipulates twelve regions (TR1, TR2, ..., TR9, TRA, TRB, TRC) with different socio-economic backgrounds. **Table 1** presents the picture of the study participants, broken down with reference to universities and regions.

Instruments

In order to investigate the teaching knowledge of future Turkish mathematics teachers, the framework of TEDS-M project was adapted which included MCK and MPCK as components of MKT. A total of 103 questions (76 MCK and 27 MPCK items) were developed and three different booklets were prepared (Tatto et al., 2008). Although the items were different for each booklet, a balanced distribution was provided in terms of content domains and cognitive levels. Original TEDS-M countries were requested to answer only one of the booklets with around 30 questions. Within the required permits, one of those booklets was employed which contains released 23 MCK and nine MPCK items and a total of 32 questions in this study (the booklet with released items is used due to copyright). The booklet was representing mathematics content domains (numbers, algebra, geometry, and data) at applicable cognitive levels (applying and reasoning for MCK; curriculum and planning, and enacting for MPCK), with different difficulties and question types (constructed-response, multiple-choice, complex multiple-choice) (Blömeke & Kaiser, 2014; Tatto et al., 2008). As a part of a large-scaled the Scientific and Technological Research Council of Turkey (TUBITAK) project, the booklet was translated into Turkish, and validity and reliability analysis were conducted (for detailed information about analysis, please see Guler, 2014). In this section, the adaptation process is presented briefly.

First of all, language validity was tested for MKT test. In this regard, some international comparative studies (e.g. Fleischman et al., 2010; Olson et al., 2008) were considered. A commission composed of mathematics educators, an English language and literature expert, and an assessment and evaluation expert was established. Multiple translations were produced independently by the commission members and revisions were provided with the discussion of members. Following the pilot study, a panel composed of the study team and commission members reviewed the items in the MKT. Finally, necessary linguistic corrections were provided and the test was prepared for the validity and reliability analysis. At the next step, the booklet was administered to a total of 63 pre-service mathematics teachers in order to check the validity and reliability of the MKT test. Since the Turkish booklet were adapted to the original TEDS-M study, it was assumed to have content validity. The construct validity of the test was investigated with item response theory (IRT) since the MKT was originally developed using this approach (Tatto et al., 2008). On the other hand, it is preferred to use Rasch model instead of IRT if the sample size is limited (Guler et al., 2014). Considering to take advantage of both analysis method, we used both approach and conducted item analysis for MKT test. Consequently, construct validity of MCK and MPCK tests regarding to two theories were presented (see Aydin, 2014). In TEDS-M study, reliability analysis was conducted separately for two components of MKT. We have remained loyal to the original analysis method and carried out reliability analysis for both MCK and MPCK tests respectively. For MCK test, Cronbach's alpha was found to be 0.76, while KR-20 was 0.72. On the other hand, for MPCK test, Cronbach's alpha and KR-20 were found to be 0.82. Each of these values met the criteria for the reliability.

Data Collection and Analysis

In the primary study, the test was applied with the participants, in a time frame of 90 minutes. Prior to application stage, teaching staff at the universities in question were contacted. Thereafter, data was gathered by the instructor the pre-service teachers were already assigned to, accompanied by a member of the study team. The goal in doing so was to increase the motivation levels of the pre-service teachers, and to make sure that the procedure was taken seriously among the participants. The data gathered were first subjected to pre-processing. The pre-processing intended to eliminate the inconsistencies (selecting more than one option, not selecting most options etc.) from the dataset. Through the process, the number of participants was reduced from an initial set of 1,431, to 1,367 by taking out the data for 64 pre-service teachers.

The rubrics used in the TEDS-M project and adapted into Turkish by the project team were used for the coding of the data gathered through the MKT test (for detailed information, see Celik et al., 2016). The test contains a variety of question types, including multiple-choice, complex multiple-choice and open constructed-response. The questions other than the open-ended ones were coded on a binary scale of 0-1, whereas for some open-ended questions, 2 was added to the coding scale. The reliability of coding was found to be 0.88 by comparing the coding by different coders, on a sample of 100 questionnaires selected randomly. Therefore, one can speak of a high level of the inter-coder reliability (Lombard et al., 2002).

Each questionnaire, and the MCK and MPCK questions in the questionnaires were scored by the researchers with reference to the rubric. The scoring process led to the expression of the data (with reference to region, university, and content domain) on a scale of 100. In other words, the success percentages served as the reference point. When calculating the success rates, a route comparable that of the TEDS-M study was applied to render the comparisons significant. The success rate for the multiple-choice and complex multiple-choice questions coded as either 0 or 1 was calculated using the formula $P_{Item} = \frac{\text{Correct response frequency}}{\text{Number of participants}} \times 100$. For open constructed-response questions coded with 0, 1, or 2; on the other hand, fully correct answers were multiplied with 1, and partially correct answers were multiplied with 0.5 to calculate the success rates through a comparable methodology. When calculating the success rates for MCK, MPCK, subdomains thereof, or specific content domains, the averages of the percentages pertaining to all items in the relevant category were calculated.

In order to establish the statistical significance of the differences observed between the regions, statistical tests were introduced. For this purpose, the normalcy of the distribution of scores within specific groups was analyzed. The review focused on the skew coefficients for the groups which were found to exhibit abnormal distribution. The positioning of these coefficients in the -1 to +1 range is a criteria used to support the assumption on whether the groups are normal or not (Buyukozturk, 2009). In this context, all groups are found to have met one of the two criteria. One-way ANOVA analysis was performed to reveal if statistical variations exist between the regions. If a significant difference was found between the groups, that analysis was followed by post-hocs analyses. Prior to those, however, the homogeneity of the variances was examined using the Levene test. Multiple comparisons with homogeneous ($p > .05$) variances determined in the Levene test were followed by a Tukey test, whereas the ones with non-homogeneous ($p < .05$) variances were followed by a Tamhane' T2 test.

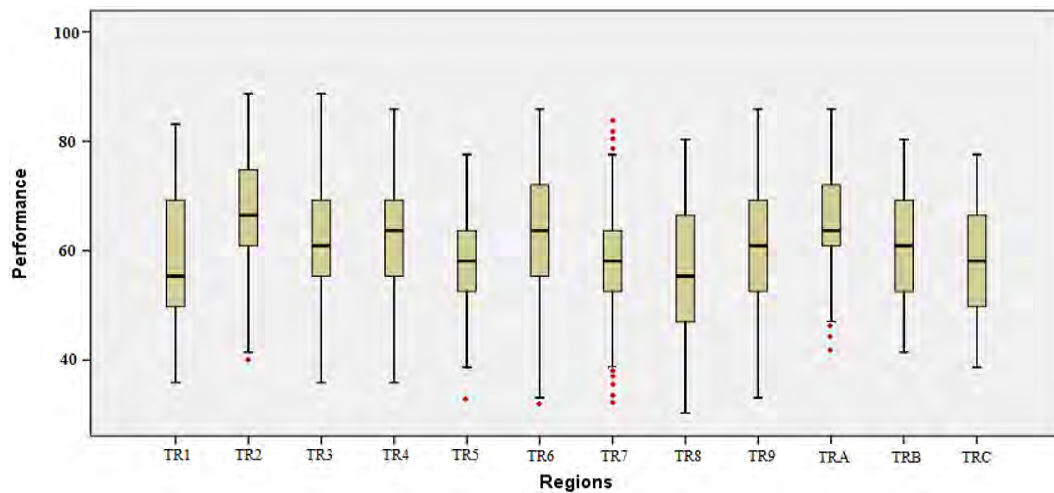


Figure 1. Distribution of PEMTs’ performance in MKT test by regions (N=1,367)

Table 2. The results of one-way ANOVA analysis for MKT test

	SS	Sd	MS	F	Sign.	Significant variation
Between groups	26.267	11	2.388	8.970	.000	TR1-TR2, TR2-TR3, TR2-TR5, TR2-TR7, TR2-TR8, TR2-TR9, TR2-TRC, TR3-TRA, TR5-TRA, TR6-TR7, TR6-TR8, TR6-TRC, TR7-TRA, TR8-TRA, TR9-TRA, TRA-TRC, TR1-TRA
Within groups	360.71	1,355	0.266			

Note. SS: Sum of squares; MS: Mean square; Statements in bold express the group favored by variation

RESULTS

This section of the study presents the results with reference to regions, universities, and content domains. With reference to the regions, the findings reached through the general MKT test, as well as through MCK and MPCK—the components of MKT. In order to assess the change in the performance levels of different universities in a given region, a similar methodology was applied, this time with reference to individual universities. Finally, the general performances of the universities were discussed with reference to content domains.

MKT Results per Region

In connection with the first sub-problem investigated in the study, the variances in PEMTs’ mathematics teaching knowledge between regions are presented in **Figure 1**.

According to the box-and-whisker plot presented in **Figure 1**, the pre-service teachers’ performance exhibits variance within regions. In other words, a given region would include not only pre-service teachers with a high level of performance, but also some with a very low level in terms of performance. In contrast to other regions, TR7 stands out as the region with the single largest number of extreme scores. In terms of variance between quintiles, TR5, TR7, and TRA exhibit rather homogeneous structures compared to other regions. It is possible to put this observation as a comment on the internal consistency of these regions on the pre-service teachers’ mathematics teaching knowledge front. **Figure 1** also makes it clear that the test scores of pre-service teachers in regions TR5, TR6, TR7, TR9, TRB, and TRC exhibit a symmetrical distribution. In terms of average MKT scores of PEMTs, TR2 region performed best, while TR8 lagged behind other regions. Also, average MKT scores also exhibit variances within regions.

One-way ANOVA analysis was applied to test the statistical significance of the variance to arise with reference to regions. The analysis revealed that the PEMTs’ performance in the MKT test exhibited significant variation with reference to regions ($F_{11, 1355}=8.97 p=.000<.05$). Levene test, on the other hand, showed that the variance in score distribution in MKT test within regions was rather homogeneous in character [$F(11.1355)=1.022, p=.424>.05$]. Against this background, Tukey test results in post-hocs analyses were also checked. The results reached are presented in **Table 2**.

A glance at **Table 2** reveals a significant difference in favor of region TR2, in comparison to regions TR1, TR3, TR5, TR7, TR8, TR9, and TRC. Moreover, significant differences were observed in favor of TR6 when compared against TR7, TR8, and TRC, and in favor of TRA when compared against TR1, TR3, TR5, TR7, TR8, TR9, and TRC.

The study goes beyond a holistic analysis of PEMTs’ performance in MKT, and investigates performance specific to sub-components of MKT –MCK and MPCK. The analysis led to the findings summarized in **Figure 2** with reference to the PEMTs’ performance in MCK and MPCK, compared between regions. A glance at **Figure 2** reveals that, across all regions, PEMTs’ performance in MPCK is higher than the performance levels in MCK. Furthermore, generally speaking, in most regions PEMTs’ performance scores for MPCK and MCK tend to rise and fall in conjunction. For instance, MPCK and MCK scores in TR2 region, when compared against TR1 region, have risen in parallel, whereas the same two scores for TR3 region, compared against those of TR2 region, fell as such. According to **Figure 2**, PEMTs’ performance levels vary from one region to another. Taking into account the average MCK scores of PEMTs, the TRA region performed best, while TR8 had the poorest performance. On the average MPCK scores front, however, TR2 region performed best while TRC region occupied the other end of the spectrum. In terms of the differences between regions, one can safely note that the variance in MCK is in excess of the variance in MPCK.

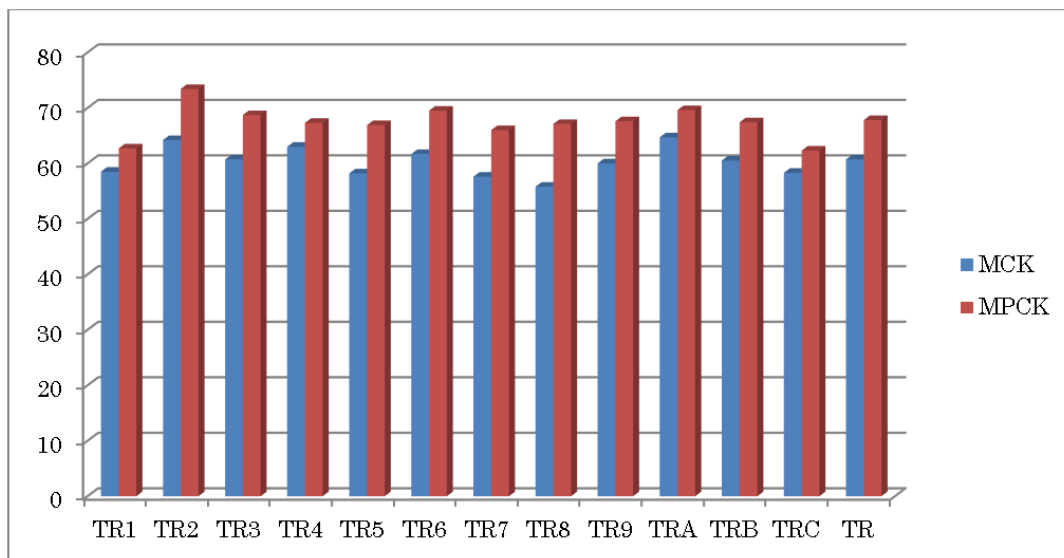


Figure 2. Performance of regions in MCK and MPCK tests (TR: Value for the whole country)

Table 3. The results of one-way ANOVA analysis for MCK test

	SS	Sd	MS	F	Sign.	Significant variation
Between groups	30.946	11	2.813	8.144	.000	TR1-TR2, TR2-TR3, TR2-TR5, TR2-TR7, TR2-TR8, TR2-TR9, TR2-TRC, TR6-TR7, TR6-TR8, TR5-TRA, TR7-TRA, TR8-TRA, TR9-TRA, TRA-TRC
Within groups	468.095	1,355	0.345			

Note. SS: Sum of squares; MS: Mean square; Statements in bold express the group favored by variation

Table 4. The results of one-way ANOVA analysis for MPCK test

	SS	Sd	MS	F	Sign.	Significant variation
Between groups	23.764	11	2.160	3.163	.000	TR1-TR2, TR2-TR7, TR2-TR9, TR2-TRC
Within groups	925.383	1,355	0.683			

Note. SS: Sum of squares; MS: Mean square; Statements in bold express the group favored by variation

One-way ANOVA analysis was applied to see if the differences between the regions in terms of the PEMTs' performances in MCK and MPCK are statistically significant or not. The one-way ANOVA analysis performed for the MCK test revealed significant differences between the regions, in terms of pre-service teachers' performances in the MCK test ($F_{11,1355}=8.144, p=.000<.05$). Levene test, on the other hand, found that the variances concerning the distribution of the regions' scores in the MCK test were homogeneous in nature ($F_{11,1355}=1.35, p=.191>.05$). That is why the multiple comparisons took the results of the Tukey test into account. The results are provided in **Table 3**. **Table 3** reveals a significant difference in favor of region TR2, in comparison to seven regions (TR1, TR3, TR5, TR7, TR8, TR9, and TRC). Moreover, significant differences in favor of TRA region when compared against regions TR5, TR7, TR8, TR9, and TRC, as well as in favor of TR6 when compared against TR7 and TR8 were observed.

The one-way-ANOVA analysis applied to see if the PEMTs' performance in the MPCK test varies statistically or not between regions led to the conclusion that the pre-service teachers' performance in the MPCK test varied significantly from one region to another ($F_{11,1355}=3.163, p=.000<.05$). On the other hand, Levene test found that the variances concerning the distribution of the regions' scores in the MPCK test were homogeneous in nature ($F_{11,1355}=1.378, p=.177>.05$). That is why the multiple comparisons took the results of the Tukey test into account. The results of the analysis are provided in **Table 4**. Tukey test shows statistically significant differences only in the context of comparing TR2 region with four regions (TR1, TR7, TR9, and TRC), in favor of TR2. Compared to MCK test, the differences between the regions were lower in MPCK test.

MKT Results per University

These differences expressed between the regions of Turkey paves the way for a second question, asking how do the performance levels of pre-service teachers vary between universities in a given region, with reference to the MKT test in general, and MCK and MPCK tests in particular. The next section of the paper will present the performance levels achieved by all universities participating in the study, in the MKT test in general, followed by a perspective on finer details regarding MCK and MPCK tests. **Figure 3** presents the MKT test performance scores achieved by 21 universities included in the sample. **Figure 3** suggests that, generally speaking, nine universities in Turkey performed better than average, while 12 were below average, with the score levels ranging in the 58%-67% range. With reference to the performance levels achieved by universities in a given region, it is evident that in some regions (TR5, TRA, and TRC) the universities in that region performed at similar levels while in others (TR3, TR6, TR7, and TR9) universities in the region exhibited varying performance levels. As an example, one can point out the fact that the universities coded TRAU1 and TRAU2 are among the top performers in the MKT Test. On the other hand, among the three universities in the Central Anatolia region, TR7U2 ranked among the nine universities, which performed above average, while TR7U1 and TR7U3 performed in the lower group.

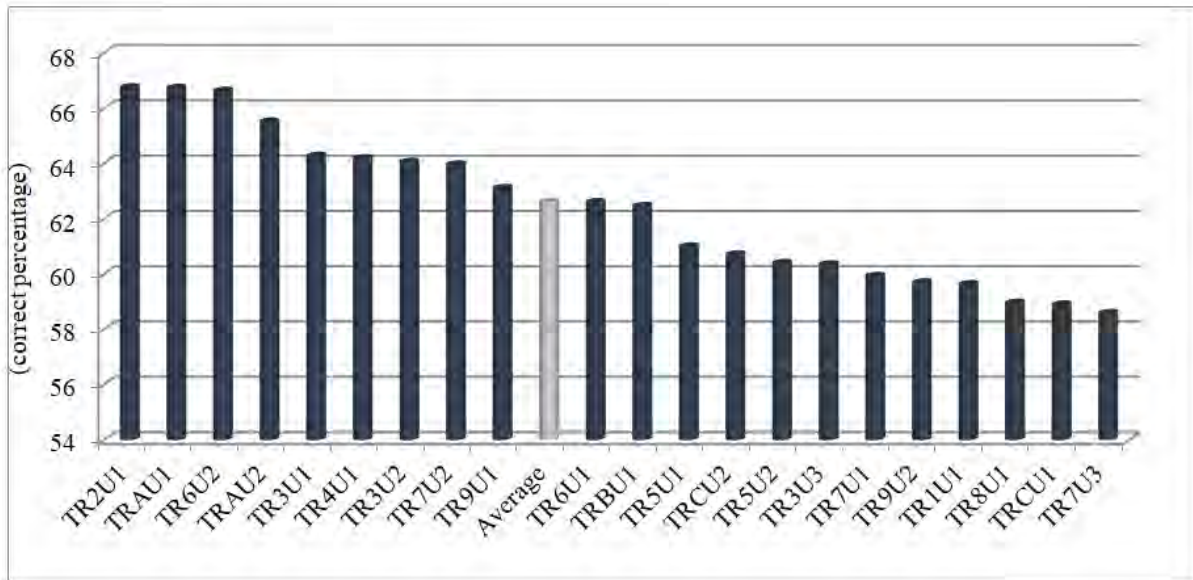


Figure 3. The universities' overall performance

Table 5. MCK & MPCK performances of universities

MCK performance of universities			MPCK performance of universities		
University	Percent correct	Standard error	University	Percent correct	Standard error
TRAU1	64.8	0.40	TR2U1	73.4	0.13
TRAU2	64.7	0.27	TR6U2	71.9	0.23
TR6U2	64.6	0.43	TRAU1	71.6	0.13
TR2U1	64.2	0.37	TR3U2	71.0	0.28
TR4U1	63.0	0.56	TRCU2	70.2	0.26
TR7U2	62.4	0.64	TR3U1	69.7	0.13
TR3U1	62.1	0.43	TR5U1	69.1	0.32
TR3U2	61.3	0.50	TR9U1	69.1	0.10
TR9U1	60.8	0.26	TR7U2	68.0	0.26
TR6U1	60.7	0.45	Average	67.8	0.04
Average	60.6	0.10	TRAU2	67.7	0.12
TRBU1	60.5	0.43	TR6U1	67.6	0.16
TRCU1	58.9	0.36	TRBU1	67.4	0.22
TR1U1	58.5	0.70	TR4U1	67.3	0.21
TR5U2	58.4	0.61	TR8U1	67.1	0.18
TR3U3	58.1	0.36	TR7U1	66.4	0.13
TR9U2	58.0	0.39	TR3U3	66.1	0.18
TR5U1	57.8	0.56	TR5U2	65.5	0.23
TR7U1	57.4	0.52	TR7U3	64.9	0.13
TRCU2	57.0	0.56	TR9U2	64.0	0.15
TR7U3	56.1	0.41	TR1U1	62.7	0.16
TR8U1	55.8	0.45	TRCU1	58.8	0.18

Table 5 presents the participating universities' performance levels in MCK and MPCK tests, the two sub-components of the MKT test. According to Table 5, for the MCK test the average percent correct of the whole country was 60.6%, with 10 universities scoring above that average and 11 scoring below. The university which performed best in the MCK test was TRAU1 (64.8%), while the one performing worst was TR8U1 (55.8%). The margin between the highest and lowest performing universities was roughly 9%; one can argue that the universities' performance in the MCK test are similar to those of others.

According to Table 5, for the MPCK test the average percent correct of the whole country was 67.8%, with nine universities scoring above that average and 12 scoring below. The margin between the performance of the highest performing university TR2U1 (73.4%) and the lowest performing university TRCU1 (58.8%) was nearly 15% for the MPCK test. In overall, the universities' performance in MCK and MPCK tests present a picture whereby the percent correct in the MPCK test was higher than those in the MCK test, whereas the variation in performance in the MPCK test was higher than that in the MCK test.

Table 5 can be used to support the argument that the universities exhibit comparable MCK performances, while their MPCK performances do not exhibit such a character. For instance, Table 5 shows that a university in TR6 region performed well above the average, while the other university of the region performed below average. Similar examples can be noted with reference to regions TRA, TR6, and TR5 as well. For the MCK test, such cases are less frequent. Compared to MPCK, the performance scores of the universities in a given region are closer in the case of MCK. For instance, in MCK test, the two universities which have the highest average score in Turkey are from the same region. In a similar vein, both universities in TR6 have scored above average. This state

Table 6. Universities' performance scores in algebra & numbers content domain

Universities' performance scores in algebra content domain			Universities' performance scores in numbers content domain		
University	Percent correct	Standard error	University	Percent correct	Standard error
TR2U1	69.0	0.17	TRAU2	70.6	0.12
TR3U1	65.0	0.19	TR6U2	69.1	0.21
TR6U1	64.8	0.24	TRBU1	68.0	0.19
TR6U2	64.4	0.35	TR2U1	68.0	0.15
TR3U2	64.3	0.32	TR4U1	67.8	0.16
TRAU1	63.5	0.20	TRAU1	66.6	0.12
TR4U1	63.2	0.29	TR9U1	66.3	0.10
TR9U1	62.2	0.15	TR3U1	65.3	0.13
TR7U2	62.2	0.31	Average	64.7	0.05
TRBU1	62.1	0.32	TR3U2	64.4	0.29
Average	61.6	0.05	TRCU2	64.3	0.18
TRAU2	61.5	0.17	TR7U1	64.3	0.14
TR9U2	60.5	0.24	TR7U2	62.6	0.22
TR1U1	59.5	0.30	TR5U1	62.5	0.30
TR5U1	59.4	0.46	TR6U1	61.9	0.18
TR7U3	59.2	0.18	TRCU1	61.9	0.15
TR8U1	58.8	0.27	TR7U3	61.5	0.13
TR5U2	58.6	0.34	TR9U2	61.5	0.15
TR7U1	58.1	0.23	TR1U1	60.6	0.22
TR3U3	57.3	0.21	TR3U3	60.2	0.18
TRCU1	54.8	0.19	TR8U1	59.6	0.19
TRCU2	54.3	0.38	TR5U2	57.2	0.21

of affairs is perhaps the fundamental reason to explain why variance between the regions is higher with MCK (see **Table 5**), and lower with MPCK (see **Table 5**).

When analyzing **Table 5**, **Table 5** reveals that certain universities exhibit different levels of performance in MCK and MPCK tests. For instance, with the MCK test TR5U1 and TRCU2 scored below the country average, they surpassed country average with the MPCK test. Similarly, with the MCK test TRAU2, TR4U1 and TR6U1 scored above the country average, they ranked just below the country average with the MPCK test.

MKT Results per Content Domains

The last question the study focused on was about the performance of the participating universities, with respect to algebra, numbers, geometry, and data content domains of the MKT test. The performance rankings of the universities in content domains algebra and numbers are presented in **Table 6**.

Table 6 reveals that all participants' average performance score in algebra content domain is 61.6%. The performance scores of individual universities in algebra content domain ranges from 54.3% to 69%. The university with the lowest performance level was TRCU2 from Southeast Anatolia, whereas TR2U1 from West Marmara achieved the highest performance level. On the other hand, 11 universities scored below the average figure for Turkey, while 10 scored above the country average. It is also evident that the average score for the numbers content domain (64.7%) is higher than that of the algebra content domain (**Table 6**).

In this content domain, eight universities performed above average, while 13 performed below. On the other hand, the correct percentage rates achieved by the universities which ranked near the average were close to each other. While the most successful university for the numbers content domain was TRAU2 from Northeast Anatolia, the least successful one was TR5U2 from West Anatolia.

The performance rankings of the universities in content domains geometry and data are presented in **Table 7**. A study of the universities performance levels in geometry content domain (**Table 7**) reveals that pre-service teachers' performance in geometry was higher than that in algebra, yet two performance levels were not very far from each other. After algebra, geometry content domain ranked second in the list of content domains where the performance is lowest. 13 universities which took part in the study ranked above the 63% figure which represented the average performance score. The range of performance levels is akin to that of the algebra content domain, while the performance figures varied between 56% and 70.4%.

The best performing university in the geometry content domain was TRAU1 in Northeast Anatolia, while TRBU1 from Central East Anatolia had the lowest performance. In general, a large majority of the participants were found to be successful in the test with respect to questions regarding the data content domain. Furthermore, the average general performance score (70.5%) is higher than those of other content domains. On the other hand, in comparison to algebra, geometry, and numbers content domains, the range of performance between the best performing university and the worst performing university was higher (25.4%) in the data content domain. As with the case of geometry content domain, the best performing university in the data content domain was once again TRAU1. TR7U3, on the other hand, had the lowest performance among all universities participating.

Table 7. Universities' performance scores in geometry & data content domain

Universities' performance scores in geometry content domain			Universities' performance scores in data content domain		
University	Percent correct	Standard error	University	Percent correct	Standard error
TRAU1	70.4	0.19	TRAU1	85.1	0.06
TR7U2	68.0	0.40	TR6U2	83.3	0.08
TR5U2	67.1	0.37	TRAU2	78.6	0.06
TR3U3	66.6	0.31	TRCU2	78.6	0.12
TRCU1	65.8	0.27	TR7U2	77.3	0.12
TR3U1	65.6	0.21	TR3U2	76.1	0.14
TR6U1	65.4	0.26	TR3U3	74.6	0.09
TR6U2	65.2	0.28	TR4U1	74.3	0.11
TR5U1	65.2	0.33	Average	70.5	0.02
TRAU2	64.8	0.23	TR5U2	69.7	0.09
TR3U2	64.0	0.42	TR7U1	69.6	0.08
TR2U1	63.4	0.22	TR1U1	69.4	0.11
TR9U1	63.3	0.17	TRBU1	69.3	0.10
Average	63.0	0.05	TR2U1	68.9	0.08
TR4U1	61.2	0.41	TR9U2	68.8	0.07
TRCU2	59.4	0.45	TRCU1	68.2	0.09
TR1U1	58.7	0.37	TR8U1	67.5	0.09
TR7U3	57.8	0.24	TR5U1	67.4	0.15
TR8U1	57.5	0.33	TR9U1	67.1	0.05
TR7U1	57.3	0.29	TR3U1	62.4	0.08
TR9U2	56.3	0.31	TR6U1	60.5	0.12
TRBU1	56.0	0.31	TR7U3	59.7	0.07

DISCUSSION, CONCLUSION AND SUGGESTIONS

Numerous studies have investigated the MCK and/or MPCK of pre-service teachers as important components of MKT. While these studies revealed the difficulties experienced by or shortcomings of future teachers on a specific topic or a content domain, the present study drew a snapshot of the general picture and reported the MKT of Turkish PEMTs as a part of a large-scale investigation. By defining what the current knowledge of future teachers, this study went beyond the test results and made comparisons among regions and universities in Turkey, with reference to MKT and its subdomains.

In this study, we used the TEDS-M framework and the data collection tools developed by the TEDS-M project team (Tatto et al., 2008). This approach, in turn, allowed us to compare the results of a large sample from Turkey, with those of TEDS-M countries. According to the results of MKT test, Turkish PEMTs had the right answers percentage over 50% in all of the test and its subcomponents. We compared this result with published international averages from TEDS-M study (Tatto et al., 2012) and found that Turkey scored appreciably higher performance on both MCK and MPCK components. Taking into consideration the results of a number of studies pointing out the positive impact of higher levels of mathematics teaching knowledge among the teachers, in terms of shaping the academic performance of the students (Olfos et al., 2014; Rowan et al., 1997), the results of the present study can be interpreted to support the view that Turkish pre-service mathematics teachers have the potential to evolve into successful teachers.

The study analyzed the MKT performance of PEMTs in 21 universities from 12 regions. Certain differences between the regions were observed with reference to PEMTs' performance in MKT. The analyses to see if such differences are statistically significant or not led to the finding that pre-service teachers in three regions (West Marmara, Northeast Anatolia, and Mediterranean) stand out from their peers in some other regions, in terms of their MKT performance. With reference to the sub-domains of the MKT test, PEMTs' scores for MCK and MPCK vary from one region to another. The variance between the regions is significant with three regions (West Marmara, Northeast Anatolia, and Mediterranean) in the case of MCK test, and with just one region (West Marmara) in the case of MPCK test. Taking these results into consideration, one can forcefully argue that the PEMTs' performance in MKT test and its subdomains do not vary immensely between regions, and any variation is concentrated in specific regions.

The results from universities located in the same region suggest that in certain regions (West Anatolia, Northeast Anatolia, and Southeast Anatolia) the universities tend to perform at similar levels, while in others (Aegean, Mediterranean, Central Anatolia, and East Black Sea) the universities performed at a wider range. However, in no case the variation between the MKT test performance levels universities in a given region achieved exceeded 5%. A similar state of affairs applies for the subdomains of the MKT test as well. Indeed, when compared against the results for the MPCK test, the universities in a given region were observed to have performed at levels closer to each other in the MCK test. These observations could lead to two fundamental conclusions: (i) with reference to MKT, PEMTs' performances do not vary much with reference to the universities and regions in Turkey and (ii) in the context of MKT, the universities more or less represent their regions, and the regions more or less represent the wider country in terms of performance levels. On the other hand, the performances of different teacher training programs in Turkey were rather close to each other.

One can forcefully argue that PEMTs' MPCK performance was close but somewhat better than their MCK performance, in terms of both regions and universities. In the case of the MPCK test, the regions exhibited a more homogeneous picture, while in the case

of MCK, the universities did so. This state of affairs making itself felt with respect to regions and universities can perhaps be explained through the differences in performance by the universities located in the same region. Indeed, in order to achieve a given standard of education in teacher training programs all around Turkey, the Higher Education Council of Turkey (HECT) implements a common content framework for faculties of education all around Turkey (HECT, 2016). In light of the results noted above, one can argue that some success is achieved towards that goal. Taking into account the shared courses required by HECT (2016), the similarities between the MCK performance levels of PEMTs from various universities are easier to explain. The basic reason leading to a higher level of difference between the MPCK performances of universities, compared to their MCK performances, could be related with the differences of in-class practices implemented at individual universities, even though common courses try and limit differences. For, it is often argued that the courses concerning MCK (analysis, linear algebra, abstract mathematics etc.) are often executed using conventional perspectives. The expression of the differences between universities, in terms of MPCK in particular, along with the causes of such differences requires some venture into the learning opportunities each university provides for its students. In this context, studies engaging in a comparative review of a number of variables, from the structure/nature/size of the universities' academic staff, to their physical equipment levels are called for. Another point that should not be ignored while interpreting the MCK and MPCK results obtained as a result is the measurement tool. MPCK questions often present students' misconceptions and difficulties by their nature, and it may be easier for prospective teachers to answer such questions (Guler & Celik, 2018). MCK questions, on the other hand, are the kind that will directly put their mathematical knowledge to work. This situation should be taken into account when interpreting the results obtained.

Many studies suggest a relation between MCK and MPCK. This is a major claim that warrants further study. Krauss et al. (2008) empirically investigated whether a correlation exists between the two most essential knowledge components of mathematics teachers' professional knowledge –MCK and MPCK– and found a strong correlation between those two knowledge bases. In contrast, TEDS-M project names only a few countries with very high correlations between MCK and MPCK (Blömeke & Kaiser, 2014). Although we have not conducted correlational analysis, the universities' performance levels in MCK and MPCK look similar. In other words, a majority of the universities which performed above average in terms of MCK had similarly above average MPCK scores, while a majority of which performed below average in MCK again ranked at the lower echelons of the MPCK ladder (see [Table 5](#)). These descriptive statistics would do with some empirical support, yet are suggestive of a correlation between MCK and MPCK.

In terms of subdomains of mathematics, the focus was not on the strengths or weakness of pre-service teachers. The results reached with reference to content domains revealed that algebra was the content domain where PEMTs performed worst. In line with this finding, Guler (2014) in a study on the pre-service mathematics teachers' algebra teaching knowledge in particular, showed that pre-service teachers failed to perform at stellar levels in this perspective. Other studies presenting a picture where Turkish students from middle school (Mullis et al., 2016) to university level (OECD, 2016) perform poor in algebra in comparison to other content domains, on the other hand, support the arguments of a number of studies expressing the relationship between the teaching knowledge and the students' performance levels (Olfos et al., 2014; Rowan et al., 1997).

Algebra is followed by geometry, as the second content domain where the pre-service teachers had most difficulty. The content domain with the highest performance level was data. In general, a 9% performance gap is observed between responses to two distinct sets of questions, one on algebra, the other on data content domain. A striking result in this context is about the variable character of the universities' performance in specific content domains. For instance, university TR7U2 scored below average in the numbers content domain, but ranked among the top universities in all other content domains. On the other hand, it is evident that the performance of different universities in a given region could also vary. For instance, a glance at the performance the universities located in region TR7 with reference to specific content domains reveals a wide range of performance levels. Taking into consideration the fact that the universities located in a given region are likely to grant admissions to students who perform at similar levels in the centralized university entrance examinations, the difference in performance levels could be associated with the education the universities themselves offer. In light of the universities' role of preparing the pre-service teachers to a professional career (Ingersoll et al., 2014; Oztay & Boz, 2022), the question "what an ideal teacher training program would look like?" merits greater attention.

Limitations and Future Directions

The present study has some limitations which suggest further interesting research questions. First of all, the MKT test adapted into Turkish within the framework of a project was used without any changes, given the goal of making comparisons between TEDS-M countries and the results concerning Turkey, in another upcoming study. Yet, the use of the tool adopted on paper media only can be considered a limitation, as it did not really allow insights into the overlap of theory with practice. Second, additional research is needed to examine how MCK and MPCK overlap quantitatively. Drawing on previous studies (e.g. Kirschner et al., 2015; Zhang, 2015), CK is defined as a necessary prerequisite of PCK. Third, although we attempted to reach the majority of the NUTS level 1 universities where the study was conducted, the number of universities from which we were able to collect data was limited. This limitation should be considered when evaluating our results. Finally, even though the average rates do not exhibit substantial variance among universities, it is still evident that the subdomains of mathematics show some difference from one university to another. Here, the characteristics of the universities (faculty, facilities etc.) should be analyzed against a background of high or low performance. And last, but not least, qualitative studies to be carried out with students can help the researchers in terms of coming up with some answers to these questions.

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