



A Teacher Training Project to Promote Mathematics Laboratory During the COVID-19 Health Crisis in Italy

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ABSTRACT

The M@t.abel2020 project supported Italian teachers during the COVID-19 pandemic period, proposing activities based on the mathematics laboratory teaching method which are suitable for distance learning situations. The project resulted in the establishment of an online community of 1,500 teachers. In this paper we present an exploratory study based on an open-ended questionnaire assigned to teachers involved in the project, with the aim of analyzing the results of this training project in terms of mathematics teacher's specialized knowledge. Results show that the project enriched teachers' knowledge not only in terms of pedagogical content knowledge but also in mathematical knowledge.

Keywords: teacher training, mathematics laboratory, distance learning, teacher knowledge

INTRODUCTION

The year 2020 was a highly critical and particular moment for everyone due to the global health emergency triggered by COVID-19. Education systems were also devastated by the arrival of the pandemic and, in Italy, schools were forced to resort to distance learning from March until the end of the school year (Krause et al., 2021). At the beginning of the new school year, a renewed spread of the virus made distance learning again necessary for secondary schools and, in some regions, even for primary schools. In this scenario, teachers had to reinvent (extremely quickly) their teaching practices and try to engage all the students locked in their houses with appealing and practical proposals to minimize the impact of this period in learning outcomes. In Italy the reorganization of teaching methods was the responsibility of each teacher and school directors. Teachers had to select which online platforms to use, and the specific software to propose to their students, while "teachers and learners had very limited prior technological and/or methodological preparation that foresaw the new settings" (Albano et al., 2021, p. 16). In other countries, the teachers' need to activate distance learning promoted the introduction of new general digital tools such as the ones for video-conferencing; however, the use of specific-mathematics digital tools decreased (Drijvers et al., 2021).

The enforced period of distance learning has led teachers of all grades to "deconstruct the usual teaching-learning paths, saving some passages and making others impossible, forcing the insertion of new elements into what were consolidated established teaching routines and sequences" (Bolondi, 2020). This is true not only in Italy but also at an international level. The need to find new ways to encourage students when classes were held in distance learning mode was a problem that teachers of all countries involved had to face instantly (Bakker & Wagner, 2020). The actual role of a teacher has been called into question and, with it, the very meaning of learning and the way of approaching pupils. A great risk of this sudden transition consisted in the falling back to less favorable pedagogical strategies, based on technology but focused solely on mere transmission of knowledge, thereby losing many important and embodied aspects of mathematics teaching and learning processes (Bakker & Wagner, 2020). In Italy, in many cases, the COVID-19 emergency has led to mere reproduction at a distance of what was previously done in the physical classroom (Bolondi, 2020), but

this is not the same in all countries as reported by Drijvers et al. (2021) considering Belgium, Germany, and Netherlands. The assessment practices in mathematics were also called for consideration: the same task, according to whether it is set in a classroom with paper and pencil or on a computer, can lead to different solution strategies and different difficulties for students (Lemmo, 2021; Puentedura, 2006). However, in many cases, the emergency has led to deep reflection and represented an opportunity for teachers to become involved and collaborate, identifying new approaches and new tools useful for teaching their subject. Albano et al. (2021), analyzing teachers' narratives regarding this period, found that some teachers "seem to discover some key aspects of the didactic system in which they are embedded, thanks to the disruption of the didactic system itself" (p. 32) and then think up new possible ways of teaching mathematics. This close attention to these issues could become a source for reinventing education and overcoming critical aspects in most of the school systems which are not in line with current society and its understanding of teaching and learning (Zhao, 2020). Otherwise, the great risk is that we do not learn from this difficult moment, as stated by Zhao (2020, p. 29):

"If we treat COVID-19 as a short-term crisis, then whatever we do to help extend learning when schools are closed will be only temporary. As soon as schools are reopened, the status quo will be restored. This seems to be the mindset and behaviors of most schools around the world."

The COVID-19 emergency could drive teachers, researchers and institutions to rethink different aspects of education. It is necessary to re-consider *what* mathematics is needed by students and differentiate learning on the basis of students' interests (Zhao, 2018). Furthermore, it is also important to think about *how* to teach mathematics to overcome a traditional and transmissive way of teaching and, finally, also *where* because the classroom is no longer the main seat of learning (Zhao, 2020), and boundaries between school and the out-of-school context have changed during these last few years (Bronkhorst & Akkerman, 2016).

Indeed, during this difficult period, numerous teacher training projects and webinars have been offered online, especially regarding the teaching of mathematics, and teachers also supported each other by sharing ideas and experiences with colleagues around the country through online communities.

In this scenario, the project M@t.abel 2020 was developed by the international center for innovation in the educational field Future Education Modena (FEM) to support teachers during this period, with the aim of proposing activities based on mathematics laboratory methodology also via distance learning. The project was very successful, with more than 1,500 teachers from all over Italy joining in from March 2020 to the present.

In this paper, we present the results of a questionnaire administered to teachers involved in the project, in order to analyze the impact of this project in terms of teacher training. We analyze the results of the questionnaire through the lenses of a mathematics teacher's specialized knowledge (MTSK) as developed by Carrillo-Yañez et al. (2018).

The M@t.abel Project

The M@t.abel project was a national project developed by the Italian Minister of Education in early 2000 to improve the teaching and learning of mathematics in all school grades. In the same years, the project was also accompanied by a process of revision of the Italian Ministerial Guidelines, which involved many experts from academic and school contexts. This process resulted in the drafting of the document 'Matematica 2001' (Mathematics 2001), which includes materials for a new mathematics curriculum and recommendations for activities and assessment tests for primary and secondary school. These materials are the foundations on which the current National Guidelines for the Teaching of Mathematics were built ten years later. The project M@t.abel had the merit of bringing these reflections on teaching into schools, improving teacher knowledge and their practices in schools. The importance of the M@t.abel project and its impact both on the Italian school system and on national research in mathematics education was also due to close collaboration within the project between mathematics teachers and researchers from the academic world.

The 116 laboratory units included in the online database of the project (<http://www.scuolavalore.indire.it/superguida/matabel/>) cover the mathematics curriculum, with materials, observations, methodology, and insights and are still a relevant reference point for all mathematics teachers in Italy. All the activities of

the database were designed by teachers and researchers and, during the last two decades, they have been validated by several research studies as well as hundreds of teachers in their classrooms. Furthermore, a university master's degree was proposed in several Italian universities while the University of Turin developed 4 MOOCs for teachers (Taranto & Arzarello, 2020). It is, therefore, a huge source of information for mathematics teachers in Italian schools.

The Mathematics Laboratory

The mathematics laboratory teaching method, which is recommended in the Italian Ministerial Guidelines (MIUR, 2012), is characterized by a constructivist and non-procedural approach to mathematics teaching and learning based on peer-interaction, class discussion, problem-based activities and manipulation of materials.

The mathematics laboratory was an approach to mathematics teaching already used in Italy in the last century (Giacardi, 2011) and discussed extensively both at national and international level (Bartolini Bussi et al., 2004; Chiappini & Reggiani, 2004; Maschietto, 2015). In early 2000, the Italian Mathematical Union (UMI) emphasised the importance of this teaching method, including a description of this approach and of its peculiarity and strengths in the 2001 mathematics curriculum (Anichini et al., 2004). In this document, the mathematics laboratory is described as being different to the idea of a laboratory in science or physics (Anichini et al., 2004, p. 28):

"The mathematics laboratory is not a physical place different from the classroom, it is rather a structured set of activities aimed at constructing meanings of mathematical objects. The laboratory, therefore, involves people (students and teachers), structures (classrooms, instruments, organization of space, and time), and ideas (projects, plans for teaching activities, experiments). The environment of the mathematics laboratory is somewhat similar to that of the Renaissance workshop, where apprentices learned by doing and seeing, communicating with each other and with experts."

The mathematics laboratory is not the place where the mathematics researcher works, it is the place where the students do mathematics and learn thanks to exploration, manipulation, resolution of significant problems and through essential interaction with peers and the teacher. Mathematical discussion plays a central role in laboratory activities; it ensures the emergence of new mathematical concepts and gives the teacher the possibility to systemize them (Bartolini Bussi, 1996). The laboratory approach is often further in the background than in the traditional classroom, which mainly consists in the exposition of theoretical content, rules and algorithms subsequently put into practice in the form of exercises. Research in mathematics education shows how important it is to offer students activities that allow for a "guided reinvention" (Freudenthal, 1991) of mathematical content and, from this perspective, "teaching mathematics is fundamentally about finding the most effective way to get pupils to do mathematics" (Bolondi, 2007).

The emphasis given to this teaching approach is found also in other countries, although the term "mathematics laboratory" assumes slightly different connotations in different contexts. For instance, Okigbo and Osuafor (2008) present the idea of the mathematics laboratory as a place where students explore new mathematical content through activities and materials, and connecting theory and practice.

Due to the relevance of this methodology and to the urgent need caused by the COVID-19 emergency, in the first months of the lockdown period in Italy, the FEM Centre decided to start a new project called M@t.abel 2020. In this project, researchers in mathematics education, teachers and experts of digital technologies, collaborated on re-examining some of the M@t.abel activities and adapting them for distance learning situations. The inclusion of specific digital technologies allowed teachers to promote laboratory activities also during the distance learning period, involving all students who were forced to stay at home.

The M@t.abel 2020 Project

The M@t.abel 2020 project was developed by the FEM Centre which is the first Italian EdTech hub; it was founded in 2019 with the aim of increasing the potential of education in society by improving the quality and impact of educational experiences through the use of digital technologies. The mathematics FEM team is composed of mathematics teachers from primary and secondary schools, researchers in mathematics

education, and experts of digital technology for education, who have worked jointly on each step of the design and implementation of the M@t.abel 2020 project. Furthermore, the project M@t.abel 2020 was developed under the collaboration of Ferdinando Arzarello, professor in mathematics education and scientific advisor of the original M@t.abel project and Giorgio Bolondi, professor of mathematics education at the Free University of Bolzano-Bozen and expert in teacher professional development.

The aim of the M@t.abel project was to support teachers in the rethinking of their didactical practices, in order to promote engaging and relevant activities also in a distance learning situation. As the COVID-19 emergency brought about an urgent need to provide proposals to teachers at very short notice, the mathematics FEM team decided to start from the previously-developed, tried and tested activities of M@t.abel. The strong educational and methodological value of M@t.abel activities allowed the team to focus their efforts on identifying the most useful digital technologies and tools to reinvent these activities within a distance learning context. In this process, another fundamental element was the collaboration of some of the original authors from the project, in order to maintain the original aim of the activities and their laboratory nature.

Seven activities were re-designed in the M@t.abel 2020 project; for each of these, a specific descriptive sheet was prepared to guide teachers in step-by-step implementation of the activity with their students. The descriptive sheet included also practical advice on how to use the digital tools with the students, didactical reflections on specific students' difficulties that emerged from mathematics education research, examples of students' results in Italian standardized assessments, and references to the Italian Ministerial Guidelines. All the activities were also presented to teachers from all over Italy through live webinars, which were each followed by more than 500 teachers. All the materials and the webinar registration were organized on an online platform in which more than 1,500 participants were registered and which offered the possibility to submit questions to the FEM mathematical team, to the original authors of the M@t.abel activity and to other teachers experimenting the activities. The collaboration on the online platform was extremely constructive; several teachers experimented the activities with their students and reported their experience to colleagues.

Seven activities were re-designed in the M@t.abel 2020 project¹, covering all grades from 1 to 11 and including different mathematical content as reported in **Table 1**.

Table 1. Description of the activities of the M@t.abel 2020 project

Activity	Mathematical contents	Grade	Digital technologies used
L'animale preferito [The favourite animal]	First elements of statistics and graphical representation of data	1-2 (age 6-9)	Jamboard & Padlet
Giochiamo con gli angoli [Play with angles]	Introduction of the concept of angles, measurement of angles	3-4-5-6 (age 8-12)	Online forms, GIFs, Mentimeter, & Padlet
L'albero maestro [The mainmast]	Perpendicularity, distance point-straight line, heights of triangles	4-5-6-7 (age 9-13)	Online forms, Padlet, slides to complete, Mentimeter, & Geogebra
Numerando [Numbering]	Operations and expressions with natural numbers	3-4-5-6 (age 8-12)	Padlet
La piazza [The square]	Estimation of large numbers in real situations	5-6-7 (age 10-13)	Mentimeter, online research, & Jamboard
La foto [The picture]	Ratio and proportionalities in real situations	5-6-7 (age 10-13)	Padlet
Le coniche [Conical curves]	Introduction to the main conic sections and their equations (circumferences, ellipses, hyperboles and parabolas)	8-9-10-11 (age 13-16)	Jamboard, Padlet, & Geogebra

One of the most popular activities was "La Foto" (The Picture), which is a problem-solving activity based on a picture of a child. In this activity, students have to hypothesize the height of a child, based on the elements present in the picture; this helps to introduce the concept of ratio in the last years of primary school or first years of secondary school. In the new version of this activity, the problem was posed through a video of the child, who is now 35 years old and who asks the students to help him to understand how tall he was in the picture. The video was included in an online virtual pin-board (using the free version of the Padlet app) and all students had the opportunity to add their hypothesis on the pin-board through a text, audio recording, video or drawing. They then had to read and comment on other students' posts, and the final classroom

¹ Original activities are available on the website: <http://www.scuolavalore.indire.it/superguida/matabel/>

discussion was proposed by the teachers in a synchronous lesson starting from the posts of the students on the virtual board.

Another activity experimented by many teachers, both in primary and secondary schools, addresses the issue of height in geometry. The original activity was based on the manipulation of concrete materials and a task to complete a specific drawing. The tasks were all integrated in the new version thanks to the use of many digital tools such as animated GIFs, slides with drawings to be completed by students, GeoGebra, and online surveys.

Theoretical Lenses: Teachers' Mathematical Knowledge

In recent years, the issue of teachers' knowledge has been widely discussed and several theoretical frameworks have been proposed also in the mathematics education field. In 1986, Shulman's (1986) distinction between subject matter knowledge (SMK), curricular knowledge (CK), and pedagogical content knowledge (PCK), underlined the importance of a new kind of knowledge specific to teaching (PCK). In this perspective, PCK includes the knowledge related to mathematical content in terms of teaching-learning and consists of two characteristic dimensions (Scheiner et al., 2019): (i) the knowledge of representation of subject matters and how these representations allow comprehension of the subject and (ii) the knowledge of specific learning difficulties and students' conceptions. Following Shulman's (1986) model, several alternatives were proposed to conceptualize teacher knowledge and growing consideration has been given to PCK (e.g., Marks, 1990). These include Ball et al. (2008), who proposed the mathematical knowledge for teaching (MKT) model, which considers PCK as divided into three areas of knowledge: content and students, content and teaching, and curriculum. Another important implication of Ball' et al.'s (2008) model is the view of SMK as divided into common content knowledge, specialized content knowledge, and knowledge on the mathematical horizon; they then underlined that teaching may require "a specialized form of pure subject matter knowledge" which is apart from PCK but specific to the teaching settings. Starting from these two key findings (importance of PCK and the need for specialized content knowledge) the mathematics teacher's specialized knowledge (MTSK) model was developed by Carrillo-Yañez et al. (2013, 2018).

The MTSK model considered mathematical knowledge (MK) and pedagogical content knowledge (PCK) as each divided into three subdomains (**Figure 1**). MK includes mathematics content itself, i.e., in-depth knowledge of mathematics objects, their definitions and relative properties, procedures and representations (KoT-knowledge of topics); the interlinking system between mathematical items and, in particular, inter-conceptual connection (KSM-knowledge of the structure of mathematics); and ways of proceeding in mathematics, e.g., demonstrating, defining, giving examples (KPM-knowledge of practices in mathematics). On the other hand, PCK is a "specific type of knowledge of pedagogy which derives chiefly from mathematics" (Carrillo-Yañez et al., 2018) and so "It is in this domain that the research literature in mathematics education has a major role as a source of knowledge for teachers" (Carrillo-Yañez et al., 2018). PCK includes three subdomains: knowledge of mathematics teaching (KMT) i.e., the teacher's awareness of potentialities and limits of teaching methods and strategies in relation to a specific content, also including a critical knowledge of materials and resources; knowledge of features of learning mathematics (KFLM), i.e., the teacher's awareness of students' difficulties and strengths, both in general and in relation to specific content, knowledge of learning theories and emotional aspects of learning mathematics; knowledge of mathematics learning standards (KMLS), which comprises knowledge of curriculum specifications and personal reflection on what students should learn. Finally, the MTSK model considered also teacher beliefs as fundamental; indeed, beliefs about maths and maths teaching/learning strongly interact with the other two domains, MK and PCK.

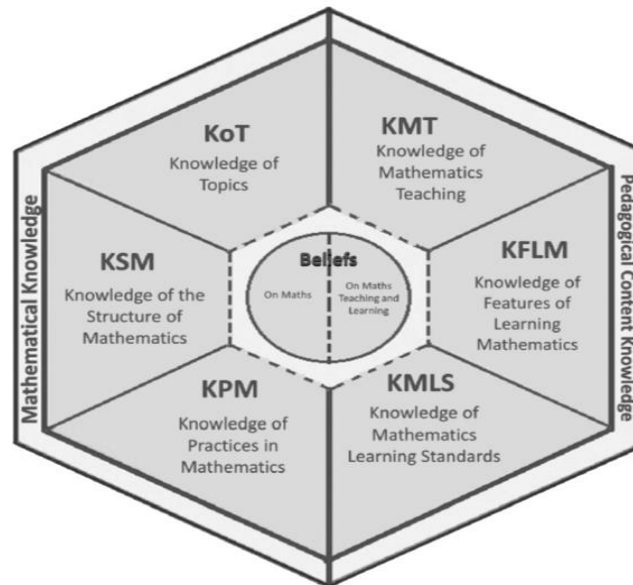


Figure 1. The mathematics teacher's specialized knowledge model (Carrillo-Yañez et al., 2018, p. 241)

Research Questions

The MTSK model is thus a useful framework to investigate and analyze in depth the impact of teacher training courses or of a project developed to support teachers (such as the M@t.abel 2020 project). The purpose of this paper is to understand how the M@t.abel 2020 project has enabled teachers to deepen their specific knowledge of mathematics and its teaching/learning processes. Indeed, even though the first aim of the project was to develop activities that teachers could use with their students, the entire pathway and all materials were designed in light of the importance of providing appropriate training and support to teachers in relation to the laboratory methodology, digital technologies, specific mathematics content and students' difficulties.

We consider teachers' declarations on their experience as an indicator of the impact of the project, and then we observe their perceived progress in mathematical knowledge due to participation in the project. Then, for the fundamental role of beliefs in the theoretical framework of Carrillo-Yañez et al. (2018) we decided to adopt this framework in this research rather than other possible models such as Ball' et al.'s (2008).

The research questions of this particular study are:

1. In which way did the teachers involved in the M@t.abel project perceive an enrichment of their knowledge?
2. In particular, which components of the MTSK model were identified by the teachers as most influenced by the project?

METHODOLOGY

In this research study, we proposed an exploratory questionnaire to teachers involved in the M@t.abel 2020 project. The aim of this study was to explore how the project had an impact on teachers' mathematical knowledge. The questionnaire, adapted from one used in a previous research study (Ferretti, 2020), was anonymous and consisted of some multiple-choice questions which allow us to analyze the sample of teachers who participated in the survey, and four open-ended questions designed to extract different aspects of MTSK (Figure 2).

Think about one activity from the M@t.bel 2020 project that interested you the most.

D1: Did your reflections on the activity (that emerged from the webinar or project materials) and/or its experimentation, enrich your knowledge from a mathematical content perspective? If so, which ones?

D2: Did the reflections on the activity and/or its experimentation, enrich your knowledge of student learning processes in relation to the content involved? If so, in what ways?

D3: Did the reflections on the activity and/or its experimentation, enrich your knowledge of teaching processes in relation to the content involved? If so, in what ways?

D4: Did the reflections on the activity and/or its experimentation, consolidate/modify your teaching practices? If so, in what ways?

Figure 2. Teachers' questionnaire

The questions were asked via an online form disseminated in the teacher community without any specific reference to the subdomains of the MTSK model. Teachers involved in the project were free to decide whether or not to answer the questionnaire and no question was mandatory.

In this paper, we present a qualitative analysis of teachers' answers on the basis of the MTSK model; indeed, we search for paradigmatic answers which allow us to investigate if, and how, the project had an impact on a specific sub-domain. The response analysis was computed following an inductive content analysis approach (Patton, 2014) and by making a categorization based on the sub-domains of the MTSK model (Carrillo-Yañez et al., 2018). The first question (D1) is aimed at exploring the MK domain, while we expect that the other three questions will provide evidence on PK and related subdomains.

Sample Description

There were 292 teachers who participated in the survey, most of whom teach in the first cycle of education (44% in primary school and 46% in lower secondary school), while 10% teach in upper secondary school (Table 2). This distinction is important in the Italian context because teachers at primary schools come from a general education background that includes many courses in pedagogy and some courses related to individual disciplines, including mathematics. On the contrary, teachers at secondary schools have a Master's degree in science, physics or mathematics and may have a strong disciplinary background but limited pedagogical education.

Table 2. Teachers' school grade and participation in the M@t.abel 2020 project

Grade	Percentage of teachers	Participation in 1 or 2 Webinars	Participation in more than 3 Webinars	Experimentation of the materials (taken)	Experimentation of the materials (planned)
Primary school	44%	58%	34%	13%	29%
Lower secondary school	46%	54%	40%	26%	34%
Upper secondary school	10%	6%	24%	10%	45%

37% of the respondents participated in the project from its beginning in spring 2020 throughout the full lockdown period in Italy, while the others started during the summer (8%) and with the new school year (55%).

Almost all the teachers (90%) participated in the online webinars, while the others consulted the online materials (didactical sheet and materials for students). A large number of teachers (35%) participated in 3 webinars or more.

All the teachers' answers were collected in a spreadsheet file, and paradigmatic teachers' answers are reported in the following section (the same number is used for the same teacher).

RESULTS

As already explained, almost all the respondents followed the webinars organized within the project, and 98% of teachers stated that the project helped improve their knowledge as mathematics teachers in general. Focusing on one of the activities proposed, more than 40% of primary teachers' state that they have already experimented the activity with their students, or are planning to do so. Teachers at lower secondary school are even more involved, with 60% stating that they have or have planned to experiment the activity with their

classes. Finally, only a few of the upper secondary school teachers have experimented one of the activities in their classes but many of them are planning to do so.

The activity most cited by the teachers was *Numerando*, but the activities *Giochiamo con gli angoli*, *Coniche*, *La foto* and *L'albero Maestro* were also indicated by many teachers.

Moreover, the classification of teachers' answers to the four open-ended questions based on MTSK subdomains was helpful in highlighting which component the project has influenced (and how). The open-ended questions in particular revealed the teachers' thoughts on their perceived enrichment due to the project. It emerged that teachers recognized an enrichment in PCK sub-domains, particularly in KMT; improvements in MK subdomains were also registered.

Knowledge of Topics (KoT)

All teachers who explicitly stated that the project had enriched their knowledge of specific mathematical content belonged to primary and lower secondary school. As already explained, in Italy, teachers in primary school might not have a strong background in mathematics and this is often true also for lower secondary school where there are few teachers with a degree in mathematics. The contents on which teachers improved their knowledge differ according to the activity they have explored more deeply. For instance, one primary school teacher stated that consulting materials of the activity *L'albero Maestro* concerning perpendicularity and height of triangles enriched his knowledge, and answered the first question with:

T_1: Definitely, yes, such as the concept of height in a polygon.

Also, the activity on angles had an impact on teachers' KoT. In this case, the answer refers to definition of angle, its properties and different possible representations:

T_2: The concept of infinity linked to the angle and all its manifestations and implications.

Angles and heights are really complex and tricky concepts, and many misconceptions and difficulties might obstruct genuine comprehension of these mathematical objects (Sbaragli & Santi, 2011; Stavy & Tirosh, 2000). This is true not only for students but also for teachers and prospective teachers, and many studies highlight the importance of improving mathematics teachers' knowledge on such concepts (e.g., Tsamir, 2007).

Similar claims also emerged in relation to arithmetical content: some primary teachers stated that the webinars and materials of one activity had clarified for them the difference between numbers and digits, numbers composition and possible different representations of numbers:

T_3: They made me think more about the composition of the number and its variations.

Almost all of the teachers who explicitly declared that the project improved his/her knowledge of mathematics topics were primary school teachers but also some teachers from lower secondary schools recognized an enrichment in their knowledge of conics, which is a topic included in the Italian Ministerial Guidelines for lower secondary school but more often developed in upper secondary school.

Knowledge of the Structure of Mathematics (KSM)

The KSM identifies the teacher's knowledge of the interconnections between different mathematical elements. This domain emerged in only a few answers. For example, one primary teacher claimed:

T_4: I really liked the insights because they offer the opportunity to create a continuum between different content from the same discipline.

More specifically, a secondary school teacher stated that a specific activity helped her to reflect on the relationship between two concepts:

T_5: I reflected on the difference between perpendicularity and verticality, and the independence of perpendicularity with regard to the position of the object.

Knowledge of Practices in Mathematics (KPM)

As already stated, in this domain we considered practices related to mathematics, as opposed to teaching mathematics, and only two of the answers might be included in this category.

One primary teacher, reflecting on her past education in mathematics, observed that the project had introduced her to the practical aspect of doing mathematics.

T_4: Your webinars certainly opened my mind to some practical aspects of mathematics previously unknown to me.

A similar statement was made by an upper secondary school teacher in relation to the activity on conics.

T_6: When I was a student, I never carried out practical activities to achieve the construction of conics. I would have had no idea where to start.

Knowledge of Mathematics Learning Standards (KMLS)

All the materials introducing the activities made explicit reference to the Italian Ministerial Guidelines: in the description sheet of each activity, the first part included a box stating the learning goals of the National Guidelines strictly related to the laboratory proposed. Despite this, due to time limits, we did not make any specific mention of these during the webinars. This might be the reason why no answer related to knowledge of curriculum specifications emerged. Moreover, we found evidence of critical reflections by teachers on what students should know, as explained by one lower secondary school teacher:

T_5: I am increasingly convinced that the “laboratory” [...] is the right way to go, but at the same time curricular demands and content to be tackled should be reduced to allow us to focus on some fundamental core issues.

Knowledge of Features of Learning Mathematics (KFLM)

The impact of the project on this domain was profound and several teachers from all grades claimed that the project enriched their knowledge about students' learning processes, both generally and in relation to specific mathematical content. Indeed, in some cases, the better knowledge and application of laboratory practice allowed teachers to observe and reflect on the learning processes of their students:

T_7: They learn the process in a hands-on way by reflecting on the activity as opposed to formulas learned by rote.

T_8: I realize that my students' preferred approach is definitely trial and error but then...contamination from their peers' work and awareness of generalizations. The collaborative work was also instrumental in the weaker group finding alternative solutions.

T_9: It allowed me to reflect on the real possibility for each pupil to learn also from observation and comparison of their own results with those of their peers.

Moreover, teachers also referred explicitly to stronger knowledge of the learning process related to specific content, and specific difficulties of students such as misconceptions, as stated by a lower secondary school teacher talking about the activity on heights:

T_10: The project helped me understand which misconceptions are more deeply rooted in students.

Again, in relation to the activity on heights, a teacher of upper secondary school stated that with the laboratory proposed:

T_10: Pupils do not focus on the 'squares' on the paper, but think about the true meaning of perpendicularity.

Finally, from some answers we found that the project helped in improving teachers' knowledge on learning processes in the specific context of distance learning, and how to manage them.

T_11: Yes, the project helped me to better understand the learning processes employed during distance learning.

Knowledge of Mathematics Teaching (KMT)

The domain in which most teachers' answers were classified is KMT. We found references to potentialities and limits of teaching methods and strategies in relation to a specific content; indeed, several teachers of all grades claimed that the project enriched their knowledge of laboratory methodology and the potentialities of laboratory activities:

T_12: By questioning "traditional methods" and experimenting with laboratory activities, I have changed the way I teach.

T_13: Yes. I'm putting a lot of emphasis on laboratory-based teaching and avoiding misconceptions.

T_14: Thanks to these activities, my teaching processes have evolved by opening my mind to the use of simple, readily-available material and dynamic geometry software (Geogebra).

Reference to critical knowledge of materials and resources emerged:

T_15: It gave me confidence in presenting the angle, avoiding stereotypes presented in books.

Also teaching methods were enriched, changing the way teachers interact with students and enhancing the importance of mathematical discussion:

T_16: The activities stimulated me and the class to practice mathematical discussion.

T_17: I learned to try not to anticipate too much, so that students can discover possible solutions.

Furthermore, teachers stated that the project had enriched their knowledge of teaching processes in relation to specific mathematical content and use of materials:

T_18: I realized that it is important to focus on the concept of perpendicularity and verticality.

T_19: They increased my personal knowledge of activities pertaining to the angle.

T_14: Thanks to these activities, my teaching processes have evolved by opening my mind to the use of simple, easily-available materials and dynamic geometry software.

Beliefs

Finally, even though no question was designed specifically to investigate beliefs, some of the teachers stated that the project and the activities proposed helped them to gain confidence in changing their way of teaching mathematics, influencing their beliefs about maths teaching and learning.

T_20: They gave me more confidence and also the opportunity to structure a more engaging way to present content, focusing more on quality rather than quantity.

T_15: They give me the strength to continue to experiment in the classroom, in making students argue... in doing maths, I would say!

T_21: [This project] strengthened my motivation!

Also, beliefs on mathematics emerged from the answers, for instance:

T_21: They reminded me that even mathematics has a laboratory and manual aspect.

CONCLUSIONS AND FURTHER PERSPECTIVES

This paper adopts the MTSK model to investigate the perceived impact of a teacher educational project developed during the COVID-19 health crisis. The aim of the project was to encourage and guide teachers in using laboratory methodology for teaching mathematics also in a distance learning context, and more than 1,500 teachers from primary to upper secondary schools participated.

The analysis of teachers' responses to an open-ended questionnaire allowed us to answer the research questions: the perceived importance of the M@t.abel project in enriching teachers' mathematical knowledge is borne out by the statements of the teachers, and all the domain and subdomains of the MTSK model have been considered in at least one response.

More specifically, it emerged that the project had a perceived impact on PCK, particularly on the subdomains KMT and KFLM: indeed, we tried to include in all webinars and materials considerations based on research results in mathematics education, which helped teachers to reflect on teaching methods, learning processes, and students' difficulties with specific mathematics content. Furthermore, we believe that the decision to revisit the activity of one of the most important projects for teacher training in Italy, which had a strong methodological and theoretical background, helped us and teachers to reflect on mathematics teaching and learning processes. Many teachers who participated in the M@t.abel 2020 project and answered the questionnaire were teachers who already adopted a laboratory approach but stated that the project enriched their knowledge and motivated them to continue with this methodology. On the other hand, there were also several teachers who claimed that, although they usually adopted a more traditional approach, the activity experienced in their classes had changed their way of teaching and helped them to reflect on the potentialities of the laboratory in mathematics.

Results also highlighted the fact that teachers perceived an enrichment of their mathematical knowledge. The subdomain of MK in which most of the teachers' answers were classified was KoT: many teachers explained that the project and reflections on the specific activities helped them to clarify some mathematical items such as perpendicularity and angle.

Finally, reports of a change in teachers' beliefs on both mathematics and mathematics teaching were highlighted.

The MTSK model was thus fundamental in analyzing the impact of the M@t.abel 2020 project and the results of this study will be useful in designing the next steps of the project. Teachers will be encouraged to answer the questionnaire also in the upcoming months and in further studies we will produce a quantitative analysis of their answers to achieve a more detailed picture of the impact that M@t.abel 2020 has on teachers' knowledge.

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