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# Examination of the Estimation Skills and Strategies of Pre-Service Elementary Mathematics Teachers

**Tuğba Baran Kaya**
Kırıkkale University, Turkey

**Sedef Çelik Demirci**
Artvin Çoruh University, Turkey

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# Examination of the Estimation Skills and Strategies of Pre-Service Elementary Mathematics Teachers

**Tuğba Baran Kaya, Sedef Çelik Demirci**

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## Abstract

The present study aimed to determine the estimation skills of elementary education pre-service mathematics teachers and the strategies they adopt in estimation. The case study methodology was employed in the study, and the participants included 37 volunteering students attending the fourth grade in the Primary Education Mathematics Instruction Department at a university located in the Central Anatolia Region in Turkey. The study data were collected with an 8-item Estimation Strategy Test developed by the authors. The test included numerosity, computational, and measurement estimation questions. In addition to making estimations, the students were asked to write the reasons behind their estimations. The descriptive analysis technique was employed to analyze the study data, and the data were coded based on the estimation strategies adopted by the pre-service teachers and similarity of their estimations with the actual values. The data are presented as MAXQDA maps, and excerpts from student responses are also presented. The study findings demonstrated that the responses of most pre-service teachers were similar to the actual value in computational estimations; however, numerosity or measurement estimations were not similar. Furthermore, certain pre-service teachers inclined to conduct the actual operation instead of estimating the results during computational estimation, and directly counted the results in numerosity estimation. Also, it was determined that a significant number of pre-service teachers made computational errors in numerosity and computational estimation.

## Introduction

The rapid development and advances led to novel efforts to adapt to daily life. The estimation skills are among the basic skills required for adaptation. Estimation only entails to the immediate recognition of the size or quantity of something without actually counting or measuring it (Micklo, 1999). Thus, estimation skills are convenient in daily life through conducting mathematical operations without pen or paper. For example, when driving, the determination of the adequate distance between our vehicle and the vehicle in front of our vehicle to prevent a possible crash requires estimation skills (Gooya et al., 2011). These are also used when estimating the total bill when shopping (Rubenstein, 1985), or the total weight in kilos of sugar cubes in a box, the square meter of a piece of land (Bulut, 2019). Henceforth, students ought to learn estimating the results of mathematical operations in real life (Berry, 1998). On the other hand, teachers need to be aware of various estimation strategies (Boz-Yaman & Bulut, 2017).

## Estimation Strategies

It was observed that various studies classified estimation skills with different methods in mathematics education literature. Whilst certain studies made two major classifications of estimation strategies, others employed three classifications. Segovia and Castro (2009) classified estimation strategies in two categories: computational estimation and measurement estimation. On the other hand, the classification by Hogan and Brezinski (2003) included three categories: computational, numerosity, and measurement estimation. Segovia and Castro (2009) categorized measurement estimation in two sub-categories: continuous and discontinuous, and considered numerosity estimation within measurement estimation.

Computational estimation entails reaching logical results with mental computations (Aslan, 2011). Dowker (1992) described it as the process of reaching reasonable and approximate solutions to mathematical problems without conducting actual calculations. For instance, computational estimation is employed when predicting the approximate bill during grocery shopping. Hildreth (1983) analyzed measurement estimation strategies within two categories: clues and foreknowledge. For example, they determined that the use of the door width to measure the hallway entails employment of a clue, while the calculation of the area of a ceiling or a floor based on the dimensions of tiles entails the employment of foreknowledge. Furthermore, benchmarking and comparison strategies are commonly used in measurement estimation (Joram et al., 2005). On the other hand, Crites (1992) classified numerosity estimation in three categories: benchmark comparison is used to number the points counted after the visual scanning of objects; decomposition/recomposition is used to divide the object of the measurement into smaller samples and reorganize them until accessing the result; and eyeball estimation is used as a perceptual strategy based on the visually scanned pieces. In brief, it was determined that previous studies on estimation skills reported various classification strategies. Tekinkır (2008) determined several estimation methods such as rounding, distributive property, mental calculations and random estimations, foreknowledge- and experience-based estimation, induction, visualization, comparison, experimental estimation, using the first and last digits, and grouping.

## Literature Review

The review of estimation literature revealed that certain studies examined the impact of estimation within mathematics course in relation to learning-teaching approaches. For instance, it was found that students tended to employ random estimation before the instruction was given with scripted conceptual cartoons, and comparison and referencing strategies that required flexible thinking skills after the instruction given (Budak & Şengül, 2021). In a study conducted with a realistic mathematics education approach, it was found that the instruction improved computational estimation achievements of the students, and the estimation strategies that they develop (Ayvalı, 2013).

Several other studies were conducted on the estimation skills of students based on their year-level (Munakata, 2002; Siegler & Booth, 2004; Tekinkır, 2008; Boz, 2009). For instance, there were studies in which the estimation skills of primary school students (Boyraz, 2017; Bulut & Taşpınar-Şener, 2017; Pilten & Yener 2009), elementary school students (Aytekin &Toluk-Uçar, 2014; Çetin & Köse, 2015; Çilingir & Türnüklü, 2009; Akkuşçi, 2019; Aydoğdu, 2020) and high school students (Boz, 2004) were examined. These studies analyzed estimation strategies adopted by the students. Furthermore, certain studies analyzed other strategies that were not described in the literature. Çilingir and Türnüklü (2009) analyzed the estimation skills of junior high school students, and revealed that students also employed the experimental strategies in measurement estimation.

It was observed that there are studies in the literature conducted with teachers and pre-service teachers who could teach estimation skills. Quite a number of studies were conducted on the employment of estimation skills by mathematics and classroom teachers (Alajmi, 2009; Bozkurt & Yavaşça, 2021). To illustrate this, there were studies in which junior high school teachers were asked about their perception of estimation skills and it was explored that those teachers agreed on the significance of such skills to use in the daily life and yet, they experienced difficulty in employing them in mathematics courses. In other words, it was observed that teachers did not prioritize computation and analysis based on approximation (Boz-Yaman & Bulut, 2017). Also, in another study that analyzed the views of fifth grade teachers on estimation skills, it was determined that difficulties were experienced in activities aiming to equip students with estimation skills (Artut & Aslan, 2014; Aslan; 2011). In a study conducted with college students on fractions, the authors concluded that the estimation skills of the students in multiplication and division were weaker when compared to their estimation skills in addition and subtraction. Moreover, Hanson and Hogan (2000) concluded that students were more successful in solving computational problems when compared to estimating the solution. On the other hand, the studies conducted with pre-service teachers on estimation skills investigated the effects of estimation skills. In a study on the estimation skills of primary school pre-service teachers, they were divided into two groups to determine the effects of estimation skills. It was determined that pre-service teachers who solved problems with estimation were less successful when compared to pre-service teachers who calculated the results of the same arithmetic problems (Gliner, 1991). Similarly, the attitudes and performances of 187 primary school pre-service teachers towards computational estimation were investigated in three groups: the control group, a group that practiced computational estimation, and a group practiced and trained in estimation techniques. The study findings demonstrated that there were significant differences between the three groups, while there was no significant difference between the experimental groups. However, it was determined that the group that was trained in estimation techniques better comprehended the estimation process (Bestgen et al., 1980). Other studies conducted with pre-service teachers on the estimation skills analyzed the estimation skills of the pre-service teachers (Sulak, 2008). However, these studies focused more on computational estimation (Seferoğlu, 2015). Özcan (2015) analyzed the computational estimation skills of primary school pre-service mathematics teachers and determined that the conceptual knowledge levels of the pre-service teachers on computational estimation were low.

**The Rationale of the Study**

Estimation skills are included in mathematics curricula in all levels of education. This is because estimation skills play a key role in the acquisition of several skills such as numerical skills, reasoning about the suitability of calculation method, and association of operations with concepts (National Council of Mathematics Teachers [NCTM], 2000). Thus, estimation skills are included in mathematics curricula in several countries (Ministry of National Education [MoNE], 2005; Department for Education and Skills [EDCTM], 2005; Australian Education Council [AEC], 1994). The analysis of the significance of estimation skills in primary school mathematics curricula (1948-2015) revealed that it included measurement estimation, computational estimation, future estimation, and problem-solving skills. It was observed that various computational estimation strategies were adopted in the 2013 mathematics curriculum, which was revised in 2009 and later (Bulut, Yavuz, & Boz-Yaman, 2017). On the other hand, although recent junior high school mathematics curricula gave particular importance to the teaching of estimation skills, they revealed that there were emphases on some other estimation strategies to be adopted (MoNE, 2005; MoNE, 2013; MoNE 2018). Thus, the number of studies on estimation has increased in recent years (Boyraz & Aygün 2017).

Certain studies were conducted with teachers and pre-service teachers who would instruct estimation skills. However, it was observed that most studies conducted with teachers in Turkey were on teacher views on estimation skills (Boz-Yaman & Bulut, 2017; Bozkurt & Yavaşça, 2021) and on the difficulties in the employment of these skills in mathematics courses. On the other hand, certain studies conducted with pre-service teachers investigated the estimation achievements or estimation strategies of pre-service teachers as well as their views on estimation skills. Boyraz and Aygün (2017) reviewed the studies on estimation skills in Turkey, and concluded that a higher number of studies were conducted with junior high school students and only a few studies were conducted with pre-service teachers. Thus, the need for further studies on pre-service teachers who will play a key role in effective use of these skills by future students is quite obvious. It is important to determine the estimation strategies of the pre-service teachers before they begin teaching. Gliner (1991) emphasized that the estimation strategies should be improved. It is argued that the pre-service teachers who employ effective estimation strategies will assist their students in the acquisition of estimation strategies. Hence, it is needed to investigate as to how pre-service teachers employ estimation strategies. Previous studies on the estimation strategies adopted by pre-service teachers demonstrated that these studies concentrated on the success of pre-service teachers in estimation (Bestgen et al., 1980; Gliner, 1991). Furthermore, the studies conducted with pre-service mathematics teachers focused on computational estimation strategies (Goodman, 1991; Seferoğlu, 2015; Özcan, 2015) or one of the estimation strategies (Yavaşça, 2021) rather than estimation skills included in the literature. Thus, it is necessary to investigate as to how pre-service teachers employ any of all estimation strategies that are addressed in the relevant literature. The present study aimed to determine the strategies that pre-service junior high school mathematics teachers employed in computational, measurement and numerosity estimation, and the similarity of their estimates with the actual values, in other words, the success of their estimates.

## Method

Since the current study aimed to investigate the estimation skills and strategies of the pre-service junior high school mathematics teachers in detail, the case study method was adopted.

### The Study Group

The study participants included 37 volunteering senior students (7 male and 30 female) in the Elementary Education Mathematics Instruction Department in a university located in the Central Anatolia in Turkey. The study was conducted with senior students since they were the closest group to start teaching.

### Data Collection Instrument

In the study, the estimation skills test developed by the authors was used to collect the data. After the test questions were developed, a pilot application was conducted with 4 pre-service teachers, who were not included in the study group. The responses of these pre-service teachers were analyzed, and they were interviewed about the test questions and their responses. Based on the views of the pre-service teachers, 1 question was removed from the test since it was similar to another question, and 2 questions were revised. Then, expert opinions were obtained from two mathematics education experts and the test was finalized. The test included 8 items, four of which were associated with numerosity estimation, two were associated with measurement estimation, and two were associated with computational estimation. A higher number of questions were associated with numerosity estimation since the said estimation was taken into limited consideration in previous studies focusing on estimation strategies. The items were selected to reflect daily life as much as possible. A sample item is presented in Figure 1.

| Question | Estimate | The reason for the estimate |
|---|---|---|
| How many apples are there in 1 kg? | | |

Figure 1. Sample Estimation Skills Test Question

### Data Analysis

Descriptive analysis was employed to analyze the study data, and content analysis was also employed in certain times. Before data analysis, a draft coding template was developed by the authors. The template included two components: estimation strategies and accuracy of the estimate. The estimation strategies component was based on the classifications reported in previous studies on estimation strategies (Crites, 1992; Köse 2013; Tekinkır, 2008). The authors randomly selected 5 pre-service teacher responses and coded these responses independently. Then these codes were compared. The coders then agreed on the codes associated with estimation strategies and accuracy of the estimates. The study data and coding template were entered in the MAXQDA qualitative data analysis software.

The accuracy of the pre-service teacher estimates, similar to estimation strategies, was classified as "quite accurate", "accurate" and “inaccurate.” There are several studies in the literature on the determination of the accuracy of the estimates. Certain studies accepted ±25% (Gatzke, 1989) or ±50% (Baroody & Gatzke, 1991; Crites, 1992; Boz, 2004; Siegel et al., 1982) as quite accurate. In the current study, Levine's (1982) accuracy criteria were employed. Thus, when the estimate was within 10% of the actual value, it was coded as “quite accurate," when the difference was between 10% and 30%, it was coded as "accurate", and when the difference was more than 30%, it was coded as "inaccurate". However, even if pre-service teachers made computational errors, the methodology they applied was taken into account during coding, not their results. For example, S19 explained the rationale of the method as follows: "There were 12x9 (1st Row), 22x9 (2nd Row), 12x9 (3rd Row), 3 1st Row, 3 2nd Row, 3 3rd Row. When we add them, it will be 846. The data on the 1st row were identified, grouped, and computed". However, if the operation mentioned is "12x9x3+22x9x3+12x9x3=1242", the result will be 1242, not 846. While 846 is inaccurate, 1242 is quite accurate. Thus, the response was coded as "quite accurate."

The authors coded the data independently based on the detailed template. After the coding, the disagreements were determined, and the authors discussed and reached a consensus. The study findings are presented as MAXQDA maps, and the views of pre-service teachers on each theme are presented with direct quote. The pre-service teachers were coded as S1, S2,…, S37.

## Results

In this section, the study findings are presented in three sub-sections on numerosity estimation, measurement estimation, and computational estimation.

### Numerosity Estimation

The first question included an estimate about the number of seats based on an image of a conference hall. In the question that requires numerosity estimation, the actual number of seats was 1254. Student responses were coded as "accurate" when within the 10% of the actual count, " quite accurate" when within 10-30%, and "inaccurate" the difference was more than 30%. Thus, when the estimates were between 1129 and 1379, they were coded as " quite accurate", when between 878 and 1128 or 1380 and 1630 they were coded as "accurate", and the rest were coded as "inaccurate". The related MAXQDA map is presented in Figure 2.

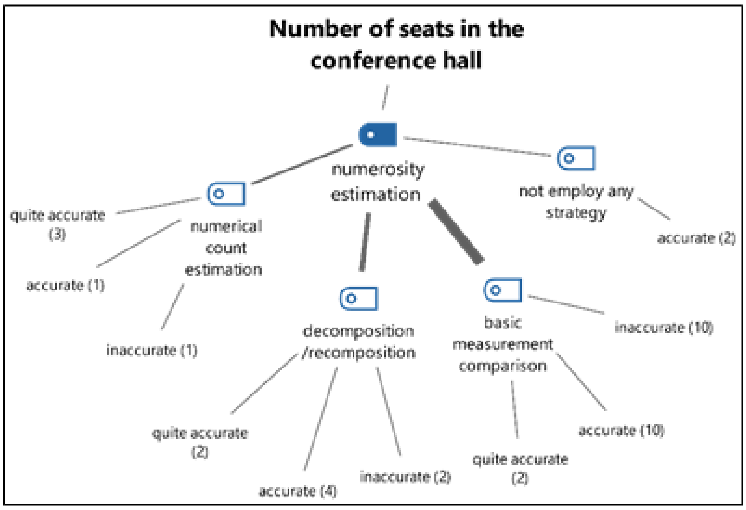


Figure 2. First Question Findings

Although the accurate answer to the question, where the number of seats in the conference hall was estimated, was 1254, it was observed the responses were between 80 and 3000. As seen in Figure 2, only 7 out of 37 pre-service teachers gave an accurate estimate within the tenth percentile. Fifteen students with various strategies and 2 students without any strategy estimated quite accurate estimates (f=17). Thirteen pre-service teachers estimated inaccurate number of seats. It was also observed that most pre-service teachers (f=22) employed the basic measurement comparison strategy.

S3 explained the most accurate estimate as follows: “I estimated that there were 12 seats in the front row, 22 seats in the middle, and 12 seats on the far right. I estimated there were 20 towards the back. 46x20=920” S26, a pre-service teacher with an inaccurate estimate stated the following: “I estimated the number of front seats. I estimated the number of seats from start to end of the stage. I reached the result of 20x20=400.” The participant’s estimation for the total number of seats based on the count of the seats in the front row could have led to an accurate result.

It was observed that all 8 pre-service teachers employed the decomposition/recomposition strategy, which required the calculation based on small samples. S7 estimated 500 seats, which is inaccurate: "I calculated how many people will be seated by estimating the number of seats in a particular area and multiplied it by the total area". S20 stated the following: “When I looked at the image, I thought that there were over 1000 seats. I accepted that there were 30 rows in the first group and 12 seats in a row.

I estimated 30 rows in 2 groups and 22 seats in the 1st row, and I multiplied them and found 1380.” This was an accurate result. S2, one of the 5 pre-service teachers who employed the numerical count estimation strategy, estimated a quite accurate figure: "If there were (at average) 45 seats in each row and 3 groups of 10, that is, if there were 30 groups, it should be between 1350 and 1500". S24 and S37 did not employ any strategy and estimated random numbers.

S37 estimated the number of seats as 1000 without any computation: “Instead of counting the seats one by one, I estimated the number of seats based on the number of people who can sit here. Because the seats were clustered”. Also, several pre-service teachers estimated inaccurate figures due to computational errors even though they employed accurate and quite accurate strategies.

The second question in the estimation skills test was also a question on numerosity estimation. An image was presented, and the participants were asked to estimate the number of books in a library. Since the authors wanted to determine whether the participants would directly count the books instead of estimation, the image included a small number of books (72). An estimate between 65 and 79 was considered "quite accurate", between 50 and 64 or 80 and 94 was considered " accurate," and other estimates were considered “inaccurate.” The analysis findings are summarized in the map presented in Figure 3.

As seen in Figure 3, the majority of pre-service teachers estimated based on the comparison of basic measurements (f=22). About two-thirds of the estimates made with this strategy were quite accurate. Only S12’s estimate was inaccurate: "I estimated 20 books in each row. I calculated each row 5x20=100 and multiplied the product by 5".

Although there were 4 shelves in the image, the participant estimated 5 shelves. The other preferred method was the numerical count estimation, and 8 pre-service teachers employed this strategy.

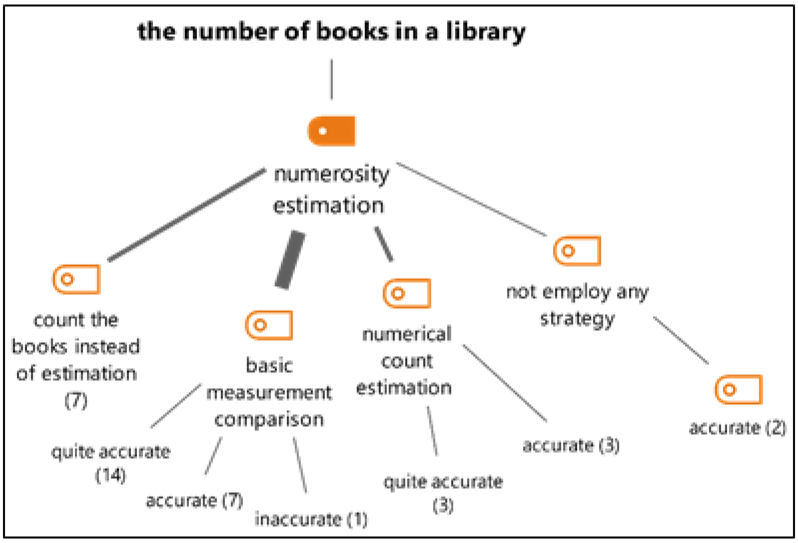


Figure 3. Second Question Findings

Their estimates were quite accurate (f=4) or accurate (f=4). S25 estimated 70 that was quite accurate with the actual value and explained the strategy in this estimation as follows: “Eyeball estimate could be beneficial. First the shelves could be estimated and then the counts could be summed. 1st shelf=20, 2nd shelf=15, 3rd shelf=15 and 4th shelf=20 total=70”. It could be noted that 7 pre-service teachers counted the books instead of estimation. S1 stated that (s)he first counted all the shelves and then estimated the total, while S11 stated the following after counting all the books: "I estimated it between 70 and 75". S14 and S37 estimated the response accurately without adopting an estimation strategy. S14: “The books on the shelf can be estimated just by looking. There were about 60 books”. It was observed that none of the pre-service teachers employed the decomposition/recomposition strategy.

In the estimation skill test, the 6th and 7th questions included numerosity estimates commonly used in the daily life without visuals. These questions were included to investigate as to how pre-service teachers made numerosity estimations without being provided with any visuals and as to which strategies about the situations in the daily life were adopted. Question 6 was about a matter of real life and required the estimation of obesity rate in Turkey. The related MAXQDA map is presented in Figure 4.

According to the Directorate of Public Health data, the prevalence of obesity in Turkey is about 30%. Based on this actual figure, it could be suggested that the estimates of most pre-service teachers (f=27) were inaccurate. The pre-service teachers mostly estimated the figure based on their prior knowledge and experiences, and most estimates were inaccurate. For example, S5 estimated 70% and S9 estimated 60%, which were quite inaccurate values. S9 stated the following: “Considering factors such as the connection of technology to homes, not doing any sports, and Turkish meals, I thought it should be more than half [of the population]”. S2, on the other hand, estimated 45% inaccurately with an unlikely strategy: "I compared the fat people I know to all people I know".

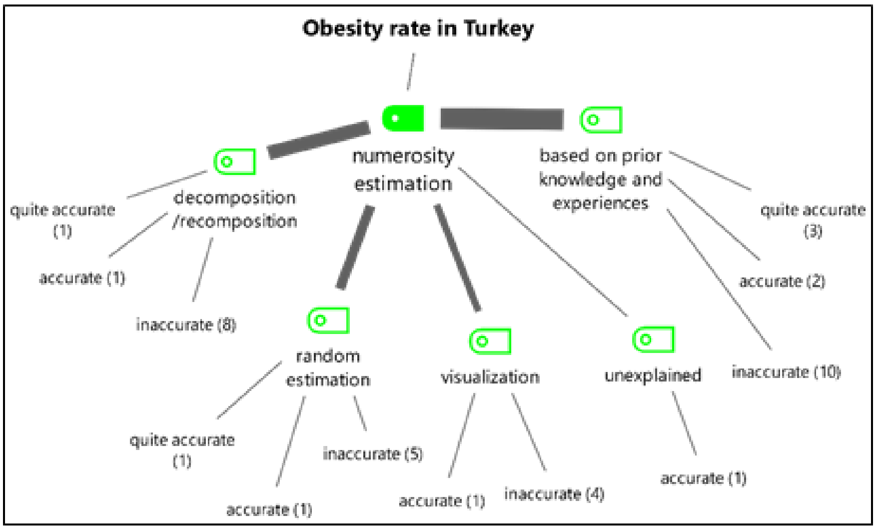


Figure 4. Sixth Question Findings

Although with a similar strategy to S2, S6 made an accurate estimate. He too rested his estimation on his own social network stating that "even if every three people out of 20 in my own family have obesity, it means 30%", and he suggested that if it was 15% among the people he knew, it should be 30% in total population, which was quite an accurate value. T1 stated that (s)he employed the visualization strategy and estimated an inaccurate 3% as follows: "I thought of a street and observed the people on that street and estimated the result". T16 estimated 38% but did not explain the estimate. S30 employed the decomposition/recomposition strategy; however, the strategy was incomprehensible: "If there are 1000 people in a city, if we multiply it by 80, it would be 80,000, which would be approximately 15%".

The final numerosity estimation question required the estimation of the population of the most crowded village in Turkey. The most crowded village in Turkey is Dirsekli village at Cizre district in Şırnak (Population: 6090) according to 2021 population data (TÜİK, 2021). The related findings are presented in Figure 5.

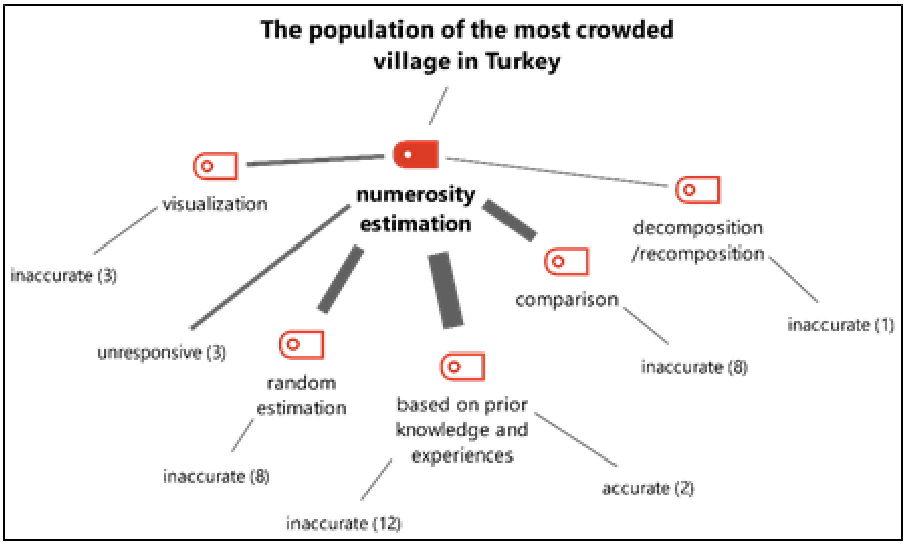


Figure 5. Seventh Question Findings

As seen in Figure 5, most teachers estimated inaccurate figures in the second numerosity estimation question that did not include a visual (f=32). It was observed that 3 participants did not answer the question and only the estimates of two were accurate. S8 employed prior knowledge and estimated 5000 that was accurate: "A friend said that his village had 3000 people.

I based my prediction on this." S20 also used prior knowledge: "I know if there are more than 500 people, it becomes a district", but the estimate was inaccurate (500). Those who replied with random estimates estimated 100,000 (S18) or 1,000,000 (S26), which were extremely inaccurate. It was observed that certain pre-service teachers compared mean populations in their provinces, villages and neighborhoods, which led to inaccurate estimates (S1:100,000, S3: 100). S19, S33, and S37 employed visualization strategy and estimated inaccurate figures.

S33 estimated 2000 with the explanation: "I envisioned a big village and thought about the number of households and people in each household". S34 employed the decomposition/recomposition strategy and yet, reached an inaccurate figure: "If there are 2000 households in the village and there are an average of 4 people in each household, it is 2000x4=8000 people in total".

**Computational Estimation**

The third question posed to the pre-service teachers was "12.6+14.7+23.8+36.4+29.2=?" in computational estimation. The related MAXQDA map is presented in Figure 6.

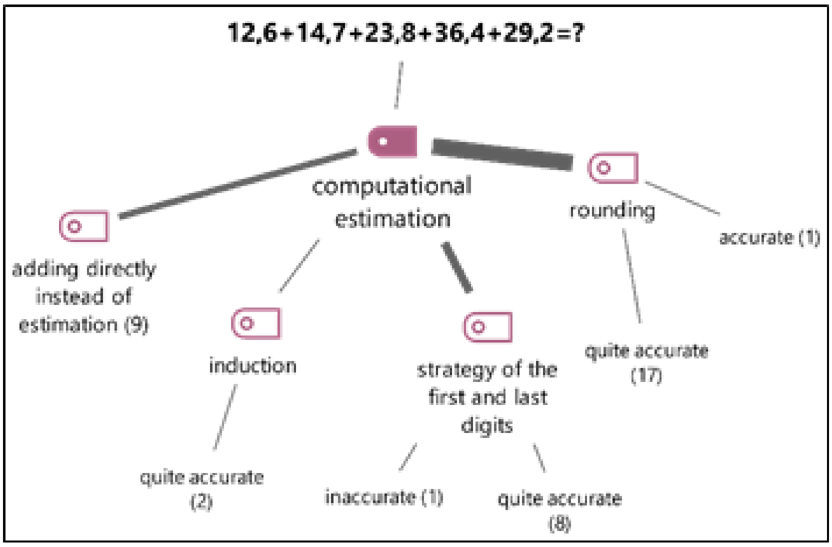


Figure 6. Third Question Findings

In question 3 that required the employment of computational estimation strategy, almost half of the pre-service teachers estimated quite accurate (f=17) or accurate (T28) figures with the rounding strategy. Nine pre-service teachers employed the strategy of the first and last digits. S1 explained the strategy as follows: "I estimated the computation by dividing decimal numbers by the digits. 12+14+23+36+29=104 0.6+0.4=1 and 0.8+0.2=1 and 0.7

was the difference, the result was 107". However, the participant made a calculation error. S10 estimated an inaccurate figure due to a mental calculation error: "I first calculated the integers, then the decimals: 79". S2 and S35 employed induction strategy and T35 stated the following "10+10+20+30+20=20, 2+4+3+6+9=24 and 0.6+0.7+0.8+0.4 +0.2=2.7; therefore, the total is 117", which showed that the calculation was made through sub-calculations. Nine pre-service teachers, including S35, preferred to add 12.6+14.7+23.8+36.4+29.2 directly instead of estimating the total. Another question on computational estimation involved multiplication (Figure 7).

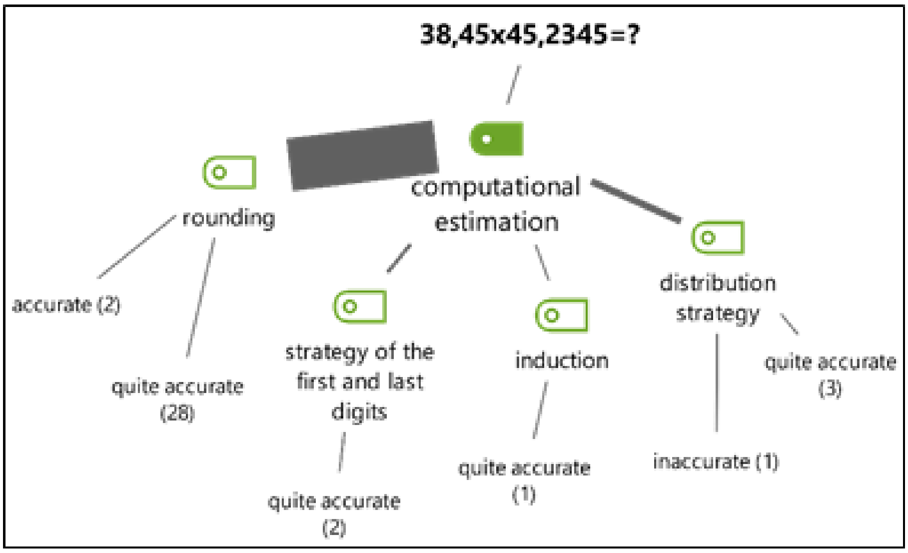


Figure 7. Fourth Question Findings

As seen in Figure 7, most participants (f=30) estimated the product of 38,45x45,2345 with rounding quite accurately or accurately. Most pre-service teachers rounded the 40x45 or 38x45. However, the responses of certain pre-service teachers included rather difficult multiplication operations that required pen and paper such as 39x46 (S10, S18) or 38.5x45 (S11). Several pre-service teachers such as S16, S19, S22, and S23 employed rounding strategy in their estimates, but made mistakes. S16 stated the following: "I did not consider the decimals in this estimation. I rounded 38 to 40. I found 180". It was not possible for the product to be 180 after rounding decimals and the relevant operation. Similarly, S19 stated 39x45=1305 with an operation mistake. On the other hand, S25 and S36 rounded the computation as 40x50=2000, an accurate estimate. S29 and S30 also employed the first or last digits, estimating the product quite accurately. S4 solved the problem of 38x45 with the operation of (30+8)x(40+5)=1710 and explained as follows: "I used the distribution property by rounding [it] to the nearest whole number". Thus, he divided the problem into sub-problems and added the results of these sub-problems. S2, S3 and S12 estimated the product quite accurately with distribution strategy: "(40x50)-(40-5)=2000-200=1800" (S3).

**Measurement Estimation**

The 5th question in the estimation skills test was a measurement estimation question, and students were asked to estimate the height of the classroom door. The actual height of the classroom door was 210 cm. As seen in Figure 8, most pre-service teachers visualized the height of the classroom door. During visualization, certain pre-service teachers employed the length of 1 m, their own hand spans or height, a 20 cm ruler, etc. as references. S37 estimated the height of the door as 2m, which was quite accurate: "I visualized the length of 1 meter." S34 estimated the figure accurately as well: "If my arm is 50 cm from my fingertip to my elbow, and if the height of the door is 5 times higher than my arm-length, then 50x5=250 cm= 2.5 m". T14 stated that "I estimated it by dividing the door into sections" and estimated 2m. S7 estimated the figure based on prior knowledge and reached the exact figure, 2.10 m: "I heard that it should be 40 cm more than the average human height. Hence, [it must be] 2,10 m". It was also observed that irrespective of the strategy adopted, including random estimates, all pre-service teachers estimated the height almost accurately.

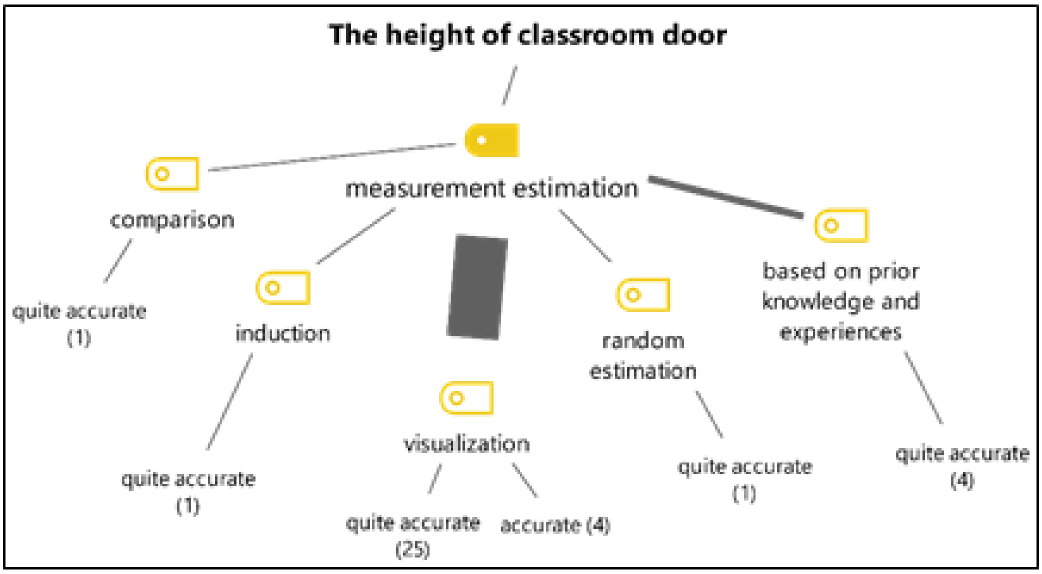


Figure 8. Fifth Question Findings

Finally, the 8th question on measurement estimation skills was "How many apples are there in a kg?" Since a large apple weigh between 180 and 200 grams, a kg should include 5-6 apples. The related MAXQDA map is presented in Figure 9.

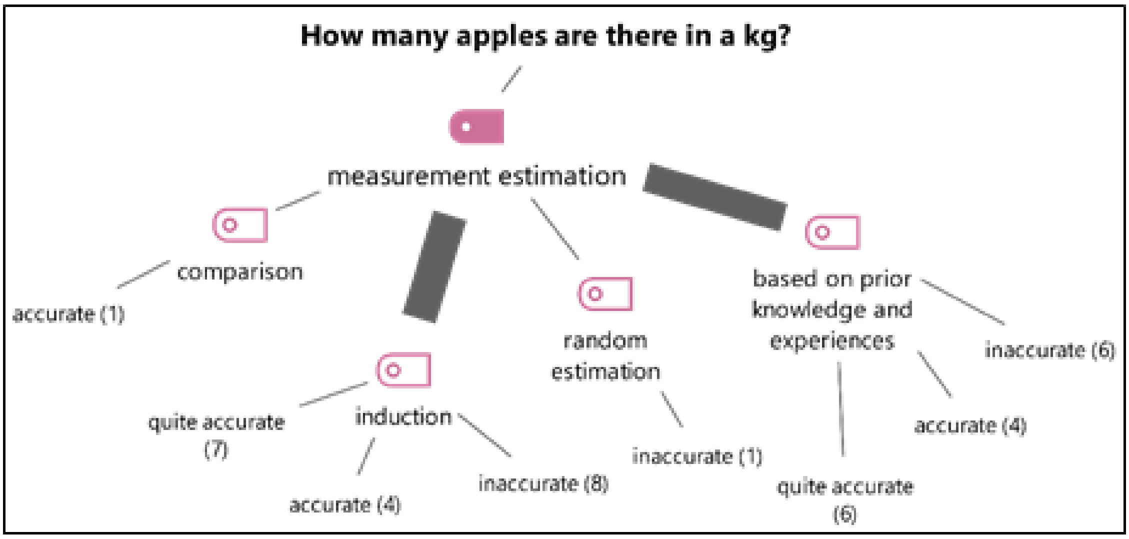


Figure 9. Eighth Question Findings

It was observed that the pre-service teachers employed the induction (f=19) and prior knowledge and experiences (f=14) strategies in the 8th question on weight estimation. It should be noted that the estimates of nearly half of the pre-service teachers (f=15) for the weight of an apple, one of the most common fruits to access in our daily life, were inaccurate. S5 employed the induction strategy; however, the estimate was inaccurate: “I assumed that an apple weighs 50 grams. So, there are 20 apples in 1 kg.” S7 employed the estimation strategy based on prior knowledge/experiences to estimate the figure inaccurately as 10 apples: "I estimated the number of units in a kg based on experience". Estimating the figure based on prior knowledge and experiences, S17 reached 5 apples and stated that: "I imagined that I would pick up apples and put them in a bag whilst shopping at the grocery, and considering the number of apples in my hand, I pictured how many of them would weigh a kilo”. S34 who estimated an accurate figure also employed the induction strategy: "If an apple is 200 grams, then there are 5 apples in a kg. 200x5=1000 grams=1 kg".

## Discussion, Conclusion and Recommendations

In the current study, the estimation strategies adopted by pre-service mathematics teachers and the accuracy of their estimates were discussed. The study findings revealed that the responses of the pre-service teachers were quite accurate in computational estimation when compared to numerosity and measurement estimation. This could be due to the fact that pre-service teachers were used to solve problems that require computational estimation in education. Indeed, several mathematics curricula emphasize computational estimation more predominantly (MoNE, 2005; AEC, 1994). These curricula, explaining computational estimation strategies, provided general information on measurement estimation. In a study by Xenofontos et al. (2022), where mathematics curricula in Eastern Mediterranean countries (Cyprus, Greece, and Turkey) were analyzed, it was reported that the curricula in these three countries emphasized computational estimation and mostly neglected numerosity and numerical axis estimation. It could be suggested that computational estimation is more prevalent in mathematics courses. Furthermore, the present study findings revealed that certain pre-service teachers preferred to calculate the result or count the items instead of adopting an estimation strategy in questions that did not require complex operations. These findings were consistent with those reported by Özcan (2015) where the estimation skills of pre-service teachers were investigated. On the other hand, Boz (2009) argued that the estimation strategies adopted by the students could be affected by cognitive and affective factors. Thus, Boz claimed that the employment of an estimation strategy could be affected by the emotions and thoughts of individuals. For example, Boz determined that students who might think that mathematics required clear answers could develop negative emotions towards estimation, and they tended to calculate the outcome precisely and abstained from the employment of estimation strategies. Gliner (1991), on the other hand, suggested that certain variables such as grade point average and students’ affinity for mathematics could affect the pre-service teachers’ employment of estimation strategies. Thus, the fact that pre-service teachers did not employ an estimation strategy in the present study could be due to their negative attitudes towards estimation. This hesitancy among pre-service teachers to employ such estimations could also be associated with the mathematical content of the test items provided in this study. Goodman (1991), in a study where the estimation skills of the pre-service teachers were investigated, reported that estimating using fractions was more difficult than estimating with whole numbers, decimals or percentages. Thus, the mathematical content of the test items should also be considered when using estimation strategies.

On the estimation strategies employed by pre-service mathematics teachers, it was determined that the most preferred strategy in computational estimation questions was the rounding strategy. Son et al. (2019) also reported that the strategy of rounding to multiples of 10 was used more frequently in a study where pre-service teachers' computational estimation skills were analyzed. Rubenstein (1985) determined that students employed rounding and substitution strategies more frequently in computational estimation and emphasized that computational estimation skills should not be limited to these strategies. Other studies on computational estimation reported that individuals employed the rounding strategy more frequently (Reys et al, 1991; Özcan, 2015), followed by grouping, (Pilten & Yener, 2009), and organization (Berry, 1998) strategies. In the current study, where the estimation strategies of the pre-service teachers were investigated, it was observed that the participants predominantly adopted distribution and counting strategies in addition to rounding in computational estimation. Thus, the employment of strategies other than rounding in computational estimation could be associated with the familiarity of the students with estimation strategies. Similarly, the non-employment of estimation strategies other than rounding could be because different strategies were not instructed in the courses. Dowker (1992) reported that there were several estimation strategies that could lead to accurate estimates, and good estimators know and use several estimation strategies.

The analysis of the pre-service teacher responses to numerosity estimation questions revealed that while the answers were accurate or quite accurate in questions that included an image, they were mostly inaccurate in questions that did not include an image. Furthermore, the basic measurement comparison method was frequently employed in numerosity estimation questions that included an image, the participants preferred prior knowledge and experiences strategy in questions that did not include an image. However, these estimates were mostly inaccurate. The presence of visual elements could facilitate the adoption of an estimate strategy. On the other hand, Goodman (1991) investigated estimation strategies adopted by pre-service teachers and concluded that the estimation skills adopted in questions presented as a story were better than those adopted in numerical questions. Thus, adoption of an estimation strategy could be associated with the quality of the questions. Furthermore, pre-service teachers, who will adopt estimation skills in their profession, should be aware of the questions where estimation skills could be employed.

Although it was not the aim of the study, it was also determined that the pre-service teachers estimated inaccurate figures due to computational mistakes in computational and numerosity estimation questions. Furthermore, it should be noted that the number of pre-service teachers who estimated inaccurate figures due to computational mistakes was quite high. Thus, it could be suggested that there was a correlation between the computational skills and estimation skills. Aytekin and Uçar (2014) determined that there was a positive and statistically significant correlation between estimation skills and computational and mathematical achievements. The analysis of the measurement estimation skills revealed that the number of pre-service mathematics teachers who estimated accurate quantity or height figures varied. For example, the participants estimated the height of the classroom door quite accurately, while almost half of the estimates were inaccurate in the question where they were asked to estimate the number of apples in a kilogram. The number of incorrect estimates in a transaction that is conducted every day was rather concerning for measurement estimation. The differences between the measurement estimation skills of pre-service teachers could be due to the adoption of different strategies in height and quantity estimations. Indeed, it was determined that the participants employed different strategies in quantity estimation and height estimation. In height estimation, visualization strategy was preferred by the majority of pre-service teachers, while induction, prior knowledge and experiences were employed in quantity estimation. It was determined that certain pre-service teachers employed references in visualization of height estimation. It could be suggested that certain participants made accurate height estimates. Joram et al. (2005) determined that students who employed references estimated more accurate standard unit and length figures when compared to students who did not employ references. Thus, it could be suggested that the employment of a reference in height estimation led to in/accurate height and quantity estimates. Subramaniam (2014) determined that pre-service teachers employed various criteria in measurement estimation that allowed them to estimate length measurements; however, these criteria were so ambiguous as to reveal the pedagogical knowledge of the participants. The present study findings were consistent with the study by Subramaniam (2014) that pre-service teachers did not employ specific strategies since they relied on prior knowledge and experiences in quantity estimates. This could be due to the lack of knowledge of the participants on estimation strategies or their inadequacy about the employment of these strategies. Furthermore, the unanswered test questions revealed that pre-service teachers experiencing difficulties in the adoption of an estimation strategy. In fact, previous studies conducted with teachers who would apply estimation skills in the real world (Alajmi, 2009; Boz-Yaman & Bulut, 2017; Son et al., 2019) unearthed that teachers did not consider that they were adequately equipped with the knowledge of such strategies and hence, did not employ effective estimation strategies in courses.

The findings associated with pre-service teachers' numerosity estimation were similar to those associated with quantity estimates in their measurement estimation. It was determined that a significant number of pre-service teachers employed random estimation strategy, and their responses were thus inaccurate. The preference of random estimation strategy and prior knowledge and experiences, as in measurement estimation, suggested that pre-service teachers had no idea about estimation strategies. Pre-service teachers who will require estimation strategies in professional life should know estimation strategies and the significance of estimation. Pre-service teachers should employ specific estimation strategies without relying on random estimations. To raise the awareness of pre-service teachers, a course where estimation strategies are instructed should be included in undergraduate education programs or estimation strategies should be instructed in existing courses such as Geometry Instruction and Instructing Numbers in undergraduate programs. Furthermore, various mathematics learning-teaching approaches could be employed to improve estimation skills of pre-service teachers in various estimation strategies. For example, activities associated with real life or models that would improve estimation skills could be employed. Furthermore, the awareness of pre-service teachers and teachers could be raised by including estimation skills in mathematics textbooks. Also, numerosity and quantity estimation strategies should be emphasized in the curricula in addition to computational estimation strategy.

## Note

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## Author Information

| Tuğba Baran Kaya | Sedef Çelik Demirci |
|---|---|
| https://orcid.org/0000-0001-9924-4352 | https://orcid.org/0000-0002-9242-8009 |
| Kırıkkale University | Artvin Çoruh University |
| Education Faculty | Education Faculty |
| Turkey | Turkey |
| Contact e-mail: *tugbabaran@kku.edu.tr* | |