Development of the Mathematical Problem-Solving Ability Using Applied Cooperative Learning and Polya’s Problem-Solving Process for Grade 9 Students

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Abstract
The purposes of the study were 1) to investigate the effectiveness of the applied cooperative learning and Polya’s problem-solving process on grade 9 students’ mathematical problem-solving ability, 2) to compare grade 9 students’ learning achievement before and after learning through the applied cooperative learning and Polya’s problem-solving process, and 3) to study the students’ satisfaction toward learning through the applied cooperative learning and Polya’s problem-solving process. The participants were 18 grade 9 students in a Thai secondary school selected by the stratified random sampling method. The instruments were 1) an applied cooperative learning and Polya’s problem-solving process learning management, 2) a mathematical problem-solving test, 3) a learning achievement test, and 4) a satisfaction questionnaire. The data were analyzed using percentage, mean score, standard deviation, one-sample t-test, paired-samples t-test, and effectiveness test with the criteria of 70. The results of the study indicate that 1) the learning management designed using the applied cooperative learning and Polya’s problem-solving process was effective in developing students’ mathematic problem-solving ability, 2) the students’ learning achievement of surface area and volume in the posttest was higher, and 3) the students were satisfied with learning with the lesson plans using the applied cooperative learning and Polya’s problem-solving process. The results could be applied in both mathematics classrooms and similar research study area of mathematics instruction.

Keywords: mathematical problem-solving ability, team assisted individualization, problem-solving processes

1. Introduction
Mathematics is a science that plays an important role in learners’ education and life in the 21st century. Science helps humans in the analysis of problems and surrounding circumstances that could guide them in prediction, planning, and finding the most appropriate way to solve the problems (Ernest, 2001). In addition, mathematics is a fundamental skill in learning subjects related to sciences and technology. To illustrate, students with a good command of calculating skills tend to be more successful in solving physics equations, analyzing chemistry formulation, writing computer programs, etc. (Bialik & Kabbach, 2014). As a result, mathematics is included in the core curriculum at the very beginning of the education path of students around the globe.

Moreover, to acquire mathematic skills, students need the mathematical problem-solving ability. The ability could be defined as the processes that real-world problems are transformed into perspective that learners could apply mathematic knowledge to find solutions (Nunokawa, 1995, 2005). With this principle, problems solvers could process the problems and apply mathematical methods. The systematic nature of mathematic thinking may reduce steps of problem solving and lead to the appropriate solution to the problems. On the other hand, practicing mathematic problem-solving skills could also benefit learners’ learning achievement. Stohlmann and Albarracin (2016) suggested that putting learners in situations that encourage them to soften difficulties using mathematical logic would illustrate to them the connection between the real-world problem and mathematic exercises leading to the understanding of class contents and learning achievement. Likewise, Beigie (2008) suggested that students can learn to calculate through problem solving as it helps in deepening their comprehension of mathematic concepts by practicing using mathematics to fix real problems. At this point, it could be claimed that practicing mathematical problem solving benefits learners both in life and in learning.

However, one does not simply acquire the mathematical problem-solving ability as complicated cognitive
processes and analytical ability are required. To clarify, practicing such skill needs the integration of mathematic knowledge and problem-solving skills. According to Simmer (2011), learners perceive mathematics as difficult. In detail, learners need to understand equations, variables, expressions, and the possibility to fix math problems. Moreover, mathematics is a continuous subject. Students who do not comprehend the content in the previous class tend to fail to understand the content that follows. With problematic teaching methods and personal beliefs, it is not a surprise that learners often rate mathematics as the most difficult problem throughout their education (Langoban, 2020). Furthermore, experience and logical thinking are needed to drive the processes (Clarke, Goos, & Morony, 2007). Therefore, learners need continuous practices and a meaningful learning environment to practice such skills.

It seems that the mathematical problem-solving ability is a complicated skill that needs to be carefully instructed to the students. Therefore, instructional methods used in developing the mathematical problem-solving ability should also be supportive in letting learners study mathematics with opportunities to practice solving problems. Cooperative learning is one of the emerging instructional methods that provide opportunities for learners to learn with a small group of peers (Kagan, 1994). They can share ideas and raise solutions through teamwork and cooperation. Moreover, teachers can apply such a technique as team assisted individualization (TAI) which allows students with a heterogeneous level of ability to cooperatively learn (Posamentier, Smith, & Stepelman, 2010). The TAI benefits the whole class in various ways: Teachers could shift their role to be an advisor. Highly skilled students could learn by practicing transferring knowledge while students with limited skills could ask for advice from other students with whom they are more encouraged to have interaction. The method seems to match the nature of mathematics which requires teachers’ advice and cooperative learning among peers.

In addition, practicing problem solving also needs specific instructional methods. Difficulties in both life and calculation are complicated. To solve problems, it takes a thorough understanding of causes and nature, a proper plan, precise execution of the plan, and the revision of outcomes. Polya (1945) proposed 4 steps of problem-solving processes including trial-and-error, analogy, generalization, working backward, and draw-a-figure. Practicing these processes would benefit students as they are encouraged to study problems, consider and execute plans, and review whether the plan leads to expected outcomes or not. With the principle, students could successfully resolve both mathematical and real-life situations.

All in all, the mathematical problem-solving ability is an essential skill contributing to success in students’ life and education. However, the ability is not simply acquired as it takes the understanding of mathematic content and practice of problem-solving skills. Cooperative learning with team assisted individualization (TAI) and Polya’s problem-solving principle are the method that could be integrated to support the mathematical problem-solving ability learning environment. Therefore, the current study applies the integration of the two methods to develop students’ mathematical problem-solving abilities. The purposes of the study were 1) to investigate the effectiveness of the applied cooperative learning and Polya’s problem-solving process on grade 9 students’ mathematical problem-solving ability, 2) to compare grade 9 students’ learning achievement of mathematical problem-solving ability before and after learning in the applied cooperative learning and Polya’s problem-solving process, and 3) to study the students’ satisfaction toward learning in the applied cooperative learning and Polya’s problem-solving process.

2. Literature Review

In this section, theories of mathematical problem-solving ability and cooperative learning and team assisted individualization (TAI), and Polya’s Processes of problem-solving are discussed to illustrate the potential effectiveness of the method in developing students’ mathematical problem-solving ability. Moreover, the examples of previous studies are demonstrated to show the uses of the principles in science education around the globe.

2.1 Mathematical Problem-Solving Ability

As discussed earlier, the mathematical problem-solving ability is the skill that benefits learners’ learning achievement in class and ability to deal with difficulties in life. However, it is not quite certain to define the skill. In this case, two dimensions could be discussed. First, if we consider problem-solving as a set of processes, mathematical problem-solving skills could be instructed by letting problem-solvers transform situations into the circumstances that they could apply mathematic knowledge. This would help learners to connect class content to what can be applied in real life (Nunokawa, 1995, 2005). On the other hand, teachers could instruct mathematical content by integrating real-life content into the class. Therefore, learners could analyze the situations, put them in equations, and find the answers. In this case, the class environment is set as teaching for problem-solving where problems are used to facilitate students’ mathematic learning in terms of connecting the
new content to the knowledge students have already had, using techniques to treat the problematic situation, managing problem-solving processes, and encouraging analytical thinking (Schroeder & Lester, 1989; Nunokawa, 2005). As a result, the mathematical problem-solving ability could be instructed based on both problem-solving and mathematical learning.

2.2 Cooperative Learning and Team Assisted Individualization (TAI)

Cooperative learning emerged from the idea that social learning experience would drive learners to learn more effectively and happily. Instead of listening to the lecture, students can work with their peers, discuss the possibilities of tasks, and achieve academic outcomes together. Kegan (1994) suggested that cooperative learning could be operated by forming a heterogeneous group consisting of students of both high and low skill levels together. The purpose of cooperative learning is to let high proficiency learners who generally learn faster assist the low-level group. Similarly, Slavin (1984) indicated that the benefits of cooperative learning rely on empowering students, improving class interaction, and bringing mutual beneficial relationships among students. Moreover, it could drive learners’ individualization as they no longer rely on only teachers. They are urged to explore their knowledge and experience in a meaningful learning environment. The author also suggested team assisted individualization (TAI) as a model to manage cooperative learning. In detail, student empowerment, diversity of the group, teamwork, and motivation are the key leading to the success of TAI. To exemplify, teachers should change their roles into facilitators and let a group of heterogeneous students work together with the same academic purposes. Learners are encouraged to share knowledge and help each other, and the scores of tasks are given to the whole group. This would benefit all stakeholders of the class as teachers could reduce their workload; high-skill students could test the hypothesis in learning by assisting their peers; the low-skill students could consult their friends in learning.

2.3 Polya’s Processes of Problem-Solving

Practicing the ability to solve problems is also crucial in instructing the mathematical problem-solving ability. Learners need to comprehend problem-solving processes to apply mathematic knowledge. Polya (1945) suggested 4 processes in solving problems including understanding the problem, devising a plan, carrying out the plan, and looking back. Information like unknown, data, condition, and possibility to fit the condition need to be analyzed. Figures could be drawn to separate parts of the condition. In addition, a plan that could connect the unknown and existing data to draw a possibility of finding a solution that can match the condition of problems should be formed. In carrying out the plan, an assessment of how the plan is executed correctly is needed. It is a carefully created plan. Therefore, the plan needs to be carried out systematically. Lastly, Such questions such as “Can you check the result?”; “Can you check the argument?”; “Can you derive the solution differently?”; “Can you see it at a glance?”; “Can you use the result, or the method, for some other problem?” should be asked in the looking back process. It can be noted that Polya’s processes of problem-solving allow learners to fix problems using systematic questioning which is beneficial in learning mathematics.

2.4 Previous Studies

As mathematical problem-solving ability has been accepted as an important skill in life and mathematics learning, and cooperative learning of the team assisted individualization (TAI) and Polya’s principles of problem-solving procedures are presented to be supportive in enhancing student skills, studies have been conducted using both TAI (e.g., Arrahim, Sugiharti, & Damayanti, 2020; Ballantine, 2007; Maryani, Pramudya, & Slamet, 2019) and Polya’s problem-solving processes (Aljaberi & Gheith, 2016; Arifin, 2014; Gopinath & Lertlit, 2017) to help students improve the ability. For example, Maryani et al. (2019) implemented team-game-tournament (TGT) and team assisted individualization (TAI) to develop eight-graded students’ mathematical problem-solving abilities in Indonesia. The results of the study on 85 students indicate that both TGT and TAI were effective in enhancing the samples’ mathematic problem-solving ability. Likewise, the results of a study on 10 grade 7 students in an international school in Thailand indicate the effectiveness of Polya 4 steps of problem-solving in developing students’ mathematical problem-solving skills. It could be noted that both TAI and Polya’s principles of problem-solving procedures have been proved to be beneficial in mathematics classrooms. Therefore, the integration of the two principles might be an alternative in solving students’ problems in learning mathematical problem-solving ability. Consequently, the current study aimed to apply the two principles in instructing mathematical problem-solving ability, and the following research questions were raised. 1) Is the applied cooperative learning and Polya’s problem-solving process effective in developing grade 9 students’ mathematical problem-solving ability? 2) Is there a difference between grade 9 students’ learning achievement before and after learning in the applied cooperative learning (TAI) and Polya’s problem-solving process? and 3) What are students’ satisfactions toward learning in the applied cooperative
learning and Polya’s problem-solving process?

3. Methodology

3.1 Participants
The participants were 18 grade 9 students in a Thai secondary school selected by purposive random sampling method. The data were collected in the second semester of the 2021 academic year. The participants were treated anonymously.

3.2 Instruments
The instruments were 1) an applied cooperative learning and Polya’s problem-solving process learning management, 2) a mathematical problem-solving test, 3) a learning achievement test, and 4) a satisfaction questionnaire. The first instrument was designed using cooperative learning and Polya’s problem-solving process. The content was related to surface areas and volume. The learning management included 12 50-minute lesson plans. The learning management was evaluated by experts and found to be appropriate in developing students’ skills. The mathematical problem-solving test was a written test aiming to evaluate students’ mathematical problem-solving ability. The test consisted of 4 items related to processes of problem solving with a total of 40 marks. The test was validated by index objective congruence (Rovinelli & Hambleton, 1977). The test was rated by 5 experts with the index objective congruences (IOC) of 0.5−1.0. The learning achievement test was a multiple-choice test aiming to evaluate students’ knowledge of surface areas and volume. The test consisted of 30 items with a total of 30 marks. The test was rated by 5 experts with the index objective congruences (IOC) of 0.5−1.0. The questionnaire consisted of 15 items aiming to investigate students’ satisfaction toward the learning management. The questionnaire was rated by 5 experts with the index objective congruences (IOC) of 0.5−1.0.

3.3 Research Design and Data Collection
The study was designed in an experimental design. The students took the pretest to investigate their knowledge on calculating surface areas and volume before learning in learning management. The treatment took place in the second semester of the 2021 academic year. The students took a learning achievement posttest, a mathematical problem-solving ability test, and a questionnaire after the treatment.

3.4 Data Analysis
The data were analyzed using percentage, mean score, standard deviation, one-sample t-test, paired-samples t-test, and effectiveness test with the criteria of 70.

4. Results
The results of the study are discussed along with the research questions below.

Is the applied cooperative learning and Polya’s problem-solving process effective in developing grade 9 students’ mathematical problem-solving ability?

<table>
<thead>
<tr>
<th>Processes of Mathematical problem-solving ability</th>
<th>n</th>
<th>Fullmark</th>
<th></th>
<th>S.D.</th>
<th>%</th>
<th>μ(70%)</th>
<th>t−test</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding the problem</td>
<td>18</td>
<td>10</td>
<td>9.56</td>
<td>0.51</td>
<td>95.56</td>
<td>7</td>
<td>21.21</td>
<td>.000*</td>
</tr>
<tr>
<td>Devising a plan</td>
<td>18</td>
<td>10</td>
<td>7.67</td>
<td>0.84</td>
<td>76.67</td>
<td>7</td>
<td>3.37</td>
<td>.004*</td>
</tr>
<tr>
<td>Carrying a plan</td>
<td>18</td>
<td>10</td>
<td>8.33</td>
<td>0.59</td>
<td>83.33</td>
<td>7</td>
<td>9.52</td>
<td>.000*</td>
</tr>
<tr>
<td>Looking back</td>
<td>18</td>
<td>10</td>
<td>7.61</td>
<td>0.92</td>
<td>76.11</td>
<td>7</td>
<td>2.83</td>
<td>.012*</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>33.17</td>
<td>1.62</td>
<td>82.93</td>
<td>28</td>
<td>13.55</td>
<td></td>
<td>.000*</td>
</tr>
</tbody>
</table>

The result of the study indicates the effectiveness of the learning management designed in applied cooperative learning and Polya’s problem-solving process on grade 9 students’ mathematical problem-solving ability. The students’ average mathematical problem-solving ability score was 33.17 (82.93%) reaching the criteria of 70. The students’ scores were significantly higher than the set criteria, t = 13.55, p = 0.00. It could be interpreted that the participants developed mathematical problem-solving ability after learning in the learning management designed in applied cooperative learning and Polya’s problem-solving process.

Is there a difference between grade 9 students’ learning achievement before and after learning in the applied cooperative learning and Polya’s problem-solving process?
Table 2. Student learning achievement of surface area and volume before and after the treatment

<table>
<thead>
<tr>
<th>Learning achievement</th>
<th>n</th>
<th>( \bar{x} )</th>
<th>S.D.</th>
<th>t</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>18</td>
<td>14.89</td>
<td>1.28</td>
<td>39.28</td>
<td>.000*</td>
</tr>
<tr>
<td>Posttest</td>
<td>18</td>
<td>21.67</td>
<td>1.42</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The result indicates the difference between the participants’ learning achievement of surface area and volume before and after learning in the learning management designed in applied cooperative learning and Polya’s problem-solving process. A paired sample t-test indicates a significant difference between students’ posttest (\( \bar{x} = 21.67, \text{S.D} = 1.42 \)) and pretest (\( \bar{x} = 14.89, \text{S.D} = 1.28 \)), \( t = 39.28, p = 0.00 \). It could be interpreted that the participants developed learning achievement of surface area and volume after learning in the learning management designed in applied cooperative learning and Polya’s problem-solving process.

What are students’ satisfactions toward learning in the applied cooperative learning and Polya’s problem-solving process?

Table 3. Students’ satisfactions toward learning in the applied cooperative learning and Polya’s problem-solving process

<table>
<thead>
<tr>
<th>Items</th>
<th>( \bar{x} )</th>
<th>S.D.</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The activities were consistent with the purposes of learning.</td>
<td>4.67</td>
<td>0.49</td>
<td>Very high</td>
</tr>
<tr>
<td>2. The activities helped students to solve problems systemically.</td>
<td>4.72</td>
<td>0.46</td>
<td>Very high</td>
</tr>
<tr>
<td>3. The activities were interesting and motivating.</td>
<td>4.11</td>
<td>0.76</td>
<td>High</td>
</tr>
<tr>
<td>4. The activities helped students to practice sharing and receiving comments</td>
<td>4.89</td>
<td>0.32</td>
<td>Very high</td>
</tr>
<tr>
<td>5. The activities helped students to reach learning goals by working cooperatively.</td>
<td>4.61</td>
<td>0.50</td>
<td>Very high</td>
</tr>
<tr>
<td>6. The activities opened opportunities for students to learn new things with peers.</td>
<td>4.56</td>
<td>0.51</td>
<td>Very high</td>
</tr>
<tr>
<td>7. The activities improved learning interaction.</td>
<td>4.83</td>
<td>0.38</td>
<td>Very high</td>
</tr>
<tr>
<td>8. The activities helped in improving the mathematical problem-solving ability</td>
<td>4.78</td>
<td>0.43</td>
<td>Very high</td>
</tr>
<tr>
<td>9. Class management supported the activities.</td>
<td>4.17</td>
<td>0.79</td>
<td>High</td>
</tr>
<tr>
<td>10. Activity time was appropriate.</td>
<td>4.72</td>
<td>0.46</td>
<td>Very high</td>
</tr>
<tr>
<td>11. Class contents were beneficial in real life.</td>
<td>4.28</td>
<td>0.75</td>
<td>High</td>
</tr>
<tr>
<td>12. Students understood more about the importance of mathematics.</td>
<td>4.39</td>
<td>0.70</td>
<td>High</td>
</tr>
<tr>
<td>13. Students learned mathematics with more pleasure.</td>
<td>4.06</td>
<td>0.94</td>
<td>High</td>
</tr>
<tr>
<td>14. Students were allowed to share opinions.</td>
<td>4.89</td>
<td>0.32</td>
<td>Very high</td>
</tr>
<tr>
<td>15. Students were treated with the consideration of individual diversity.</td>
<td>4.94</td>
<td>0.24</td>
<td>Very high</td>
</tr>
<tr>
<td>Overall</td>
<td>4.57</td>
<td>0.54</td>
<td>Very high</td>
</tr>
</tbody>
</table>

The result indicates the students’ positive attitude toward learning in the learning management designed in applied cooperative learning and Polya’s problem-solving process. The overall satisfaction toward learning in the learning management was at a very high level (\( \bar{x} = 4.57, \text{S.D} = 0.54 \)). It could be noted that the students agreed with the positive statements regarding to the instructional method. It could be interpreted that the students were satisfied with learning in the learning management designed in applied cooperative learning and Polya’s problem-solving process.

5. Discussions

The results of the study lead to the issues discussed below.

5.1 The Effectiveness of the Applied Cooperative Learning and Polya’s Problem-Solving Process on Student Mathematical Problem-Solving Ability

The results indicate that the students could develop mathematical problem-solving ability after learning in the learning management designed in applied cooperative learning and Polya’s problem-solving process. The results are consistent with Aljaberi and Gheith (2016), Arifin (2014), Arrahim, Sugiharti and Damayanti (2020), Ballantine (2007), Gopinath and Lertlit (2017), and Maryani, Pramudya and Slamet (2019) who also found the benefits of cooperative learning of the TAI model and Polya’s problem-solving process. It seems that cooperative learning using heterogeneous groups of students who are encouraged to help each other and work as a team is beneficial in instructing a complex process of mathematic problem-solving ability. In the current study, problems in daily routine were brought into mathematics. Students in each group shared ideas and assisted each other to transform the problems into mathematics. Groups also worked cooperatively to analyze problems, create plans, manage plans, and reflex the results. This led to success in terms of both group work and individual
development as evidenced in the result of the study.

5.2 The Effectiveness of the Applied Cooperative Learning and Polya’s Problem-Solving Process on the Student Learning Achievement of Surface Area and Volume

The results show the effectiveness of the applied cooperative learning and Polya’s problem-solving process on the student learning achievement of surface area and volume as evidenced by the comparison of students’ pretest and posttest scores. The results of the study confirmed the benefit of TAI in instructing mathematics contents (Arifin, 2014; Ballantine, 2007; Gopinath & Lertlit, 2017). In the current study, the implementation of TAI resulted in the improvement of students’ mathematical knowledge of surface area and volume. Furthermore, students’ mathematical problem-solving ability also contributes to learning achievement. The results of the study went in line with Aljaberi and Gheith (2016), Arifin (2014), Arrahim, Sugiharti and Damayanti (2020), and Gopinath and Lertlit (2017) who also found the benefits of mathematical problem-solving ability in teaching mathematics. As it also suggested that the students could develop the mathematical problem-solving ability, it could be interpreted that the ability was also used in learning mathematics.

5.3 Students’ Satisfaction Toward the Applied Cooperative Learning and Polya’s Problem-Solving Process Learning Management

The results also indicate that the students were satisfied learning in the applied cooperative learning (TAI) and Polya’s problem-solving process learning management. It was reported that the learning management brought about a meaningful learning environment, motivating class atmosphere, supportive learning for sharing ideas in learning, and joyful class activities. The results also confirm the benefit of cooperative learning with the TAI model. According to Slavin (1984), the advantages of the team assisted individualization (TAI) rely on the way it drives out student empowerment, idea sharing, and meaningful learning. Moreover, Polya’s problem-solving process (Polya, 1945) also focuses on fixing problems systematically. In the current study, it seems that apart from the benefit of the integration of the two principles in developing student academic skills, they also put learners in a preferable class.

6. Conclusion

In conclusion, the research questions of the current study could be answered that 1) the learning management designed in toward the applied cooperative learning and Polya’s problem-solving process was effective in developing students’ mathematic problem-solving ability, 2) Students’ learning achievement of surface area and volume in posttest was higher than in pretest, and 3) students were satisfied learning in the learning management designed in toward the applied cooperative learning and Polya’s problem-solving process. It could be seen that the integration of cooperative learning in the model of the team assisted individualization (TAI) and Polya’s problem-solving process could lead to success in developing students’ mathematical problem-solving ability and learning achievement. It also contributed to a satisfied class. Therefore, the idea should be implemented in mathematics classes to develop students’ abilities. Moreover, further studies could employ learning management to develop other mathematic content. In addition, further studies could include qualitative aspects into the data collection to provide a broader picture of how the integration of the two principles affects students’ mathematical learning processes. As mathematics is the fundamental of education, mathematical problem-solving ability should be focused to bring about success in the subject which could also pave ways for students toward education and life.

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