Journal of Research and Advances in Mathematics Education

Volume 7, Issue 2, April 2022, pp. 77-92
DOI: 10.23917/jramathedu.v7i2.16325
p-ISSN: 2503-3697, e-ISSN: 2541-2590

Students' semantic reasoning characteristics on solving double discount problem

Lydia Lia Prayitno[1*], Purwanto[2], Subanji[2], Susiswo[2], Ninik Mutianingsih[1]

[1] *Department of Mathematics Education, Universitas PGRI Adi Buana Surabaya, Indonesia*
[2] *Department of Mathematics Education, State University of Malang, Indonesia*

Citation: Prayitno, L. L., Purwanto, P., Subanji, S., Susiswo, S., & Mutianingsih, N. (2022). Students' semantic reasoning characteristics on solving double discount problem. *JRAMathEdu (Journal of Research and Advances in Mathematics Education), 7*(2), 77-92. https://doi.org/10.23917/jramathedu.v7i2.16325

ARTICLE HISTORY:
Received 16 November 2021
Revised 17 February 2022
Accepted 17 February 2022
Published 30 April 2022

KEYWORDS:
Double discount question
Ill-structured problems
Problem-based learning
Semantic reasoning

ABSTRACT

Semantic is associated with the relationship between symbol, reference, and the problem's context involved in the problem-solving process which also involves reasoning and decision-making. Hence, this study describes the characteristics of students' semantic reasoning to solve the double discounts problem. 51 high school students in Sidoarjo participated in this qualitative study. The data were collected through 15-20 minutes problem-solving tests. The students' answers were grouped into correct and wrong answers. The correct answers were then regrouped once more based on the strategies used by the students to answer the test and to identify their semantic reasoning characteristics. The data were analyzed by reducing, classifying the think-aloud and also by observing. Then the similarity of characteristics of students' semantic reasoning when solving the double discount problem was identified. To test the accuracy of the data, a triangulation method was used. This semantic reasoning was identified by (1) giving the problem situation, (2) stating the keywords and their meaning, (3) stating the relationship, (4) transforming it into a mathematics statement, (5) calculating based on their strategies, (6) decision making, and (7) completing the answer interpretation. This study contributes to developing basic knowledge in interpreting each process of solving ill-structured problems until finding a solution.

INTRODUCTION

Reasoning is one of the goals of learning mathematics at school as it is a major component in mathematics, especially problem-solving (Bergqvist et al., 2004; Cai & Nie, 2007; Hwang et al., 2007; Lithner, 2008; NCTM, 2000). Problem-solving is not only useful in solving mathematical problems but also in daily life situations. The primary component of the problem-solving process is to understand the context of the presented problem. Understanding the problem's context building new knowledge begins by looking at the meaning of the problem (Hegarty et al., 1995; Pape, 2004). The understanding is then related to the problem-solvers knowledge and experience to determine the strategies to solve it (Clement, 2008). Therefore, students' understanding of the meaning of the problem is important in the process of problem-solving, because it involves reasoning (Uhden et al., 2012).

Experts use different words to express meaning or semantics. According to the context, meaning is associated with the relationship between the symbol and the reference. In addition, the meaning of context is involved in the problem-solving process (Hegarty et al., 1995; Jonassen, 1997; Liang et al., 2016). Problem solvers take the problem-solving process to solve the problem. The

*Corresponding author: lydialia@unipasby.ac.id

sequence of problem-solving processes involves reasoning and decision-making (Davis, 2013; Liang et al., 2016). Problem solvers should interpret the stages they do (Davis, 2013; Pape, 2004).

Reasoning has an important role in the problem-solving process. One reasoning that involves problem-solving is semantic. Semantic reasoning is the process of thinking to understand the meaning of each stage of the problem-solving process to find a solution to the problem (Davis, 2013; Liang et al., 2016; Littlefield & Rieser, 1993; Murphy, 2015; Prayitno et al., 2018). The study about semantic reasoning has been focused on different things, such as semantic reasoning in computer programs (Patelli et al., 2014; Shi et al., 2015), physics education (Mao & Sen, 2018; Uhden et al., 2012), proofing (Alcock & Inglis, 2008), analyzing errors or obstacles (Kaur & Yeap, 2001; Nesher & Katriel, 1977; Prayitno et al., 2018), and problem-solving (Cirino et al., 2007; Davis, 2013; Liang et al., 2016; Littlefield & Rieser, 1993; Murphy, 2015). Besides, there are also semantic reasoning in word numeracy problems (Davis, 2013), statistical math word problems (Liang et al., 2016), puzzles (Murphy, 2015), word problems with irrelevant information (Littlefield & Rieser, 1993) number word problems (Cirino et al., 2007; Shi et al., 2015), and calculation (Cirino et al., 2007; Patelli et al., 2014).

The studies on semantic reasoning in solving problems examined the proven and routine problems while problems with incomplete information have not been deeply studied. Yet, many problems are considered to have incomplete information (Abdillah et al., 2016; Littlefield & Rieser, 1993). In addition, the study of semantic reasoning is only limited to the factors that affect the process of proving while the characteristics of semantic reasoning have not been studied in detail. The previous study on semantic reasoning in problem-solving focused on the process of solving routine problems. As in Pape's study (2004), the problem-solving process of solving routine problems uses a meaning-based approach. The characteristics of Pape's study are recording information, using the context of the problem, and completing it with an explanation or proof of the performed mathematical operations. In this process, problem solvers can directly translate into mathematical equations and then interpret each process. But in non-routine problems, such as the form of incomplete information, this cannot be done immediately because students need special strategies to solve them.

This situation raises a gap to examine characteristics of semantic reasoning in solving problems with incomplete information. Problem-solvers need to maintain the semantic structure of the problems (Cirino et al., 2007; Davis, 2013; Liang et al., 2016; Littlefield & Rieser, 1993; Murphy, 2015) to solve problems. The goal is to maintain the selected representation meaning (Bassok et al., 1998; Prayitno et al., 2018) and get the right solution. The results of the observation to high school students when studying social arithmetic often give different meanings to double discounts. The double discount cuts the price of an item twice in a succession.

To avoid the wrong solution, students need to involve semantic reasoning to obtain the appropriate results based on the problem. In this case, they involve their semantic reasoning to understand the meaning of the problem and plan the strategy to find the final solution to the problem (Bassok et al., 1998; Bergqvist et al., 2004; Hegarty et al., 1995). The reasoning is proof that the process of thinking is needed to be taught to students to adapt them to the problem-solving process (Alcock & Inglis, 2008; Prayitno et al., 2018).

Problems with incomplete information are encountered in daily life situations (Abdillah et al., 2016; Littlefield & Rieser, 1993). When solving problems with incomplete information, problem solvers must maintain a semantic structure to maintain the meaning of the chosen representation (Bassok et al., 1998; Prayitno et al., 2018). This reason is the basis for identifying the characteristics of semantic reasoning in detail to develop basic knowledge in interpreting each problem-solving stage. It is also important to know the characteristics of the semantic reasoning of the students in the problem-solving process when solving the incomplete problem. Because of the different problem situations, problem solvers have different characteristics when solving these. Thus, the findings of this study are the characteristics of semantic reasoning when solving incomplete information. It contributes to the knowledge development from each problem-solving stage when solving non-routine problems.

Problem solving in mathematics learning

Problem-solving is an integral part of learning mathematics in schools and it should not be separated from the learning process in the classroom (NCTM, 2000). In the problem-solving process, students are involved to find a solution to a previously unknown task. In order to find it to the problem, the students must be able to apply related knowledge and be required to develop a mathematical understanding of the new situation.

Students who succeed in solving the problem will have the ability to identify the word (Clement, 2008) on the issue, explain the crucial information to solve the problem, and make a mental representation of the problem scheme (Swastika et al., 2020; Xin et al., 2005). The mental representation or mental model is self-constructed by students. The students must actively make changes to each basic text element of the problem (Hegarty et al., 1995; Pape, 2004), activate various schemes involved in the problem, and integrate all the problem's elements with the scheme that the student has (Pape, 2004). In addition, they also need to create a summary of specific mathematical representations according to the problem and develop plans to find the solution to the problem (Hegarty et al., 1995).

The stages of problem-solving are expressed by experts such as Hegarty et al. (1995) and Polya (1973). Hegarty et al. (1995) reported that there are a few things that must be constructed by the problem solver in the problem-solving process, i.e. the basic text, specific mathematical representation, and plan to get the solutions. In constructing basic text, the problem solver should read the statement and integrate new information with the specified base text. In the next stage, the problem solver is guided to achieve the goal of the problem-solving process by constructing a representation referring to a more specific mathematical representation according to the problem. Furthermore, the problem solver presents back information from the problem and then designs the counting process which is also important in the problem-solving process.

Polya (1973) describes the four stages of problem-solving, (1) understanding the problem, (2) making a problem-solving plan, (3) implementing the plan, and (4) checking back on the work. The expressed stages indicate the suitability of the two experts. The difference is in the re-examining stage as Polya (1973) has expressed, while Hegarty et al. (1995) do not perform it. The suitability can be seen in Table 1.

Reasoning in problem solving process

The standard of mathematical learning objectives is to teach the process of problem-solving, reasoning and proofing, connection, communication, and representation (NCTM, 2000). Reasoning and problem-solving are important components in the school's mathematical learning process. Lithner (2008) explains reasoning as a thinking process that is used to generate relevant statements relating to the problem and define conclusions in the task of problem-solving.

To extend a reasoned custom, the students are required to have the appropriate tools and a grasp of the problem situation (Hwang et al., 2007; Panasuk & Beyranevand, 2011). It will begin by involving specific objects. This object is the most basic form so that it can be a function, a statement, a diagram, or another form that fits the situation of the problem (Lithner, 2008).

Reasoning is a logical thinking process to produce a statement and reach a conclusion. Logical thinking is a mental activity based on behavior or the result of problem-solving implicating prior and related knowledge. Bergqvist et al. (2004) emphasize the structure of reasoning to solve problems; 1) Problem situation (a situation that must be managed by problem solvers to proceed to the next stage); 2) Selection of strategies (determine the strategies used to solve the problem through activities to recall, build, find, and so on); 3) Implement the strategy (verification is needed for each step conducted); 4) Conclusions (results obtained from implementing the strategy).

Similar to the opinion by Lithner (2008), there are four stages of reasoning structure when solving problems: 1) Problem situation; 2) Selection of strategies (determine the strategies that will be used to solve the problem); 3) Application of the strategy (the strategy used must be supported by verification of the given arguments); and 4) Get conclusions.

Table 1
Problem-solving stage

Polya (1973)	Hegarty et al. (1995)
Understanding the problem	Construct the basic text
Make a plan	Construct a specific mathematical representation
Implementing the planning	Construct a plan
Checking back	Implement a plan to find solutions

Table 2
The component of semantic reasoning based on stage problem solving

Stage problem solving (Polya, 1973)	Reasoning in problem-solving (Bergqvist et al., 2004; Lithner, 2008)	Meaning based approach (Pape, 2004)	Component of semantic reasoning in this study
Understanding the problem	Problem context	Record information Problem context	Record information
Make a plan	Strategy selection		Strategy election
Implementing planning Checking back	Implement strategies	Proof and explanation	Implement strategies and explanation
	Conclusion		Conclusion

The two opinions above show the similarities in the structure of reasoning when solving problems. The two experts began by recognizing the given problem situation, which depends on the problem solver's ability. Then, it is followed by choosing a strategy that would be used to solve the problem and implement the selected strategy. Finally, the conclusion is arranged as the final answer to the problem presented. So, in this study, researchers will use the mathematical reasoning structure of the two experts to identify students' semantic reasoning in solving double discount problems.

Semantic reasoning in problem solving

Semantic is a branch of linguistics that learns about the meaning or a form of representation with the presented context. In learning mathematics, semantics have an important role in the problem-solving process. It considers the character of a representation system obtained from the other forms of representation, as desired by problem solvers when facing problems (Alcock & Inglis, 2008). The presented problems must be translated into the other forms of representation that are known by the problem solvers. This aims to help problem solvers to find the final solution.

In the problem-solving process, Pape (2004) uses the term meaning-based approach rather than semantic. This means that it gives meaning and explanation at each stage of the problem-solving. Pape (2004) uses the term meaning-based approach consisting of 3 characteristics, (1) recording information, (2) using the context of the problem, and (3) completing the proof and explanation of the performed mathematical operations.

Semantic reasoning is the process of thinking in understanding the meaning of each stage of the problem-solving process to find a solution to the problem. So, the semantic reasoning component used in this study is adapted from Bergqvist et al. (2004) and Pape (2004), which can be seen in Table 2. Those four components of the semantic reasoning above were used by researchers to identify the characteristics of students' semantic reasoning in solving double discount problems. The problem used in this study is non-routine problems, specifically double discounts. Non-routine problems are more complex than routine problems, so strategies that are used to solve them may not appear directly. This is certainly different between students because it is influenced by the level of the students' creativity and originality. Double discount is one of the sub materials of social arithmetic. Double discount problems are problems with incomplete information, allowing students to use different strategies to find solutions.

Students often give different meanings to double discounts based on the results of the observation conducted by researchers in high school students when studying social arithmetic. The double discount cuts the item's price twice in a succession, but they often sum the two numbers of the discounts. Summing both discount numbers is not compatible with the actual meaning of the double discount. This condition happens not only in students but also in the broader community. It indicates that an error in the semantic reasoning is wrong in giving the meaning of a double discount. Consequently, the final solution is not appropriate in the actual condition. Therefore, the double discount is evaluated in this study.

The problem of double discounts is often found in malls as they often give double discounts on certain events. The wrong meaning of students will certainly carry over in daily life situations at any time. Differences in meaning by students are affected by student abilities, experiences, and the surrounding environment (Adu-Gyamfi et al., 2012; Bossé et al., 2011, 2014; Gagatsis & Shiakalli, 2004). Students understand the process well because it reduces errors in the interpreting process and translates keywords (Bossé et al., 2011) to get the proper understanding. It is substantial to know the characteristics of the semantic reasoning of the students in the problem-solving process when solving the double discounts problem. Therefore, problem solvers have different characteristics in this process. Thus, this study identifies the characteristics of semantic reasoning when solving incomplete information.

METHODS

Study design

This study was qualitative research. It was intended to produce a clear and detailed description of the students' semantic reasoning in solving double discount problems (Creswell, 2012; Miles et al., 2014; Miles & Huberman, 1994). The data were collected through mathematical problems, observation, and interviews. The three ways of data collection were done to strengthen the characteristics of students' semantic reasoning. Mathematical problems were used as the tool to describe students' semantic reasoning in solving double discount problems. During the process of solving problems, researchers observed and recorded their behavior in the process of giving meaning. Furthermore, in-depth interviews were conducted with selected subjects to find out their semantic reasoning in problem-solving and the interview process was recorded in videos. The results of student work when solving the problem of double discounts were collected. The collected data were analyzed using Miles and Huberman's data analysis principles, namely reduction, presentation, and conclusion (Miles et al., 2014; Miles & Huberman, 1994).

Participant

The participants in this study were 51 senior high school students in 10th grade at one of Senior High School in Sidoarjo, East Java, who voluntarily participated in this study. The activity to acknowledge the double discount problem was done before the test was given to the students. The test was presented to students who have attended the social arithmetic material. The double-discount problem was selected since it can be used as a problem with incomplete information. In addition, misconceptions about double discounts are often encountered in everyday life. The students' answers were grouped based on the strategies to find solutions to the problem. For each problem-solving strategy, the researcher assigned one subject to be interviewed in-depth to find out the semantic reasoning in solving the double discount problem. The three strategies used by students in solving multiple discount problems were using (1) variables, (2) numbers, and (3) the combination of variables and numbers. The selection of subjects was based on the use of problem-solving strategies, complete problem-solving processes, and the consideration of their mathematics teacher, such as students with good verbal communication skills. The results were then used to identify the characteristics of the semantic reasoning of students in each group based on the used strategies. The similarity of these characteristics is the reason researchers choose one subject. Students who were not included in the criteria were not designated as research subjects. Thus, the selection has resulted in three students who fit the research criteria.

Instruments and procedure

The problem used was a double discount problem with incomplete information, which was adapted from Abdillah et al. (2016): "A pair of shoes at the Sogo and the Matahari has the same price. Sogo provides a discount of 50% then 30%, while the Matahari gives a discount of 70%. In your opinion, which shop gives the bigger discount?".

First, the problem was consulted with a high school math teacher to obtain validation and revision. Second, the students were given 15–20 minutes to solve the problems. After the test was handed out, the researcher collects student answers then groups them based on strategies given by the students. From the grouping, the researcher then consulted with the teacher to determine the students who were able to communicate well from each grouping category. Three students who used different strategies and had good communication skills were interviewed in-depth to clarify their work and find out their semantic reasoning in solving the double discount problems. The question guidelines for the in-depth interview in this study are: (1) Can you explain what you want from the question?, (2) What did you do to solve the problem?, (3) Why did you use it?, (4) After that, what did you do using analogy?, (5) Can you explain what you did?, (6) What was your conclusion?, and (7) Why did you choose them?

Data analysis

The data analysis method used in this study was descriptive qualitative analysis, namely reduction, presentation, and conclusion (Miles et al., 2014; Miles & Huberman, 1994). The reduction was done after collecting data and then grouping the students based on the correct and wrong answers. The correct answers were then regrouped based on the strategies that were used by the students to answer the problem. There were three groups, namely using (1) variables, (2) numbers, and (3) a combination of both. The presentation was done by reducing and classifying data and then presenting the data narratively. From these groups, the researcher played back the results of students' observations when solving problems, then identified the similarities in the characteristics of semantic reasoning in each group. From the similarity of these characteristics, the researcher asked the mathematics teacher's consideration to determine one subject for in-depth interviews. The collected and analyzed data can describe students' semantic reasoning in solving double discount problems.

Triangulation was a process to strengthen the evidence in many ways including through different subjects, types of data, and data collection methods (Creswell, 2012; Miles et al., 2014; Miles & Huberman, 1994). In this study, the triangulation of data collection methods was used to test the data accuracy. Researchers compared the characteristics of semantic reasoning based on students' work data, observations, and interviews. The three methods showed the same characteristics, so the data satisfied the valid criteria.

FINDINGS

Based on the grouping, there are three characteristics of the answers: (1) semantic reasoning using variables, (2) semantic reasoning using numbers, and (3) semantic reasoning using variables and numbers. The analysis to obtain the three subjects is described in Table 3. The following are the interview results that could indicate the answers' characteristics based on the selected subject.

Semantic reasoning using variables

Record information

The subjects that solve problems with semantic reasoning using variables were called S1. Based on the semantic reasoning component, S1 started working by recording information based on the problem's context. S1 wrote the information that was known from the problem using its own language. S1's activities indicate that the information presented in the problem is well understood (Mairing, 2017; Meyer, 2014; Özcan et al., 2017). This initial step determines the next steps that must be taken. The work of S1 in writing information can be seen in Figure 1. To explore the students' understanding of the context of the problem, the researchers matched the interview data. S1 wrote, "Sogo gives a discount 50% + 30%" and S1 means Sogo giving a 50% discount and then continuing with another discount of 30%. The meaning of the sentence is following the interview's quote:

R : You wrote Sogo giving a 50% + 30% discount. Can you explain what that means?
S1 : Sogo gives a 50% discount first and then from there gets a discount of 30%.
R : How about in Matahari?
S1 : If in the Matahari the discount is directly 70%

At this stage, the students' activities include the determination of the keywords regarding the problem and giving them meaning. This activity is the next step in the problem-solving process (Clement, 2008; Hegarty et al., 1995; Liang et al., 2016; Pape, 2004; Uhden et al., 2012; Xin et al., 2005) before determining the strategy to use. Giving meaning to keywords certainly involves prior knowledge related to the problems. In this case, S1 assigned keywords that Sogo gives a 50% + 30% discount and then gave the keyword a meaning.

Selection of strategy

S1 selected a strategy to answer the problem. From the results of the work, S1 used a shoe price example as variable x, as the following interview quote:

R : What will you do?
S1 : The price is not known mam, so I'm using the letter x
R : Why do you use letters?
S1 : You are free to use letters or use the price of the shoes

Problem-solving strategies must be determined in advance so that the desired solution is suitable to answer the problem (Davis, 2013). Interpreted strategy is influenced by the problem solver's ability to solve problems (Cirino et al., 2007; Davis, 2013; Littlefield & Rieser, 1993; Polya, 1973) and prior mastered knowledge. In this case, S1 gave meaning to variable x as the shoe price, which showed S1's abstract thinking.

Implement strategies and explanation

S1 stated variable x as the price of shoes in both stores. From the chosen strategy, S1 implemented it to find a solution. The results of implementing it by S1 can be seen in Figure 2. From the picture above, S1 calculated the shoe price example using the variable x by multiplying the given discount. At Sogo's stores, variable x was multiplied by a 50% discount then the initial price of the shoe was reduced by the discount. To give a second discount of 30%, S1 multiplied the price after getting a 50% discount by a 30% discount. Then, the results were used to reduce the price of shoes after deducting a 50% discount, which was 0.35x. At the Matahari store, S1 multiplied variable x by 70% and then deducted the initial price of the shoe to obtain the 0.3x.

S1 provided the proof of calculation for each store to help S1 to get the solution and continued implementing the strategy through a calculation process (Cirino et al., 2007; Hegarty et al., 1995). S1 gave meaning to every problem-solving stage as in previous studies reported (Mairing, 2017; Pape, 2004). Thus, it is important to integrate context-related meanings and involve meaningful mathematics in daily life situations.

Conclusion

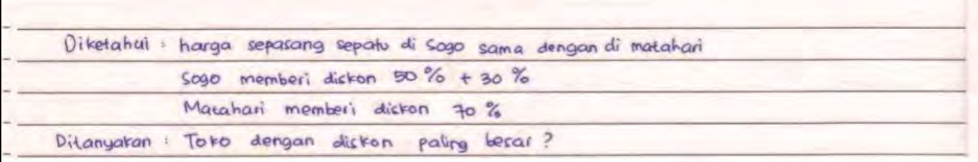
After calculating and providing evidence, S1 provided conclusions to the problem. S1's work result can be seen in Figure 3. It showed that after concluding, S1 also gave the meaning of the conclusions. The interview with S1 showed the following:

R : So what is the conclusion?
S1 : The bigger discount store is in Matahari mam...
R : Why can you conclude like that?
S1 : In Sogo, the shoe is 0.35x, if Matahari is 0.3x. The difference is 0.5x between Sogo and Matahari store
R : Meaning the difference between Sogo and Matahari?
S1 : The price of paying Sogo is clearly more expensive, so I choose Matahari

From the interview above, the answer showed that S1 stated that 0.35x is greater than 0.3x. It appears that S1 explained that there is a 0.5x difference between the two stores, so S1 immediately concluded that Matahari provides a cheaper price than Sogo. The conclusion is the solution to the given problem (Davis, 2013). It is reinforced by an explanation of the meaning of the final solution (Holisin et al., 2017). In this case, S1 gave meaning that paying for shoes in Sogo is more expensive than in Matahari. Furthermore, a conclusion certainly involves a dynamic process known as thinking (Bakry, 2015). This is certainly influenced by the existing information on problems and experiences in everyday life.

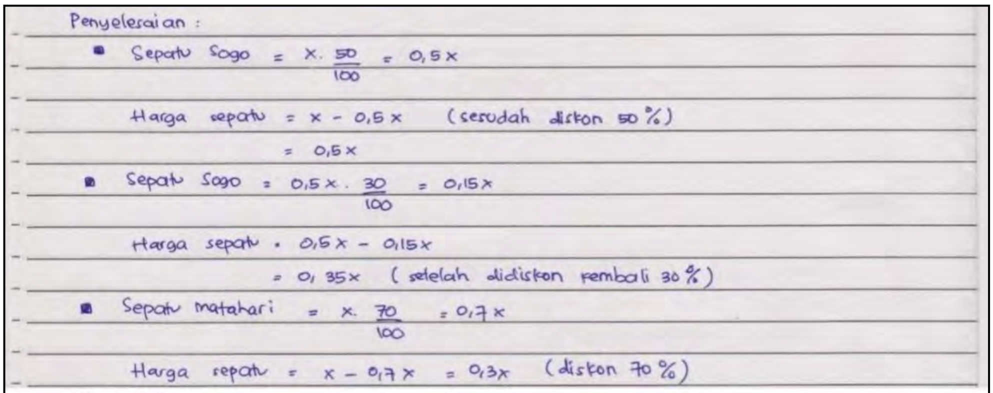
Table 3
The distribution of subjects based on their work

Characteristics	Number of Students	Selected Subject
Using variables	2	1
Using numbers	30	1
Using variables and numbers	6	1
Directly answers	13	-
Total	51	3


Diketahui : harga sepasang sepatu di Sogo sama dengan di matahari
Sogo memberi diskon 50 % + 30 %
Matahari memberi diskon 70 %
Ditanyakan : Toko dengan diskon paling besar ?

Translate
Known : the price of a pair of shoes in sogo same as in Matahari
Sogo gives discount 50% + 30%
Matahari gives discount 70%
Problem : shop given a big discount?

Figure 1. S1's writing regarding the information based on the given problems

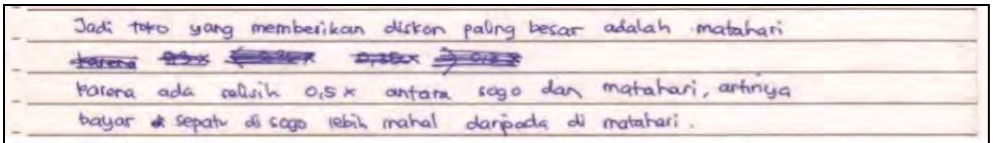

Penyelesaian :
Sepatu Sogo = x . 50/100 = 0,5 x
Harga sepatu = x - 0,5 x (sesudah diskon 50 %)
= 0,5 x
Sepatu Sogo = 0,5 x . 30/100 = 0,15 x
Harga sepatu = 0,5 x - 0,15 x
= 0,35 x (setelah didiskon kembali 30 %)
Sepatu matahari = x . 70/100 = 0,7 x
Harga sepatu = x - 0,7 x = 0,3 x (diskon 70 %)

Translate
Solve:
Sogo's shoes = $x\frac{50}{100} = 0,5$
Price shoes = *x* – *0,5x* = *0,5x* (after discount 50%)

Sogo's shoes = $0,5\ x\frac{30}{100} = 0,15\ x$
Price shoes = 0,5*x* – *0, 15x* = *0,35x* (after discount 30%)

Matahari's shoes = $x\frac{70}{100} = 0,7x$
Price shoes = *x* – *0,7x* = *0,3x* (after discount 70%)

Figure 2. S1's implementation of the chosen problem-solving strategies with mathematical calculation as evidence


Jadi toko yang memberikan diskon paling besar adalah matahari
karena ada selisih 0,5 x antara sogo dan matahari, artinya bayar sepatu di sogo lebih mahal daripada di matahari.

Translate
So, a store that provides the biggest discount is Matahari. Because there is a difference of 0.5x between Sogo and Matahari, meaning that paying shoes in Sogo is more expensive than in Matahari.

Figure 3. S1's conclusion

Semantic reasoning using numbers

Record information

The subject who solved the problem with semantic reasoning using numbers was called S2. Based on the semantic reasoning component used in this study, S2 began by recording information related to the context of the problem. This is done by S2 by writing information that was known from the problem by his understanding. The results of the S2's work in writing information can be seen in Figure 4. From the S2 works, S2 did not write down what was asked according to the problem. To clarify this, the researcher asked through an interview, quoted as the following:

R : What problem did you know?
S2 : Discounts at two shops. Prices in Sogo and Matahari are the same
R : Why about the discount?
S2 : In the Sogo, the discount is 50% and further 30%, Matahari only 70%
R : What's the difference?
S2 : If there are two Sogo discounts, the price is cut by 50% and then cut 30% more, then Matahari just 70%
R : What do you think is (priced) less than you explained just now?
S2 : Hhmmm ... (the subject pauses)
S2 : Asked the store who gave the cheapest price for what store?

Exploring problems in the problem-solving process certainly involves thinking skills (Bakry, 2015). In this case, S2 explained the information in the problems using their language and analogous with previous studies by Clement (2008); Uhden et al. (2012); Xin et al. (2005), etc. that stated the problem context is key in the problem-solving process. S2 assigned keywords comparing a single discount and double discount, then determine the store that provides the lowest price.

Selection of strategy

S2 made a selection of strategies using the price of shoes at a certain price. S2 considered the price of shoes, which was 150,000, but S2 does not write it on the results of his work. The researcher clarified it through interviews as quoted below:

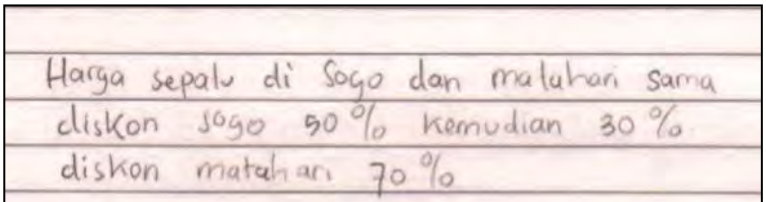
R : What will you do?
S2 : I made the shoes (priced) for 150,000 ma'am...
R : Why is the price of the shoes made?
S2 : The price of the shoes isn't in the question. (the price) Let me easily calculate it

S2 means that the shoes' price is a certain number so it can be operated with the number of the given discounts. The idea of the shoe price by S2 is analogous to previous studies by Abdillah et al. (2016) and Stylianou (2013), which used the current number as the shoe price. This activity shows the ability of students who are accustomed to the problem-solving process, especially in daily life situations.

Implement strategies and explanations

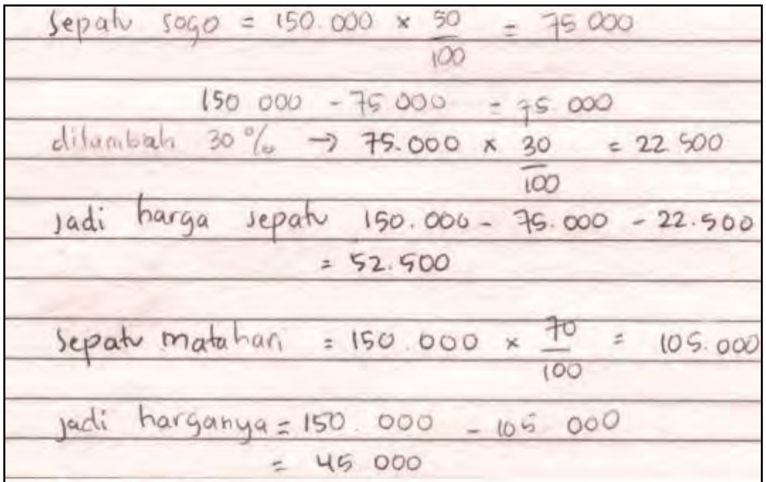
Based on the results of the work and interviews, the results showed that S2 intended to give a 50% discount and then 30% as giving two sequence discounts. That is, the initial price was given a 50% discount, then cut again by 30%, and the result is the price to be paid. Furthermore, S2 implemented the chosen strategy to answer the problem. The results of implementing the chosen strategy can be seen in Figure 5.

The chosen price of shoes was 150,000, then S2 proceeded to process the discounts by doing a calculation to determine the store that provides the lowest price. S2 began with calculations for the Sogo shop followed by Matahari to determine the amount of money to be paid after discount(s). This is analogous to previous studies such as Abdillah et al. (2016) and Stylianou (2013) that used an initial price to determine the price to be paid. S2 used a different strategy from S1 as an effort to get a solution to the problem (Davis, 2013; Stylianou, 2013) and then proceed to the calculation process.



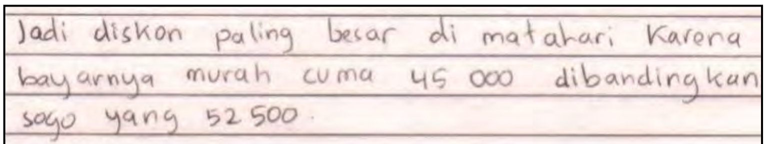
Translate
Price shoes in Sogo and Matahari are same
Sogo's discount 50% then 30%
Matahari's discount is 70%

Figure 4. S2's writing regarding the information of the problem



Translate
Solve:
Sogo's shoes = $150.000.\frac{50}{100} = 75.000$
150.000 – 75.000 = 75.000
Adding discount 30% = $75.000.\frac{30}{100} = 22.500$
So, the shoes's price = 150.000 – 75.000 – 22.500
= 52.500
Matahari's shoes = $150.000.\frac{70}{100} = 105.000$
Price shoes = *150.000 – 105.000 = 45.000*

Figure 5. S2's implementation of the selected problem-solving strategies with mathematical calculation as evidence



Translate
Biggest discount in Matahari because cheaper than Sogo

Figure 6. S2's conclusion

Conclusion

After completing the proof of calculation, S2 compared the cheapest store and arranged the conclusion. The conclusion of S2's work results can be seen in Figure 6. To confirm the answers given, the researcher matches the results of the interview as follows:

R : What is the conclusion of your work?
S2 : The store with the most discounts is the Matahari, ma'am

R : Are you sure your answer is correct?
S2 : I'm sure ma'am, if in Matahari, it costs 45,000. But on Sogo, it costs 52,500. I believe that the price in Sogo is more expensive.

S2 stated the shop that provides the biggest discount is the store with the cheapest price. Here, S2 showed the price to be paid to Matahari and Sogo will be 45,000 and 52,500, respectively. By comparing prices, S2 could determine the final solution to the problem precisely (Littlefield & Rieser, 1993; Özcan et al., 2017). In this case, S2 put the relationship between two conditions and assigns their opinion (Bakry, 2015) to solve the problems.

Semantic reasoning using variables and numbers

Record information

The subjects who solve problems by involving semantic reasoning using variables and numbers were called S3. Based on the semantic reasoning component used in this study, S3 directly leads to the calculation process involving variables. To clarify that S3 recorded information before solving a problem, the researcher interviewed S3. The following are the results of the interview data:

R : What do you understand from the problem you were working on?
S3 : The problem is about discounts at the mall, ma'am. So there are two other stores, Matahari and Sogo.
R : continue...
S3 : There are shoes with the same price but the discount is different. If there is a discount in Sogo, (it is) 50% and continues with 30%. Ah ... if in Matahari the discount is only one that is 70%
R : What is the problem?
S3 : In the matter of being asked, it is about which store that gives a lot of discounts
R : When you answer, why isn't it written?
S3 : Yes ma'am

From the interview above, the results show that S3 was able to record the information in the problem according to the context of the problem. S3 explained his own opinion about the problem. S3 stated that the problem is which store that gives more discounts. This is analogous to a previous study by Bakry (2015). In this case, S3 interpreted the information of multiple discounts as giving two discounts, 50%, and 30%. Giving meaning to the problem is also influenced by prior knowledge related to the problems. This is followed by processing each text element from the problem based on the mental model of the situation (Murphy, 2015; Pape, 2004).

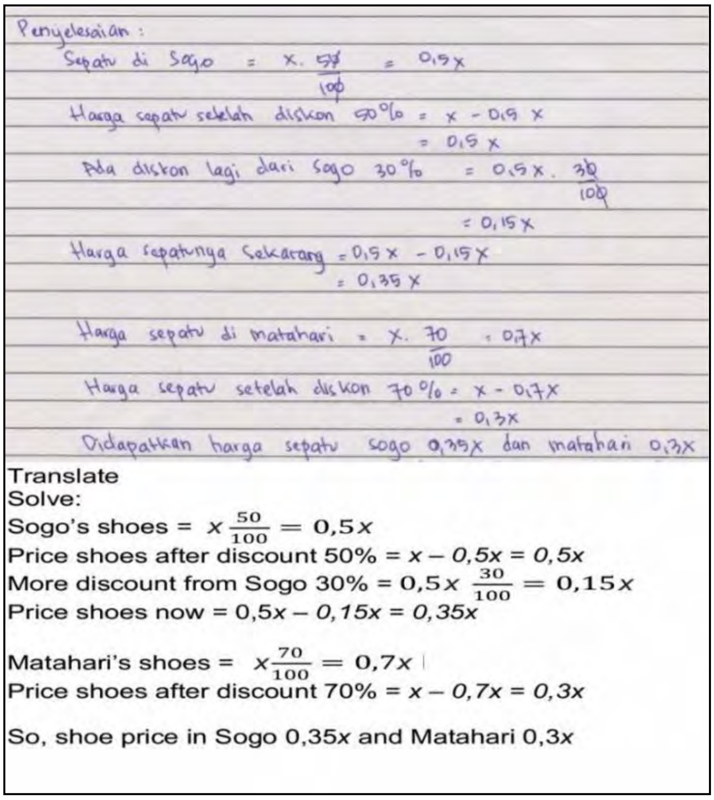
Selection of problem

In solving the problems, S3 conducted a selection of strategies using the example of shoe prices using the variable x. From the example given by S3, it is shown that S3 intended the price of shoes in both stores as free numbers, thus the variable x. S3's strategy selection can be seen in Figure 7. Problem-solving strategies must be determined to get a suitable solution to the problem (Davis, 2013). In this case, S3 used variable x as the shoes' price which showed S3 abstract thinking. However, to ensure the answer, S3 used numbers that represent the price of the shoe. This showed that S3 used alternative strategies to solve the problems (Paradesa, 2018; Sardi et al., 2018).

Implement strategies and explanation

S3 conducted the process of applying the chosen strategy to solve the problem and provides the proof of calculation. From Figure 7, S3 implemented the selected problem-solving strategies for each store. S3 gave discounts at both stores by starting the counting process for the Sogo shop, followed by the Matahari until S3 determined the price to be paid on Sogo was 0.35x and 0.3x on Matahari.

Before concluding the store that provides the most discount between the two, S3 changed the variable x with a certain number, which was 100,000. After that, S3 continued with the concluding process. S3's work results can be seen as shown in Figure 8. From figure 8, S3 used alternative strategies to solve the problems (Paradesa, 2018; Sardi et al., 2018). In this case, S3 substituted variable x with 100,000 then S3 determine the price to be paid at each store. From the calculation process, S3 assigned that the price to be paid in Sogo would be 35,000 and 30,000 in Matahari. This condition helps S3 to conclude the solution to the problems (Davis, 2013).

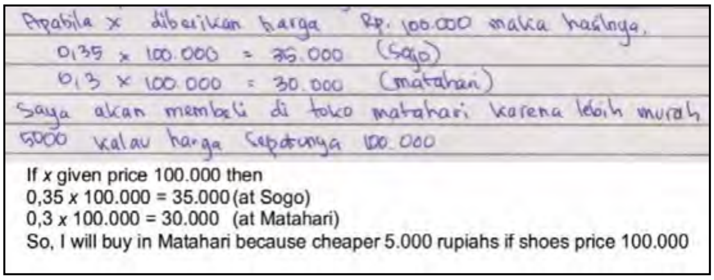

Penyelesaian :
Sepatu di Sogo = x . 50/100 = 0,5x
Harga sepatu setelah diskon 50% = x − 0,5 x
= 0,5 x
Ada diskon lagi dari Sogo 30% = 0,5 x . 30/100
= 0,15 x
Harga sepatunya sekarang = 0,5 x − 0,15 x
= 0,35 x
Harga sepatu di matahari = x . 70/100 = 0,7x
Harga sepatu setelah diskon 70% = x − 0,7x
= 0,3x
Didapatkan harga sepatu sogo 0,35x dan matahari 0,3x

Translate
Solve:
Sogo's shoes = $x\frac{50}{100} = 0,5x$
Price shoes after discount 50% = $x - 0,5x = 0,5x$
More discount from Sogo 30% = $0,5x\ \frac{30}{100} = 0,15x$
Price shoes now = $0,5x - 0,15x = 0,35x$

Matahari's shoes = $x\frac{70}{100} = 0,7x$
Price shoes after discount 70% = $x - 0,7x = 0,3x$

So, shoe price in Sogo 0,35x and Matahari 0,3x

Figure 7. S3's strategy implementation


Apabila x diberikan harga Rp. 100.000 maka hasilnya,
0,35 x 100.000 = 35.000 (Sogo)
0,3 x 100.000 = 30.000 (matahari)
Saya akan membeli di toko matahari karena lebih murah 5000 kalau harga sepatunya 100.000

If x given price 100.000 then
0,35 x 100.000 = 35.000 (at Sogo)
0,3 x 100.000 = 30.000 (at Matahari)
So, I will buy in Matahari because cheaper 5.000 rupiahs if shoes price 100.000

Figure 8. S3's variable x substitution and conclusion

Conclusion

Figure 8 shows that in the conclusion, S3 also provided an explanation that the Matahari store is cheaper than Sogo. Here, S3 stated the difference between the price of Sogo and Matahari is 5,000 if the initial price of the shoe is 100,000. Thus, S3 concluded that the store that gave the biggest discount was Matahari. The given conclusion is the solution to the given problem (Davis, 2013), this is reinforced by an explanation of the meaning of the final solution of the subject during the interview. A conclusion certainly involves a process known as thinking (Bakry, 2015). This is certainly influenced by the existing information on problems and experiences that have been had in everyday life.

DISCUSSION

The works of S1, S2, and S3 showed that the characteristics of students' semantic reasoning in solving problems are divergent. Each subject gives meaning to the problem with their own language and it is key in the problem-solving process (Clement, 2008; Hegarty et al., 1995; Liang et al., 2016; Pape, 2004; Xin et al., 2005). Giving meaning to information related to the problems also involves the role of system representation (Alcock & Inglis, 2008; Clement, 2008; Dawkins, 2012; Hwang et al., 2007; Shi et al., 2015; Sukoriyanto et al., 2016). It is also important to note that the ability of problem solvers is the key to success in problem-solving.

Table 4
The characteristic of semantics reasoning in each strategy

Semantic reasoning using variable	Semantic reasoning using number	Semantic reasoning using variables and number
Identify the information, interpret terms according to the context of the problem, and build the relationships that are expressed in writing.	Identify the information, interpret the terms according to the context of the problem, and build the relationships that are expressed in writing.	Identify the information, interpret the terms according to the context of the problem, and build the relationships that are expressed in oral.
Paying attention to the relationships based on existing information and using variables to assume the price of shoes.	Paying attention to the relationships based on the existing information and using numbers to assume the price of shoes.	Paying attention to the relationships based on the existing information and using variables to assume the price of shoes.
Provide evidence and explanation of the calculation process involving variables.	Provide evidence and explanation of the calculation process involving numbers.	Provide evidence and explanation of the calculation process involving variables. Then, substitute variable to number to get shoe price.
Final answers to the problems	Final answers to the problems	Final answers to the problems

In addition to involving the representation system, giving meaning to the problem is also influenced by prior knowledge and followed by the processing of each text element from the problem based on the mental model of the situation (Murphy, 2015; Pape, 2004). After interpreting the given problem, each subject uses his strategy as an effort to get a solution to the problem (Davis, 2013; Hegarty et al., 1995; Polya, 1973) then proceed with applying the chosen strategy, such as carrying out the calculation process (Cirino et al., 2007; Hegarty et al., 1995; Polya, 1973). Each subject interpreted this strategy in their own ways (Cirino et al., 2007; Davis, 2013; Littlefield & Rieser, 1993; Polya, 1973). They interpreted it using variables for S1, numbers for S2, and both variables and numbers for S3. After the calculation process, each concludes the process that has been done. The given conclusion is the solution to the given problem (Davis, 2013). This is reinforced by an explanation of the meaning of the final solution by the subject during the interview.

The incomplete problem presented in this study is ill-structured problem. The problem-solving process involves ill-structured problems, an activity that implicates a metacognitive strategy. Metacognitive strategies are involved when the subject monitors the information presented from the problem, creates plans, and determines the solutions to answer the problem (Shin et al., 2003). This shows that applying semantic reasoning in problem-solving involves metacognitive strategies.

From the students' work, it shows that most students use numbers as the chosen strategy in the problem-solving process. According to Piaget's theory, high school students should be in the formal operational stage. At this stage, their way of thinking is more abstract. This is where the need to create good problems in accordance with the stages of cognitive development lies. In addition, it is important to integrate various topics related to context and involve meaningful mathematics in daily life situations (Bassok, Chase, & Martin, 1998; NCTM, 2000) (Bassok et al., 1998; NCTM, 2000). By paying attention to the components of semantic reasoning and processes carried out by students in solving problems, the characteristics of students' semantic reasoning in solving the problem of double discounts obtained in this study can be seen in Table 4.

Therefore, to develop students' semantic reasoning in solving problems, the teacher has an important role in the classroom. These processes can be led by the teacher to develop the semantic reasoning of students. It could be developed through familiarizing students with: various representations that are felt easy to use, connecting between representations, and presenting their ideas; with the semantic reasoning they have in the problem-solving process; and explaining what is done in the problem-solving process. For this reason, further research is required to identify the semantic reasoning of students in solving problems in different materials. The hope is to know the general characteristics of students' semantic reasoning in solving diverse problems.

CONCLUSIONS

The characteristic of semantic reasoning-based strategies used by students is using a variable, using numbers, and using the combination of variables and numbers. Interestingly, the process is almost similar, such as 1) giving the problem situation a meaning with their own understanding, (2) stating the keywords and giving them their meaning, (3) stating the relationship for the current keywords, (4) transforming them into a mathematical statement, (5) calculating the mathematical statement using their strategies, (6) making the decision, and (7) interpreting the final answer. The limitation of this study was only one student for each strategy was chosen as a subject for further analysis using in-depth interviews and they were also chosen from the group with the right answers. Thus, it is imperative to use more subjects for each strategy to confirm the semantic reasoning in solving incomplete information problems on a variant math subject. Besides that, the semantic reasoning in students who have the wrong answer and their obstacles in solving the incomplete information problems are needed to be identified.

BIBLIOGRAPHY

Abdillah, A., Nusantara, T., Subanj, S., Susanto, H., & Abadyo, A. (2016). The students decision making in solving discount problem. *International Education Studies*, *9*(7), 57–63. https://doi.org/10.5539/ies.v9n7p57

Adu-Gyamfi, K., Stiff, L. V., & Bossé, M. J. (2012). Lost in translation: Examining translation errors associated with mathematical representations. *School Science and Mathematics*, *112*(3), 159–170. https://doi.org/10.1111/j.1949-8594.2011.00129.x

Alcock, L., & Inglis, M. (2008). Doctoral students' use of examples in evaluating and proving conjectures. *Educational Studies in Mathematics*, *69*(2), 111–129. https://doi.org/10.1007/s10649-008-9149-x

Bakry, M. N. B. B. (2015). The process of thinking among junior high school student in solving HOTS question. *International Journal of Evaluation and Research in Education*, *4*(3), 138–145. https://doi.org/10.11591/ijere.v4i3.4504

Bassok, M., Chase, V. M., & Martin, S. A. (1998). Adding apples and oranges: Alignment of semantic and formal knowledge. *Cognitive Psychology*, *35*(2), 99–134. https://doi.org/10.1006/cogp.1998.0675

Bergqvist, T., Lithner, J., & Sumpter, L. (2004). Reasoning characteristics in upper secondary school students' task solving. In C. Bergsten & B. Grevholm (Eds.), *Mathematics and language* (pp. 71–77). Malmö: Svensk Förening för Matematik Didaktisk Forskning.

Bossé, M. J., Adu-Gyamfi, K., & Chandler, K. (2014). Students' differentiated translation processes. *International Journal for Mathematics Teaching & Learning*, *828*, 1–28.

Bossé, M. J., Adu-Gyamfi, K., & Cheetham, M. (2011). Translations among mathematical representations: Teacher beliefs and practices. *International Journal for Mathematics Teaching & Learning*, *12*(2), 1–23.

Cai, J., & Nie, B. (2007). Problem solving in Chinese mathematics education: Research and practice. *ZDM Mathematics Education*, *39*(5–6), 459–473. https://doi.org/10.1007/s11858-007-0042-3

Cirino, P. T., Morris, M. K., & Morris, R. D. (2007). Semantic, executive, and visuospatial abilities in mathematical reasoning of referred college students. *Assessment*, *14*(1), 94–104. https://doi.org/10.1177/1073191106291487

Clement, J. J. (2008). Does decoding increase word problem solving skills? *Action Research Projects*, *32*. https://digitalcommons.unl.edu/mathmidactionresearch/32

Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (4th ed.). Boston: Pearson.

Davis, J. (2013). Student understandings of numeracy problems: Semantic alignment and analogical reasoning. *The Australian Mathematics Teacher*, *69*(2), 19–26. https://doi.org/10.3316/informit.642989128192841

Dawkins, P. C. (2012). Extensions of the semantic/syntactic reasoning framework. *For the Learning of Mathematics*, *32*(3), 39–45.

Gagatsis, A., & Shiakalli, M. (2004). Ability to translate from one representation of the concept of function to another and mathematical problem solving. *Educational Psychology*, *24*(5), 645–657. https://doi.org/10.1080/0144341042000262953

Hegarty, M., Mayer, R. E., & Monk, C. A. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of Educational Psychology*, *87*(1), 18–32. https://doi.org/10.1037/0022-0663.87.1.18

Holisin, I., Budayasa, I. K., & Suwarsono, St. (2017). Comparison of male and female primary school student reasoning profiles in solving fractional problems. *International Journal of Environmental and Science Education*, *12*(6), 1553–1565.

Hwang, W.-Y., Chen, N.-S., Dung, J.-J., & Yang, Y.-L. (2007). Multiple representation skills and creativity effects on mathematical problem solving using a multimedia whiteboard system. *Journal of Educational Technology & Society*, *10*(2), 191–212.

Jonassen, D. H. (1997). Instructional design models for well-structured and III-structured problem-solving learning outcomes. *Educational Technology Research and Development*, *45*(1), 65–94. https://doi.org/10.1007/BF02299613

Kaur, B., & Yeap, B. H. (2001). Semantic characteristics that make arithmetic word problems difficult. *Numeracy and Beyond: Proceedings of the 24th Annual Conference of the Mathematics Education Research Group of Australasia Incorporated*, 555–562.

Liang, C.-C., Tsai, S.-H., Chang, T.-Y., Lin, Y.-C., & Su, K.-Y. (2016). A meaning-based english math word problem solver with understanding, reasoning and explanation. *Proceedings of COLING 2016, the 26th International Conference on Computational Linguistics: System Demonstrations* (pp. 151–155). Osaka, Japan: The COLING 2016 Organizing Committee.

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*, *67*(3), 255–276. https://doi.org/10.1007/s10649-007-9104-2

Littlefield, J., & Rieser, J. J. (1993). Semantic features of similarity and children's strategies for identifying relevant information in mathematical story problems. *Cognition and Instruction*, *11*(2), 133–188. https://doi.org/10.1207/s1532690xci1102_2

Mairing, J. P. (2017). Thinking process of naive problem solvers to solve mathematical problems. *International Education Studies*, *10*(1), 1–11. https://doi.org/10.5539/ies.v10n1p1

Mao, X., & Sen, C. (2018). Physics-based semantic reasoning for function model decomposition. *Volume 1A: 38th Computers and Information in Engineering Conference*, 1–12. https://doi.org/10.1115/DETC2018-86273

Meyer, K. (2014). Making meaning in mathematics problem-solving using the Reciprocal Teaching approach. *Literacy Learning : The Middle Years*, *22*(2), 7–14. https://doi.org/10.3316/aeipt.204235

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: A sourcebook of new methods*. Baverly Hills: SAGE Publications.

Miles, M. B., Huberman, A. M., & Saldana, J. (2014). *Qualitative data analysis* (3rd ed.). California: SAGE Publications.

Murphy, C. (2015). Authority and agency in young children's early number work: A functional linguistic perspective. *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia*, 453–460. https://eric.ed.gov/?id=ED572496

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. United States of America: The National Council of Teachers of Mathematics Inc.

Nesher, P., & Katriel, T. (1977). A semantic analysis of addition and subtraction word problems in arithmetic. *Educational Studies in Mathematics*, *8*(3), 251–269. https://doi.org/10.1007/BF00385925

Özcan, Z. Ç., İmamoğlu, Y., & Bayraklı, V. K. (2017). Analysis of sixth grade students' think-aloud processes while solving a non-routine mathematical problem. *Educational Sciences: Theory & Practice*, *17*(1), 129–144. https://doi.org/10.12738/estp.2017.1.2680

Panasuk, R. M., & Beyranevand, M. L. (2011). Preferred representations of middle school algebra students when solving problems. *The Mathematics Educator*, *13*(1), 32–52.

Pape, S. J. (2004). Middle school children's problem-solving behavior: A cognitive analysis from a reading comprehension perspective. *Journal for Research in Mathematics Education*, *35*(3), 187–219. https://doi.org/10.2307/30034912

Paradesa, R. (2018). Pre-service mathematics teachers' ability in solving well-structured problem. *Journal of Physics: Conference Series*, *948*(1), 012015. https://doi.org/10.1088/1742-6596/948/1/012015

Patelli, A., Calinescu, R., & Wang, H. (2014). Semantic reasoning for autonomic IT systems. *Proceedings of the 19th International Doctoral Symposium on Components and Architecture*, 13–18. https://doi.org/10.1145/2601328.2601331

Polya, G. (1973). *How to solve it* (2nd ed.). Princeton, N.J: Princeton University Press.

Prayitno, L. L., Purwanto, P., Subanji, S., & Susiswo, S. (2018). Identification errors of problem posed by prospective teachers about fraction-based meaning structure. *International Journal of Insights for Mathematics Teaching (IJOIMT)*, *1*(1), 76–84.

Sardi, Rizal, M., & Mansyur, J. (2018). Behaviour of mathematics and physics students in solving problem of vector-physics context. *Journal of Physics: Conference Series*, *1006*, 012019. https://doi.org/10.1088/1742-6596/1006/1/012019

Shi, S., Wang, Y., Lin, C.-Y., Liu, X., & Rui, Y. (2015). Automatically solving number word problems by semantic parsing and reasoning. *Proceedings of the 2015 Conference on Empirical Methods in Natural Language Processing*, 1132–1142. https://doi.org/10.18653/v1/D15-1135

Shin, N., Jonassen, D. H., & McGee, S. (2003). Predictors of well-structured and ill-structured problem solving in an astronomy simulation. *Journal of Research in Science Teaching*, *40*(1), 6–33. https://doi.org/10.1002/tea.10058

Stylianou, D. A. (2013). An examination of connections in mathematical processes in students' problem solving: Connections between representing and justifying. *Journal of Education and Learning*, *2*(2), 23–35. https://doi.org/10.5539/jel.v2n2p23

Sukoriyanto, J., Nusantara, T., Subanji, S., & Chandra, T. D. (2016). Students thinking process in solving combination problems considered from assimilation and accommodation framework. *Educational Research and Reviews*, *11*(16), 1494–1499. https://doi.org/10.5897/ERR2016.2811

Swastika, G. T., Nusantara, T., Subanji, & Irawati, S. (2020). Alteration representation in the process of translation graphic to graphic. *Humanities & Social Sciences Reviews*, *8*(1), 334–343. https://doi.org/10.18510/hssr.2020.8144

Uhden, O., Karam, R., Pietrocola, M., & Pospiech, G. (2012). Modelling mathematical reasoning in physics education. *Science & Education*, *21*(4), 485–506. https://doi.org/10.1007/s11191-011-9396-6

Xin, Y. P., Jitendra, A. K., & Deatline-Buchman, A. (2005). Effects of mathematical word problem—solving instruction on middle school students with learning problems. *The Journal of Special Education*, *39*(3), 181–192. https://doi.org/10.1177/00224669050390030501