Journal of Research and Advances in Mathematics Education

Volume 7, Issue 2, April 2022, pp. 105-115
DOI: 10.23917/jramathedu.v7i2.16394
p-ISSN: 2503-3697, e-ISSN: 2541-2590

Students' understanding of a geometric theorem: A case of grade 9 problem posing

Adriano Villarosa Patac, Jr[1*], Louida Penera Patac[2], Nicolas Ensomo Crispo, Jr[3]

[1] *College of Teacher Education, Surigao State College of Technology, Surigao City, Philippines*
[2] *College of Arts and Sciences, Surigao State College of Technology, Surigao City, Philippines*
[3] *Department of Education, Surigao City Division, Philippines*

Citation: Patac, A., Patac, L., & Crispo, N. (2022). Students' understanding of a geometric theorem: A case of grade 9 problem posing. *JRAMathEdu (Journal of Research and Advances in Mathematics Education), 7*(2), 105-115. https://doi.org/10.23917/jramathedu.v7i2.16394

ARTICLE HISTORY:
Received 24 November 2021
Revised 26 April 2022
Accepted 28 April 2022
Published 30 April 2022

KEYWORDS:
Problem posing
Student's understanding
Teaching problem posing
Geometric interpretation
Natural language mathematics

ABSTRACT

Teaching axiomatic representation of mathematical objects in all grades can and should be done. The paper analyzes students' understanding and how they perceive theorems using problem posing. We looked at how English-language learners create questions about four geometric theorems from a 9th-grade math textbook. The analysis looks at the question's distinctiveness, its elements' relationships, and sentence structure flaws. These lines, angle, and triangle theorems were chosen to exemplify problem scenarios when a theorem is conveyed in words but not explicitly symbolized. The difficulty of posing mathematically relevant problems stems from the required process of simultaneously changing the theorem language, home language, and formal mathematics language. In Van Hiele's methodology, the pupils' issues aren't classified as a formal or informal deduction. Questions either deduce from a formal system or emphasize theorems. Mastering the required representation registers can assist students in posing problems that reflect, at the very least, at the formal deduction level. The absence of symbolic representation increases the difficulty in posing original problems involving geometric theorems. As a result, how problems are made, especially how they are written, shows how well students understand math through problem-posing.

INTRODUCTION

Geometric notions are a necessary component of mathematical reasoning. Not only can geometric concepts illustrate how we define and visualize the items around us, but geometric thinking, particularly spatial reasoning, also pertains to the way we view non-observable objects, such as representations of objects and the object itself. Additionally, this characteristic of geometric thinking predicts later success in mathematics (Dindyal, 2015). Geometry's importance as a subject in secondary education in the Philippines is mirrored in the K–12 curriculum, which begins in grade 3 and progresses upward in a spiral development (Adarlo and Jackson, 2017; Montebon, 2014; Orbe, Espinosa, and Datukan, 2018). The spiraling growth strategy ensures that concepts and abilities are addressed with greater depth and rigor at each stage. Similar emphasis was placed on the subject's importance. When you look at the curricula for Malaysian schools, you'll see that the subject is taught in a more learner-centered way. This means that instruction isn't directed toward the teacher or the students' needs.

*Corresponding author: apatac@ssct.edu.ph

Problem-posing and mathematical thinking

Research in geometry and geometric thinking includes van Hiele's level of thinking (Prayito, Suryadi, and Mulyana, 2019; Skrbec and Cadez, 2015); and ways of increasing students' performance in geometry using various instructional approaches (Altakhyneh, 2018). A recent trend in mathematics instructional approaches is to teach mathematics through problem posing (Cai and Hwang, 2021). Problem posing encompasses both the invention of difficulties in response to a given scenario and the re-formulation of existing problems that may occur prior to during, or following action, such as problem resolution (Silver, 1994). Successful problem posing in mathematics has been demonstrated globally, for example, through research that examines the relationship between mathematical content knowledge and problem-posing ability (Leavy and Hourigan, 2020; Mestre, 2002). Given the paucity of research on the practice of problem-posing in mathematics education, it is unusual to find a mathematics classroom activity that involves students in problem-posing (Xu, Cai, Liu, and Hwang, 2020). In certain mathematics classes, students' participation in problem development is limited to answering the teacher's problems. When teachers employ problems in the classroom, most of the problems are drawn from textbooks, and the teacher's role is to present the students with these problems to solve (Silver, 1994). Prospective instructors who later become mathematics teachers have insufficient experience in developing problems (Crespo and Sinclair, 2008). Leiken and Elgrably introduce problem-posing investigation strategies in their study of prospective teachers in a dynamic geometry environment in order to assist prospective teachers in developing proficiency in posing mathematics problems (Leikin and Elgrably, 2020).

Language, task, and mathematics knowledge

Language, mathematics knowledge, and the way these concepts are understood are among the issues teachers confront when teaching mathematics in general. Kilpatrick (1987) identifies problem-posing tasks as a necessary component of mathematical thinking. Lin (2004) developed a problem-posing job for instructors to aid them in establishing evaluations that are integrated with instruction. Teachers create problems to be used as artifacts of conversation in their respective classes to determine whether these problems can provide information about students' mathematical understanding. The application of problem posing as a strategy for comprehending mathematical information is predicated on the premise that representation is an outcome of an object's experience (Matus, 2018). Additionally, pupils' performance in posing problems is proportional to their mathematical understanding (Leung & Silver, 1997). This means that the way problems are made, especially the way they are written, shows how students think about the math they are learning.

Problem solving and problem posing are critical in the teaching and learning of mathematics (Ayllon et al., 2016). Solving problems needs new methods of thinking about possibilities and the use of various types of mathematical knowledge (Bolden et al., 2010). While problem-posing necessitates a subjective assessment of the circumstance in order to establish what is required (Koichu & Kontorovich, 2012), numerous scholars have connected problem-solving with categories such as flexibility, fluency, and inventiveness. Yuan and Sriraman (2011) argue that difficulties provided by students promote student innovation. Indeed, most research on the association between problem posing and mathematical content knowledge has been conducted on pre-service teachers (e.g., Akay and Boz, 2009; Voica and Pelczer, 2010), as well as on high school students enrolled in advanced mathematics courses (e.g., Van Harpen and Presmeg, 2013). Chances to master various mathematical strategies should be made available to everyone, not just a chosen few. Teaching of Geometry often combines symbols and words, virtual manipulative and physical manipulatives, words and visual representations (Gecu - Parmaksiz and Delialioglu, 2019; Lavy and Bershadsky, 2003; Tacgin, 2020) However, little is known about the problem-solving abilities and mathematical subject knowledge of high school English-language learners when textual tasks are presented for geometric theorem. The purpose of this paper is to examine students' comprehension, difficulty, and problem-posing abilities regarding written geometric theorems that lack a symbolic representation. Analysis of problem-posing performances of students focuses on a dynamic environment (Lavy and Bershadsky, 2003). The purpose of this paper is to examine students' comprehension, difficulty, and problem-posing abilities regarding written geometric theorems that lack a symbolic representation. Analysis of problem-posing performances of students focuses on a dynamic environment (Lavy and Bershadsky, 2003). This inquiry seeks to elucidate the following queries: "What are the difficulties encountered by the students as shown in the posed problems? What are the errors in students' posed problems? and, In terms of Van Hiele, how do we classify the problems the students came up with?

This study's goal is to better understand ESL students' cognitive processes while posing problems in a geometric context. This understanding of students' problem-solving processes was founded on the semiotic portrayal of mathematical operations (Duval, 2006). We distinguish between representations and signs. These concepts all share a common purpose: they "stand for" what they "show" or "denote". They are both subject to the same fundamental epistemic requirement as signs and objects. It is important to remember this when interpreting geometric theorems. is necessary to distinguish the representation of mathematical objects from the mathematical objects they represent. Even if figures in geometry are formed precisely and assigned a specific value, they are merely representations and cannot be accepted as proof. Dindyal (2015) asserts that geometry makes use of words, symbols, and figures classified as the "register of the tongue," "the register of symbolic language," and the "figurative register."

Figure 1 illustrates how the various representation systems should interact for the problem posing activity to be successful. This figure is critical for students whose native language is not English, as it illustrates the difficulty of translating the mathematical statement's language into the student's native language and then back into the language of mathematics, such as formal notation or algebraic notation, simultaneously. The difficulty of converting the Geometric theorem to a problem is further explained by the operation's non-algorithmic nature. To complete the exercise successfully, students must change each cognitive activity simultaneously in order to formulate a mathematical problem. A strong foundation in mathematics is required to generate a mathematical problem that integrates multiple mathematical concepts, for example, a problem of generalization about the properties of a quadrilateral. The dominance of mathematics over everyday language creates a big gap in terms of knowledge acquisition. The decision to teach elementary school students in English as a second language contributed to their low performance on international assessment tests (Robertson & Graven, 2020). Students in a bilingual classroom frequently engage in the process of translanguaging in order to process both the mathematics language and the medium of instruction utilized at home. This procedure is occasionally insufficient considering the intricacy of the process necessary to address both languages concurrently (Mielicki et al., 2017; Hahn et al., 2019; Tyler, 2016). Understanding entails grasping the entire structure and processing both linguistic and symbolic registers concurrently (Duval, 2006). Conceptual comprehension is required to bolster one's ability to solve problems. This involves the capacity to recognize and represent the same mathematical notions in a variety of ways. It is not sufficient to represent the same concept in multiple ways; this also requires the ability to switch between representations flexibly. Even so, understanding a notion requires an understanding of the context in which the problem is presented, as well as the methods in which representation is used (Even, 1998).

Students' difficulties that accurately describe the object they describe are further classified into levels according to van Hiele's classification (van Hiele, 1999). Visualization, analysis, informal deduction, formal deduction, and rigor are the five stages of development. Students can discern and recognize the properties of a geometric item at the visualization level and can even draw generalizations about the properties of a geometric object. This can be accomplished either by observation or by conducting experiments. Students can build competences at the second stage of development that enable them to determine the links between the properties of a given geometric object. As a result, determining the classifications to which a certain geometric object belongs is a sign that thinking has reached this level. However, students continue to be unsure about the role of abstract reasoning, axiom relationships, and formal proof in geometry. For the third level of geometric reasoning, students can now appreciate the critical role of formal deduction in deepening their understanding of geometric ideas. Students can employ a variety of strategies for determining the correctness of a mathematical statement. The ultimate degree of geometric thinking is rigor; at this level, pupils are comfortable with a variety of axiomatic system techniques. They can now study non-Euclidean geometry (Fuys, Geddes, and Tischler, 1988; Shaughnessy and Burger, 1985; Van Hiele, 1999).

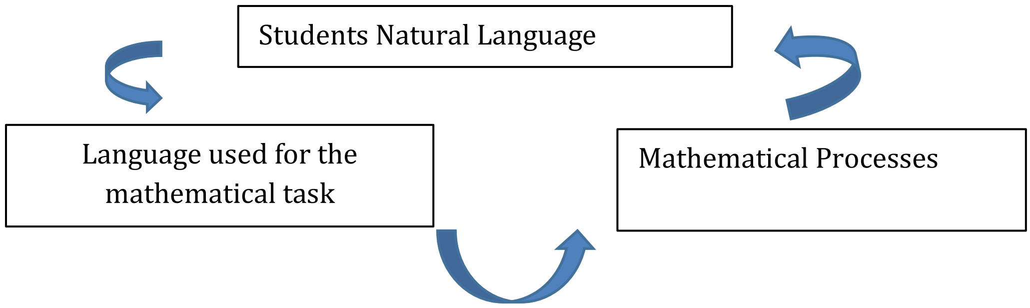

Figure 1. The production system used by the students

METHODS

To synthesize components of problem-posing in smaller groups into a larger class of related events (Gerring, 2006) when given with geometric theorems without expressly creating a symbolic representation. In this case, data should be collected for management, not comparison. Before classifying the students' issues, they are filtered as assertions or problems. Only issues are investigated, with the exception of incorrect classification. Early analysis classifies student issues according to fluency, adaptability, and creativity. A task's fluency is defined by Wechsler-Kashi et al. (2014) the number of student-posed difficulties was simply operationalized. They called it fluency (Plucker and Makel, 2010). Fluency was calculated by dividing the range of scores in each task by three (high, moderate, and low). A student's total fluency was estimated using the modal value of four activities.

The problem statement's flexibility is linked to the number of ideas. The number of discrete response clusters or attention changes. They could be factual, logical, or open (Vacc, 1993). A practical technique or procedure is encouraged rather than just counting. An open question combines both. An issue solution and explanation are required rather than just listing, differentiating, or naming. An open question elicits data that has already been identified but allows for a wide variety of acceptable solutions (e.g., what do you observe about these shapes?) Vacc claims open inquiry allows students to "explain witnessed events without memorizing names." Questions with open answers have high fluency, while questions with reasoning questions have intermediate fluency. Each student's flexibility was graded (high, moderate, or low) and documented. Overall flexibility was assessed by comparing the four tasks' modes of flexibility.

A sample's rarity defines its uniqueness. Each activity's originality score is the total of individual respondents' originality ratings. The rare proportion was calculated by categorizing the posed issues. A concept is considered unique if it is the only one in the group to have thought of it. This study modified the Torrance Test of Creativity. Student creativity was assessed using fluency, adaptability, and originality.

A similarity in the symbolic representation of the relationships between quantities as defined by Cañadas et al. (2018). In this work, we used left-to-right congruence to compare the problem and the theorem. "Same" meant the students' linguistic challenges (translation and starting problem) matched. In other words, the transformation from words to symbols is used to compare them. We called issues "equivalent" when their equations were different but worked in the same domain. In other circumstances, the verbal issue had a "different" grammatical structure than the delivered item. These three categories were "incomplete" for some issues. But mathematical and practical errors are classified. Peng and Luo's (2006) method helped us classify errors into two groups: informational and logical.

The study included 38 Grade 9 students from Northern Mindanao, Philippines, for the 2019-2020 academic year. There were 22 males and 16 females. Mathematics is one of the courses offered to these children in K-12 (K to 12 Mathematics Curriculum Guide, 2016). This grade level expects students to generate and solve issues with circles and other related concepts. Using geometric figures requires patience and precision while forming and solving problems. One of the authors also taught the course in a classroom. He also helps collect data. The three researchers and three research assistants work collaboratively to ensure inter-rater reliability when coding participant replies.

Coded questions: We did this for each theorem and got 85%, 80%, 84, and 84 percent interrater agreement on the questions that the participants asked. For theorem 5, 87 percent of the coders agreed with the coders.

A one-hour class intervention session was used to collect data from students utilizing four written activities. One of the researchers uses the intervention class once a month to track students' progress and discuss school difficulties connected to their enrolled subjects. To begin, students were instructed to write five (5) problems for each activity. The written task contains four (4) mathematical theorems (see Table 1). Each activity uses the same theorems as the mathematics learner's handbook.

Data Analysis

Some modifications to the Torrance Criteria specifically for determining originality were used in the analysis of the results, following the Silver (1997) definition (see Table 2). Originality is based on the rarity of the problem created in the set of all the responses.

FINDINGS

Table 3 shows the fluency level of students in each task and in general. As previously stated, fluency was operationalized by counting the number of problem statements generated by a participant. In this study, the number of problems created in each task was determined to analyze the fluency level of students. It was task number two that demonstrated students' fluency in producing difficulties in comparison to the remainder of the assignment. This task is about lines and transversals. Lines are a topic that students frequently encounter in daily life; for example, pedestrian lanes and parking lots are examples of parallel lines in real life. From the four tasks offered as prompts, task number two contains the fewest words (i.e., 18), followed by the first task (26 words), the third task (30 words), and the fourth task (36 words) (21 words). The concepts involved in task two are elementary, and their organization is based on simply two operators: intersecting and equal measures. Although we deemed task one to be the most familiar of the tasks offered, the omission of the task's famous name, the Pythagorean theorem, adds a new level of difficulty or familiarity. Making it so that the task receives the fewest responses possible.

According to Table 4, task number one generates the most difficulties in comparison to the rest of the tasks, while task number four generates the fewest new problems. This distinction can be attributed to the task's familiarity and intricacy. The familiarity with the Pythagorean Theorem for the first exercise and the difficulty of the language used to present the third task.

The syntactic structure of students' posed problems was determined through the transformation of word problems developed by the students and the equivalent symbolic representation of the problem. As a way to look at the syntactic structure, when possible, a left-to-right congruence was looked at. Then, the written equation and the problem were compared.

When comparing across tasks using Table 5, many problems created by students that are equivalent when converted to equations originate from the first task. While the fourth task generates the fewest identical problems generated by students, When comparing the varied structures for all tasks, there are a greater number of incomplete tasks uploaded by students in the second and third tasks than in the first and fourth tasks. This is because the task (task 1) and the assignment (task 4) are both well-known.

Students' stated difficulties would be categorized as having insufficient information, mistakes, or technical errors based on the examination of the error type. The distinction between these two sorts of errors is in the degree to which the formed problem requires correction. When extra words or groups of words are entered to indicate that the problem has been solved or that a solution process is possible, it is considered an inadequate error; otherwise, it is a technical error.

The higher number of technical errors occurred in the developed problems by the students for tasks 1, 2, and 4 as shown in Table 6. Information errors occur at a higher percentage at task 3, but the difference is slightly small compared to the technical error difference among tasks 1, 2, and 4. Note that the number of errors occurred only for those that were not considered problems. In other words, these data sets are responses to the problem-posing task that are disregarded as problems.

Table 1

List of theorems

1. In any right triangle, the square of the length of the hypotenuse is equal to the sum of the square of the lengths of the legs.
2. If two parallel lines are intersected by a transversal, then the alternate exterior angles are equal in measure.
3. If two secants or chords intersect in the interior of a circle, then the measure of each angle formed is half the sum of the measures of the intercepted arcs.
4. An exterior of a triangle is equal in measure to the sum of the measures of its two remote interior angles

Table 2

Criteria for originality of posed problems

Level of Originality	Percent	Interpretation
High	0%-10%	The question formulated by a participant is less than 10% structurally like the total questions made compared by the entire group.
Moderate	10%-25.0%	The question formulated by a participant falls within 10% to 25% structural similarity to the total questions made by the entire group.
Low	25%-above	The question formulated by the participant is more than 25% structurally like the total questions made compared to the entire group.

Table 3

Fluency level of students for each task

Fluency Level	Task 1		Task 2		Task 3		Task 4		General	
	n	%	n	%	n	%	n	%	n	%
High	17	44.7	4	10.5	15	39.5	17	44.7	14	36.8
Moderate	18	47.4	14	36.8	6	15.8	7	18.4	5	13.2
Low	3	7.9	20	52.6	17	44.7	14	36.8	12	31.6

Table 4

Originality Level of Students

Level	Task 1	Task 2	Task 3	Task 4	General
	n (%)	n (%)	n (%)	n (%)	n (%)
High	14 (11)	21 (38.9)	13 (14)	16 (15.9)	64 (17.1)
Moderate	26 (20.5)	10 (18.5)	0 (0)	0 (0)	36 (9.6)
Low	87 (68.5)	23 (42.6)	80 (86)	85 (84.2)	275 (73.3)
Total	127	100	54	100	93

Table 5
Syntactic structure of students posed problems

Task	Total	Same n (%)	Equivalent n (%)	Different n (%)	Incomplete n (%)
1	127	79 (62.2)	29 (22.8)	14 (11.0)	15 (11.8)
2	54	2 (3.7)	6 (11.1)	17 (31.5)	29 (53.7)
3	93	3 (3.2)	0 0	5 (5.4)	85 (91.4)
4	100	0 0	88 (87.1)	0 0	13
Total	374 100	84 (22.4)	123 (32.8)	36 (9.6)	142 (37.8)

Table 6
Distribution of errors in the problem posed

Task	Lack of information N (%)	Technical error N (%)	Other N (%)	Total
1	29 (15.1)	159 (82.8)	4 (2.1)	192
2	48 (50.3)	56 (52.8)	2 (1.9)	106
3	93 (50.3)	89 (48.1)	3 (1.6)	185
4	8 (5.5)	134 (92.4)	3 (2.1)	145
Total	178 (28.3)	438 (69.7)	12 (1.9)	628

DISCUSSION

According to the comments of pupils, mathematical reasoning skills, as demonstrated by the problems they posed, remain low. Thus, students must become accustomed to problem posing in order to express mathematical reasoning through developed questions. Because mathematical thinking is a multifaceted process, the study of mathematical thinking and problem-posing has numerous limits. It is the result of numerous definitions, criteria, or concepts. We situate the concept of mathematical thinking in this section by posing problems. This study's findings indicate that pupils' abilities to pose mathematical problems vary. This finding corroborated Van Harpen and Sriraman's (2012) assertion that problem-posing activities help students develop their mathematical content knowledge and attitudes toward mathematics. Researchers have been chastized for hitting a deadlock in their studies on problem solving (English & Sriraman, 2010). Even though problem posing offers considerable benefits and has big mean effects on ability-based, skill-based, and attitude-based learning outcomes (Rosli et al., 2014), it has received less attention than the other study strands.

Additionally, students' difficulties were found when they were asked to explain mathematical theorems. The inconsistencies in the students' formulations in terms of syntactic patterns reflect certain gaps in their understanding, not just in geometry, but also in other branches of mathematics such as algebra and arithmetic. Students' representations of the object and its representations demonstrate poor knowledge of mathematical theorems, limiting their capacity to frame problems using the same grammatical structure as the provided theorems, as demonstrated by the number of errors. The activities in this study focus students' attention on the richness of meaning associated with mathematical theorems and their geometric representations, which should be incorporated into routine secondary mathematics practice to avoid students' competence in the use of mathematical theorems becoming disconnected from their meanings. To ensure that students become familiar with the process, the concept of problem-posing practice must be included in routine instruction.

Students' difficulties have a high level of technical mistakes; students utilize inappropriate terminology; the symbol is incorrectly shown or omitted; the problem is logically contradictory; or the problem contains insufficient information for debugging. Because they are outside the domain of mathematics, problems cannot be classified as mathematical problems. The absence of information and logical errors was linked to students' difficulties with thought and language building in Task 4. Students used a variety of cognitive methods while completing the problem-posing exercise. The pupils' translanguaging technique for processing the mathematical notion provided by the prompt and the needed process by the problem-posing task was insufficient. Although translanguaging occurs entirely in the mind, the cognitive difference between the English language and the students' native tongue added an additional layer of difficulty to completing the problem-posing exercise successfully. Tyler's work also considers the difficulty of comprehending the language and the successful accomplishment of the task (2016). The cognitive cost of prolonged reaction times and decreased accuracy occurs when the student's and task's languages (i.e., the student's medium and the task's language) and retrieval do not match (Volmer et al., 2018). Another reason students struggle with the job inside the realm of language is the absence of a second representation register for the prompt and a common name for the theorem. The geometric symbol enables simultaneous processing of symbols and words, which is required for comprehending geometric concepts and mathematical sense-making (Pohl and Doppler Haider, 2017). The geometrical version of the theorem can assist in comprehending the operation required for the issue-posing assignment. Understanding entails familiarity with the contents and grasping the entire structure while processing both language and symbol registers concurrently. Because mathematics material is acquired concurrently with the acquisition of a new language, additionally, the significant cognitive distance between the native language, mathematical language, and English language complicates matters further. Similar to Cuevas, the added complexity of the interaction of language and mathematics registers in terms of meaning styles and modes of presentation within the setting of mathematics (1984).

CONCLUSIONS

Mathematical understanding of a mathematical idea can be gathered and the value of problem-posing tasks in mathematical thinking can be quantified. When using problem posing to test students' background knowledge and levels of thinking, geometric thinking fluency is important. The lack of understanding of mathematical theorems is immediately reflected in the lack of syntactic organization of students' difficulties. Students' errors in problem-solving originate from a lack of proficiency in both English and geometric languages. Students' difficulties visualizing and describing their thinking may be due to a lack of experience and mental imagery. Emphasis on using problem posing to teach not only geometry but all levels of mathematics, adds to students' burden of learning geometric thinking. Mathematicians rarely employ problem posing as a teaching approach. It is also worth noting that the activity requires "processing" rather than "conversion" cognitive processes. That is, students must develop an issue within the same system of representation (use of language within the mathematics language domain). Despite the limited sample size, this study provides insight into how problem posing is employed in a specific classroom. The lack of symbolic representation makes it harder to pose new questions involving geometric theorems. Thus, the way problems are formed, especially phrased, reflects how well pupils comprehend math. This study helps teachers in the classroom understand how ESL students who haven't had a lot of experience with advanced math problems are asked to pose a geometric activity.

ACKNOWLEDGMENT

Special thanks to the Department of Education, Surigao City Division, for support during data collection and to the students who participated in this research.

BIBLIOGRAPHY

Abdullah, A. H., & Zakaria, E. (2013). Enhancing students' level of geometric thinking through van Hiele's phase-based learning. *Indian Journal of Science and Technology*, 6(5), 4432-4446.

Adarlo, G., & Jackson, L. (2017). For whom is K-12 education: A critical look into twenty-first century educational policy and curriculum in the Philippines. In *Educating for the 21st Century* (pp. 207-223). Springer, Singapore. https://doi.org/10.1007/978-981-10-1673-8_11

Akay, H., & Boz, N. (2009). Prospective teachers' views about problem-posing activities. *Procedia-Social and Behavioral Sciences*, 1(1), 1192-1198. https://doi.org/10.1016/j.sbspro.2009.01.215

Altakhyneh, B. H. (2018). Levels of Geometrical Thinking of Students Receiving Blended Learning in Jordan. *Journal of Education and Learning (EduLearn)*, 12(2), 159-165. https://doi.org/10.11591/edulearn.v12i2.8289

Anderson, L. W. (2003). *Classroom assessment: Enhancing the quality of teacher decision making*. Routledge.

Ayllón, M. F., Gómez, I. A., & Ballesta-Claver, J. (2016). Mathematical Thinking and Creativity through Mathematical Problem Posing and Solving. *Journal of Educational Psychology-Propósitos y Representaciones*, 4(1), 195-218. http://dx.doi.org/10.20511/pyr2016.v4n1.89

Bolden, D. S., Harries, A. V., & Newton, D. P. (2010). Preservice primary teachers' conceptions of creativity in mathematics. *Educational Studies in Mathematics*, 73(2), 143-157. https://doi.org/10.1007/s10649-009-9207-z

Cai, J., & Hwang, S. (2021). Teachers as redesigners of curriculum to teach mathematics through problem posing: conceptualization and initial findings of a problem-posing project. *ZDM–Mathematics Education*, 1-14. https://doi.org/10.1007/s11858-021-01252-3

Cañadas, M. C., Molina, M., & Del Rio, A. (2018). Meanings given to algebraic symbolism in problem-posing. *Educational Studies in Mathematics*, 98(1), 19-37. https://doi.org/10.1007/s10649-017-9797-9

Crespo, S., & Sinclair, N. (2008). What makes a problem mathematically interesting? Inviting prospective teachers to pose better problems. *Journal of Mathematics Teacher Education*, 11(5), 395-415. https://doi.org/10.1007/s10857-008-9081-0

Cuevas, G. J. (1984). Mathematics learning in English as a second language. *Journal for research in mathematics education*, 15(2), 134-144. https://doi.org/10.5951/jresematheduc.15.2.0134

Diezmann, C., & Lowrie, T. (2009). Primary students' spatial visualization and spatial orientation: an evidence base for instruction. In *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (pp. 417-424). PME.

Dindyal, J. (2015). Geometry in the early years: A commentary. *ZDM*, 47(3), 519-529. https://doi.org/10.1007/s11858-015-0700-9

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational studies in mathematics*, 61(1), 103-131. https://doi.org/10.1007/s10649-006-0400-z

English, L. D., & Sriraman, B. (2010). Problem solving for the 21st century. In B. Sriraman & L. D. English (Eds.), *Advances in Mathematics Education. Theories of mathematics education: Seeking new frontiers* (pp. 263–285). New York: Springer. https://doi.org/10.1007/978-3-642-00742-2_27

Even, R. (1998). Factors involved in linking representations of functions. *The Journal of Mathematical Behavior*, 17(1), 105-121. https://doi.org/10.1016/S0732-3123(99)80063-7

Fuys, D., Geddes, D., & Tischler, R. (1988). The van Hiele model of thinking in geometry among adolescents. *Journal for Research in Mathematics Education*. Monograph, 3, i-196. https://doi.org/10.2307/749957

Gecu-Parmaksiz, Z., & Delialioglu, O. (2019). Augmented reality-based virtual manipulatives versus physical manipulatives for teaching geometric shapes to preschool children. *British Journal of Educational Technology*, 50(6), 3376-3390. https://doi.org/10.1111/bjet.12740

Gerring, J. (2004). What is a case study and what is it good for?. *American political science review*, 341-354. https://doi.org/10.1017/S0003055404001182

Gerring, J. (2006). Single-outcome studies: A methodological primer. *International sociology*, 21(5), 707-734. https://doi.org/10.1177/0268580906067837

Hahn, C. G., Saalbach, H., & Grabner, R. H. (2019). Language-dependent knowledge acquisition: investigating bilingual arithmetic learning. *Bilingualism: Language and Cognition*, 22(1), 47-57. https://doi.org/10.1017/S1366728917000530

K to 12 Mathematics Curriculum Guide. (2016). Retrieved from Learning Resources Management and Development System: https://lrmds.deped.gov.ph/search?query=curriculum+guide

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from. *Cognitive science and mathematics education*, 123-147.

Koichu, B., &Kontorovich, I. (2012). Dissecting success stories on mathematical problem posing: A case of the billiard task. *Educational Studies in Mathematics*, 83(1), 71–86. https://doi.org/10.1007/s10649-012-9431-9

Kontorovich, I., Koichu, B., Leikin, R., & Berman, A. (2012). An exploratory framework for handling the complexity of mathematical problem posing in small groups. *The Journal of Mathematical Behavior*, 31(1), 149-161. https://doi.org/10.1016/j.jmathb.2011.11.002

Lavy, I., & Bershadsky, I. (2003). Problem posing via "what if not?" strategy in solid geometry—a case study. *The Journal of Mathematical Behavior*, 22(4), 369-387.

Leavy, A., & Hourigan, M. (2020). Posing mathematically worthwhile problems: developing the problem-posing skills of prospective teachers. *Journal of Mathematics Teacher Education*, 23(4), 341-361. https://doi.org/10.1007/s10857-018-09425-w

Leikin, R., & Elgrably, H. (2020). Problem posing through investigations for the development and evaluation of proof-related skills and creativity skills of prospective high school mathematics teachers. *International Journal of Educational Research*, 102, 101424. https://doi.org/10.1016/j.ijer.2019.04.002

Leung, S. S., & Silver, E. A. (1997). The role of task format, mathematics knowledge, and creative thinking on the arithmetic problem posing of prospective elementary school teachers. *Mathematics Education Research Journal*, 9(1), 5-24. https://doi.org/10.1007/BF03217299

Lin, P. J. (2004). Supporting Teachers on Designing Problem-Posing Tasks as a Tool of Assessment to Understand Students' Mathematical Learning. *International Group for the Psychology of Mathematics Education*. https://files.eric.ed.gov/fulltext/ED489580.pdf

Ma, H. L., Lee, D. C., Lin, S. H., & Wu, D. B. (2015). A Study of Van Hiele of Geometric Thinking among 1st through 6th Graders. *Eurasia Journal of Mathematics, Science and Technology Education*, 11(5), 1181-1196. https://doi.org/10.12973/eurasia.2015.1412a

Matus, P. (2018). Discursive representation: Semiotics, theory, and method. *Semiotica*, 2018(225), 103-127. https://doi.org/10.1515/sem-2017-0019

Mestre, J. P. (2002). Probing adults' conceptual understanding and transfer of learning via problem posing. *Journal of Applied Developmental Psychology*, 23(1), 9-50. https://doi.org/10.1016/S0193-3973(01)00101-0

Mielicki, M. K., Kacinik, N. A., & Wiley, J. (2017). Bilingualism and symbolic abstraction: Implications for algebra learning. *Learning and Instruction*, 49, 242-250. https://doi.org/10.1016/j.learninstruc.2017.03.002

Montebon, D. T. (2014). K12 science program in the Philippines: Student perception on its implementation. *International Journal of Education and Research*, 2(12), 153-164.

Nadjafikhah, M., Yaftian, N., & Bakhshalizadeh, S. (2012). Mathematical creativity: some definitions and characteristics. *Procedia-Social and Behavioral Sciences*, 31, 285-291. https://doi.org/10.1016/j.sbspro.2011.12.056

Nazari, A., & Allahyar, N. (2012). Grammar Teaching Revisited: EFL Teachers between Grammar Abstinence and Formal Grammar Teaching. *Australian Journal of Teacher Education*, 37(2). https://doi.org/10.14221/ajte.2012v37n2.6

Orbe, J. R., Espinosa, A. A., & Datukan, J. T. (2018). Teaching chemistry in a spiral progression approach: Lessons from science teachers in the Philippines. *Australian Journal of Teacher Education* (Online), 43(4), 17-30. https://ro.ecu.edu.au/ajte/vol43/iss4/2/

Peng, A., & Luo, Z. (2009). A framework for examining mathematics teacher knowledge as used in error analysis. *For the learning of mathematics*, 29(3), 22-25.

Plucker, J. A., & Makel, M. C. (2010). Assessment of creativity. In J.C. Kaufman & R. J. Sternberg (Eds.), *The Cambridge handbook of creativity*. New York, NY: Cambridge University Press.

Prayitno, L. L., Purwanto, P., Subanji, S., & Susiswo, S. (2018). Identification Errors of Problem Posed by Prospective Teachers About Fraction Based Meaning Structure. *International Journal of Insights for Mathematics Teaching,* 01(1), 76–84.

Pohl, M., & Doppler Haider, J. (2017). Sense-making strategies for the interpretation of visualizations—bridging the gap between theory and empirical research. *Multimodal Technologies and Interaction*, 1(3), 16. https://doi.org/10.3390/mti1030016

Prayito, M., Suryadi, D., & Mulyana, E. (2019). Geometric thinking level of the Indonesian seventh grade students of junior high school. *In Journal of Physics: Conference Series* (Vol. 1188, No. 1, p. 012036). IOP Publishing. https://iopscience.iop.org/article/10.1088/1742-6596/1188/1/012036/pdf

Robertson, S. A., & Graven, M. (2020). Language as an including or excluding factor in mathematics teaching and learning. *Mathematics Education Research Journal*, 32(1), 77-101. https://doi.org/10.1007/s13394-019-00302-0

Rosli, R., Capraro, M. M., & Capraro, R. M. (2014). The Effects of Problem Posing on Student Mathematical Learning: A Meta-Analysis. *International Education Studies*, 7(13). https://doi.org/10.5539/ies.v7n13p227.

Shaughnessy, J. M., & Burger, W. F. (1985). Spadework prior to deduction in geometry. *The Mathematics Teacher*, 78(6), 419-428.

Silver, E. A. (1994). On mathematical problem posing. *For the learning of mathematics*, 14(1), 19-28. https://www.jstor.org/stable/40248099

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM*, 29(3), 75-80. https://doi.org/10.1007/s11858-997-0003-x

Škrbec, M., & Čadež, T. H. (2015). Identifying and fostering higher levels of geometric thinking. *EURASIA Journal of Mathematics, Science and Technology Education,* 11(3), 601-617. https://doi.org/10.12973/eurasia.2015.1339a

Tacgin, Z. (2020). *Virtual and Augmented Reality: An Educational Handbook*. Cambridge Scholars Publishing.

Tyler, R. (2016). Discourse-shifting practices of a teacher and learning facilitator in a bilingual mathematics classroom. *Per Linguam: a Journal of Language Learning*= Per Linguam: Tydskrif vir Taalaanleer, 32(3), 13-27. https://hdl.handle.net/10520/EJC-80edafa7e

Vacc, N. (1993). Questioning in the mathematics classroom. *Arithmetic Teacher*, 41(2), 88–91.

Van Harpen, X. Y., & Presmeg, N. C. (2013). An investigation of relationships between students' mathematical problem-posing abilities and their mathematical content knowledge. *Educational Studies in Mathematics*, 83(1), 117-132. https://doi.org/10.1007/s10649-012-9456-0

Van Harpen, X. Y., & Sriraman, B. (2012). Creativity and mathematical problem posing: an analysis of high school students' mathematical problem posing in China and the USA. *Educational Studies in Mathematics*, 82(2), 201–221. https://doi.org/10.1007/s10649-012-9419-5

Van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. *Teaching children mathematics*, 5(6), 310-317. https://doi.org/10.5951/TCM.5.6.0310

Voica, C., & Pelczer, I. (2010). Problem posing by novice and experts: Comparison between students and teachers. CERME 6–WORKING GROUP 12, 2356.

Volmer, E., Grabner, R. H., & Saalbach, H. (2018). Language switching costs in bilingual mathematics learning: Transfer effects and individual differences. *Zeitschrift für Erziehungswissenschaft*, 21(1), 71-96. https://doi.org/10.1007/s11618-017-0795-6

Wechsler-Kashi, D., Schwartz, R. G., & Cleary, M. (2014). Picture naming and verbal fluency in children with cochlear implants. *Journal of Speech, Language, and Hearing Research*, 57(5), 1870-1882. https://doi.org/10.1044/2014_JSLHR-L-13-0321

Xu, B., Cai, J., Liu, Q., & Hwang, S. (2020). Teachers' predictions of students' mathematical thinking related to problem posing. *International Journal of Educational Research*, 102, 101427. https://doi.org/10.1016/j.ijer.2019.04.005

Yuan, X., & Sriraman, B. (2011). An exploratory study of relationships between students' creativity and mathematical problem-posing abilities: Comparing Chinese and US students. In *The elements of creativity and giftedness in mathematics* (pp. 5-28). Brill Sense.

Yunus, M., Suraya, A., Ayub, M., & Fauzi, A. (2019). Geometric Thinking of Malaysian Elementary School Students. *International Journal of Instruction*, 12(1), 1095-1112. https://files.eric.ed.gov/fulltext/EJ1201177.pdf