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COMPARISON OF PROBLEMS RELATED TO THE MULTIPLICATION OF FRACTIONS IN MATHEMATICS TEXTBOOKS: THE CASE OF SINGAPORE AND TURKEY

Research article

Nazan Mersin http://orcid.org/0000-0002-4208-3807
(Corresponding Author)

Department of Mathematics and Science Education, Çanakkale Onsekiz Mart University

nazan09gunduz@gmail.com

Elif Nur Akkaş http://orcid.org/0000-0002-8286-8203

Department of Mathematics and Science Education, Bolu Abant İzzet Baysal University

elifakkas@ibu.edu.tr

Biodatas:

Nazan Mersin is Doctor Research Assistant at the Department of Mathematics and Science Education, Faculty of Education, Çanakkale Onsekiz Mart University in Turkey. Her research interests are mathematics education, math textbook analysis, history of mathematics, problem posing, activity-based learning.

Elif Nur Akkaş is Assistant Professor at the Department of Mathematics and Science Education, Faculty of Education, Bolu Abant İzzet Baysal University. She carries out studies in fields such as entrepreneurship skills in math education, teacher training, geometry teaching.

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Nazan Mersin

nazan09gunduz@gmail.com

Elif Nur Akkaş

elifakkas@ibu.edu.tr

Abstract

In this study, it is aimed to comparatively examine practice questions and sample solved questions about the multiplication of fractions in mathematics textbooks in Turkey and Singapore. In this direction, document analysis was used as a research method. Descriptive analysis was used in the analysis and the questions in the books were examined in seven categories: the number of steps, contextual properties, representation type, model type, meaning of multiplication, fraction type, and cognitive expectation. From the results of the analysis, it was seen that there were more questions with pure mathematical context that were solved in one step in the books in both countries. However, all kinds of fraction models are used in the questions in the Singapore textbook, whereas the length model is not used in Turkey. In addition, in fraction multiplication, the repeated addition meaning of multiplication is used only in Turkey, whereas in Singapore the operator meaning is mostly used. When examined in terms of fraction types, it was concluded that there are no examples in the form of natural numbers × fractions in the Singapore books, but that there are more problems about fraction × fraction in Singapore. When the results are evaluated in general, it is observed that the books in both countries have advantages and disadvantages compared to each other, but in terms of the categories studied the distribution in the Singapore textbook is more balanced than it is in Turkey.

Keywords: comparative education, textbook comparison, multiplication of fractions, Singapore, Turkey

1. Introduction

Various studies are carried out throughout the world in order to determine the achievements of primary and secondary school students in the fields of mathematics and science and to evaluate their progress in these fields. The Trends in International Mathematics and Science Study (TIMSS) and Progress in International Reading Literacy Study (PIRLS), which are conducted by the International Association for the Evaluation of Educational Achievement (IEA), and the Programme for International Student Assessment (PISA), held among Organization for Economic Cooperation and Development (OECD) countries, can be given as examples of these studies. When the results of the PISA and TIMSS exams, which are the most popular internationally among these studies, are examined, it is seen that while East Asian countries are in first place, Turkish students' mathematics achievement is alarmingly low.

Similarly, in the TIMSS 2015 results, it is seen that the mathematics achievement of 4th and 8th grade Turkish students is below the international average, and 4th graders rank 36th among 49 countries and 8th graders 24th among 39 countries (Yıldırım et al., 2016). According to the TIMSS result in 2019, Singapore ranked first in mathematics with a score of 625 at the level of eighth graders, while according to the PISA result in 2018, it ranked second with a score of 569 in mathematics. Turkey, on the other hand, is 23rd with 523 points in the 2019 TIMSS. It was ranked 42nd with 454 points according to the 2018 PISA results. Based on the results, it is seen that there is a significant difference in success in mathematics in two different exams between the two countries. There can be many causes of this achievement gap between countries, one of which can be differences in the content of the textbooks used (Fan, Zhu & Miao, 2013; Fan & Zhu, 2007; Kulm & Capraro, 2008; Mesa, 2004; Zhu & Fan, 2006). The reason for this situation is thought to be that the textbook analyses can provide predictions about the differences in success of students in the international exams due to textbooks' being significant regarding learning and teaching progress and determining what to teach to the students (Fuson, Stigler & Bartsch, 1988; Kwon & Kim, 2014; Zhu & Fan, 2006). However, comparing textbooks from different countries provides a way of comparison as to what opportunities are given to students to learn mathematical concepts since textbook presentation has a major impact on learning, including the contexts of problems in the textbooks, the types of problems, and the order in which they are presented (Li, Ding, Capraro & Capraro, 2008; McNeil et al., 2006) (Ding & Li, 2010).

In the present study, it is focused on how multiplication of fractions, which many students have difficulty understanding, is covered in secondary school mathematics textbooks in Singapore and Turkey, and the similarities and differences between them. There are several reasons for choosing multiplication of fractions in the study. The first of these is that fractions is a subject that challenges students, and students especially have more difficulty in multiplying and dividing fractions compared to addition and subtraction, and they have many misconceptions about fractions (Huang, Liu & Lin, 2009; Lin et al., 2013). The second reason is that fractions form the basis of topics such as proportion-proportion, rational numbers, and algebra in mathematics and other topics will be negatively affected if they cannot be learned (Saxe, Gearhart & Nasir, 2001; Yang, Gearhart & Nasir, 2010). Thirdly, the multiplication of fractions plays a key role in the teaching of division of fractions. As a matter of fact, the inverse multiplication algorithm is used in the division of fractions (Kar et al., 2018). Finally, the purpose of contributing to the development of the content of "multiplication by decimals" in textbooks in our country through the comparison of textbooks of different countries is also among the objectives of the present study.

It is also aimed to determine the strengths and weaknesses of the books in both countries by examining the lessons, solved examples, and practice questions related to the multiplication of fractions in Singapore and Turkish secondary school mathematics textbooks. It is thought that the results obtained will assist the designers of the program and the authors of textbooks and will lay the groundwork for significant improvements. In the next section, information about the multiplication process for fractions and the criteria studied is presented.

1.1. The process of multiplying fractions

The concept of fractions, which is the basis for the teaching of many subjects in mathematics, is one of the most important subjects in elementary and secondary school mathematics (Gurefe, 2020). A meaningful learning of the concept of fractions and computational fluency in fractions provide a basis for advanced mathematical studies (Yang, Reys & Wu, 2010). From another point of view, many researchers state that if a definite and correct concept of fractions has not been formed in children, this seriously restricts their ability to learn other related mathematical concepts (Behr et al., 1984; Cramer, Post & delMas, 2002). Therefore, using various models and representations, solving problems with more than one method, using different meanings of fractions and four operations, and using different kinds of fractions will help students better understand these concepts during the teaching of fractions and operations in fractions (Grant, Lo & Flowers, 2007; Gurefe, 2020). However, it is also reported that direct rule learning of students without learning the meaning of operations does not help students, but rather reduces their success (Aksu, 1997; English & Halford, 1995). In this sense, it is emphasized that while students learn operations in fractions, learning conceptual knowledge in addition to transactional knowledge will provide deeper learning (Lamon, 1999).

Fractions are taught in operational knowledge: first addition and subtraction and then multiplication and division. As in the teaching of the four operations performed on natural numbers, addition and subtraction are taught first; multiplication is taken into account in the aspect that repeated addition and division help to use the meanings of repeated subtraction (Gurefe, 2020). However, knowing only these meanings is not enough for multiplication and division operations. As a matter of fact, when considering multiplication in fractions, it is stated that in addition to the repeated addition meaning of multiplication, the processor meaning of fractions and area models should be used that also support conceptual knowledge (Toluk Ucar, 2020). When the achievements in the mathematics curriculum of the two countries that are the subject of the research are examined, it is seen that the students in Turkey encounter the multiplication of fractions only in the sixth grade, while the students in Singapore first learn them in the fifth grade (Turkey Ministry of Education, 2018; Singapore Ministry of Education, 2013). Looking at the current secondary school mathematics curriculum in Turkey, it is known that there are achievements that include the multiplication of a natural number by a fraction and the multiplication of two fractions by each other (Turkey Ministry of Education, 2018). When these gains are examined, three different situations arise. The first of these is the case when the first multiplier is a natural number and the second multiplier is a fraction. In such operations, the meaning of repeated addition of multiplication is used (Toluk Uçar, 2020). For example, the operation 5×3/8 means the sum of 5 3/8. Since students are familiar with this meaning of multiplication, it is recommended that this situation be addressed first in the curriculum. The second case is the case when the first multiplier is a fraction and the second multiplier is a natural number. In this type of operation, the processor meaning of fractions is used. For example, a 3/8 × 5 operation means finding 3/8 of 5. In order to use the repeated addition meaning of multiplication in this process, it is necessary to use the change property of multiplication first (Toluk Uçar, 2020). Another method of such operations is the use of the area model. In the third case, there is a multiplication of two fractions. In the multiplication of two fractions, the repeated addition meaning of the algorithm cannot be used, but the processor meaning of the fractions and the area model can be used, as in the second case. In order for students to understand all three types of multiplication operations, it is considered important to include various and sufficient examples both in classroom settings and in textbooks.

In addition to transactional and conceptual situations, it has been stated that various representations can also be used to reveal ideas that may not be obvious (Son & Lee, 2016). For example, it is emphasized that the use of pictorial representations is important in guiding both students and teachers to reason about problems, validate their own thoughts, and convince others that their thoughts are correct (Wu, 2001). Also for mathematics, the Common Core State Standards (National Governors Association Center for best practices (NGA) & Council of Chief State School Officers (CCSSO), 2010) emphasized the importance of using visual fraction models to help students discover and learn about fractions and multiplication of fractions by fractions in terms of deepening their understanding of the underlying ideas. As a matter of fact, models that can be seen as a communication tool are effective in helping communication between individuals and creating a suitable environment for mathematical discourse, as well as helping students concentrate on the manipulation of symbols and problems (Ervin, 2017; Zazkis & Liljedahl, 2004). However, the NCTM (2000) stated that models containing various representations can be used in organizing mathematical ideas, choosing between representations to solve problems, and interpreting them from preschool to 12th grade. Zazkis and Liljedahl (2004) stated that models can be useful if they can establish connection between the ideas represented and the ideas intended to be represented. In addition, it is suggested to use different models because they may offer students the opportunity to see the problems in different ways and from different perspectives (Ervin, 2017). In the studies conducted in this sense, it is seen that different models are used in the representation of fractions. One of them, the area model, which represents fractions as parts of a area or region, has several advantages in terms of being a familiar way to interpret the product of integers and seeing the relationship between decimals (Graeber & Tanenhaus, 1993; Olkun & Toluk-Ucar, 2009; Thompson & Saldanha, 2003; Van de Walle et al., 2010). The length model, on the other hand, provides a context for students to associate fractions with measures on a ruler and to see fractions as numbers. In particular, the numerical axis model provides an opportunity to compare the relative size of a fraction with other numbers and to show that there is always another fraction between any two given fractions (Ervin, 2017; Lannin, Chval & Jones, 2013). When looking at cluster models, on the other hand, all the elements in the cluster represent a whole, while each subset of the cluster is used to denote a fractional part of the whole. Although cluster models are useful for establishing relationships in terms of the concept of proportion and real-life applications, the fact that all objects in a cluster form a whole can often be confusing for students (Van de Walle et al., 2010). For this reason, it is the least preferred model among fraction models.

In addition to transactional and conceptual knowledge, models, and representations, students also need to acquire various mathematical knowledge and skills to solve mathematical problems (Son & Senk, 2010). This feature, called cognitive expectation, also reveals the depth of students' understanding of the subject. Therefore, problems with different cognitive expectations contained in textbooks will make students master the topic in a broad perspective. Cognitive expectation includes all the features of transactional knowledge, conceptual knowledge, type of representation, mathematical reasoning, problem-solving, and problem-building. Each of these properties is considered necessary to make sense of the multiplication process in fractions, and it is considered important to have a solved example or exercise question in textbooks for each of them. As a matter of fact, problems containing transactional information require students to use the multiplication algorithm in fractions (Swaafford & Findell, 2001), while problems measuring conceptual knowledge consist of questions that directly question the meaning of multiplication of fractions (Kar et al., 2018). If the problem with the multiplication of fractions expects the student to explain or justify the solution or involves guessing the answer to the problem, questions that require mathematical reasoning can be considered as those that requires problem-solving if they include a daily life problem (Son & Senk, 2010). In addition, according to Bloom's taxonomy, problem-setting, which is a high-level skill and is included in the creation step, consists of cognitive expectations that ask students to create problems related to multiplication of fractions. All this organization actually signals an association process. It is argued that the concept of association is also important in mathematics teaching, and the more students can relate a concept to everyday life, the better their level of understanding will be (Cooper & Harries, 2003; Post et al., 2008). In this sense, it is thought that it is necessary to examine the solved examples and practice questions related to the multiplication process in fractions according to whether they are related to daily life. Evaluations were made that the questions examined in the category called contextual feature show a contextual feature if they are related to everyday life situations, but when there is no context, it is a question with pure mathematical content.

Considering the importance of multiplication by fractions, it is curious how teachers and students are treated in textbooks, which are the main source, what examples of solutions are given, and what kinds of practice questions are used. Especially in Singapore, such as TIMMS and PISA, as the analysis of the mathematics textbook of a country that has achieved high success in international exams examines the learning opportunities offered by the curriculum in high-achieving countries to students, it is thought that it may contribute to the development of students in other compared countries. In addition, Hiebert et al. (2003) and Stigler and Hiebert (2004) argue that through international comparative studies, individuals will be able to learn the advantages and disadvantages of textbooks of their own country. It is thought that this study, which aims to investigate and analyze the multiplication of fractions in mathematics textbooks in two different countries, will contribute to the future reform of textbooks and will help those interested in carrying out this reform.

When the literature is examined, it is seen that various studies have been conducted on fractions (Alajmi, 2012; Kar et al., 2018; Son & Senk, 2010; Watanabe, Lo & Son, 2017; Yang, Reys & Wu, 2010). In Yang et al. (2010), the topic of fractions in American, Taiwanese, and Singapore mathematics textbooks; the contextual structure of problems; and the representations used were compared according to fraction models. Accordingly, it was concluded that the problems in Taiwanese and Singapore books are more transactional-oriented, while the activities in American books are focused on conceptual development rather than transactional development. However, it was also concluded that the teaching of multiplication by fractions in Singapore books began earlier than in the other two countries. In addition, it was observed that American textbooks give more space to real-world contexts than Taiwanese and Singapore books do.

Son and Senk (2010) analyzed and compared the problems related to multiplication and division of fractions in American and Korean mathematics textbooks. It was determined that multiplication by fractions was covered in Korean books earlier than in American ones, but there are similar hours of lesson time in the two countries. In addition, while conceptual understanding was given importance before transactional fluency in American textbooks, it was observed that both developed simultaneously in Korean textbooks. However, it was concluded that most of the problems of multiplication by fractions in both countries require operational knowledge. In addition, it was observed that multistep calculation problems and types of answers are more common in Korean books than in American ones.

Alajmi (2012) compared the subject of fractions in mathematics textbooks in Japan, Kuwait, and America. It was observed that the Kuwaiti mathematics textbook focuses on pure mathematical contexts, introduces mathematical rules, and then provides a wide range of exercises. It was seen that the Japanese textbooks discuss fractions in the sense of measurement and address real life in the context of fraction problems. It was found that linear models are used more to represent fractions, area models are used in a limited number of cases, and the cluster model is not used at all. In American books, it was seen that length, set, and area models are used to represent fractions.

Watanabe et al. (2017) compared Japanese, Korean, and Taiwanese mathematics textbooks in terms of their content of multiplying fractions. It was seen that the Japanese textbooks differed from the Korean and Taiwanese books. For example, in the Korean and Taiwanese books, the topic of multiplication of fractions is not switched to division of fractions before the end, while in the Japanese textbooks, the topic of dividing fractions into integers was discussed before multiplication of fractions. It was also observed that unit fractions play a fundamental role in the Taiwanese textbooks. As a matter of fact, the product of a natural number and a fraction and a natural number and two fractions is started with unit fractions. However, in the Korean textbooks, unit fractions in multiplying fractions are treated as a special case of simple fractions. Therefore, exercises related to the multiplication of integer and unit fraction, and fraction and unit fraction are discussed as examples of multiplying a natural number and simple fraction, or a simple fraction and natural number. Kar et al. (2018) analyzed the problems related to multiplication of fractions in Turkish and American textbooks. In that study, in which the content of problems related to multiplication of fractions was examined, it was seen that American textbooks aim to develop conceptual understanding first and then transactional fluency, while Turkish textbooks aim to develop both at the same time. It was determined that American textbooks provide more opportunities for different computational strategies and contain more problems aimed at higher-level thinking skills, such as mathematical reasoning. However, it was observed that there are one-step calculations in the textbooks of both countries and that there are questions that require transactional knowledge.

In the present study, it is aimed to examine and compare the exercises for multiplication of fractions and solved sample questions in Turkish and Singapore mathematics textbooks. In this direction, the study problem takes the form "How do exercises for multiplication by fractions and solved sample questions in mathematics textbooks in Turkey and Singapore vary?".

The subproblems are: Exercises and sample solved questions on multiplication of fractions found in mathematics textbooks in Turkey and Singapore:

1. How does the question vary by type?
2. How does it vary according to the number of steps?
3. How does it vary according to its contextual characteristics?
4. How does it vary depending on the type of representation used?
5. How does it vary depending on the type of model used?
6. How does it vary according to the meanings of multiplication of fractions?
7. How does it vary according to the types of fractions used?
8. How does it vary according to cognitive expectations?

2. Methodology

In the present study, in which solved examples and practice questions for fraction multiplication in mathematics textbooks in Turkey (TR) and Singapore (SGP) are examined and compared, document analysis, which is a qualitative research method that is used to analyze the content of written documents regularly and systematically, was employed (Wach & Ward, 2013; Bowen, 2009). As a source of data analysis, elementary and secondary school mathematics textbooks that deal with the multiplication of fractions in both countries were selected.

2.1. Selection of Textbooks in Turkey and Singapore

In the present study, in which the questions related to multiplication of fractions are examined and compared, mathematics textbooks in TR and SGP are examined. The reason for choosing these two countries is that SGP shows higher success than TR in international exams that measure mathematics achievement such as TIMSS and PISA, and it is aimed to investigate what advantages or disadvantages Turkish books have by comparing them with the textbooks of this country, and to make suggestions in this direction. Textbooks in TR are prepared by two types of company: private publishing houses (authorized by the Ministry of Education) and those affiliated to the Ministry of Education. However, in both cases, in order for textbooks to be used in schools, they must be prepared according to the secondary school mathematics course curriculum updated in 2018 in Turkey and approved by the Board of Education. In the present study, textbooks prepared by private publishing houses (Öğün Publications) that are more up-to-date than the Turkey Ministry of Education publications approved by the Ministry of Education and Training in 2019 were used. In TR, operations on fractions are taught in the fourth and fifth grade according to the curriculum of the secondary school mathematics course, and first of all, addition and subtraction operations on fractions are focused on. Multiplication and division operations in fractions are first taught in the sixth grade. Therefore, in the present study, as the multiplication process in fractions is discussed, the sixthgrade mathematics textbook is examined. Textbooks in SGP are prepared by private publishing houses, but are taught in schools depending on the approval of the Ministry of Education of Singapore. For this reason, the New Syllabus series, published by a private publishing house Shing Lee Publishers and approved for reading by the Singapore Ministry of Education, used as one of the two most common publishing houses in Singapore, was selected as the books representing SGP. In SGP, operations on fractions, similar to in TR, are first taught in the fourth grade and addition and subtraction operations on fractions are focused on. Multiplication of fractions is taught in the fifth grade and division in fractions in the sixth grade. Therefore, in the present study, the fifth grade SGP mathematics textbook containing multiplication of fractions was examined. In addition, the Turkish textbook was accessed through the EBA (Educational Information Network), while the SGP textbook was accessed by requesting a sample book from Shing Lee Publishers, the publisher of the book. Information concerning the books examined in the study is given in Table 1.

Table 1. *The Books Examined in the Study*

Class/Country	Textbook Information
Turkey	Middle School Math 6. Classroom Textbook, ÖĞÜN Publications, 2019
Singapore	New Syllabus Primary Mathematics 5A, Shing Lee, 2018

2.2. Data Analysis and Process

When the literature is examined, it is seen that there are international studies in which various topics in mathematics textbooks are compared. In the present study, an analysis framework created by using textbook analysis studies in the literature was used (Alajmi, 2011; Li, 2002; Kar et al., 2018; Son & Senk, 2010; Stigler et al., 1986; Watanabe et al., 2017). The analysis framework in question is included in Table 2.

Table 2. *Data Analysis Framework*

Property	Category
The number of steps	One-step operation (OS) Multistep operation (MS)
Context	Pure mathematics (PM) Real life (RL)
Type of representation	Symbolic (S) Verbal (VR) Visual (VS) Mixed (M)
Model type	Area(A) Set (S) Length (L)
Meaning	Repeated Addition (RA) Processor (P) Area(A)
Types of Fractions	Whole number × Fraction (W×F) Fraction × Whole number (F×W) Fraction × Fraction (F×F)
Cognitive Expectation	Operational Knowledge (OK) Conceptual Knowledge (CK) Representation (R) Problem Solving (PS) Reasoning (R) Problem Setting (PSE)

In the process of data analysis, firstly, sample questions and practice questions with solutions for multiplication of fractions in textbooks in TR and SGP were determined. All of these questions determined are numbered according to the book name, page number, and question number. After that, each category in the data analysis framework was entered into an Excel file and two different researchers evaluated each question according to these categories. In the data analysis, first of all, the number of steps required for solving the problem in the number of steps category was taken into account. According to this category, if the examined questions are solved in one step or solved in one step in the solved examples, it is one-step; if there has been a solution in more than one step, it is encoded as multistep (Kar et al., 2018; Son & Senk, 2010). In the multiplication of integer fractions, the translation of the integer into a compound fraction was also regarded as a step, so questions involving the multiplication of integer fractions were coded in a multistep manner. In addition, some questions measuring conceptual knowledge were excluded from this category by not being coded according to the number of steps. An example of this is figure 1 given below.

Aşağıdaki ifadelerden doğru olanın başına "D", yanlış olanın başına "Y" yazınız.

(...) Kesirler ile çarpma işlemi yapılırken tam sayılı kesir varsa bileşik kesre çevrilir.

(...) Kesirlerle çarpma işleminde sadeleştirme yapılmaz.

(...) Bir doğal sayı ile bir kesir çarpılırken doğal sayı ile kesrin paydası çarpılır.

(...) Bir doğal sayı ile bir bileşik kesir çarpıldığında sonuç bu doğal sayıdan küçük olur.

(...) Bir kesrin diğer bir kesir kadarı bulunurken bu iki kesir toplanır.

Turkish-English translation

From the following statements, type "D" at the beginning of the correct one and "Y" at the beginning of the incorrect one.

(….) When multiplying fractions, the compound fraction is translated if there is an integer fraction.

(….) Simplification is not performed in the process of multiplying by fractions.

(….) A natural number is multiplied by a fraction, while the denominator of a fraction is multiplied by a natural number.

(….) When a compound fraction is multiplied by a natural number, the result is less than this natural number.

(….) A fraction is a fraction of another fraction, while these two fractions are added up.

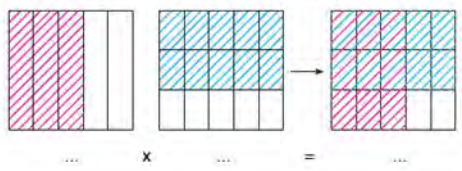
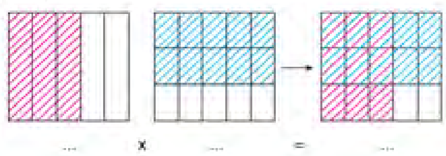
Figure 1. Öğün Publications, page 110

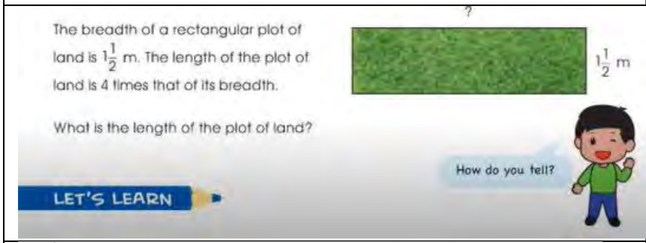
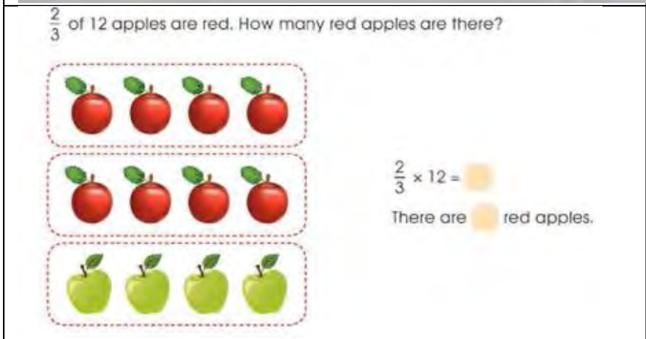
In the analysis carried out according to the context type, which is the second category, the situation of the questions being related to a purely mathematical form (pure mathematics) or to a daily life situation was taken into account (Alajmi, 2011; Kar et al., 2018; Son & Senk, 2010). Accordingly, the question examined was coded in a real-life context if it included real-life situations and as purely mathematically if it contained only mathematical expressions. According to the type of representation, which is the third category, the question statements of the exercise questions and solved examples were coded as symbolic if they only contained mathematical expressions, verbal if expressed verbally, visual representation if any visual was used in the question, and mixed representation if any two or three of them were used. However, it was also examined whether any fraction model was included in the question statements of both exercise questions and solved examples. In addition, the use of any model in the solutions of the solved examples was examined. Accordingly, if any model was used in exercise questions or solved examples, they were encoded as "area model", "cluster model", or "length model"; if it was not used, it was encoded as "no model".

Since multiplication of fractions has broader meanings than multiplication of natural numbers, whether the questions in the aforementioned textbooks are included in the meanings of repeated addition, operator, and area was also examined as another category. However, the types of fractions used in the teaching of multiplication of fractions and the order of fractions in the multiplication process are considered important for students to perform meaningful learning (Watanabe et al., 2017). In this context, the questions related to multiplication of fractions were examined according to the types of fractions used in the first and second multipliers in the process as the sixth category. In this case, three different situations arose according to the fact that the first and second factors are natural numbers and fractions. These are the cases where the fraction of the first multiplier is the natural number of the second multiplier, the natural number of the first multiplier, the fraction of the second multiplier, and both multipliers are fractions. Another category studied is the cognitive expectations of questions aimed at multiplication of fractions. Cognitive expectation has been defined as the mathematical knowledge and skills that students should gain while doing mathematics (Son & Senk, 2010). Therefore, while the questions were examined in this category, they were examined for what cognitive needs the student should have in order to solve the question. These cognitive expectations are composed of the categories of transactional knowledge, conceptual knowledge, problem-solving, problem-setting, reasoning, and representation. Under the transactional information category, there are problems that students solve using only operations and algorithms, while the conceptual information category includes problems that question the meaning underlying the multiplication rule in fractions (Kar et al., 2016). The category of representations includes problems whose solutions require diagrams, pictures, or their interpretation. The problem-solving category includes situations that require solving problems related to everyday life (Son, 2012; Son & Senk, 2010), while the mathematical reasoning category includes problems in which it is desirable to explain solutions or justify the strategies used (Son & Hu, 2016). The problem-forming category was also included in the cognitive expectation feature by Kar et al. (2018). The category of problem-setting, which is also considered in this work, includes problems that require creating new problems or organizing an already existing problem based on a specific situation or experience. It has been determined that the questions examined may require more than one cognitive skill. In this case, the studied problem is examined in the category of which cognitive level is the highest. For example, if a problem requires both transactional skills and reasoning skills, this problem is classified in the category of mathematical reasoning (mathematical reasoning), which requires a higher cognitive demand.

The following table gives an example analysis of some of the questions contained in the textbooks.

Table 3. *Sample Problem Analysis*

	The number of Steps	Context	Type of Representation	Model Type	Meaning	Type of Fraction	Cognitive Exp.
3 Aşağıda modellenen çarpma işleminin matematik cümlesini yazınız.  **Turkish - English translation** Write the mathematical sentence of the multiplication operation modeled below. 	OS	P M	M VR+V S	A	A	F× F	PSE
• Bir günün $\frac{3}{4}$'ünü uyuyarak geçiren bir bebek, günde .... saat uyur. **Turkish - English translation** The baby, who spends three quarters of a day sleeping, sleeps ..... hours a day.	OS	R L	S	-	P	F× W	OK, PS

Türkân, bir romanın $\frac{3}{12}$'ünü okuyor. Sonra da romanın kalan $\frac{2}{3}$'sini okuyor. Buna göre Türkân romanın toplam kaçta kaçını okumuştur? **Turkish - English translation** Türkan reads three twelfths of a novel, and then reads two thirds of the remaining novel. According to this, how many times has Türkan read the novel?	MS	R L	S	-	P	F× F	OK, PS
Hangi sayının $\frac{2}{3}$'sinin $\frac{1}{2}$'i 12'dir?	MS	P M	S	-	P	F× F	OK, PS
	MS	R L	M VR+V S	A	R A	W ×F	OK, PS
	OS	R L	M VS+V R	S	P	F× W	OK, PS
3. What is $\frac{2}{5}$ of $\frac{1}{6}$? Express your answer in its simplest form.	OS	P M	VR	-	P, A	F× F	OK
6. What is $\frac{9}{5} \times \frac{5}{4}$?	OS	P M	M VR+S	-	P, A	F× F	OK

2.3. Reliability

Solved examples and practice questions about multiplication of fractions in the books of the two countries were coded separately by two mathematics educators. There was no difference of opinion between the decoders regarding the number of steps, context, model type, or fraction type. Therefore, the compatibility between the decoders was one hundred percent. According to the reliability formula of Miles and Huberman (1994) consistency of 94.3% (Consensus / Consensus + Disagreement = 115/115+7) was observed regarding the type of representation along with a consistency of (28/28+6) 88.2% regarding the meaning of multiplication of fractions and a consistency of (120/120+9) 93% regarding cognitive expectation. In this aspect, the findings of the study are reliable.

3. Results

In this study, in which the problems related to multiplication of fractions in mathematics textbooks in TR and SGP were examined and compared, the findings obtained are presented respectively in accordance with the research questions.

3.1. Results on the distribution of questions in books

In Table 4, the findings on the distribution of solved sample and exercise questions in mathematics textbooks in TR and SGP related to the multiplication operation in fractions are given.

Table 4**.** *Distribution of the Studied Questions on Multiplication in Fractions by Country*

	TR		SGP		Total	
	f	%	f	%	f	%
Solved Examples	13	38.2	21	61.8	34	100
Practice Questions	49	55.7	39	44.3	88	100

According to Table 4, there are 13 solved examples in the Turkish textbook and 21 solved examples in the SGP book. In both books, there are a total of 34 (61.8%) solved examples related to multiplication of fractions, the majority of which are in the SGP book.

When the practice questions are examined, it is seen that there are 88 problem situations in total. Of these problems, 49 (55.7%) are present in the textbook in TR and 39 (44.3%) are present in that in SGP. When the books are evaluated in general on the basis of question distribution, it can be said that SGP stands out in terms of solved examples and TR stands out in terms of practice questions regarding quantity.

3.2. Results for comparison of solutions by number of steps

Table 5 shows the results on how many steps in which the multiplication operation questions can be solved in fractions contained in the books of both countries.

Table 5. *Comparison of the Solutions of the Question of Multiplication in Fractions by the Number of Steps*

		TR		SGP		Total
		f	%	f	%	f
Solved Examples	One Step	8	61.5	14	66.7	22
	Multistep	5	38.5	7	33.3	12
Practice Questions	One step	20	40.8	20	51.3	40
	Multistep	20	40.8	19	48.7	39
	Conceptual	9	18.4	-	-	9

According to Table 5, the vast majority of the solved examples in the textbooks in both TR and SGP are one-step. When the practice questions were examined, it was seen that 20 (40.8%) of the problems in TR were one-step and 20 (40.8%) were multistep. In addition, since there are non-procedural problems, they cannot be evaluated according to the number of steps; they are examined under the heading “conceptual”. In this case, 9 exercise questions were encountered in the book in TR. While 20 of the exercise questions in the SGP textbook consisted of one-step problems, 19 of them consisted of multistep problems. There was no question only measuring conceptual knowledge in the SGP book.

3.3. Results related to contextual understanding

In Table 6, the contextual meanings of the questions related to the multiplication operation in fractions are examined and compared.

Table 6. *Comparison of Questions on Multiplication by Fractions by Type of Context*

		TR		SGP		Total
		f	%	f	%	
Solved Examples	Pure Mathematics	8	61.5	10	47.6	18
	Real Life	5	38.5	11	52.4	16
Practice Questions	Pure Mathematics	37	75.5	35	89.7	72
	Real Life	12	24.5	4	10.3	16

According to Table 6, 8 (61.5%) of the solved examples in the Turkish textbook are pure mathematical and 5 (38.5%) are in the context of real life. When the practice questions were examined, it was determined that there were 37 (75.5%) pure mathematical problems and 12 (24.5%) real-life problems in the textbook in TR, with 35 (89.7%) purely mathematical and 4 (10.3%) real-life problems in that in SGP. It has been concluded that the pure mathematical context is in the textbooks of both countries in both solved examples and practice questions related to multiplication of fractions.

3.4. Results on the Type of Representation

In Table 7, a comparison of the representations used in the question part of the problems related to the multiplication of fractions is given.

Table 7. *Comparison of Questions Related to Multiplication in Fractions by Type of Representation*

		TR		SGP		Total
		f	%	f	%	f
Solved Examples	Symbolic	-	-	-	-	-
	Verbal	2	15.4	14	66.7	16
	Image	-	-	-	-	-
	Verbal+Symbolic	9	69.2	3	14.3	12
	Verbal+Visual	2	15.4	4	19	6
Practice Questions	Symbolic	1	2	24	61.5	25
	Verbal	22	44.9	4	10.3	26
	Image	1	2	-	-	1
	Verbal+Symbolic	22	44.9	10	25.6	32
	Verbal+Visual	3	6.1	1	2.6	4

According to Table 7, in the solved examples the maximum number verbal-symbolic representations used was 9 (69.2%), in TR; the maximum number of verbal representations used was in SGP. There is no question that visual representation is used in both SGP and TR in the solved examples. When the practice questions were examined, it was seen that the most verbal representations were used in TR and the most symbolic representations were used in SGP. While all the representations were used in the practice questions in TR, no practice questions with visual representations were encountered in the SGP textbook.

3.5. Results for the model type

In Table 8, the findings regarding the types of models used in the question statements of the questions in the textbooks in TR and SGP on multiplication of fractions are given.

Table 8. *Comparison of Questions Related to Multiplication of Fractions by Model Type*

		TR		SGP		Total
		f	%	f	%	
Solved Examples	Clustering	-	-	-	-	-
	Area	-	-	3	14.2	3
	Length	-	-	1	4.8	1
	None	13	100	17	81	30
Practice Questions	Clustering	-	-	-	-	-
	Area	1	2	1	2.6	2
	Length	2	4.1	-	-	2
	None	46	93.9	38	97.4	84

According to Table 8, while no model was used in any of the solved examples in the Turkish textbook, three problems were used in the SGP book and the area model was used in the question statement. When the practice questions were examined, two questions were encountered in which the area model was used and the length model was used in the Turkish mathematics textbook. In contrast, the area model was used in only one question in SGP. In the vast majority of the questions in the books of both countries, no model was used. Table 9 shows the models used to solve the solved examples.

Table 9. *Comparison of Models Used to Solve the Solved Examples of Multiplication of Fractions*

		TR		SGP		Total
		f	%	f	%	f
Solved Examples	Clustering	1	7.7	2	9.5	3
	Area	4	30.8	5	23.8	9
	Length	-	-	4	19	4
	Set+Length	-	-	1	4.8	1
	None	8	61.5	9	42.9	17

According to Table 9, while 1 of the sample solutions in the TR textbook uses a set and 4 use an area model, 5 area models, 4 length models, 2 sets, and one set and length model were used together in the SGP textbook. The most used model in both countries is the area model. In addition, it is seen that no model is used to solve 8 problems in TR and 9 problems in SGP.

3.6. Results on the meaning of multiplication of fractions

In Table 10, it is investigated in which meanings of multiplication of fractions these questions are used, since there is only the solution of solved examples.

Table 10. *Comparison of the Meanings of Multiplication of Fractions Used in Solved Examples*

		TR		SGP		Total
		f	%	f	%	f
Solved Examples	Repeated Addition (RA)	4	30.8	2	9.5	6
	Processor (P)	6	46.2	16	76.2	22
	Area (A)	3	23	3	14.3	6

According to Table 10, the processor meaning of fractions was used in the majority (46.2%) of the samples analyzed in TR, and the repeated addition and area meanings were used in numbers close to each other. Similarly, in the SGP book, the processor meaning was used in 16 (70.6%) solved examples, the repeated addition meaning of multiplication in 2, and the area meaning of fractions in 3. In these solved examples, the various forms of fractions used in the multiplication of fractions are also compared, as shown in Table 11.

Table 11. *Comparison of the Types of Fractions Used in the Multiplication of Fractions*

			TR		SGP		Total
			f	%	f	%	f
Solved Examples	Whole Number × Fraction	Whole number × simple fraction	3	23.1	1	4.8	4
		Whole number × integer fraction	-	-	-	-	-
		Whole number × Compound fraction	1	7.7	-	-	1
	Fraction × Whole Number	simple fraction × whole number	2	15.4	5	23.8	7
		Compound fraction × Whole number	-	-	1	4.8	1
		integer fraction × Whole number	-	-	4	19	4
	Fraction × Fraction	Simple fraction × simple fraction	4	30.8	4	19	8
		Simple fraction × compound fraction	-	-	2	9.5	2
		Simple fraction × integer fraction	-	-	2	9.5	2
		Compound fraction × simple fraction	-	-	-	-	-
		Compound fraction × integer fraction	-	-	-	-	-
		Compound fraction × compound fraction	-	-	2	9.5	2
		Integer fraction × simple fraction	1	7.7	-	-	1
		Integer fraction × compound fraction	-	-	-	-	-
		Integer fraction × integer fraction	2	15.4	-	-	2
Practice Questions*	Whole number × Fraction	Whole number × simple fraction	3	7.5	-	-	3
		Whole number × integer fraction	1	2.5	1	2.6	2
		Whole number × Compound fraction	1	2.5	-	-	1
	Fraction × Whole Number	simple fraction × whole number	9	22.5	11		20
		Compound fraction × Whole number	1	2.5	2	5.1	3
		integer fraction × Whole number	-	-	14		14
	Fraction × Fraction	Simple fraction × simple fraction	9	22.5	4	10.3	13
		Simple fraction × compound fraction	2	5	3	7.7	5
		Simple fraction × integer fraction	6	15	-	-	6
		Compound fraction × simple fraction	-	-	-	-	-
		Compound fraction × integer fraction	-	-	-	-	-
		Compound fraction × compound fraction	-	-	4	10.3	4
		Integer fraction × simple fraction	3	7.5	-	-	3
		Integer fraction × compound fraction	-	-	-	-	-
		Integer fraction × integer fraction	5	12.5	-	-	5

*Conceptual ones are excluded.

In Table 11, the types of fractions of the first and second multipliers used in problems related to the multiplication of fractions are investigated. In TR, in the solved examples in the textbook, the number of problems in which both the first and the second multiplier are fractions is the largest. It was determined that there are more problems related to the product of two simple fractions in these. In addition, we encountered 4 solved examples where the first factor is a natural number and second factor is fraction and 2 where the first factor is a fraction and the second factor a natural number. In the SGP mathematics textbook, it has been determined that the problems where the first multiplier is a fraction and the second multiplier is a whole number, unlike in TR, are the most numerous. However, while a problem was encountered where the first multiplier was a whole number and the second multiplier was a fraction, 10 problems were encountered where both multipliers were fractions. Among them, the number of problems related to the multiplication of two simple fractions, similar to in TR, was greater.

When the types of fractions in the exercise questions in Table 11 were examined, it was determined that the problems related to the multiplication of two fractions in TR were the most common. This was followed by problems where the first multiplier is a fraction and the second multiplier is a whole number and problems where the first multiplier is a whole number and the second multiplier is a fraction, in that order.

In the practice questions in the SGP book, it was determined that the most common ones were the problems in which the first factor is a fraction and second factor is a natural number, and of these problems the ones with a mixed fraction as the first factor were the most common problems. Similar to the solved examples, a problem was encountered where the integer of the first multiplier is the fraction of the second multiplier, while 27 problems were found where the fraction of the first multiplier is the integer of the second multiplier. It has been concluded that the types of fractions used in TR and SGP in both solved examples and practice questions are similar to each other.

3.7. Results regarding the level of cognitive expectation

In Table 12, the cognitive expectation levels of multiplication operation questions in fractions contained in the textbooks in TR and SGP are indicated.

Table 12. *Comparison according to the level of cognitive expectation expected from multiplication of fractions*

	Cognitive Expectation	TR		SGP		Total
		f	%	f	%	f
Solved Examples	Operational knowledge (OK)	5	35.7	11	52.3	16
	Conceptual Knowledge (CK)	-	-	-	-	-
	Representation (R)	-	-	1	4.8	1
	Problem-Solving (PS)	5	35.7	9	42.9	14
	Reasoning (R)	4	28.6	-	-	4
	Problem-Setting (PSE)	-		-	-	-
Practice Questions	Operational knowledge (OK)	21	45.7	35	85.4	56

	Conceptual Knowledge (CK)	9	19.5	-	-	9
	Representation (R)	3	6.5	1	2.4	4
	Problem-Solving (PS)	11	23.9	5	12.2	16
	Reasoning (R)	1	2.2	-	-	1
	Problem-Setting (PSE)	1	2.2	-	-	1

In some problems, the expectation of more than one cognitive level was formed, but whichever cognitive level was the highest in the problem, that level was accepted. While it was determined that the vast majority of the problems in the SGP book required operational knowledge, it was observed that there were equal numbers of operational knowledge and problem-solving level questions in TR.

In addition, 5 of the questions included in the book in TR required problem-solving and 4 required mathematical reasoning, while those in SGP included solved examples that required only problem-solving skills, except for operational knowledge. This ratio is higher than in TR. When the practice questions were examined, it was seen that the cognitive expectations of the questions in TR were distributed more evenly than in SGP. As a matter of fact, while there are examples of practice questions related to multiplication of fractions in TR at all levels of cognitive expectations, it was determined that SGP has operational knowledge, problem-solving, representations, and cognitive expectations in the practice questions. In TR, the practice questions with conceptual knowledge and problem-solving expectations were included after operational knowledge.

4. Discussion, Conclusions and Recommendations

In the present study, a comparative analysis of the questions related to the multiplication of fractions in mathematics textbooks in TR and SGP was carried out. In this sense, first of all, it was investigated at what grade level decimation is considered in schools in SGP and TR, and the first differences between the two countries were encountered. As a matter of fact, multiplication of fractions in TR begins to be taught in the sixth grade and in SGP in the fifth grade. Son and Senk (2010), Kar et al. (2017), and Ding and Li (2010) stated that multiplication operations in fractions are considered at an early level in countries such as America, Korea, and China in their studies. For example, multiplication of fractions is taught in the fifth grade in Japanese and Korean textbooks and in the fourth grade in Taiwanese textbooks (Watanabe et al., 2017). Based on this, it can be said that countries with high success in international exams tend to start learning concepts earlier. However, it is noticeable that there are more solved examples of multiplication by fractions in the textbook in SGP than in TR, but fewer practice questions. In this sense, it can be thought that in SGP it may be the aim to teach the subject in more depth by seeing more and different resolved examples of students compared to in TR.

It is suggested that multiplication of fractions should be taught for the first time through the meaning of repeated addition used in multiplication of natural numbers (Toluk Uçar, 2020; Musser, Peterson & Burger, 2014). However, when the mathematics textbooks of the two countries are examined, the difference between the countries in introducing multiplication of fractions is seen. This difference manifests itself at the point where this ordering is observed in the Turkish book, but the SGP book directly enter the multiplication process using the processor meaning of fractions. In TR, the number of both solved examples and questions for the repeated addition of multiplication in practice questions is higher than in SGP. In this sense, it can be said that TR's teaching style of multiplication of fractions is similar to that in Korean textbooks (Son & Senk, 2010; Kar et al., 2017). However, it was observed that the processor meaning is used more intensively in SGP than in TR.

When the literature is examined, it is recommended that multiplication of fractions begin with the repeated addition meaning of multiplication, therefore starting with the multiplication of a natural number and a fraction (Musser et al., 2014). From this point of view, the types of fractions used in the first and second multiplier cases in the multiplication process are also considered important. Therefore, the types of fractions used in the multiplication of fractions should be carefully selected during the teaching of fractions. Firstly, starting with the types that students can most easily learn and are familiar with will make it easy for them to learn multiplication by fractions. From this point of view, it is proposed to teach the multiplication of a fraction and a whole number first in textbooks. It was observed that both countries have adopted this rule and started teaching by multiplying a fraction and a natural number. However, it was observed that the Turkish textbook takes into consideration the “natural number × simple fraction” order for the repeated addition meaning of multiplication, where the first fraction is a natural number and the second fraction is a simple fraction, while in SGP the multiplication operation is taught directly by taking into consideration the “simple fraction × natural number” in both the repeated addition meaning and operand meaning of multiplication. It can be thought that this situation may force students to understand the multiplication process of fractions, because the symbolic representations and visuals in the book in SGP related to repeated aggregation do not show consistency with each other. Of course, since multiplication has a commutative property, the form “simple fraction × whole number” can also be used in repeated addition. However, it has been stated that it is important for students to learn that the meaning of each multiplication case is shown in different symbolic forms (Van de Walle, 2013). When the solved examples were examined, it was determined that the book in TR contains questions about the multiplication of a whole number by fractions, fraction by a whole number, and fraction by a fraction, but that in SGP does not contain any practice questions about the multiplication of a whole number and fraction. When the practice questions were examined, it was seen that there was no multiplication operation in the form of "natural number × fraction" in the SGP textbook, similar to the solved examples, and there were more examples for the "fraction × natural number" operation. In TR, in addition to the fact that there are examples of all types of multiplication in the textbook, it was determined that the questions in the form of multiplication of two fractions are weighted. In general, it seems that simple fractions are preferred as a multiplier.

The use of fraction models in the teaching of multiplication of fractions is considered important for students to achieve deeper learning and gain different perspectives. When the models used in textbooks were examined, it was interestingly determined that no models were used in the presentation of the solved examples or the vast majority of the practice questions. It was noted that in the solved examples only the area model was used in the SGP book, while in the practice questions one area and two lengths were used in TR, and one area model was used in SGP. In the questions where the model is used in the presentation, it is usually noted that the processor meaning of fractions or the area meaning is also used. When the models used in the solutions of solved examples were examined, only the set and area models were used in TR, and the set and length models were used together in addition to the set area and length models in SGP. It was observed that the processor meaning of fractions is used in solutions where the cluster model is used in both countries, and the area or repeated addition meanings of multiplication are used in solutions where the area model is used. It was determined that the processor meaning of fractions is used in the solution where the length model is used in SGP and the set and length model are used together. Proceeding from this, it can be said that the book in SGP uses a more diverse and multiple model in solving solved questions than that in TR. This situation is considered important in terms of the fact that more than one method of representation has the potential to complement each other in the learning environment, and one method does not show the direction in concepts; the other makes it visible and provides meaningful learning (Ainsworth, 2006; Gagatsis & Shiakalli, 2004). The different representations used in the presentation of the questions are considered important because they will allow students to understand the questions better and more comprehensively (Ainsworth, 2006). In this sense, when the presentation representations of the questions related to multiplication operations in fractions are examined, it can be said that the distribution in TR is more balanced than in SGP. However, in the resolved examples, verbal and verbal+symbolic representations were used predominantly in TR, while symbolic representation was determined to be predominant in SGP.

When the number of steps required to solve the questions contained in the textbooks is examined, it is noted that the questions that can be solved in one step in SGP in terms of solved examples are more numerous than multistep ones and compared to single-step questions in TR. In TR, the one-step sample solved questions were more than the multistep ones, but the multistep questions took up more space than in SGP. However, it was determined that the repeated addition meaning of multiplication is usually used in one-step sample solved questions in TR, and the processor and area meanings are used in multistep questions. In SGP, in contrast to TR, it was observed that the processor meaning of fractions is used in the vast majority of single-step solved examples; in addition to this, the area meaning is also included. In parallel with the examples with one-step solutions, the processes used in multistep solutions in SGP also differ from those in TR in terms of their meaning. Moreover, it is noteworthy that repeated addition and processing steps are more common in multistep solution examples. It was determined that the distribution of the practice questions was similar for the two countries and that the questions solved in one step in both were more than those solved in multiple steps. In addition, in both single- and multistep practice questions, processor and area meanings were used predominantly in the multiplication of fractions in both countries, and the repeated addition meaning was used only in TR. In addition to one-step and multistep questions, it was observed that there are questions in TR that emphasize the conceptual aspect of multiplication in fractions, unlike in SGP.

When the examined questions were evaluated from a contextual point of view, it was seen that pure mathematical questions constituted the majority in both countries. Real-life contextual questions were included more in SGP in terms of solved examples and in TR in terms of practice questions. Considering that real-life oriented teaching has been emphasized in mathematics education in the twenty-first century (Yang, Resy & Wu, 2010), it can be said that this ratio may not be sufficient. There is also conclusive evidence that mathematical activities should be connected with the state of everyday life (Lesh & Lamon, 1992; NCTM, 2000; OECD, 2004). In addition, the results of various studies (Cooper & Harries, october003; Post et al., 2008; Tarr et al., 2008) suggested that integrating authentic mathematical activities into teaching can not only increase the desire to learn, but also increase children's achievement in mathematics and provide meaningful learning.

The diversity in the cognitive expectation levels of the questions is considered important for students to learn the subject more deeply. Based on this, it was seen that the vast majority of the solved examples in the textbooks in TR and SGP contain operational knowledge. However, while there are questions about solving problems that require high-level cognitive skills in both countries, it was determined that there is only a solved example in the Turkish book for reasoning. In the practice questions, it was found that almost all of the questions in SGP require operational knowledge, but the questions in TR have a balanced distribution and contain questions for each cognitive expectation. It is thought that the questions in the Turkish book may be due to the fact that SGP has more diverse and high-level cognitive expectations, and that TR presents multiplication of fractions to students in a more advanced class than SGP does. In addition, the fact that there is great interest in the operational knowledge in the SGP book may be a result of the culture of Asian countries, where the structure and rules are highly respected, a value that is also reflected in textbooks (Yang et al., 2010). Comparing the multiplication of fractions in American and Turkish mathematics textbooks, Kar et al. (2017) obtained findings that support these results.

According to the conclusion of the study, it has been revealed that SGP and TR textbooks have advantages and disadvantages compared to each other in terms of dealing with multiplication in fractions. To summarize; multiplication of fractions in SGP begins to be taught earlier than TR and it is noticeable that there are more solved examples of multiplication by fractions in the textbook in SGP than in TR, but fewer practice questions. However, it was concluded that the use of the fraction model was more homogeneous in the SGP, and that TR used the questions with higher cognitive expectations in the practice questions.

Textbooks, which are the concrete means of implementing the curriculum in the classroom, are an important teaching material. Therefore, the results of the present study provide a perspective for program developers, researchers, and teachers to evaluate textbooks. However, our study is considered to be limited due to its focus on multiplication of fractions only, and therefore it is envisaged that different research is needed in order to obtain comprehensive findings about mathematics textbooks. In addition, the present study is limited to the analysis of mathematics textbooks of the two countries. In later studies, it is thought that inclusive findings can be obtained by analyzing textbooks from a larger number of different countries.

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