



# European Journal of Educational Research

Volume 11, Issue 2, 821 - 834.

ISSN: 2165-8714

<http://www.eu-jer.com/>

## How Students Use Cognitive Structures to Process Information in the Algebraic Reasoning?

Mochamad Abdul

Basir\* 

Universitas Negeri Semarang,  
INDONESIA

S.B. Waluya 

Universitas Negeri Semarang,  
INDONESIA

Dwijanto 

Universitas Negeri Semarang,  
INDONESIA

Isnarto 

Universitas Negeri Semarang,  
INDONESIA

Received: September 7, 2021 • Revised: December 8, 2021 • Accepted: January 23, 2022

**Abstract:** Cognitive processes are procedures for using existing knowledge to combine it with new knowledge and make decisions based on that knowledge. This study aims to identify the cognitive structure of students during information processing based on the level of algebraic reasoning ability. This type of research is qualitative with exploratory methods. The data collection technique used began by providing a valid and reliable test instrument for algebraic reasoning abilities for six mathematics education student programs at the Islamic University of Sultan Agung Indonesia. Subjects were selected based on the level of upper, middle, and lower algebraic reasoning abilities. The results showed that (1) students with the highest level of algebraic reasoning ability meet the logical structure of Logical Reasoning which shows that students at the upper level can find patterns and can generalize; (2) Students at the intermediate level understand the cognitive structure of Symbolic Representations, where students can make connections between knowledge and experience and look for patterns and relationships but have difficulty making rules and generalizations; (3) students at lower levels understand the cognitive structure of Comparative Thinking, where students are only able to make connections between prior knowledge and experience.

**Keywords:** Algebraic reasoning, cognitive psychology, cognitive structure, information processing.

**To cite this article:** Basir, M. A., Waluya, S. B., Dwijanto, & Isnarto. (2022). How students use cognitive structures to process information in the algebraic reasoning? *European Journal of Educational Research*, 11(2), 821-836. <https://doi.org/10.12973/eu-jer.11.2.821>

### Introduction

The rapid development of science and technology is inseparable from the role of mathematics as one of the basic sciences in various fields. This development triggers the emergence of new challenges in the world of education so that the reasoning abilities of every human being are needed. If the knowledge of reason is integrated into the curriculum, it will certainly have a positive effect on learning mathematics. However, most teachers do not know how students learn and master complex concepts, rules, procedures, or processes in mathematics (Thuneberg et al., 2018). For this reason, Teachers don't just deliver math material but can understand how students learn mathematics, including understanding students' cognitive processes. One indicator of a quality mathematics teacher is that teachers can understand students' thinking processes in problem-solving and expand students' abilities (Siagian et al., 2019). If the teacher pays attention to how students think, the teacher can teach mathematics (Kusmaryono et al., 2021). Cognitive psychology becomes the focus and concern in learning mathematics because it is related to the nature and characteristics of students. Axiomatic deductive reasoning makes mathematics material have an important role in training and improving thinking skills (Li et al., 2019).

In learning mathematics, the concept of algebra is a generalization of arithmetic, so algebra plays an important role in mathematics. However, there are still students who have difficulty in learning algebra. The algebraic difficulties faced by students are mainly in mathematical modelling, understanding algebraic expressions, applying arithmetic operations, understanding the different meanings of the equal sign, and understanding variable notation (Jupri et al., 2014). Most students have difficulty in generalizing arithmetic using algebraic symbols (Ayber & Tanışlı, 2017). Cognitive changes are needed by students in learning algebra with the transition from one form to another so that epistemological barriers arise (Loibl & Leuders, 2019). Students prefer to explain rhetorically rather than give symbolic

\* Corresponding author:

Mochamad Abdul Basir, Building A Postgraduate Campus, Semarang State University, Semarang, Indonesia. ✉ [abdulbasir@unissula.ac.id](mailto:abdulbasir@unissula.ac.id)

© 2022 The Author(s). Open Access - This article is under the CC BY license (<https://creativecommons.org/licenses/by/4.0/>).



answers (Zayyadi et al., 2019). To overcome student difficulties, teachers need to train students' algebraic reasoning abilities.

The emphasis in algebraic learning does not lie in algebraically qualified activities but rather on students' thinking and reasoning processes (Cai et al., 2005). Algebraic reasoning is more important than procedural skills which tend to be mechanistic because the material in mathematics is easier to understand through reasoning. The reasoning is a thought process that tries to connect known facts or evidence to a conclusion (Hawes & Ansari, 2020). In line with this, algebraic reasoning is a process of generalizing mathematical ideas from a set of examples, proving these generalizations through argumentative discourse, and expressing them formally according to the level of student age development (Cañadas et al., 2016).

Algebraic reasoning encompasses all mathematical thinking because of its use in exploring mathematical structures. Algebraic reasoning also requires students to explore a relationship and build generalizations to support conceptual understanding of the relationship in a formula. Indicators of algebraic reasoning (National Council of Teachers of Mathematics, 2016).

- a. Meaningful use of symbols. Selecting variables and constructing expressions and equations in context.
- b. Connecting algebra with geometry. Use connections to solve problems.
- c. Linking expressions and functions. Connect expressions and functions.
- d. Manipulative consciousness. Doing mental calculations.
- e. Reasoned solving. View the solution steps as a logical understanding of the relationship; can find patterns, recognize patterns, and generalize.

The thinking process has a close relationship with cognition to identify students' mental structures such as perception, memory, reasoning, decision selection, problem-solving, and the methods used for introspection (Matlin, 2009). The problems of students of the Sultan Agung Islamic University Mathematics Education Study Program in studying matrix algebra material are very diverse and complex. When students respond to non-routine math problems there is a cognitive process of students in obtaining new information. The teacher records and analyses student behavior to determine students' thinking processes in linking concepts that are appropriate to the problem. Mental processes in processing and understanding information and creating meaning are called cognitive structures. There are three stages of students' cognitive structure: comparative thinking, symbolic representation, and logical reasoning (Garner, 2007). To develop cognitive structures, students need to store information storage memories and visualize information processing. Teachers can use their knowledge and experience to build students' cognitive structures. Characteristics of cognitive structures in processing information and creating meaning by (a) making connections; (b) finding relationships and patterns; (c) formulating rules; (d) generalizations (Garner, 2007).

Strategies to develop cognitive structures by encouraging someone to reflective awareness and imagination in visualization to develop regularity in learning, creating, and changing. reflective awareness means awareness that makes unconscious awareness into conscious awareness. The process of reflective awareness helps develop cognitive structures in processing information and creating meaning. Visualization is the mental representation and manipulation of information, ideas, feelings, and sensory experiences. Without visualization, a person relies on specific information within the sensory range and has difficulty thinking abstractly. In visualization, there are images, symbols, and other forms of mental coding to represent the receipt of information (Garner, 2007).

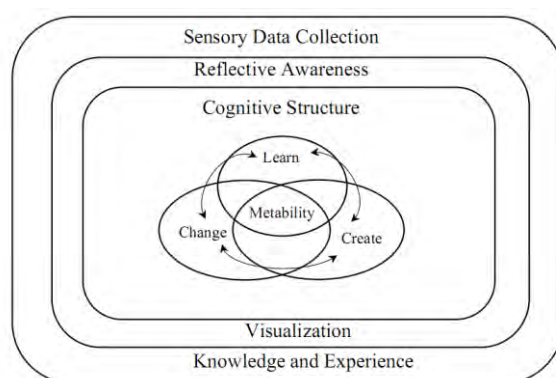


Figure 1. The Cognitive Structure Development Process

In addition, the cognitive structure can also develop through the relationship of attention with students; Encourage the use of cognitive structures through relationships with prior knowledge, explain problem-solving processes and their rationality, look for patterns that connect information automatically and quickly, abstract principles that can generalize ideas from one context to another; Determine what cognitive structures are needed to master standards and scaffolding as basic skills; Encourage students to ask questions; and Instead of providing information, use open clues. *Metabilitas*

can be developed by someone who uses his cognitive structure. An order that describes the dynamic and interactive cycle of *Learn, Create, and Change* (see in figure 1). Learning is more than just collecting facts or skills, which are created by students themselves to produce self-reinforcing energy through a continuous cycle of innovation and change. If there is no change by gaining new understanding, considering new ideas, obtaining additional data, or learning new behaviors because of their interaction with information, then it is not said to have learned. The more students are involved in creating meaning, the more they change and learn.

## Methodology

### Research Design

This qualitative research design uses an exploratory method that aims to identify and describe students' cognitive structures in processing information at the level of algebraic reasoning abilities (upper, middle, and lower) in solving matrix algebra problems.

### Sample and Data Collection

The population in this study were all students of the Mathematics Education Study Program, Sultan Agung Islamic University, Indonesia. The algebraic reasoning test instrument is based on the Marzano taxonomy cognitive system on matrix algebra with an average expert validation score of 4.47 or 89.4% with excellent criteria, the validity test of the two questions is 0.844 and 0.936, which means the test instrument has high criteria and a reliability level of 0.749 which indicates the criteria are feasible (Basir et al., 2021). The question on the algebraic reasoning ability test.

*Every second, a mixture of 220 ml NO (Nitrogen Monoxide) and 20 ml N<sub>2</sub>O<sub>2</sub> (Dinitrogen Dioxide) changes in concentration after heating, i.e., 10% of NO changes to N<sub>2</sub>O<sub>2</sub> and 20% of N<sub>2</sub>O<sub>2</sub> changes to NO.*

- Draw a diagram that represents the information!*
- Write a matrix equation that represents the information!*
- What is the amount of NO and N<sub>2</sub>O<sub>2</sub> concentrates after one second, two seconds and 10 seconds?*
- What is the amount of NO and N<sub>2</sub>O<sub>2</sub> concentrates after n seconds?*

The research data is sourced from the answers to the algebraic reasoning ability test and interview transcripts of the research subject. The research sample was taken by purposive sampling based on the level of algebraic reasoning ability in solving matrix problems (upper, middle, and lower). In this study, the research sample was obtained, namely students who have characteristics according to the research objectives as many as three people, each of which represents a group of algebraic reasoning abilities. The list of research subjects that became the research sample is as shown in table 1.

Table 1. List of Research Samples

Levels of Algebraic Reasoning Ability	Code
Lower	BJH
Middle	TRS
Upper	AMK

### Analyzing of Data

The process of data processing and analysis of qualitative data is carried out interactively and continues until it is complete until the data is saturated. This research emphasizes the power of data analysis, the source of the data obtained, and the theory developed. In qualitative research, the first thing to do is analyses the data, which is described as interwoven between data, namely between links before, during, and after data collection in the form of an interactive cycle process as shown in Figure 2 following (Miles et al., 2014).

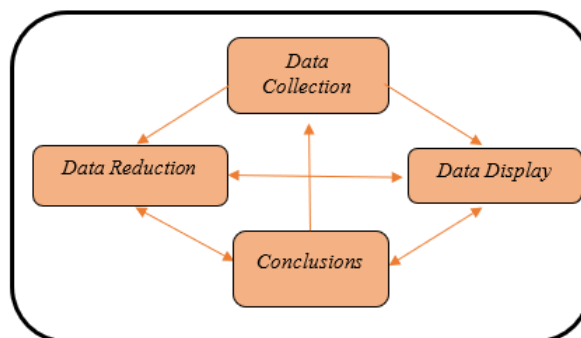


Figure 2. Interactive Model Data Analysis Component

Based on Figure 2, the interactive model qualitative data analysis contains three simultaneous activities: data reduction, data presentation, and conclusion drawing/verification. The interactive model data analysis steps are as follows:

(1) Data Reduction

Data condensation is a process of selecting, simplifying, abstracting, and/or transforming rough data that emerges from written notes in the field or interview transcripts. Researchers simplify through data collection that supports this research. So that the data can be accounted for and concluded. The matrix algebra problem studied in this study is a problem that is related to the concepts that have been studied previously. The structure of the matrix algebra problem is illustrated in Figure 3 and explanatory information in table 2.

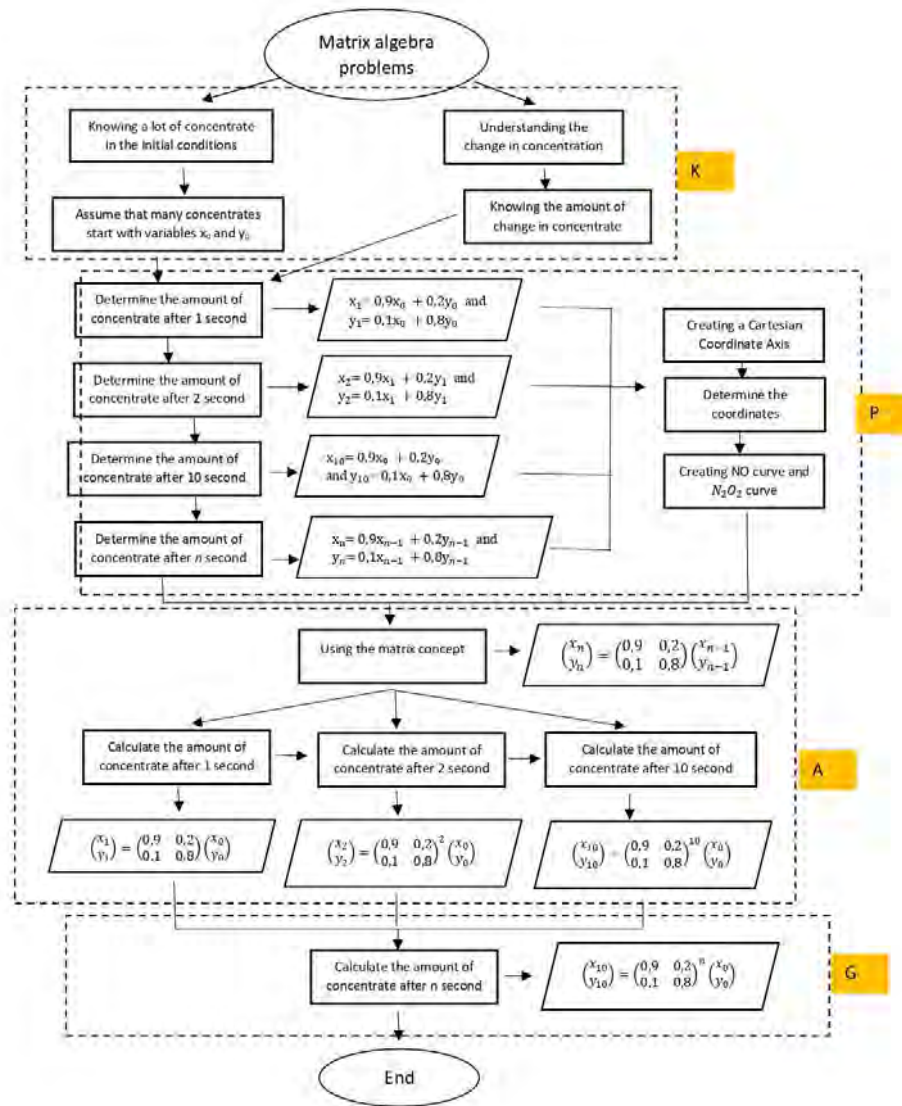


Figure 3. Matrix Algebra Problem Structure

Table 2. Coding Structure Problem

Graphics Code	Description
K	Making Connections
P	Finding Patterns and Relationships
A	Formulating Rules
G	Abstracting Generalizable Principles

## (2) Data Presentation

Data presentation is the process of collecting organized and compressed information that allows drawing conclusions and verification. In qualitative research, the presentation of data is presented in the form of brief descriptions, flowcharts, charts, and the like.

## (3) Drawing Conclusion

The final step in the interactive analysis is conclusion drawing and verification. From the beginning of data collection, qualitative analysis interprets what is meant by noting patterns, explanations, causal paths, and propositions. So that drawing conclusions is one of the efforts to give meaning to the presentation of the data. Thus, conclusion is only part of the activities of the complete configuration. Conclusions were also verified during the study.

## Findings / Results

## (a) Cognitive Structure Lower-Level Algebraic Reasoning Ability

In dealing with matrix algebra problems, some of the problem structures are already known by BJH. BJH subjects are not complete in making connections, are not aware of the process of searching for patterns and relationships, so that the subject cannot formulate rules. BJH uses existing cognitive structures to make connections. This is indicated by the statement from BJH as follows.

BJH: *I imagined this was a chemistry problem that could be solved with math, and I remembered the solution was related to the system of linear equations of two variables material so, I did an example with the variables  $x$  and  $y$ .*

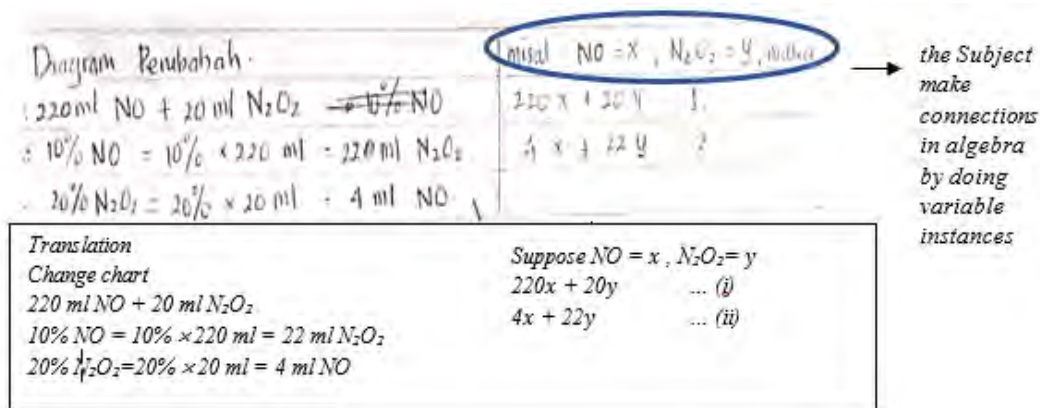


Figure 4. Results of BJH Subjects in Making Connections

From the results of the answers in Figure 4, BJH can think algebraically but does not understand the meaning of the algebraic symbol. This is also clarified by the change in the amount of concentrate that BJH understands is only limited to the percentage that has changed, even though there are still concentrates that still contain the original compounds. Because BJH has not fully understood the problem, resulting in BJH is unable to make connections with the matrix material. In the stage of looking for patterns and relationships, BJH finds it difficult to compare the changes in concentration every second. this can be seen in the graphical representation made by BJH (figure 5).

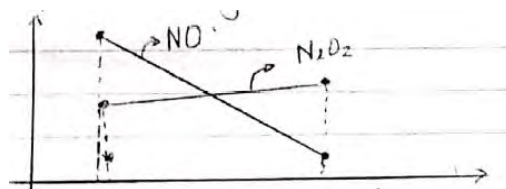


Figure 5. Graph of the Change in Concentration Over Time

From BJH's explanation at the time of the interview, the graphs made did not represent the description of the problem. This is because the line that is connected to represent the change in NO is drawn from the amount of initial concentrate (220 ml) with the number of changes in the concentration of  $N_2O_2$  which turns into NO after just one second. Meanwhile, the amount of NO concentrate that did not turn into  $N_2O_2$  and the change after one second was not considered again. In addition, BJH also cannot analyses mathematical equation models that can be formed and cannot organize changes in concentration through an approach with a matrix concept.



At the stage of formulating the rules, BJH could not correctly predict through a pattern. In the process of completion, BJH is only limited to revealing the calculation procedure to obtain the number of concentrates per second even though there are still conceptual errors by writing (Figure 6).

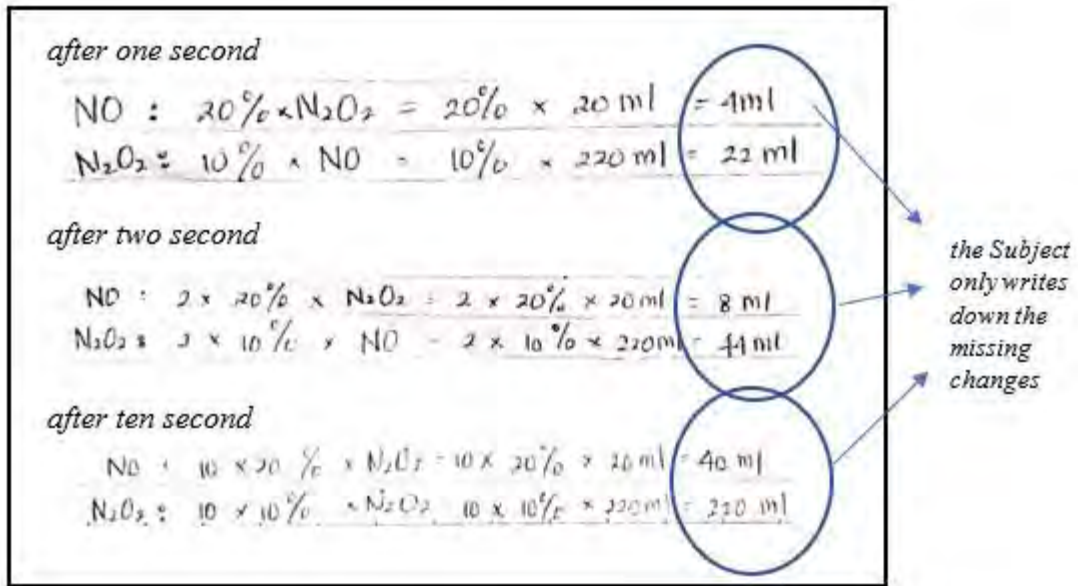


Figure 6. the Subject's Error in Determining the Pattern

BJH writes that the amount of NO compound concentrates after one second is 20% multiplied by the initial amount of N<sub>2</sub>O<sub>2</sub> concentrate, which is 20 ml to obtain 4 ml of NO compound. With the wrong prediction, BJH made a pattern for the number of concentrates after ten seconds, which means ten times the number of concentrates after one second. Therefore, the assumptions given are wrong, so the generalizations made by BJH are also wrong. Furthermore, the cognitive structure of BJH in solving problems can be described as shown in Figure 7.

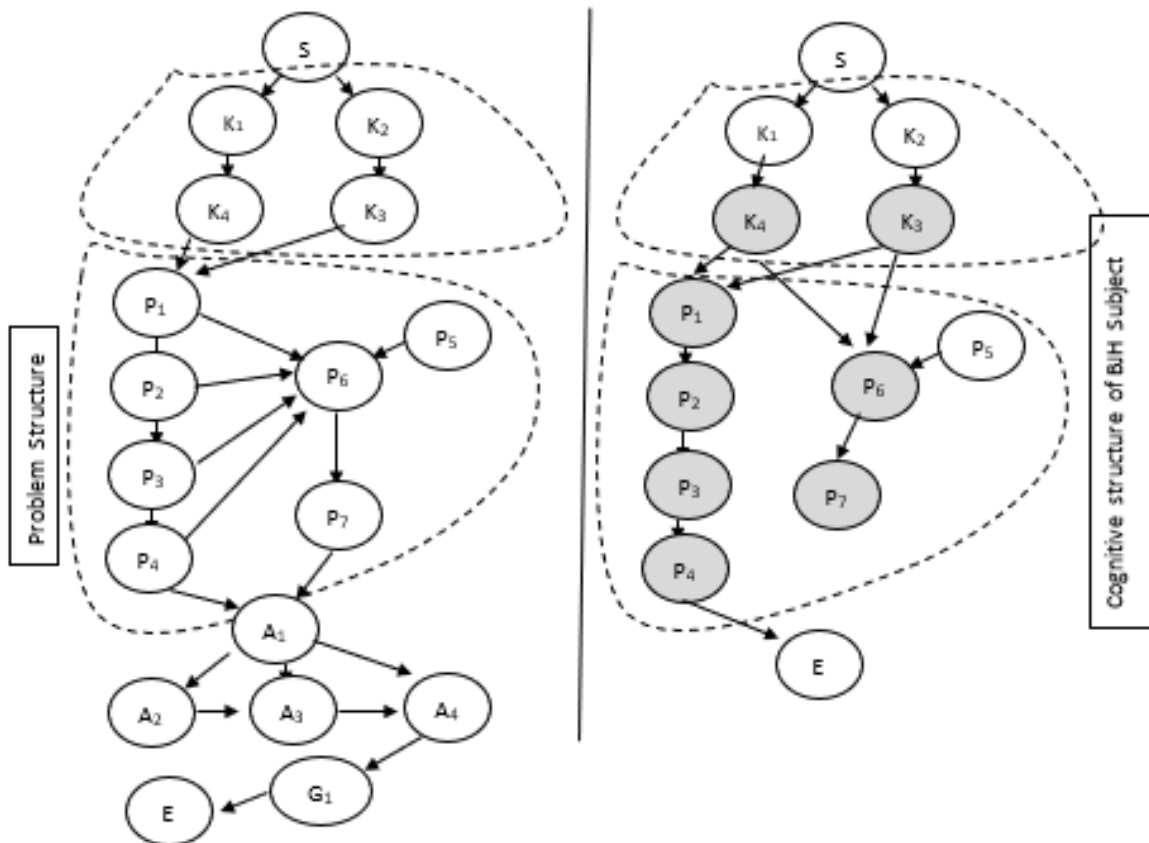


Figure 7. Problem Structure and the Cognitive Structure of BJH Subjects.

The following is an explanation of the coding and thinking structure of BJH in solving matrix algebra problems in table 3.

Table 3. The Remarks of the Notes in Figure 7

Graphic Codes	Remarks
S	Can understand the main problem, namely matrix algebra
K <sub>1</sub>	Can find out a lot of concentrate at the initial condition
K <sub>2</sub>	Can understand the change in concentration
K <sub>3</sub>	Unable to correctly determine the amount of concentrate change that occurred
K <sub>4</sub>	Cannot perform multi-concentrate splits with variables $x_0$ and $y_0$
P <sub>1</sub>	Cannot determine the amount of concentrate after 1 second i.e., $x_1 = 0.9x_0 + 0.2y_0$ and $y_1 = 0.1x_0 + 0.8y_0$ because they cannot compare concentrate changes
P <sub>2</sub>	Unable to determine the amount of concentrate after 2 seconds i.e., $x_2 = 0.9x_1 + 0.2y_1$ and $y_2 = 0.1x_1 + 0.8y_1$
P <sub>3</sub>	Unable to determine the amount of concentrate after 10 seconds i.e., $x_{10} = 0.9x_9 + 0.2y_9$ and $y_{10} = 0.1x_9 + 0.8y_9$
P <sub>4</sub>	Cannot determine the amount of concentrate after n seconds i.e., $x_n = 0.9x_{n-1} + 0.2y_{n-1}$ and $y_n = 0.1x_{n-1} + 0.8y_{n-1}$
P <sub>5</sub>	Can make Cartesian coordinate axes
P <sub>6</sub>	Unable to determine coordinates due to misperception of value
P <sub>7</sub>	Unable to connect coordinate points to form a curve due to incorrect coordinates entered
E	Finished
-----	The concepts are not well connected
—————>	connected concepts

#### (b) Cognitive Structure Middle-Level Algebraic Reasoning Ability

In matrix algebra problems, the structure of the TRS problem is more complete than the structure of the BJH problems. TRS can make connections and can find patterns and relationships but cannot formulate rules. TRS uses the existing cognitive structure to make connections algebraically by excluding NO and N<sub>2</sub>O<sub>2</sub> with variables x and y. TRS subjects were able to know a lot of concentrate at first and understand the change in concentration every second. TRS can also know the amount of change in the concentration of each compound in every second correctly. This can be seen in Figure 8 which shows that the TRS subject knows a lot of the initial concentrate and the change in concentration in the first second and can understand the magnitude of the change in a concentrate that occurs correctly, and can-do equations with variables so that the appropriate mathematical model is obtained, as shown in Figure 8.

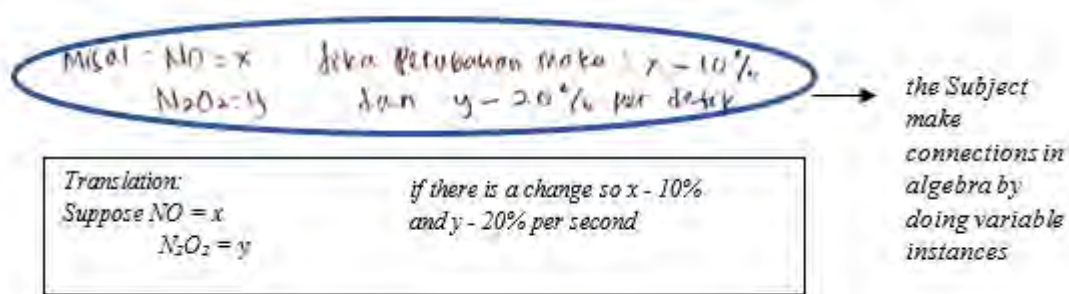


Figure 8. Result of TRS Subjects in Making Connections

From the results, it appears that TRS can compare and analyses the changes in a concentrate that occur every second where every second there is a change that is 10% of NO turns into N<sub>2</sub>O<sub>2</sub>, while a 20% change from N<sub>2</sub>O<sub>2</sub> turns into NO. At the stage of looking for patterns and relationships, TRS begins to compare the many concentrates after the first and second seconds by performing calculations as shown in Figure 9:

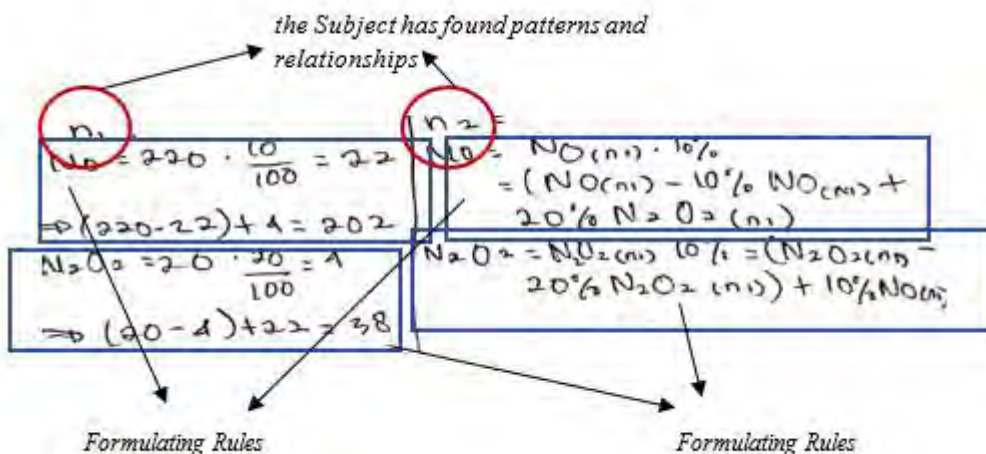


Figure 9. TRS answer results in Formulating Rules

Initially, TRS first looked for many changes that occurred in each compound so that in the first second, the change in NO was 22 ml, and the change in N<sub>2</sub>O<sub>2</sub> was 4 ml. However, TRS has not been able to formulate rules because it cannot make predictions through a pattern correctly. Furthermore, the cognitive structure of TRS in solving matrix algebra problems can be described as shown in Figure 10.

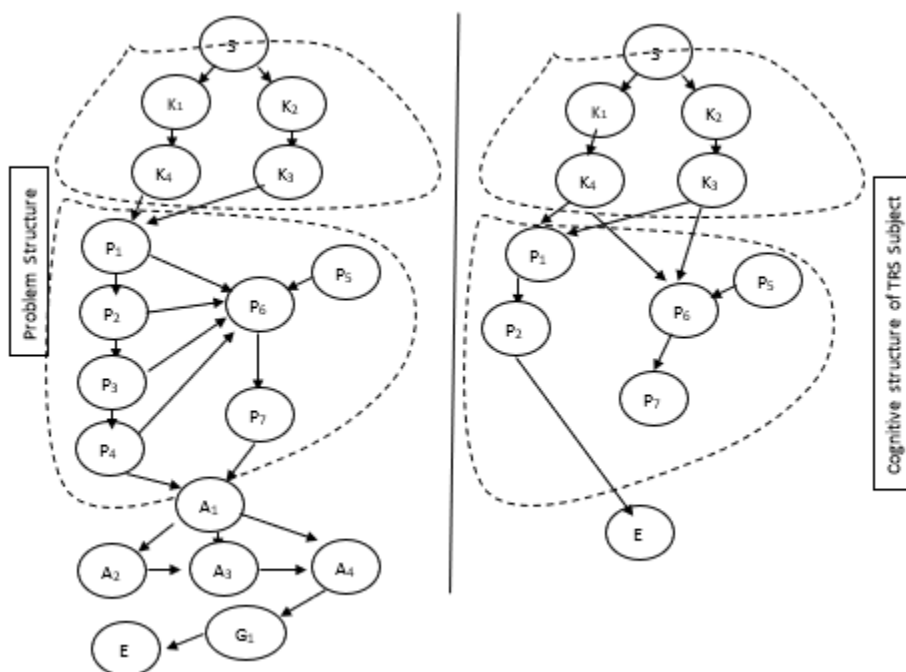


Figure 10. Problem Structure and Cognitive Structure of TRS Subjects.

The following is an explanation of the coding and thinking structure of TRS in solving matrix algebra problems in table 4.

Table 4. The Remarks of the Notes in Figure 10

Graphic Codes	Remarks
S	Can understand the main problem, namely matrix algebra
K <sub>1</sub>	Can find out a lot of concentrate at the initial condition
K <sub>2</sub>	Can understand the change in concentration
K <sub>3</sub>	Can know the amount of concentrate change that occurs correctly
K <sub>4</sub>	Can perform multi-concentrate separation starting with variables $x_0$ and $y_0$
P <sub>1</sub>	Can determine the amount of concentrate after 1 second, namely $x_1 = 0,9x_0 + 0,2y_0$ and $y_1 = 0,1x_0 + 0,8y_0$



Table 4. Continued

Graphic Codes	Remarks
P <sub>2</sub>	Can determine the amount of concentrate after 2 seconds, namely $x_2 = 0,9x_1 + 0,2y_1$ and $y_2 = 0,1x_1 + 0,8y_1$
P <sub>5</sub>	Can make Cartesian coordinate axes
P <sub>6</sub>	Can determine point coordinates due to misperception of value
P <sub>7</sub>	Can connect the coordinate points to form a curve due to the error of the entered coordinates
E	Finished
-----	The concepts are not well connected
—————→	connected concepts

(c). Cognitive Structure Upper-Level Algebraic Reasoning Ability

In the matrix algebra problem, the problem structure of the AMK subject is more complete than the structure of the previous subject problem. AMK can make connections, can identify search patterns and relationships so that it can formulate rules. AMK uses existing cognitive structures to make connections and search for patterns and relationships. This is indicated by the statement from AMK as follows.

AMK: *I find it quite easy to know what is known and asked. because the question has provided an adequate explanation. Then I assume the variable x for NO and variable y for N2O2 to make it easier to pronounce. At first, I wasn't sure how to finish it, but I tried to find the pattern while working on the number of changes in concentrate per second.*

When solving this matrix algebra problem, AMK can already know the initial concentration and understand the change in concentration every second and understand the meaning of the problem given correctly. AMK subjects assume that there are many initial concentrates of NO and N<sub>2</sub>O<sub>2</sub> with variables x and y. The initial steps for completing the AMK can be seen in the following figure 11.

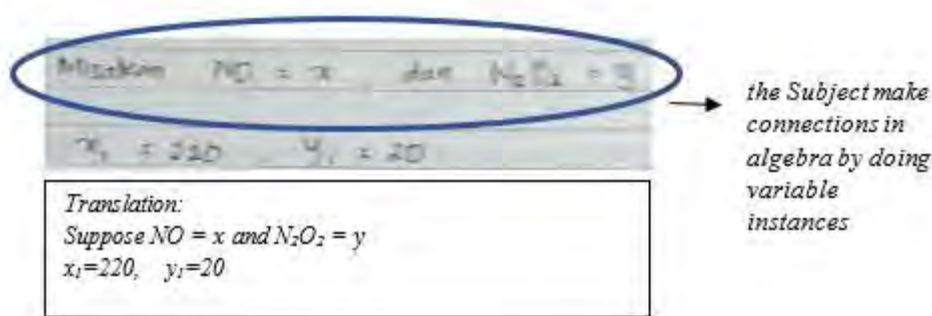


Figure 11. Results of AMK Subjects in Making Connections

From the results of these answers, it appears that the AMK assumes the quantity of the variable x is a lot of NO concentrates, and the variable y is a lot of N<sub>2</sub>O<sub>2</sub> concentrates. Then AMK continued the settlement process by looking for a pattern of many changes in concentrate after one second, there was a change where 10% NO turned into N<sub>2</sub>O<sub>2</sub>, and a 20% change in N<sub>2</sub>O<sub>2</sub> turned into NO. At the stage of looking for patterns and relationships, AMK wrote down the initial amount of concentrate for 220 ml of NO and 20 ml of N<sub>2</sub>O<sub>2</sub>. This can be seen in the results of AMK's answer in Figure 12.

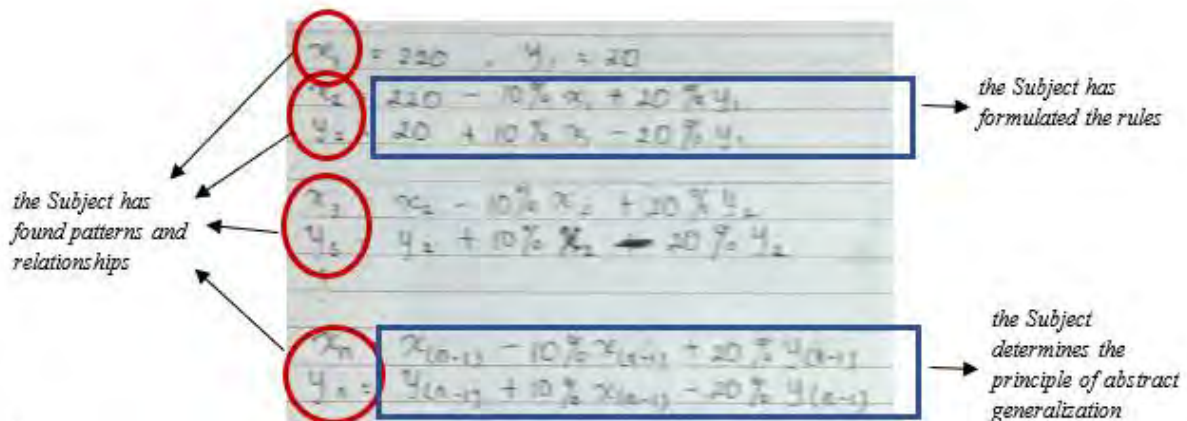


Figure 12. AMK answer results in Formulating Rules and Abstracting Generalizable Principles

From the AMK explanation at the time of the interview, the subject has not simplified the pattern. so that the pattern for calculating the number of NO concentrates, which was originally  $x_2 = x_1 - 10\%x_1 + 20\%y_1$  becomes  $x_2 = 90\%x_1 + 20\%y_1 \leftrightarrow x_2 = 0.9x_1 + 0.2y_1$ . AMK subjects can formulate rules for calculating the number of concentrates after the nth second by correctly predicting a pattern, namely  $x_n = 0.9x_{n-1} + 0.2y_{n-1}$ . Furthermore, the cognitive structure of AMK in solving matrix algebra problems can be described as shown in Figure 13.

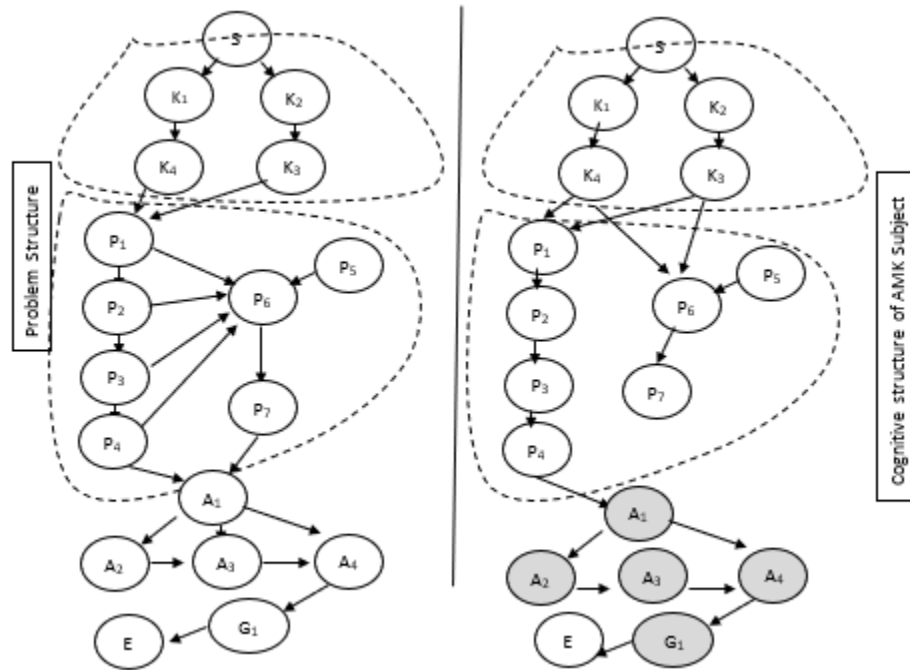


Figure 13. Problem Structure and the Cognitive Structure of AMK Subjects

The following is an explanation of the coding and thinking structure of AMK in solving matrix algebra problems in table 5.

Table 5. The Remarks of the Notes in Figure 13

Graphic Codes	Remarks
S	Can understand the main problem, namely matrix algebra
K <sub>1</sub>	Can find out a lot of concentrate at the initial condition
K <sub>2</sub>	Can understand the change in concentration
K <sub>3</sub>	Can know the amount of concentrate change that occurs correctly
K <sub>4</sub>	Can perform multi-concentrate separation starting with variables $x_0$ and $y_0$
P <sub>1</sub>	Can determine the amount of concentrate after 1 second, namely $x_1 = 0.9x_0 + 0.2y_0$ and $y_1 = 0.1x_0 + 0.8y_0$
P <sub>2</sub>	Can determine the amount of concentrate after 2 seconds, namely $x_2 = 0.9x_1 + 0.2y_1$ and $y_2 = 0.1x_1 + 0.8y_1$
P <sub>3</sub>	Can determine the amount of concentrate after 10 seconds, namely $x_{10} = 0.9x_9 + 0.2y_9$ and $y_{10} = 0.1x_9 + 0.8y_9$
P <sub>4</sub>	Can determine the amount of concentrate after 2 seconds, namely $x_n = 0.9x_{n-1} + 0.2y_{n-1}$ and $y_n = 0.1x_{n-1} + 0.8y_{n-1}$
P <sub>5</sub>	Can make Cartesian coordinate axes
P <sub>6</sub>	Can determine point coordinates due to misperception of value
P <sub>7</sub>	Can connect the coordinate points to form a curve due to the error of the entered coordinates
A <sub>1</sub>	Can calculate the amount of concentrate after 1 second
A <sub>2</sub>	Can calculate the amount of concentrate after 2 second
A <sub>3</sub>	Can calculate the amount of concentrate after 10 second
A <sub>4</sub>	Can calculate the amount of concentrate after n second
E	Finished
-----	The concepts are not well connected
—————>	connected concepts

Based on the results of the study, the researcher described the cognitive structure used by students in processing information in solving matrix algebra problems which can be seen in Table 6.

Table 6. The Cognitive Structures of Students are Reviewed based on the Level of Algebraic Reasoning Ability

Information Process Indicator	Algebraic Reasoning Ability Level		
	Lower	Middle	Upper
Making Connections	imperfect in processing information making connections between prior knowledge and experience as an intermediary between the known and the unknown	can process information make connections between prior knowledge and experience as an intermediary between the known and the unknown	can process information make connections between prior knowledge and experience as an intermediary between the known and the unknown
Finding Patterns and Relationships	not yet aware of the process of converting information into a coding system in search of patterns and relationships	can turn information into a coding system in search of patterns and relationships	can turn information into a coding system in search of patterns and relationships
Formulating Rules	can't formulate rules	can formulate rules	can formulate rules
Abstracting Generalizable Principles	unable to use abstract thinking strategies to generate information systematically	unable to use abstract thinking strategies to generate information systematically	can use abstract thinking strategies to generate information systematically
Cognitive Structures	<i>Comparative thinking</i>	<i>Symbolic Representations</i>	<i>Logical Reasoning</i>

Following table 6, Students with upper-level algebraic reasoning abilities use information processes from making connections, finding patterns and relationships, formulating rules, and abstracting principles to solve matrix algebra problems. Students with upper-level algebraic reasoning abilities have the category of logical reasoning cognitive structure. Meanwhile, students with middle algebraic reasoning abilities only process information from making connections, looking for patterns and relationships, and formulating rules. Thus, cognitive structures fall into the category of symbolic representations. Finally, students with lower algebraic reasoning abilities can only make connections. So, students with lower-level algebraic reasoning abilities have the category of comparative thinking cognitive structure.

### Discussion

The cognitive structure is a basic mental process for understanding information, both in terms of collecting, organizing, and processing information (Garner, 2007). Categorization of cognitive structures include:

- Comparative thinking structures. Process information through the identification of how the data bits are the same and different. This stage is the basis for learning as well as a prerequisite for more complex cognitive structures in the other two categories.
- Symbolic representation structure. Information is transformed into a generally accepted coding system, including verbal and nonverbal language; mathematics; music and rhythm; body move; interpersonal interactions; graphics (two-dimensional drawings, paintings, logos); construction; and simulations, as well as multimedia.
- Logical reasoning structures. Abstract thinking strategies are used to process and generate information systematically, including deductive and inductive reasoning, analogical thinking and hypotheses, causal relationships, analysis, synthesis, evaluation, problem formulation, and problem-solving.

As an effort to develop the cognitive structure of comparative thinking, students need to keep a record of the information received and visualize the information for processing. Although cognitive structures cannot be taught directly, teachers can use the curriculum and their experiences to help develop cognitive structures so that students can develop skills in processing information. Characteristics of cognitive structures include processing information and creating meaning in different ways (Garner, 2007):

#### a. Make connection

Cognitive structures help students make connections between prior knowledge and experience as intermediaries between the known and the unknown. It is very important to ask students what the information that educators share with students means.

#### b. Finding relationships and patterns

Cognitive structures help students compare, analyze, and organize information into patterns and relationships. All learning is based on two relationships, i.e., something has meaning when compared and harmonized with others.

### c. Formulating the rules

Cognitive structures help students formulate rules that make information processing predictable. Most educators teach the rules first and then ask students to apply the rules by making connections and patterns. However, this makes it difficult for students to use the rules when the problem given is different from the example given. For this reason, it is advisable to make connections and find patterns and relationships before formulating rules. This can make the information received by students last longer to remember the rules because they were created by themselves.

### d. Generalization

Cognitive structures help students abstract generalizable principles that apply or change to situations other than the original learning context. Principles that can be generalized can certainly clarify understanding and can be applied to various situations.

Strategies to develop cognitive structures can be done by encouraging someone to reflective awareness and imagination in visualization to develop students' algebraic reasoning abilities. Students with upper-level algebraic reasoning abilities can process the information obtained to solve matrix algebra problems. Subjects with upper-level algebraic reasoning abilities use cognitive structures to make connections, identify search patterns and relationships, to formulate rules. At the stage of making connections, students with upper-level algebraic reasoning abilities already know the initial concentration and understand the occurrence of changes every second to make examples with variables. In the stage of looking for patterns and relationships, the subject can be comparing analyzed information into patterns and relationships. Students can formulate rules to get concentration every second with fast and precise predictions. In the last stage in the information process, students with advanced algebraic reasoning abilities can generalize matrix equations to obtain the number of concentrates in general. In essence, students with upper-level algebraic reasoning abilities can solve matrix algebra problems well and can write down the steps of completion in a coherent manner according to the given problem. Based on the results of interviews, it is known that students with upper-level algebraic reasoning abilities use cognitive structures to process information and solve to get the right results.

Students with middle-algebraic reasoning abilities have not processed all the information obtained to solve matrix algebra problems. Subjects capable of middle-level algebraic reasoning use cognitive structures only at the stage of making connections and identifying search patterns and relationships. At the stage of making connections, students with middle-level algebraic reasoning abilities already know the initial concentration and understand the occurrence of changes every second to make examples with variables. In the stage of finding patterns and relationships, the Subject can compare the changes in concentration every second but cannot analyze the equations formed. The Subject tries to predict but cannot formulate rules to make the information concentrate on the amount of concentration per second. So that the generalization process also cannot be generated. In essence, students with middle-level algebraic reasoning abilities can understand the process of information obtained, but they are not perfect in making the right predictions.

Students with lower-level algebraic reasoning abilities cannot process information appropriately to solve matrix algebra problems. Subjects with lower-level algebraic reasoning abilities use cognitive structures that they already must make connections but are not yet perfect in processing connections. At the stage of making connections, students with lower-level algebraic reasoning abilities already know the initial concentration but have difficulty imagining the initial concentration with variables. The subject cannot compare the changes in concentration every second and cannot analyze the equations that are formed by the mathematical model. The next stage is that students cannot formulate rules to get concentration every second with fast and precise predictions. In essence, students with lower-level algebraic reasoning abilities can only process information at the stage of making connections but are not perfect.

Vygotsky put forward several ideas about the Zone of Proximal Development (ZPD). Vygotsky argues that a series of tasks that are too difficult for students to master independently but can be learned with the help of adults or more capable students (Taber, 2020). To understand the limits of a child's ZPD there is an upper limit, namely the level of responsibility or additional tasks that a child can do with the help of a capable instructor. Which the child can solve alone. ZPD according to Vygotsky, shows the importance of social influence especially the influence of instruction or teaching on children's cognitive development (Basir & Wijayanti, 2020).



Figure 14. ZPD Concept Illustration

The shaded area describes the area in figure 14 of development that students get when learning independently without the help of others. Everyone's ZPD is developing but, there are limitations in its development. Zone of Proximal Development students is individual so that the provision of teacher assistance to students in the classroom will vary depending on the level of students' cognitive structure. Provision of various types and levels by teachers to students to facilitate them in solving problems (scaffolding).

Forms of effort that teachers can do to improve algebraic reasoning abilities include the teacher delivering the material in stages, the teacher providing explanations from concrete to abstract, the teacher giving examples of questions from simple things to more complex things, or teachers starting from easy concepts. Every time student learn a new concept, student need to pay attention to the previous concept or material (Keller et al., 2020). In this case, it can also repeat the concept by expanding and deepening the learning material. In addition, teachers can also manage classrooms by conducting teaching cycles according to the target. In conducting the assessment, the teacher must prepare the material, provide direction to students, and provide sufficient time to invite students to practice and adjust student performance to the learning progress zone.

### Conclusion

Students with upper-level Algebraic Reasoning Abilities have a logical reasoning cognitive structure, where students can make connections, find patterns and relationships, formulate rules, and generalize. Students with middle-level algebraic reasoning abilities have a symbolic representations cognitive structure. Students can make connections and find patterns and relationships but have not been able to formulate rules and generalize. Students with lower-level algebraic reasoning abilities have a comparative thinking cognitive structure. Students are only able to make connections and perform mechanistic actions without looking for patterns and relationships, establishing rules, and generalizing.

### Recommendations

Based on the results of the research, the researcher provides suggestions for the development of further research. First, it is necessary to research the structure of pseudo thinking in solving algebraic reasoning problems. second, appropriate learning strategies to improve algebraic reasoning abilities. Third, the development of teaching materials as a facility in written form scaffolding to defragment cognitive structures.

### Limitations

This study is limited to subjects with upper, middle, and lower-level algebraic reasoning abilities in solving algebraic problems that aim to identify cognitive structures. Thus, it is still very open to conducting research related to algebraic reasoning in terms of cognitive style or learning style. Algebraic thinking processes can also be identified using the theory of assimilation and accommodation.

### Acknowledgements

Thank you to the Semarang State University and Sultan Agung Islamic University, Indonesia, for supporting this research.

### Funding

Thank you to the Minister of Education, Culture, Research, and Technology of the Republic of Indonesia for providing financial support in the form of a dissertation research grants number 044/SP2H/LT/DRPM/2021.

### Authorship Contribution Statement

Basir: Conceptualization and design, drafting manuscript, writing. Waluya: reviewing, supervision and final approval. Dwijanto: critical revision of manuscript. Isnarto: data acquisition and data analysis.

### References

- Ayber, G., & Tanışlı, D. (2017). An analysis of middle school mathematics textbooks from the perspective of fostering algebraic thinking through generalization. *Educational Sciences: Theory & Practice*, 17(6), 2001–2030. <https://doi.org/10.12738/estp.2017.6.0506>
- Basir, M., & Wijayanti, D. (2020). Strategies to provide scaffolding when teaching mathematical reasoning. In B. Santoso, H.P. Adi, H. Sulistyono, D. Wijayanti, C. Anwar, & A. F. Ogunbaado (Eds.), *Proceedings of the 1st International Conference on Islamic Civilization, ICIC 2020, Indonesia, 201-208*. European Alliance for Innovation (EAI). <https://doi.org/10.4108/eai.27-8-2020.2303266>



- Basir, M. A., Waluya, S., Dwijanto, & Isnarto. (2021). Development and use of test instruments to measure algebraic reasoning based on cognitive systems in Marzano's taxonomy. *European Journal of Mathematics and Science Education*, 2(2), 163-175. <https://doi.org/10.12973/ejmse.2.2.163>
- Cai, J., Lew, H. C., Morries, A., Moyer, J. C., Ng, S. F., & Schmittau, J. (2005). The development of students' algebraic thinking in earlier grades. *ZDM - International Journal on Mathematics Education*, 37(1), 5-15. <https://doi.org/10.1007/BF02655892>
- Cañadas, M. C., Brizuela, B. M., & Blanton, M. (2016). Second graders articulating ideas about linear functional relationships. *Journal of Mathematical Behavior*, 41(3), 87-103. <https://doi.org/10.1016/j.jmathb.2015.10.004>
- Garner, B. (2007). *Getting to "got it!": Helping struggling students learn how to learn*. Association for Supervision and Curriculum Development.
- Hawes, Z., & Ansari, D. (2020). What explains the relationship between spatial and mathematical skills? A review of evidence from brain and behavior. *Psychonomic Bulletin and Review*, 27(3), 465-482. <https://doi.org/10.3758/s13423-019-01694-7>
- Jupri, A., Drijvers, P., & van den Heuvel-Panhuizen, M. (2014). Difficulties in initial algebra learning in Indonesia. *Mathematics Education Research Journal*, 26(4), 683-710. <https://doi.org/10.1007/s13394-013-0097-0>
- Keller, A. S., Davidesco, I., & Tanner, K. D. (2020). Attention matters: How orchestrating attention may relate to classroom learning. *CBE Life Sciences Education*, 19(3), 1-9. <https://doi.org/10.1187/cbe.20-05-0106>
- Kusmaryono, I., Jupriyanto, & Kusumaningsih, W. (2021). Construction of students' mathematical knowledge in the zone of proximal development and zone of potential construction. *European Journal of Educational Research*, 10(1), 341-351. <https://doi.org/10.12973/eu-jer.10.1.341>
- Li, N., Mok, I. A. C., & Cao, Y. (2019). The evolution of mathematical thinking in Chinese mathematics education. *Mathematics*, 7(3), 1-18. <https://doi.org/10.3390/math7030297>
- Loibl, K., & Leuders, T. (2019). How to make failure productive: Fostering learning from errors through elaboration prompts. *Learning and Instruction*, 62, 1-10. <https://doi.org/10.1016/j.learninstruc.2019.03.002>
- Matlin, M. W. (2009). *Cognition* (7th Ed.). John Wiley & Sons.
- Miles, M. B., Huberman, A. M., & Saldana, J. (2014). *Qualitative data analysis: A methods sourcebook* (3rd ed.). SAGE Publications.
- National Council of Teachers of Mathematics. (2016). *Focus in high school mathematics: Reasoning and sense making*. United States of America.
- Siagian, M. V., Saragih, S., & Sinaga, B. (2019). Development of learning materials oriented on problem-based learning model to improve students' mathematical problem solving ability and metacognition ability. *International Electronic Journal of Mathematics Education*, 14(2), 331-340. <https://doi.org/10.29333/iejme/5717>
- Taber, K. S. (2020). Mediated learning leading development: the social development theory of lev vygotsky. In B. Akpan & T. J. Kennedy (Eds.), *Science Education in Theory and Practice: An Introductory Guide to Learning Theory* (pp. 277-291). Springer. [https://doi.org/10.1007/978-3-030-43620-9\\_19](https://doi.org/10.1007/978-3-030-43620-9_19)
- Thuneberg, H. M., Salmi, H. S., & Bogner, F. X. (2018). How creativity, autonomy and visual reasoning contribute to cognitive learning in a STEAM hands-on inquiry-based math module. *Thinking Skills and Creativity*, 29(1), 153-160. <https://doi.org/10.1016/j.tsc.2018.07.003>
- Zayyadi, M., Nusantara, T., Hidayanto, E., Sulandra, I. M., & As'ari, A. R. (2019). Exploring prospective student teacher's question on mathematics teaching practice. *Journal of Technology and Science Education*, 9(2), 228-237. <https://doi.org/10.3926/jotse.465>