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Geogebra in the Correction and Understanding of Errors Regarding the Concept of Asymptote

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Abstract
The aim of this study was to investigate the effectiveness of GeoGebra software in preservice mathematics teachers' process of correcting and making sense of errors regarding the concept of the asymptote. Case study method, one of the qualitative approaches, was adopted in the study. The study group consisted of 11 preservice teachers studying in the mathematics teaching program. This study comprised of three parts: pre-GeoGebra application, during the GeoGebra application and after the GeoGebra application. Data were analyzed using descriptive analysis method. The participants' written answers before and after the GeoGebra-assisted application revealed that their approaches to possible errors regarding the concept of asymptote changed positively. Moreover, interviews with three participants who showed "development" "a partial development" and "no development " supported this result.

Keywords: Geogebra, Asymptote, Error Approach, Preservice Mathematics Teacher

Introduction

The dominant role of technology in the education has increased the importance of visualization in learning and teaching. While visualization opens up new ways for mathematical thinking, it also supports the students' visual reasoning process (Tall, 1991; Zimmermann & Cunningham, 1991). Visualizing a mathematical concept makes the understanding of a mathematical concept easier for students. The use of visualization, particularly, increases permanence and improves problem-solving skills while teaching concepts (Jencks & Peck, 1972). Today, computer technologies are commonly used for visualization in mathematics education. Instead of using technology merely as a presentation tool, designing learning environments in which students interact with mathematical software at various levels is considered significant (Kabaca, 2016). Several dynamic software enabling more effective computer use is encouraged by the predominant role of information and communication technologies. GeoGebra, one of these pieces of dynamic software, supports effective mathematics teaching (Hohenwarter, Preiner & Yi, 2007). The use of GeoGebra software in mathematics education allows the visualization of concepts (Guncaga & Majherova, 2012). In fact, due to the algebra and graphics functions, not only representations of mathematical objects are provided but also a change made in one of these functions also occurs in the other (Hohenwarter & Preiner, 2007b). GeoGebra, the aim of which is to teach and learn mathematics, is designed to help students understand mathematics better. It is a software that can be used from elementary school to university level (Hohenwarter & Preiner, 2007a). Particularly, it is a versatile tool for mathematics education in elementary schools. Teachers use it for demonstration, visualization and preparation of teaching materials. Students can also use mathematics as a dynamic tool to explore (Hohenwarter, 2004; Hohenwarter & Fuchs, 2004).

The studies investigating GeoGebra software have shown that it supports visualization and can facilitate error correction. The studies on error detection regarding any concept in mathematics covers issues such as preservice teachers' approaches to student errors (Baştürk, 2009; Crespo, 2000; Crespo, 2003) and teachers' responses to student errors and their analysis of errors (Haydar, Vatuk & Angulo, 2009; Kafoussi & Skoumpourdi, 2006; Abu Mokh, Othman & Shahbari, 2019; Peng & Song, 2008).

However, some studies also have dealt with professional errors. For example, Borasi (1989) used professional errors to determine students' approaches to errors. She concluded that since students were not able respond to solutions containing professional error immediately, they could be supported to produce new ideas about the concept. In another study in which professional errors were used, Demirci, Özkaya and Konyalıoğlu (2017) examined the approaches of preservice mathematics teachers towards incorrect solutions of probability problems. Özkaya (2015) attempted to improve the general knowledge of mathematics teachers with the professional errors based experiment.

GeoGebra software is considered to be effective in dealing with errors and professional errors. Karakuş and Konyalıoğlu (2018) used GeoGebra-assisted instruction to develop preservice mathematics teachers' ability to explain professional errors made by researchers concerning the concepts of extremum and turning point. They found a significant increase in the ability to detect errors. Considering their study, which took into account the errors related to extremum and turning points and correcting them, the concept of asymptote associated with these concepts stands out. The concept of asymptote is included mathematics teaching program at university. Furthermore, a full understanding of this concept will facilitate a number of topics such as function graphing. In the study conducted by Özgen and Alkan (2012), two vertical and one horizontal asymptote were provided and the students were expected to find the rule of the function appropriate to these conditions and draw the graph. The aim was that the students sought answers to questions such as "what are the conditions for vertical and horizontal asymptotes, how does asymptotes occur in the graph?" and that they were encouraged to review, manage and shape their knowledge. Besides, Duran, Doruk and Kaplan (2017) investigated the content knowledge of high school mathematics teachers about the concept of the asymptote. In addition, Nair (2010) aimed to investigate students' perceptions about asymptotes of rational functions and to understand the relationship that students developed between the concepts of asymptote, continuity, and limit which are closely related to each other. Furthermore, Kidron (2011) focused on the process of structuring knowledge related to horizontal asymptote through different tasks.

The asymptote is one of the significant concepts at the university level for mathematics education students. As mentioned above, it forms the basis of many subjects, especially drawing function graphs. Furthermore, it is among the subjects that will contribute to the development of content knowledge of pre-service teachers. Hence, it is essential for pre-service teachers to understand the concept of asymptote and to be able to put this concept into practice. It is anticipated that GeoGebra software will contribute to understanding of the asymptote concept and even to make sense of errors related to this concept. To the best of the researchers, a GeoGebra-assisted study aimed at improving the ability to explain professionally made errors about the concept of asymptote cannot be found in the literature. In this sense, the present study aimed to investigate the effectiveness of GeoGebra software in preservice mathematics teachers' process of correcting and making sense of errors regarding the concept of the asymptote. In line with this purpose, answers to the following questions were sought:

- How do the preservice mathematics teachers deal with asymptote problems before GeoGebra assisted application?

- How do the preservice mathematics teachers deal with asymptote problems after GeoGebra assisted application?
- What are the views of preservice mathematics teachers about GeoGebra assisted application?

Method

Research Method

In qualitative studies, the situations, events or phenomena under investigation are examined in a holistic way from the perspectives of the individuals constituting the sample (Ekiz, 2009; Metin, 2014). The case study method provides the opportunity to profoundly collect the information obtained from the sample and to present it effectively (Creswell, 2015). Since the preservice teachers' approaches to possible errors related to the concept of asymptote were examined in detail before and after the GeoGebra application, the case study method was used in this study. In addition, as the phenomena were explained comprehensively and longitudinally, on the basis of the GeoGebra application process, Program Implementation Case Study (cited in Aytaçlı, 2012), one of the six different types of case studies defined by Datta (1990), was adopted.

Study Group

This study was carried out with 11 preservice mathematics teachers, 10 female and 1 male, studying in the third grade of a state university in Turkey. All of them took the courses such as Analysis-I, Analysis-II, which were expected to contribute to the formation of the concept of the asymptote. They also had a knowledge of how to use the GeoGebra software since they attented the Computer-Aided Mathematics Teaching-I course. The participation to the study was on a voluntarily basis and the participants were anonymized and coded as T1, T2, T3..., T11 instead of using their real names.

Data Collection Tool

A data collection tool, used both before and after the application, was developed by the researchers. The data collection tool consisted of 14 question which were either correct or incorrect. Each question consisted of a space that the participants can encode as T (true) or F (false), and can explain the reason for their answers. In order to develop the questions in the data collection tool, different sources were used and a question pool containing questions about asymptotes was formed. Questions were selected from this pool in line with the aim of the study. To prevent the fact that the participants may think that each question was wrong, question that were correct within the framework of the research purpose were also included. Expert opinion for the questions were obtained from two experts in the field or mathematics education and the final version of the data collection tool, whose validity and reliability studies were conducted, was developed. The data collection tool is presented in the appendix.

Data Collection Process

The study consisted of three stages. In the first stage, to determine the participants' knowledge levels about asymptotes before the GeoGebra application, the data collection tool was applied to the participants and their written answers were obtained.

In the second stage of the study, a GeoGebra assisted application on the topic of asymptotes was performed. This application was carried out in a computer laboratory with a smart board and a computer in front of each participant, through a GeoGebra-supported lecture by the instructor and mutual question and answer session. An example of the application is as follows:

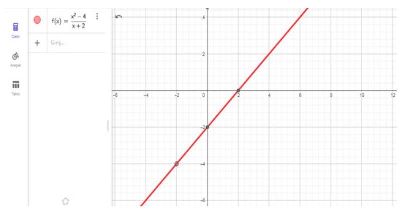


Figure 1: The Examination of the Asymptotes of the $(x^2-4)/(x+2)$ Function

In this example, a discussion environment was created by making the participants think about how the graph and asymptotes of the f (x) $=(x^2-4)/(x+2)$ function would be. They were then expected to draw the function in GeoGebra. In the discussion section, they were asked to comment on the graphics they thought initially and they drew. The purpose of this quesiton was to make them aware of that x=2 is not a vertical asymptote since both the denominator and the numerator are zero, and also that the oblique asymptote coincides with the graph. Another example of implementation is shown in Figure 2.

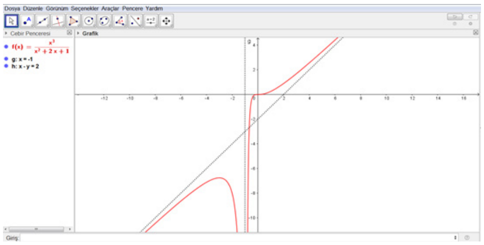


Figure 2: The Examination of the Asymptotes of $x^3/(x^2+2x+1)$ Function

In this example, a classroom discussion was carried out through making the participants think about the asymptotes of the f (x) $=x^3/(x^2+2x+1)$ function. They were then asked to draw the graph in GeoGebra and question which asymptotes the function had and whether their thoughts about the function and its asymptotes were correct. They were also expected to conclude that the oblique asymptote could interrupt the graph and that the oblique asymptote could intersect with the vertical asymptote and to question the reason for this

The third stage was performed after the GeoGebra application. Three weeks after the application, the data collection tool used in the first stage were re-applied to the participants. In this manner, the effectiveness of the GeoGebra application in determining and correctly explaining the errors related to the concept of asymptote was investigated.

In addition, on the basis of the pre- and post-application answers, preservice teachers were divided into three categories: "no development", "partial development" and "development". Two separate interviews were conducted with 3 participants in each category. One of these interviews was related to the written explanations they provided in the data collection tool (Interview 1) and the other (Interview 2) focused on questioning the effectiveness of the application. In Interview 1, the preservice teachers verbally explained their statements on paper. In Interview 2, questions such as "What impact does GeoGebra have on your final answers?", "What and how did GeoGebra help you see?" and "What are the pros and cons of teaching with GeoGebra?" were asked to participants. Similarly, the effectiveness of teaching was questioned in the second interviews. The interviews were audio-recorded.

Data Analysis

Data were analyzed using descriptive analysis method. In descriptive analysis, data are examined in line with the predetermined themes and direct quotations are presented (Yıldırım & Şİmşek, 2013). The categories and codes in Karakuş and Konyalıoğlu (2018) were used in the study. For each statement, "correct detection", "incorrect detection" and "unanswered" categories were established and the codes "correct explanation", "incorrect explanation", "incomplete explanation" and "no explanation" were formed under these categories.

Validity and Reliability

Validity and reliability are different in qualitative studies (Yıldırım & Şimşek, 2013). In this context, the studies carried out to ensure the validity and reliability of the present study are presented in Table 1 below.

Table 1: Validity and Reliability Studies of the Present Study

Validity and Reliability	Strategy Used	Explanation
Validity	Long term interaction	The study lasted five weeks with the application of the data collection tools.
	Expert review	The research findings were reviewed by two experts.
	Peer checking	Transcripts of the interviews were examined by the preservice teachers. The preservice teachers gave feedback about the accuracy of what were written.
	Detailed description	Direct quotations from the data obtained were included.

Validity	Purposive sampling	The preservice teachers who took Analysis-I, Analysis-II and Computer-Aided Mathematics Teaching-I course were selected.
Reliability	Consistency review	After the findings obtained from the data collection tools were re-evaluated by two researchers, the final version of the findings was presented in the study.
	The role of the researcher	Considering the possibility of bias of the researcher, the interviews were audio-recorded.
	Detailed description of the sample	Detailed information about the sample can be found in the study group section.

Results

In this section, the findings before and after the application is presented in a table and compared with each other. The results of the interviews with the participants are also included. The frequency of the answers obtained in the pre-application stage is presented in Table 2.

Table 2: The Frequency of the Answers in Pre-application Stage

Correct Detection				Incorrect Detection				Unanswered
C.E	I.E	M. E	N. E	C.E	I.E	M.E	N.E	
7(T1,T2,T4, T5,T6,T7,T11)	0	1(T9)	1(T8)	0	0	0	2(T3,T10)	0
0	0	2(T6,T7)	2(T2,T9)	0	3(T1,T10,T11)	0	3(T3,T4,T8)	1(T5)
1(T7)	1(T10)	0	3(T2,T3, T6)	0	3(T1,T5,T11)	0	3(T4,T8,T9)	0
2(T7,T11)	0	0	4(T4,T6,T 8,T9)	0	2(T5,T10)	0	3(T1,T2,T3)	0
1(T6)	0	0	2(T4,T8)	0	3(T7,T10,T11)	0	5(T1,T2,T3, T5,T9)	0
1(T6)	1(T3)	0	3(T1,T5,T8)	0	3(T4,T9,T11)	0	3(T2,T7,T10)	0
2(T2,T8)	1(T11)	2(T1,T4)	3(T6,T9,T10)	0	0	0	2(T3,T5)	1(T7)
3(T1,T6,T8)	0	3(T5,T7, T11)	4(T2,T3, T9,T10)	0	0	0	1(T4)	0
0	2(T1,T5)	0	2(T2,T6)	0	4(T4,T9,T10, T11)	0	2(T3,T7)	1(T8)
1(T11)	0	1(T9)	3(T4,T6,T10)	0	3(T3,T5,T7)	0	3(T1,T2,T8)	0
2(T1,T7)	0	0	0	0	9(T2,T3,T4,T5,T6, T8,T9,T10,T11)	0	0	0
5(T1,T7,T8, T9,T11)	1(T3)	1(T2)	0	0	4(T4,T5,T6,T10)	0	0	0
3(T6,T7,T8)	3(T3, T4,T9)	1(T2)	1(T1)	0	3(T5,T10,T11)	0	0	0
0	1(T9)	0	0	0	8(T1,T2,T3,T4, T6,T7,T8, T10)	0	1(T11)	1(T5)

*C.E: Correct Explanation, I.E: Incorrect Explanation, M.E: Incomplete (Missing) Explanation, N.E: No Explanation

Table 2 showed that a number of participants misidentified the error and provided an incorrect explanation. In addition, the number of participants who identified the error correctly or incorrectly but did not make a statement was quite high. In other words, there were many questions that the participants did not explain despite making a determination. Besides, regarding some questions, it was found that the number of correct detections and correct explanations was high. For example, in Question 1, nine participants provided a correct detection and seven of them provided the correct explanation. Two participants who made an incorrect detection did not provide any explanations. Especially, regarding the Questions 11 and 14, it was observed that 9 preservice teachers provided an incorrect detection and incorrect explanation.

The frequency of the answers obtained in the post-application stage is shown in Table 3.

Table 3: The Frequency of the Answers in Post-application Stage

Correct Detection				Incorrect Detection				Unanswered
C.E	I.E	M.E	N.E	C.E	I.E	M.E	N.E	
7(T1,T2,T3, T4,T5,T6,T7)	0	1(T8)	2(T9,T11)	0	0	0	0	1(T10)
3(T4,T6,T8)	0	2(T1,T7)	5(T2,T3, T5,T9,T11)	0	1(T10)	0	0	0
3(T4,T6,T8)	0	2(T1,T7)	4(T2,T5, T9,T11)	0	0	0	2(T3,T10)	0
4(T4,T6,T7,T8)	0	2(T1,T10)	5(T2,T3, T5,T9,T11)	0	0	0	0	0
5(T1,T2, T6,T7,T8)	0	1(T4)	2(T3,T10)	0	0	1(T5)	2(T9,T11)	0
6(T1,T2,T5, T6,T7,T8)	0	1(T4)	1(T3)	0	1(T9)	0	2(T10,T11)	0
4(T1,T2,T6,T8)	0	3(T4, T5,T7)	3(T9,T10,T11)	0	0	0	1(T3)	0
7(T1,T2,T4, T5,T6,T7,T8)	0	0	3(T3,T9,T11)	0	0	0	1(T10)	0
0	0	0	0	0	5(T5,T6, T7,T9,T10)	0	6(T1,T2,T3, T4,T8,T11)	0
2(T7,T11)	0	0	6(T1,T2,T4, T5,T8,T10)	1(T6)	0	0	1(T9)	1(T3)
3(T1,T2,T7)	0	0	0	0	8(T3,T4,T5,T6, T8,T9,T10,T11)	0	0	0
7(T1,T4,T6, T7,T8,T9,T10)	0	3(T2,T5,T11)	0	0	0	0	1(T3)	0
4(T1,T3, T6,T10)	1(T11)	4(T2,T4, T7,T8)	0	0	2(T5,T9)	0	0	0
4(T2,T3, T6,T10)	0	0	0	0	6(T1,T4,T7, T8,T9,T11)	0	0	1(T5)

*C.E: Correct Explanation, I.E: Incorrect Explanation, M.E: Incomplete (Missing) Explanation, N.E: No Explanation

Table 3 revealed that the participants generally provided a correct detection and a correct explanation after the application. In the first question, all of the participants provided a correct detection and a correct explanation, except for one participant who did not answer the question. The number of the participants providing an incorrect detection exceeded the number of those providing the correct detection only in Questions 9, 11, and 14. Regarding Question 9, no preservice teacher provided the correct detection and the preservice teachers either provided an incorrect explanation or did not provide an explanation at all. For Question 10, most of the participants who made the correct detection did not provide any explanation. In Question 13, although a participant correctly detect the error, he/she came up with an incorrect explanation. Regarding Question 10, a participant made an incorrect detection, however, he/

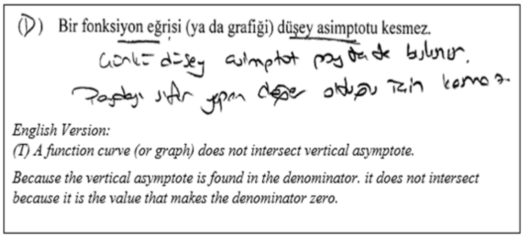
she provided a correct explanation. The participants almost did not left any questions unanswered.

The examination of the frequency of the answers before and after the application showed that there was an increase in the number of the participants providing the correct detection except for two questions. The number of the participants providing the correct detection remained constant in Question 8, and decreased in Question 9. However, it should be noted that in Question 9, although there were some participants who correctly detected the error before the application, none of the participants provided a correct or missing explanations. For Question 8, the number of the participants providing the correct detection remained constant; however, the number of those providing the correct explanation increased.

In general, the number of the participants who provided the incorrect detection and incorrect explanation decreased after the application. However, there was not any change in some questions. Only in Question 9, there was an increase in the number of the participants providing the incorrect detection and incorrect explanation. There was also an increase in the number of those who provided an incorrect detection and did not provide an explanation.

No significant change was observed in the questions remained unanswered.

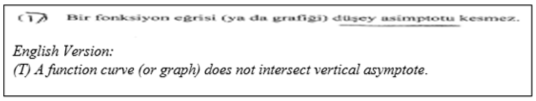
Some excerpts of the participants' answers before and after the application are presented below:


(D) Bir fonksiyon eğrisi (ya da grafiği) düşey asimptotu kesmez.
Çünkü düşey asimptot paydada bulunur. Paydayı sıfır yapan değer olduğu için kesmez.

English Version:
(T) A function curve (or graph) does not intersect vertical asymptote.
Because the vertical asymptote is found in the denominator. it does not intersect because it is the value that makes the denominator zero.

Figure 3: T_9's Answer for Question 1 Before the Application

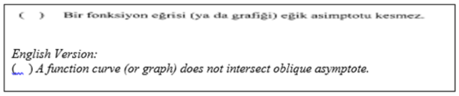
T_9 provided a "correct detection" by saying that the statement was correct given before the application. He/she explained that the vertical asymptote was in the denominator, emphasizing that it would not intersect because it was the value that made the denominator zero. However, here it was seen that the participant did not consider whether the numerator would or would not be zero when the denominator is zero and thus he/she provided a "missing explanation."


(T) Bir fonksiyon eğrisi (ya da grafiği) düşey asimptotu kesmez.

English Version:
(T) A function curve (or graph) does not intersect vertical asymptote.

Figure 4: T_9's Answer for Question 1 After the Application

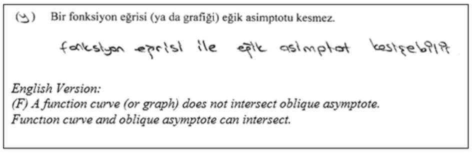
T_9 also stated that the statement was correct after the application and made a "correct detection." However, he/she did not provide an explanation and thus it was evaluated under the code "no explanation".


( ) Bir fonksiyon eğrisi (ya da grafiği) eğik asimptotu kesmez.

English Version:
( ) A function curve (or graph) does not intersect oblique asymptote.

Figure 5: T_5's Answer for Question 2 Before the Application

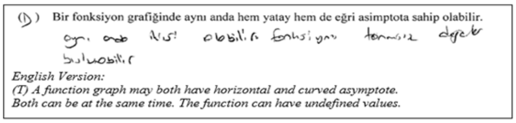
T_5 did not provide an answer and an explanation to Question 2 before the application w. Therefore, T_5 was evaluated in the "unanswered" category.


(Y) Bir fonksiyon eğrisi (ya da grafiği) eğik asimptotu kesmez.
fonksiyon eğrisi ile eğik asimptot kesişebilir

English Version:
(F) A function curve (or graph) does not intersect oblique asymptote.
Function curve and oblique asymptote can intersect.

Figure 6: T_5's Answer for the Second Expression After the Application

After the application, T_5 provided a "correct detection", stating that the statement was incorrect. However, T_5 did not provide any explanation regarding why the statement was incorrect. Therefore, the explanation of the preservice teacher was evaluated under the "no explanation" code.


(D) Bir fonksiyon grafiğinde aynı anda hem yatay hem de eğri asimptota sahip olabilir.
aynı anda ikisi olabilir. fonksiyon tanımsız değer bulunabilir

English Version:
(T) A function graph may both have horizontal and curved asymptote.
Both can be at the same time. The function can have undefined values.

Figure 7: T_4's Answer for Question 6 Before the Application

T_4 stated that the statement was correct and thus made an "incorrect detection" before the application. In the explanation, T_4 mentioned that there can be both horizontal and curved asymptote in a function graph. Then, he/she provided an explanation for the vertical asymptote by emphasizing the values for which the function is undefined. Therefore, it was found that the preservice teacher made an "incorrect explanation."

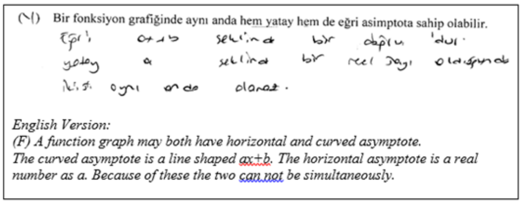


Figure 8: T_4's Answer for the Sixth Expression After the Application

After the application, T_4 stated that the statement was incorrect and thus made the "correct detection". Although his/her explanation showed that his/her thinking was correct in general, he/she mixed the oblique and curved asymptotes and provided an expression for oblique asymptote instead of the curved asymptote. His/her logic was correct; nevertheless, he/she could not exactly state the expected answer. Therefore, it was decided that he/she made a "missing explanation".

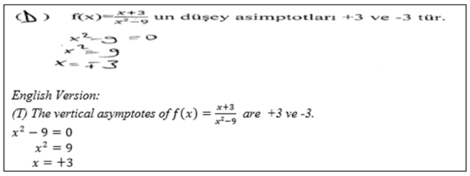


Figure 9: T_2's Answer for Question 11 Before the Application

T_2 said that the statement was correct and thus made the "incorrect detection" before the application. He/she explained that he/she found the vertical asymptotes of the function as -3 and +3 as a result of his/ her operations. Here, T_2 ignored the fact that the numerator should not be zero and made an "incorrect explanation".

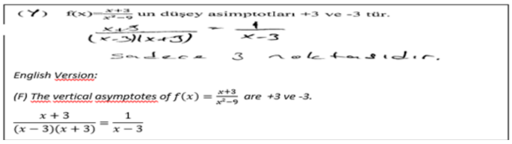


Figure 10: T_2's Answer for the Eleventh Expression After the Application

After the application, T_2 made a "correct detection", saying that the statement was incorrect. Besides, based on his/her operations regarding why he/she could not obtain the value -3, he/she provided a "correct explanation." In addition, in the interview on the answers given, T_2 supported the statements he wrote by saying that "Here, -3 does not make it undefined. Therefore, when simplification is made, it becomes 1/(x-3). Here, the value of 3 is just undefined. At that point, there is a vertical asymptote."

In Interview 1, conducted on the answers the participants provided in the data collection tool, it was found that the participants generally re-expressed the explanations they wrote on paper.

In Interview 2, conducted to examine the effect of GeoGebra in the teaching of the asymptote, the opinions of the participants who exhibited "no development", "partial development" and "development" were obtained. Their answers were as follows:

T_7, who showed a development, stated that mathematics was better understood when visualized and that the image came to his/her mind after the application with GeoGebra. He/she maintained that, in this way, he/she did not memorize and instead developed a logic. T_7 also stated that knowledge was more permanent with GeoGebra supported teaching. A statement of T_7 about visualization and permanence of the knowledge is presented below.

> "Before we learned asymptote traditionally, not computer-aided. With GeoGebra, from where exactly that asymptote passes, for instance, let's say the vertical asymptote's going to infinity, its looking better is more beneficial. Because we can't draw them all with our hands. But, we see it more clearly with the support of computer."

T_7 expressed that after what he/she learned in GeoGebra, he/she had the opportunity to generalize. However, he/she stated that regarding application in high schools, it could not be applied everywhere due to problems related to the lack of knowledge of GeoGebra and lack of sufficient computers and that it could pose a problem in terms of it being uneconomical. T_7 also emphasized that there was no computer available for everyone and that even if the teacher presented it on the interactive board, it may not be different than the traditional black board teaching because the students did not practice it themselves. He/she also highlighted the necessity of making use of the board when necessary, instead of completely teaching with GeoGebra. He/she stated that he/she wanted to use GeoGebra in his/her teaching life both regarding asymptotes and other mathematics subjects.

T_2, who exhibited partial development, stated that they realized that the things they thought were simple before the application were not simple when they were asked a question, and they confused some things with each other. . He/she said that he/she partially corrected his/her errors after the application. He/she stated that the reason he/she could not completely correct the errors was that he/she did not revise after the application. He/she expressed that GeoGebra provided visualization, saying that "We were actually able to see and think of the graphics better because it was visualized."

He/she also stated that GeoGebra increased permanency. He/she said that a schema formed in his/her mind with GeoGebra and that he/she could then develop ideas by deliberating on the subject. He/she believed that GeoGebra had all kinds of contributions to teaching and did not have any negative effects on teaching. He/she maintained that GeoGebra helped them overcome their shortcomings and that he/she would use GeoGebra in his/her teaching life. He/she emphasized that using GeoGebra for every topic would be difficult and that it would be better to use GeoGebra in convenient topics. Finally, he/she concluded that visual materials attracted the attention of students and that they would eliminate their prejudices against mathematics.

T_8, who exhibited no development, stated that he/she did not know whether there was a difference between the answers he/she provided before and after the application but that he/she was somehow aware that something changed. In other words, he/she maintained that there was a change in his/her thoughts but could not be sure about whether he/she could reflect his/her thoughts on paper or not. He/she said that the teaching of the topic with GeoGebra attracted his/her attention. However, he/she expressed that he/she did not practice and revise after the application. He/she emphasized that he/she knew something about the subject, were able to visualize what was taught but could not completely internalize them. He/she also stated that he/she understood better with GeoGebra's visuality, that Geogebra helped him/her in drawing graphs similar to the graphic examples given in GeoGebra and helped him/her to make comments. Finally, he/she emphasized that it would be easier to teach graphics to the students using GeoGebra. Some of the excerpts of T_8 are given below:

> "It is better for teaching the asymptotes to students. For example, when we draw on the blackboard, we can only show it to a certain point, there's nothing after that. The student should visualize it and most of them cannot; but with GeoGebra, for instance, we can show a lot of its length; for instance, asymptote proceeds but does not coincide. They go to infinity together, but they do not coincide."

He/she stated that GeoGebra should be used but it would not be appropriate to use it all the time and that the lesson should be supported with the use of paper and pencil. He/she said that the important points of the subject should be taught using technology and there was no need to use the technology afterward. He/she maintained that he/she would prefer to use GeoGebra until the topic was understood and then would not prefer to use it in examples. He/she emphasized that GeoGebra cannot be used in every subject and that he/she would use GeoGebra only in the internalization phase of the subject.

Conclusion and Discussion

The examination of participants' written answers before and after the GeoGebra-assisted application aimed at asymptotes showed that their approaches to possible errors regarding the concept of asymptote change positively. In addition, interviews with three participants who exhibited "development", "a partial development" and "no development" supported this finding. The findings of the present study are similar to those of Karakuş and Konyalıoğlu (2018) who used GeoGebra for extremum points and milestones.

Nair (2010) emphasized the importance of group work by stating that the redefinition of asymptotes developed as a result of teamwork and meaning reconciliation. Birgin and Acar (2020) stated that computer-supported collaborative learning using GeoGebra software was effective on students' mathematics achievement in exponential and logarithmic functions. In addition Radović et al. (2020) drew similar conclusions for another mathematical subjects. In the present study, it is thought that GeoGebra supported learning through mutual interaction with preservice teachers, contributes to the learning of preservice teachers. During the application, it was observed that participants tried different examples in GeoGebra in line with their curiosity and that they reached certain conclusions themselves.

An increase was observed in the number of the participants who made correct detection and correct explanation and a decrease was observed in the number of the participants who provided incorrect detection and incorrect explanation. In addition, it was also determined that the number of the participants who made correct detection decreased (Question 9). In this question, four participants provided correct detection before the application. A detailed investigation showed that 2 participants made incorrect explanation while 2 did not provide any explanation. However, none of the participants provided a correct detection in Question 9 after the application. At first sight, this can be interpreted as a negative finding for pre-service teachers. However, it should be noted that there was no decline after the application and that the participants who made a correct detection before the application made incorrect explanations or were unable to explain.

In the interviews, the preservice teachers generally stated that they understood the concept of asymptote better due to the visuality provided by GeoGebra and that it increased the permanency. In fact, the visual feature of GeoGebra is frequently emphasized in the literature (Hohenwarter, 2004; Hohenwarter, Preiner, & Yi, 2007; Guncaga & Majherova, 2012; Tatar & Zengin, 2016; Zengin, 2017). However, participants stated that teaching only with GeoGebra would be incorrect just as they did not think that teaching only with the blackboard was correct. They stated that a GeoGebra-assisted instruction where GeoGebra could be used when necessary would be useful. One of the participants, who did not exhibit any development, stated that there was a change in his/ her thoughts about the subject but he/she could not fully internalize the subject and was not sure whether she could reflect his/her thoughts on the paper.

Based on the answers given before the application and the interviews conducted after the application, it was found that preservice teachers generally had some kind of ideas about the concept of asymptote and asymptote types before the application, but they could not internalize these definitions. Nair (2010) argues that students do not really establish a connection between the concepts they should learn. For instance, although the students had previously learned the asymptotes of rational functions before, they did not even know that rational functions can have asymptotes. In the present study, it was found that although most of the preservice teachers knew the asymptotes, they could comment on the vertical asymptote's state of intersecting function graph, but generally could not make a correct detection and correct explanation about the horizontal, oblique and curved asymptotes' state of intersecting the graph. After the application, a certain increase was observed in the number of the participants who provided the correct detection and the correct explanation. Although correct detection and correct explanation was usually made before and after the intervention regarding the vertical asymptote cutting the graph, it was found that the participants generally made a mistake when it was asked to find the vertical asymptote of a given function (Question 11). Nair (2010) stated that this situation shows a deficiency in recognizing what constitutes an indefinite form or what an undefined form is in the behavior of the function. In a study conducted by Kidron (2011) with a high school student, the definition of horizontal asymptote was given after obtaining student's idea about asymptote, vertical asymptote, and horizontal asymptote. In the first task, the student was given an example of a function that did not contradict the definition of horizontal asymptote and was asked to draw the asymptote with the graph of this function. In the second task, when the student drew the desired function, he/she saw that the asymptote interrupted the function and re-examined the function he/she drew, thinking that he/she made an error. He/she formed a hole in cut points by removing the intersection points. Similarly, Nair (2010) observed that students think of the point where the horizontal asymptote intersects the graph of the function as a hole. Kidron (2011) found that at the end of certain stages in the second task, the horizontal asymptote could intersect the function and this was not contrary to the definition, concluding that the vertical asymptote might not intersect the function graph. In the third task, Kidron saw that the horizontal asymptote could intersect with the function at an infinite point. In this way, it was ensured that the student understood the concept of the horizontal asymptote.

In the present study, it was observed that GeoGebra assisted teaching through mutual interaction was beneficial in preventing possible errors regarding the subject of the asymptote. Therefore, similar studies can be conducted on different subjects. A similar study can also be conducted by making a comparison with a control group in which a different teaching is delivered. This study is limited to the GeoGebra software as an instructional technology, the answers given by 11 pre-service mathematics teachers, the knowledge and opinions of the pre-service teachers and the experiences of the researchers. Considering these limitations, future studies should focus on in-service teachers and using more than one instructional technology. Both a wide variety of instructional technologies and the knowledge and experience of in-service teachers can also contribute to students.

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Appendix

There are 14 expressions about asymptotes below. Mark the expressions as true (T) or false (F) and write the reason of the answer you gave in the blanks.

( ) A function curve (or graph) does not intersect vertical asymptote.

( ) A function curve (or graph) does not intersect oblique asymptote.

( ) A function curve (or graph) does not intersect curved asymptote.

( ) A function curve (or graph) does not intersect horizontal asymptote.

( ) A function graph may both have horizontal and oblique asymptote.

( ) A function graph may both have horizontal and curved asymptote.

( ) A function graph may both have oblique and curved asymptotes.

( ) There may be both vertical and horizontal asymptote in a function graph.

( ) A tangent drawn towards a function graph from a point may be asymptote for the function in question.

( ) A function's graph does not coincide with any asymptote at an infinite point.

( ) The vertical asymptotes of $f(x)=(x+3)/(x^2-9)$ are +3 ve -3.

( ) The horizontal asymptote of $f(x)=(x^2-16)/(x^3+1)$ is 0.

( ) The horizontal asymptote of $f(x)=(x^3-1)/(x+1)$ is -1.

( ) The curved asymptote of $f(x)=(3x^2+x)/(x+1)$ is 3x-2.

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