Journal of Urban Mathematics Education
December 2021, Vol. 14, No. 2, pp. 71–104

Translanguaging to Persevere Is Key for Latinx Bilinguals' Mathematical Success

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Recent reform efforts state the importance of providing students with opportunities to persevere with challenging mathematics to make meaning. We posit translanguaging practice as a vital option by which Latinx bilingual students can sustain collective perseverance during problem solving. In this paper, we employ a constant-comparative overlay analysis to simultaneously study the discursive translanguaging and perseverance practices of Latinx bilingual students and the corresponding classroom supports. We observed collaborative problem solving in two classrooms of 12th-grade Latinx bilinguals working to make sense of an exponential function and the involvement of the same monolingual English-speaking teacher. Working within similar, supportive classroom environments, we describe how one group of students spontaneously and dialogically leveraged communicative resources to help persevere with in-the-moment obstacles, while another group of students worked together across languages but did not engage in a translanguaging mathematical practice to persevere. We suggest that only establishing a classroom environment conducive for translanguaging and perseverance practice is insufficient and that teachers should not solely rely on students spontaneously engaging in these practices. To complement this environment, we recommend specific teacher moves and scaffolds that could help Latinx bilingual students initiate collaborative translanguaging and support their ongoing perseverance to make meaning of mathematics.

KEYWORDS: exponential functions, Latinx multilingual learners, mathematics discourse, perseverance, translanguaging

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There is a pressing need to rehumanize mathematics education for Latinx bilingual[1] students in the United States. The act of rehumanizing fosters respect and dignity through privileging the viewpoint and experiences of the Latinx student and the ways in which the student perseveres to develop personal understandings through their own disciplinary perspective on mathematics. In this context, a rehumanizing perspective positions a Latinx student as central to the meaning-making process while engaging in the practice of doing mathematics (Gutiérrez, 2018). A rehumanizing perspective rejects the notion that bilingual students must solely reproduce the teacher's idea of productive mathematical activity (Lawler, 2016; Matthews, 2018). Instead, a rehumanizing perspective adopts a social-cultural lens on learning, which positions bilinguals as agents in their language use capable of interacting and communicating while working collaboratively with a challenging mathematical task (Cross et al., 2012; Khisty & Chval, 2002; Vomvoridi-Ivanović, 2012; Waddell, 2010). In stark contrast to this rehumanizing perspective is a deficit perspective often applied to Latinx bilingual learners. This deficit perspective has been widespread in education around the United States and suggests that bilingual students are passive recipients of mathematical knowledge and slowed by language barriers (Razfar et al., 2011; Rubel, 2017). Educators who ascribe to this model tend to employ classroom practices that marginalize rather than privilege linguistic, social, and cultural capital, thus creating dehumanizing school norms (Langer-Osuna et al., 2016). These perspectives have persisted for years (cf. Moll, 2001) and continue to create a distance between Latinx students' language, cultural knowledge, and opportunities to develop mathematical meaning. As mathematics educators, we are interested in researching ways to support Latinx bilinguals in leveraging their bilingualism to persevere to make meaning of mathematical ideas.

Developing personal mathematical meanings requires student perseverance and teacher support. This notion of student perseverance exists in the moment at specific times during problem solving when productive struggle is required. Productive struggle, or grappling with mathematical ideas that are familiar but not yet well formed (Hiebert & Grouws, 2007), is necessary to overcome obstacles along the path toward developing conceptual knowledge. However, not all struggle is guaranteed to be productive. Students can struggle unproductively when working without a teacher support system in place to offer feedback and guidance. Such unproductive struggle can

[1] We use the term *Latinx bilinguals* to refer to our research participants to encompass a more inclusive identifier that reflects the diversity of gender, sexuality, gender-nonconforming, and trans people who identify with a Latin descent. We also use this term to bring attention to the complexity of the intersectionality of language, culture, and identity commonly associated with speaking named languages (e.g., Spanish and English). We use the term *bilinguals* to encompass multilingual students who have access to or are in the process of acquiring access to multiple languages. We acknowledge that researchers should be purposeful about the labels they use and contribute to the conversation while paying attention to the political and social nature of these important conversations.

spur frustration, anxiety, and discouragement toward perseverance in problem solving (DiNapoli, 2018; Star, 2015). Perseverance is so vital to the learning process that guiding texts like *Principles to Actions: Ensuring Mathematical Success for All* (National Council of Teachers of Mathematics [NCTM], 2014) have asserted that all students must have access, opportunity, and support to persevere in their study of mathematics. Further, the Common Core State Standards Initiative (CCSSI, 2010) has explicitly noted the importance of cultivating perseverance for all students to encourage mathematical expertise. However, there is a concern that mathematics reforms have largely ignored the needs of Latinx students (Martin, 2015; Moschkovich, 2000, 2015). Supporting students to persevere to learn mathematics with understanding has often involved more social and verbal activities that require students and teachers to engage in more substantive mathematical discussions and collective practice (Bass & Ball, 2015). Such communicative practice dangerously assumes a shared collaborative language between students and teachers (Chval & Khisty, 2009) and alternately requires careful consideration of all language repertoires in the mathematics classroom to truly support Latinx bilingual students' perseverance. While this is a generalized perspective of classrooms with Latinx bilingual students, it nevertheless raises questions about marginalization and undervaluing Latinx bilingual students' learning resources in mathematics.

Much of the research concerning Latinx bilingual students learning mathematics has focused on only what students cannot do and largely ignores what students can do. This kind of deficit perspective is often manifested by focusing on the relationship between Spanish-speaking students' struggles with English and their difficulties in learning mathematics (e.g., Langer-Osuna et al., 2016). This deficit perspective has also manifested by detailing the barriers faced by Latinx bilinguals learning mathematics across the Spanish and English languages (e.g., MacSwan & Faltis, 2019). Deficit perspectives emerge when the affordances of bilinguals' linguistic resources are ignored in the classroom while only privileging the dominant school language (Langer-Osuna et al., 2016). García et al. (2017) argued that this view of language and academic discourse in schools acts as a hurdle to knowledge (e.g., mathematical knowledge) and only empowers those students whose linguistic repertoire mirrors the dominant school language. Combatting this deficit perspective, research has shown that Latinx students can use a wide variety of cultural resources to construct, negotiate, and communicate (verbally or in writing) about mathematics (Chval & Khisty, 2009; Morales, 2012). Cultural resources include linguistic scaffolds like a mathematics register and mathematical discourse, everyday experiences, life histories, and community funds of knowledge (Celedón-Pattichis, 2003; Gutiérrez, 2002, 2017; Moll, 2001; Morales et al., 2011; Moschkovich, 2015). More recent research has privileged the complex personal perspective of the Latinx bilingual mathematics student to demonstrate that involving family and/or community members' knowledge and experiences in school mathematics can play a critical role in a child's learning (e.g., Maldonado et al., 2018; Mazzanti Valencia & Allexsaht-Snider, 2018).

In this paper, we extended prior research through our investigation of the nature of Latinx bilingual students' perseverance in problem solving and how it can be encouraged and supported. To do so, we employed the construct of translanguaging in our conceptual framework. Translanguaging is a theory of language that shifts ideologies from language separation perspectives to one that values the complex and interrelated communicative practice that makes up bilingual students' linguistic repertoires (Cenoz, 2017). Translanguaging illuminates the ways in which Latinx bilinguals can be empowered by their linguistic and cultural resources and how leveraging such resources can inform persevering to make meaning of mathematics. As such, this study is grounded in the conceptual perspectives of both translanguaging and perseverance practice.

Frameworks for Translanguaging and Perseverance Practice

We have drawn on the concept of translanguaging to explore and reconceptualize bilingualism as an empowering and liberating practice. Translanguaging is an ideological stance on bilingualism that goes beyond transitioning bilinguals to the dominant school language (García et al, 2017). García (2017) defined translanguaging as the authentic meaning-making practices of bilinguals; she posited, "…speakers use their languaging, bodies, multimodal resources, tools and artifacts in dynamically entangled, interconnected and coordinated ways to make meaning" (p. 258). Thus, the act of translanguaging looks and sounds like bilinguals seamlessly leveraging two languages and linguistic features to make sense of concepts, contexts, relationships, and representations. Adopting a translanguaging stance shifts us ideologically toward viewing bilingualism holistically and understanding that bilinguals have a single language repertoire on which they draw as a resource for learning. This is in contrast to the notion that bilinguals simply shift between two language repertoires as if their language use is mutually exclusive (Cenoz, 2017). Instead, "the act of translanguaging is in itself transformative, having the potential to infuse creative bilingual meaning into utterances" (García et al., 2017, p. 20). In other words, the symbiosis between or across languages becomes a mediating tool that bilinguals can use as an affordance to make meanings.

Some researchers have used a translanguaging framework to better understand language practices of multilingual persons in mathematics classrooms. For instance, Latinx kindergarteners' use of English and Spanish occurred simultaneously as a way to make mathematical meanings (Mazzanti Valencia & Allexsaht-Snider, 2018). Their meanings were not limited to just linguistic accomplishments but also used symbolic representations of numbers and visual models of the number problems to leverage these students' communicative resources. Other studies have focused on purposeful translanguaging of Latinx bilingual mathematics students. Morales and DiNapoli (2018) found that when students were given the freedom to explore mathematics via their dynamic bilingualism, they were able to dialogically leverage communicative resources to help them overcome in-the-moment obstacles and make meaning. Illuminating these translanguaging practices repositions these students as competent problem solvers and agents of their own learning while leveraging bilingual identities as learners of mathematics, which reflects a rehumanizing perspective for all students in the group.

Pedagogies That Support Translanguaging Practice

Mathematics teachers can enact translanguaging pedagogies in their classrooms to help create powerful spaces in which Latinx bilinguals can make meaning. For instance, consider Ramirez and Celedón-Pattichis' (2012) five guiding principles for teaching mathematics to Latinx bilinguals. These principles articulated effective ways to engage Latinx bilinguals to persevere with rigorous mathematics while engaging in translanguaging mathematical practice. Ramirez and Celedón-Pattichis first recommend that teachers enact challenging mathematical tasks that provide students multiple entry points but non-obvious solution paths. Within engagement with such tasks, teachers can offer mathematical tools and even model their use as a resource. While these pedagogical recommendations allow for and support student meaning-making with mathematics, Ramirez and Celedón-Pattichis also urge for cultural and linguistic teaching moves to support Latinx bilingual students. Teachers can help students leverage their cultural and linguistic differences as intellectual resources, such as by approaching a mathematical task in their native language to initiate engagement. Teachers can also offer support for learning English while learning mathematics by juxtaposing certain mathematical terms in both English and Spanish, in the context of Latinx bilingual students' problem solving. In addition, Ramirez and Celedón-Pattichis also urge for teachers to build a linguistically sensitive social environment by celebrating bilingualism and removing language separation. Some ways teachers can do this is by encouraging students to present their work in several languages, using translators to help facilitate whole-class bilingual discussions, and by celebrating how bilingualism can be a mathematical resource used to better understand concepts and connections.

Teachers who enact translanguaging pedagogies challenge the deficit perspectives commonly associated with language separation (Moschkovich, 2019). These teachers can leverage students' linguistic repertoires to engage with complex mathematics content and texts, strengthen students' linguistic repertoires in mathematical contexts, draw on students' bilingualism for the purpose of expanding their ways of knowing, and support students' bilingual identities that counter English-only ideologies (García et al., 2017). In conjunction with Ramirez and Celedón-Pattichis' (2012) call for translanguaging pedagogies, researchers (e.g., García-Mateus & Palmer, 2017; Maldonado et al., 2018) have urged teachers to develop a translanguaging stance to help enact these practices in their classrooms. An authentic embrace of a translanguaging stance involves the teacher's belief that bilingual students have one holistic language repertoire on which they draw, not two mutually exclusive repertoires. When teachers create a climate where translanguaging is embraced, it can help reconceptualize mathematics learning and language practices of Latinx bilinguals. Still, more research is needed in bilingual mathematics classrooms to study how flexibility and fluidity with students' translanguaging repertoires is centrally aligned to mathematical meaning-making practices.

Translanguaging Mathematical Practice

The construct of translanguaging mathematical practice describes the ways in which flexibility and fluidity with students' translanguaging repertoires is central to meaning-making practices in mathematics contexts. Translanguaging mathematical practice captures how bilingual students holistically use their linguistic, multimodal, and mathematical artifacts repertoires to make meanings as they persevere with challenging mathematical ideas. Translanguaging mathematical practice was informed by Maldonado et al.'s (2018) work on a translanguaging stance. We extended García's (2017) translanguaging practice framework to accommodate how bilingual students engage in mathematics discourse, including repertoires like everyday language, mathematics register, and mathematical artifacts, such as visual representations and mathematical notations (Avalos et al., 2018; O'Brien & Long, 2012). Bilingual students do not learn best through passive, one-dimensional mathematics instruction (Moschkovich, 2015). Instead, a multimodal approach can incorporate several dimensions of a powerful learning environment. Such an approach incorporates mathematical symbols and visual images to enhance the view of understanding language relative to teaching and learning mathematics. The meanings of mathematical symbols are encoded with precise and condensed representations of mathematical ideas. Similarly, visual images help us represent mathematical ideas and apply knowledge (O'Halloran, 2015). O'Halloran (2015) argued for a multimodal literacy, "a literacy which extends beyond language to include mathematical symbolic notation and mathematical images, and the relations between and integration of these three resources" (p. 73). Translanguaging mathematical practice describes how bilinguals are engaged in this kind of learning environment. Within the framework of translanguaging mathematical practice (see Figure 1), students engaging within the mathematics discourse deploy fluid movement between mathematical and everyday speaking across Spanish ($_S$) and English ($_E$) by using everyday linguistic features (ER_S and ER_E) and mathematics register resources (MR_S and MR_E) in dialogically entangled ways. Additionally, students draw on non-verbal mathematical representations, including visuals (MV), notations (MN), and gestures (MG), to complement their mathematics and everyday register. In translanguaging mathematical practice, these linguistic and semiotic resources work together with the intention to help bilingual students persevere with challenging ideas to make meaning. Next, we introduce the conceptual perspective of perseverance in problem solving, which is an ideal outcome of bilingual students engaging in translanguaging mathematical practice.

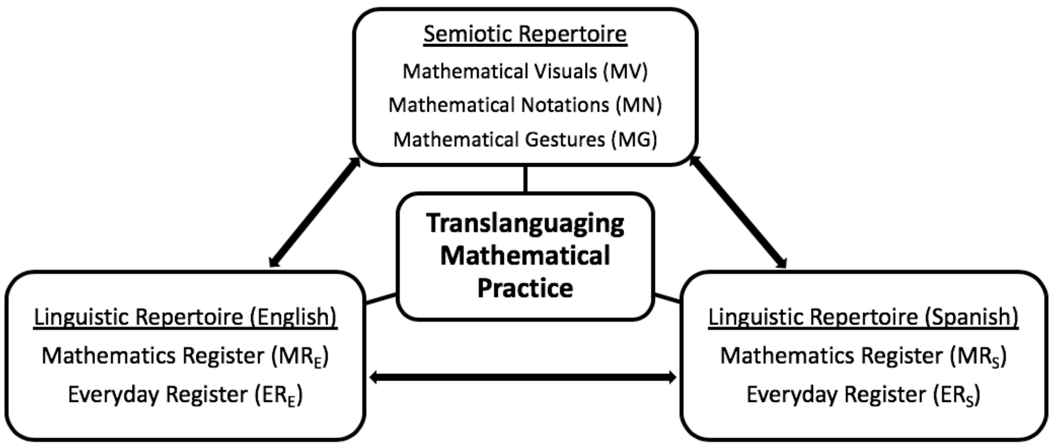


Figure 1. A Framework for Translanguaging Mathematical Practice

Perseverance in Problem Solving

In the context of working on a challenging mathematical task, perseverance is defined as "initiating and sustaining in-the-moment productive struggle in the face of one or more obstacles, setbacks, or discouragements" (DiNapoli, 2018, p. 890). Perseverance during problem solving is especially important for learning mathematics because students develop conceptual understanding as they wrestle with ideas that are not immediately apparent (Hiebert & Grouws, 2007). The self-regulatory actions amidst uncertainty as students navigate an obstacle during problem solving helps describe perseverance, and analysis of such engagement should consider the ways in which students first explore an uncertain mathematical situation and how (if necessary) they amend their initial plan to find a way to continue to make progress toward building understanding.

Perseverance in problem solving can be operationalized using the Three-Phase Perseverance Framework (3PP; DiNapoli, 2018; see Table 1 for a description of the framework and corresponding analytic codes for each component). The 3PP is an analytical perspective by which perseverance can be qualitatively described and measured. The framework reflects perspectives of concept (Middleton et al., 2015; Sengupta-Irving & Agarwal, 2017), problem-solving actions (Pólya, 1971; Schoenfeld & Sloane, 2016; Silver, 2013), self-regulation (Carver & Scheier, 2001; Zimmerman & Schunk, 2011), and making and recognizing mathematical progress (Gresalfi & Barnes, 2015). The 3PP has been used in several studies across different contexts to make explicit the perseverance process for an individual and/or group of students working towards a mathematical goal. Findings have helped inform several outcomes relevant to mathematics education, including productive strategies for student disposition development (DiNapoli, 2019), support systems for impasse learning (DiNapoli, 2018), and making learning visible in non-formal science, technology, engineering, and mathematics learning environments (Satyam et al., 2021).

Using the 3PP focuses the analysis on three chronological phases of perseverance: the Entrance Phase, the Initial Attempt Phase, and the Additional Attempt Phase. The Entrance Phase was designed to consider first if the task at hand warranted perseverance for students. Essentially, the analysis of this phase establishes the appropriateness of a task for perseverance analysis with a particular group. For perseverance to be necessary, students must have understood the objectives of the task (C-0) but did not immediately know how to achieve them (IO-0). Next, the Initial Attempt Phase was designed to consider the ways in which students initiated and sustained productive struggle. Because the students did not immediately know a solution pathway, evidence of perseverance includes deciding to engage with the task at all (IE-1). If the students decided to pursue solving the problem, evidence of perseverance includes the use of a problem-solving strategy to diligently explore the uncertain nature of the mathematical situation (SE-1). As a result of these diligent efforts, evidence of perseverance includes mathematical progress that was made toward better understanding the mathematical relationships or the problem being solved outright (OE-1). Last, the Additional Attempt Phase was designed to consider the ways in which students reinitiated and resustained productive struggle if they encountered a perceived setback as a result of their initial attempt. To enter this phase (marking the end of a first attempt and the beginning of a second attempt), the group must first have encountered a perceived setback. Evidence of a perceived setback occurs when a group collectively affirmed that they were substantially stuck and unsure how to continue with the task (VanLehn et al., 2003). At this point, evidence of perseverance includes deciding to reengage with the task using a different strategy—one that was not used during the first attempt (IE-2). Assuming the students decided to pursue solving the problem with a new plan, additional evidence of perseverance includes diligent exploration of the mathematical situation with the new strategy (SE-2) and new progress being made toward better understanding the mathematics involved in the task or the problem being solved outright (OE-2). Students could continue on to start a new Additional Attempt Phase(s) if their efforts produce another perceived setback(s) and require initiating a new effort(s) with a new strategy(ies).

Table 1
Three-Phase Perseverance Framework

Entrance Phase	
Clarity (C-0)	Objectives were understood
Initial Obstacle (IO-0)	Expressed or implied that a solution pathway was not immediately apparent
Initial Attempt Phase	
Initiated Effort (IE-1)	Expressed intent to engage with task
Sustained Effort (SE-1)	Used problem-solving heuristics to explore task
Outcome of Effort (OE-1)	Made mathematical progress toward a solution
Additional Attempt Phase (after *n* perceived setback(s))	
Re-initiated Effort (IE-n+1)	Expressed intent to reengage with task
Re-sustained Effort (SE-n+1)	Used problem-solving heuristics to explore task
Outcome of Effort (OE-n+1)	Made additional mathematical progress toward a solution

n = the number of perceived setbacks

Because student perseverance is vital for learning mathematics with understanding, it is important for educators to encourage perseverance via their classroom practice. As such, NCTM (2014) called for teaching practices that "consistently provide students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships" (p. 48). One such practice specific to working with Latinx bilingual students considers the structure of tasks. For Latinx bilingual students, mathematical tasks necessitating perseverance should be complex enough such that students engage at all levels of language proficiency to help make their own connections (Driscoll et al., 2012) yet invite engagement with multiple entry points and resources. However, it is still unclear how perseverance might manifest in students working across two languages. Further, outside of task design, more research is needed to consider the ways in which teachers can provide opportunities for Latinx bilingual students to leverage or capitalize their communicative and linguistic repertoires to build understandings.

Research Questions

In this paper, we addressed calls to action to examine translanguaging and perseverance practice in classrooms largely from the student point of view (e.g., Maldonado et al., 2018; Martinez et al., 2017; NCTM, 2014). Our research questions are grounded in the belief that translanguaging mathematical practice can encourage perseverance practice in Latinx bilinguals when given the right pedagogical conditions. We carefully considered if and how Latinx bilingual students used translanguaging practice while collaborating (as per Maldonado et al., 2018; Martinez et al., 2017) and if and how these students persevered with challenging mathematical ideas (as per NTCM, 2014) as they worked together. By studying these practices closely, we used inductive reasoning to discern important tenets of translanguaging pedagogy that can support these practices in mathematics classrooms. The research questions used to guide this study were as follows:

1. In what ways, if any, do secondary Latinx bilingual students demonstrate translanguaging mathematical practice while collaboratively engaging with a mathematical task?
2. In what ways, if any, do secondary Latinx bilingual students demonstrate perseverance practice while collaboratively engaging with a mathematical task?
3. By comparing the evidence of Latinx bilingual students' translanguaging mathematics and perseverance practice, what pedagogical insights can we glean the about creating productive (or unproductive) learning environments for Latinx bilingual students?

Methods

Our observations and interviews come from a larger study of 12th-grade Latinx students in mathematics classes in a school with an 80% Latinx student population. Each class was using Year 4 of the Interactive Mathematics Program (IMP; Fendel et al., 2015b) curriculum, which focused on topics such as statistical sampling, systems of linear equations and inequalities, geometric transformations, integral and derivative concepts, and deeper exploration of previously encountered mathematical ideas. We studied two 12th-grade mathematics classes taught by the same teacher.

Participants

We focused on two purposeful groups of students—one group from each class. We chose to focus on these particular groups because they had collaborated together for the majority of the school year, were bilingual, had similar past experiences with mathematics, and demonstrated average achievement in mathematics as informed by their past grades and standardized test scores.

The first group of students consisted of Carina, Jessica, Elena, and Ines (pseudonyms). These students were all children of Mexican parents and grew up speaking both English and Spanish at home. Carina, Elena, and Jessica were born and have lived in the United States for their entire lives. These three students had been enrolled in bilingual programs during their elementary grades and had transitioned into mainstream classrooms by the time they were in middle school. They all stated that they felt comfortable speaking Spanish but were not as comfortable reading or writing in Spanish. Ines arrived to the United States from Mexico at age 12, returned to Mexico at age 14, and ultimately came back to the United States to complete her high school education. Ines had formal educational experiences around speaking, reading, and writing in Spanish and reported being more comfortable speaking Spanish than English. These students represented typical students in the school with a history of average achievement in mathematics. Each student reported that Spanish had always played a major role in their meaning-making process for mathematics.

The second group of students consisted of Yasmin, Julia, Bernardo, and Lorenzo (pseudonyms). Lorenzo, Yasmin, and Bernardo all arrived to the United States at ages 12, three, and seven, respectively, and all three of them stated that they speak only Spanish at home. Julia was born in the United States to an American mother and Mexican father. Julia reported some comfort in speaking Spanish but was not as comfortable reading or writing in Spanish. Lorenzo was the only student who reported some discomfort reading words or sentences in English. Similar to the first group, these students represented typical, average achieving mathematics students for whom Spanish had always played a major role in their process for understanding mathematics.

The teacher for both groups was Ms. Patrick (pseudonym). Ms. Patrick was a monolingual English speaker with 20 years of mathematics teaching experience. She had seven years of experience enacting the IMP curriculum and was a proponent of student-centered mathematical activity and problem solving. Via our preliminary observations, Ms. Patrick's teaching philosophy incorporated many aspects of Smith and Stein's (2011) *5 Practices for Orchestrating Productive Mathematical Discussions*, and she often explicitly encouraged her students to collaborate with challenging mathematics in their native language and use mathematical tools to explore multiple resources and representations while doing so. We selected her classroom because she had many bilingual students and often had them working in groups—a perfect setting to learn more about how Latinx bilingual students use translanguaging to persevere.

Data Sources and Context

We observed two groups of students with the same teacher every day for six weeks each for a total of 50 class sessions. We accumulated over 40 hours of video-recorded observation with accompanying field notes. We also collected copies of student-generated mathematical artifacts. Our observations largely concentrated on the students' discourse patterns. We paid particular attention to how students used their linguistic resources (linguistic, visual-graphic, or gestures) while negotiating meaning. We also focused on the ways in which students interacted with available resources as a method for aiding the development of meaning.

Each participant was interviewed after the observations were completed. The main purpose of these interviews was to member-check the field notes of our impressions of the observation data (Guba & Lincoln, 1994). Such member-check interviews helped ensure that participants' voices were prevalent and accurate in the findings (Guba & Lincoln, 1994). These interviews also helped us gather information about the students' family background, language proficiency in both English and Spanish, academic history, and impressions of their mathematical identity.

To select the cases to report in this paper, we drew from a thematic analysis of the entire data set. A saturation of themes emerged from our analysis, and we chose vignettes that illustrated those themes. Additionally, we purposely selected vignettes from each group that concerned the same mathematical task and that demonstrated students' typical interactions with each other and with Ms. Patrick.

The findings reported in this paper stem from each group working on an activity designed to necessitate perseverance. Each group was assigned the Function Analysis Task: a task of discerning ways in which mathematical functions were helpful (Figure 2). This activity was part of the IMP Year 4 goal of deeper exploration of previously encountered mathematical ideas. Each group collaboratively selected a unit and a function they had encountered from a past unit. Groups could choose a past function with which they experienced success and possessed prior knowledge. This made it more accessible for group members to initially engage in the meaning-making suggested by the Function Analysis Task. Each group would eventually be asked to share about their work on the Function Analysis Task with the class.

Function Analysis Task

Select a specific function from a past unit.

1. Describe the problem context in which the function was used and explain what the input and the output for the function represent in terms of the problem context
2. Describe how the function was helpful to you in solving the central unit problem or some other problem in the unit.
3. If possible, determine what family the function is from.

Figure 2. Function Analysis Task (Fendel et al., 2015b)

Both groups chose "All About Alice" (Figure 3) as their function, selected from a Year 2 IMP unit with which both groups reported that they experienced success and understanding. Ms. Patrick corroborated this report. This unit started with a modeling task based on Lewis Carroll's *Alice's Adventures in Wonderland.* From this situation came the basic principles for working with exponents and an introduction to exponential growth and decay problems. This task of analyzing a previously encountered function necessitated perseverance because the All About Alice problem is familiar to each group, yet the analysis is open to student exploration.

All About Alice

Alice's height changes when she eats the cake. Assume as before that her height doubles for each ounce of cake she eats.

1. Find out what Alice's height is multiplied by when she eats 1, 2, 3, 4, 5, or 6 ounces of cake.
2. Make a graph of this information.

Figure 3. "All About Alice" (Fendel et al., 2015a, p. 385)

To solve this problem, one could represent this relationship in the form of a table of values and a graph and consequently discover that if Alice's height is doubling, then her height is multiplied by powers of two depending on how many ounces of cake she eats. For example, if Alice's height is originally four feet and she ate three ounces of cake, her height is now 32 feet:

$$\begin{aligned} 4(2 \cdot 2 \cdot 2) &= 4(2^3) \\ &= 4(8) \\ &= 32 \end{aligned}$$

This mathematical problem can also be represented by an equation of the form $y = a \cdot b^x$. In this case, Alice's height is doubling, so the base b is 2 and Alice's original height is represented by the variable a given the equation $y = a \cdot 2^x$. This problem can be difficult because students have to grapple with the distinction between doubling and multiplying by two, an often-confusing idea because they seem so similar (Confrey, 1991). Also, the key variable x is an exponent, which is uniquely different from all of the other functions they have studied thus far.

Data Analysis

We employed a three-tier analytic method designed to recognize the ways the two groups of Latinx bilingual students used translanguaging practice to collaboratively persevere and construct meaning while engaging with a challenging mathematical task (Morales & DiNapoli, 2019). By studying the simultaneous evidence of translanguaging and perseverance practices, we used inductive reasoning to make inferences about creating productive mathematics learning environments for Latinx bilingual students. Video transcriptions of students' dialogic interactions were the center of our analysis. This analysis helped reveal how the groups of students used translanguaging to persevere by identifying translanguaging mathematical practice, perseverance practice, and the overlay of both practices simultaneously (see Figure 4). The first tier of analysis captured evidence of students' translanguaging via their verbal and non-verbal communications. The second tier of analysis captured evidence of students' perseverance in problem solving. The third tier of analysis captured how evidence of students' translanguaging occurred alongside their perseverance (see Morales & DiNapoli, 2019).

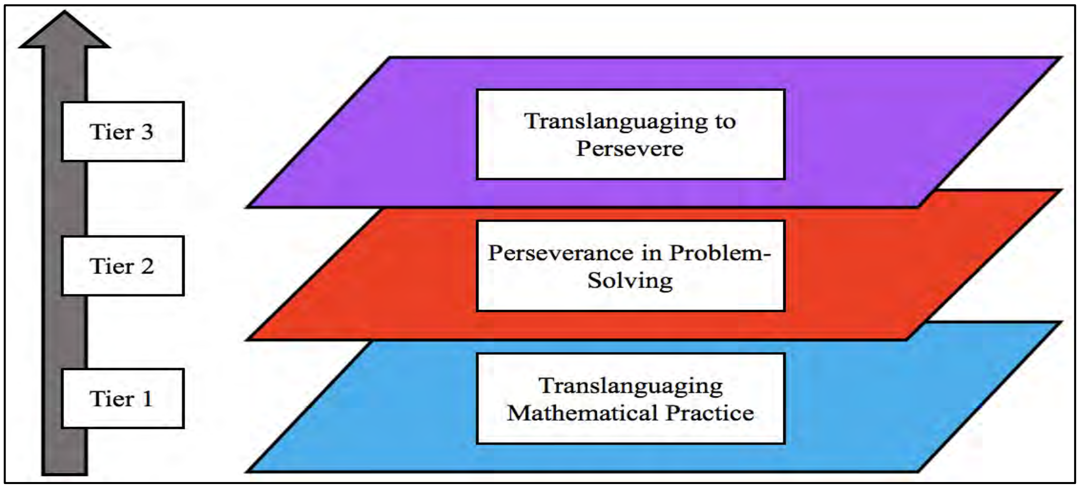


Figure 4. Translanguaging to Persevere: Three-Tier Analytic Method

In Tier 1, the main objective was to understand the meaning-making process and to identify the translanguaging resources to which students made connections a result of this process. We coded video transcripts using the translanguaging mathematical practice framework (see Figure 1) to identify which linguistic resources students used across both English and Spanish as well as non-verbal resources. These resources included the use of the everyday register (ER) and mathematics register (MR) in both English ($_E$) and Spanish ($_S$). Given the nature of communicating mathematically to negotiate meanings, identifying moments when students transitioned to non-verbal mathematical representations that include visual-graphic (MV) and symbolic representations or notations (MN) are also key components to the meaning-making process.

In Tier 2, the main objective was to capture the ways in which students collectively persevered, or did not, during their engagement with a challenging mathematical task. We used the 3PP to do so (DiNapoli, 2018; see Table 1). The Entrance Phase captured whether the group of students understood the entirety of what a task was asking (C-0) and if the group immediately knew how to solve the problem (IO-0). The Initial Attempt Phase examined whether and how a group of students initiated (IE-1) and sustained (SE-1) their effort and the outcome (OE-1) of such effort as they worked toward solving all parts of the problem. In the event the group did not solve the problem after making a first attempt, the Additional Attempt Phase aimed to capture if (IE-2) and how (SE-2) the students amended their original problem-solving plan and the outcome (OE-2) of such efforts as they worked to overcome any setbacks and worked toward solving all parts of the problem. If at this point the problem was not yet solved, a group of students could continue their effort in the Additional Attempt Phase—making a second, third, or fourth additional attempt, and so on.

In Tier 3, we conducted a constant comparative overlay analysis to simultaneously analyze the Latinx bilingual students' translanguaging practice during key moments of their problem solving when perseverance was necessary. Inductively, we analyzed the ways in which evidence of translanguaging practice occupied different phases and components of perseverance. Such analysis results in a theoretical sampling of patterns and themes that help describe the meaning-making actions in those moments of collective problem solving. Two primary themes concerning students' translanguaging and perseverance emerged through this constant comparative overlay analysis, and such qualitative saturation directly informed the selection of two illustrative cases to report in the findings.

Findings

In this section we will first describe aspects of the learning environment established in Ms. Patrick's classroom as evidenced by our observations and interactions with her to help the reader interpret student data. Then, we will present two cases of two different groups of students in two different classrooms exemplifying Latinx students' meaning-making processes and perseverance while engaging in advanced mathematics (with and without Ms. Patrick present). Case 1 depicts Carina, Jessica, Elena, and Ines, and Case 2 depicts Yasmin, Julia, Bernardo, and Lorenzo. The mathematical task at hand spanned three days. For both groups of students, Ms. Patrick introduced the task for about 15 minutes during Day 1, provided about 30 minutes of work time during Day 2, and provided about 30 minutes of work time during Day 3.

Alongside the transcripts of the vignettes, we will depict our Tier 1 and Tier 2 impressions of the group's translanguaging and perseverance, respectively (see Tables 2, 3, 4, and 5 below). We then will unpack the translanguaging mathematical practice that took place across the groups' phases of perseverance in our Tier 3 analysis in the paragraphs that follow each table. We will share these vignettes from both cases to compare how students may or may not persevere and make meaning of a complex mathematical situation involving exponential growth.

Aspects of the Learning Environment in Ms. Patrick's Mathematics Classroom

Through our observations and interactions with Ms. Patrick, we were able to make inferences about her opinions and expectations that helped describe aspects of the learning environment in her classroom. Across the three days of both groups of students engaging with the Function Analysis Task (Fendel et al., 2015b) with "All About Alice" (Fendel et al., 2015a, p. 385), much of Ms. Patrick's careful efforts were in preparation with intent to build a classroom culture that was conducive to exploratory problem solving with bilingual learners. The thoughtful selection of a challenging mathematical task designed with several supports for bilingual learners demonstrated Ms. Patrick's commitment to nurturing her bilingual students toward mathematical understanding. Assigning the Function Analysis Task was an important decision because it allowed students to access mathematical ideas through their prior work and created an environment conducive for productive struggle. Ms. Patrick often encouraged her students to first read their textbook in English and then discuss the mathematical meaning in both English and Spanish. She also encouraged her students to work in their native language when it felt necessary. This offered support for their language and literacy development by helping make the mathematical content more comprehensible.

Collaborative learning was a commonly used pedagogical strategy. Ms. Patrick frequently used collaborative learning strategies to allow her students to communicate with each other in their groups. Such emphasis on student-to-student communication in any language evidenced Ms. Patrick's belief that cultural and linguistic differences are an intellectual resource and valuable to the meaning-making process when learning mathematics. Further, there was ample evidence that Ms. Patrick created a learning environment where using mathematical tools was the norm. Students' understandings were often mediated by the use of the graphing calculator, In/Out tables, and other multimodal visual and symbolic representations. Access to these mathematical tools were always available and encouraged in Ms. Patrick's classroom.

Vignettes From Case 1 – Carina, Jessica, Elena, and Ines

Consider the following vignettes from this group's three-day collaboration around the All About Alice problem. These exchanges exemplify Latinx bilinguals' translanguaging meaning-making processes and perseverance while doing mathematics. When student utterances are spoken in Spanish, we set apart the English translation using italics within parentheses. Just prior to this vignette during Day 1, Ms. Patrick introduced the task at hand. The group demonstrated evidence of understanding the goal of the task (C-0), which was to describe the mathematical relationship at hand, but an immediate solution pathway was not known (IO-0). Thus, our analytic process began on Day 2, when this group already passed through the Entrance Phase of the 3PP and were ready to enter and work within their Initial Attempt Phase (see Table 2).

Table 2
Translanguaging Practice in the Initial Attempt Phase of Perseverance for Case 1

Transcript *(English Translation)*	**Evidence of Translanguaging**	**Evidence of Perseverance**
JESSICA: ¿Qué era la primera, se hace así? *(What was the first one, do you do it like this?)* If [Alice] eats one ounce, that means that she grows twice, dos ¿qué? *(two, what?)* Double, no double, two…See, so when two is four, and then three is six, and four is eight, y así, y así vamos hacer la graph *(like this, and this is how we are going to make the graph).* Going like that (gesturing), para arriba *(up)*. You get it? **ELENA**: Um hmm. Pero *(But)*, how to times it?	**MR_E & MR_S**: Jessica uses her mathematics register to describe and represent how height grows: “grows twice,” “dos,” “double, two,” “two is four, three is six, four is eight.” **MG**: Jessica gestures graphically about the nature of doubling. **MR_S**: Jessica describes the shape verbally: “para arriba.”	**IE-1**: Making sense of context **SE-1**: Exploring doubling
JESSICA: Porque mira *(look)*, two, times two. Well no… Double it by, nomas *(just)* double the number of ounces, so if she takes… **ELENA**: Two times two, y luego *(and then)* four times two, y luego *(and then)* six times two, is that what you are saying?	**ER_S & MR_E**: Jessica combines everyday and mathematics registers: “Porque mira”, “nomas double the number of ounces.” **MR_E & ER_S**: Elena combines everyday and mathematics registers: “Two times two, y luego, four times two…”	**IE-1**: Making sense of context **SE-1**: Exploring doubling
JESSICA: Más o menos como sumando el mismo número. *(More or less like adding the same number.)* **CARINA**: Pero es lo mismo de sumando si lo multiplicas por dos. *(But it is the same as adding if you multiply by two.)*	**MR_S**: Jessica represents doubling with operations: “como sumando el mismo número”.	**SE-1**: Exploring doubling **OE-1**: Better understanding doubling
INES: Lo que parece escomo hicimos un in/out table y ya lo sacamos. *(It looks like we just did an In/Out table and that’s it).* [see Figure 5] **CARINA**: Yeah. In times two equal out… ¿Ya no tenemos que hacer su altura? *(We don’t have to use her height?)*	**MV**: The group draws on a tabular representation of the doubling relationship [see Figure 5]. **MN**: Carina expresses the equation symbolically.	**SE-1**: Revisiting representations of function **OE-1**: Recognizing interpretation mistake

This group entered the Initial Attempt Phase of the 3PP by initiating effort (IE-1) toward differently expressing what doubling meant to them. Jessica used her full linguistic and multimodal repertoire as she expressed what doubling meant to her. She moved fluidly between English and Spanish, drawing on a variety of registers to express her understanding of doubling. She had multiple equivalent ways to describe the relationship: grows twice, dos, double, two, a sequence of number relationships (two is four, three is six, etc.), and finally made an upward, increasing hand gesture to represent the shape of the graph. The group did not, however, realize immediately that they were multiplying the number of ounces of cake by two instead of Alice's height. Not sure how exactly to represent their doubling relationship, Jessica and Elena dialogically moved between everyday and mathematics registers (ER & MR) to try to represent the relationship algebraically. Carina and Jessica finally agreed that it was similar to adding the same number or multiplying it by two, utilizing the mathematics register in Spanish (MR_S). They also quickly represented the relationship symbolically as an equation. Exploration of these points of view is evidence of sustained effort (SE-1) to make sense of the All About Alice function.

Their equation correctly spanned the table of values (see Figure 5), yet these representations did not model an exponential function. This provided the group a metacognitive opportunity to become aware of their error, empowering them to engage more deeply to recognize their error, overcome the setback, and make an additional attempt to make sense of the function. Not completely agreeing with the other students' mathematical representation, Ines specifically studied the table of values and helped her peers realize that they were doubling the number of ounces of cake instead of Alice's height. Speaking entirely in Spanish, Ines persuaded the group back to the problem context and reminded them that they must start with Alice's initial height. Ines used her mathematics register in Spanish (MR_S) to describe how she doubled Alice's height as each ounce of cake was eaten. Ines' revelation showed how perseverance can emerge from a group dynamic.

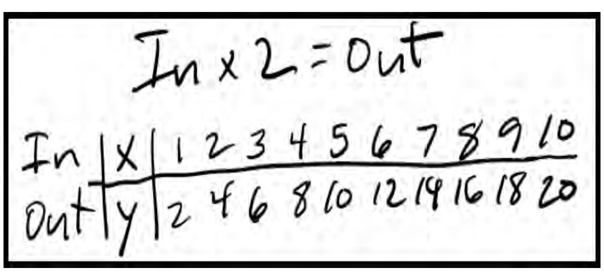


Figure 5. Student-Made In/Out Table From Case 1

Near the end of Day 2, Ines' individual metacognitive awareness of the mistake during their Initial Attempt Phase of perseverance helped collectively move the discussion in a different direction that considered Alice's initial height. The rest of the group began to buy into Ines' point of view amidst moments of struggle, which was essential for perseverance and building equitable and effective learning communities. This change in strategy marked the exit of the Initial Attempt Phase and entrance into the Additional Attempt Phase (see Table 3), which took place during Day 3.

Table 3

Translanguaging Practice in the Additional Attempt Phase of Perseverance for Case 1

Transcript *(English Translation)*	Evidence of Translanguaging	Evidence of Perseverance
INES: Empezamos de cuatro pies. Si toma si come un pedacito son ocho, si come un pedacito son dieciseis, el tercer pedazo dieciseis y dieciseis. Treintaidos ¿no? *(We start at four feet. If she drinks, if she eats one piece it becomes eight, if she eats one piece it becomes sixteen, the third piece, sixteen and sixteen, thirty-two, no?)* **JESSICA**: Pero, ¿cómo sacastes eso? *(But how did you get that?)*	**MRs**: Ines rethinks the problem in relation to the problem context representing a different doubling relationship.	**IE-2**: Changing strategies to consider initial height **SE-2**: Exploring how her height changes
INES: Porque si empezamos con cuatro pies, como yo les digo, si come un pedacito y sale, aumenta de altura de doble (gesturing up). *(Because, if we start at four feet, like I'm telling you, if she eats one piece and it comes out to, her height grows double (gesturing up)*). **JESSICA**: Ohh, her height doubles!	**MRs**: Ines begins with an initial height and doubles that instead of the ounces of cake—"aumenta de altura de doble."	**SE-2**: Exploring how her height changes **OE-2**: Understanding how her height changes
ELENA: You know it's the same thing, mira *(look)*. Dos *(two)*, you multiply one times two is two, two times four is eight, y si pones *(and if you put)* two times two is four, four times four is sixteen. **CARINA:** In squared, times two is equal to your out. [see Figure 6]	**MV & MN**: Students express the new relationship with a table and new equation [see Figure 6].	**SE-2:** Revising a representation of the function

At the start of Day 3, this group began to reinitiate their effort (IE-2) to rethink about the mathematics, crossed out their table, and began to think about starting a new table. Ines helped resustain effort (SE-2) in their Additional Attempt Phase of the 3PP to understand that Alice's initial height is necessary to compute subsequent heights as she ate each ounce of cake. It is important to note that the problem was written in English, yet Ines leveraged her home language of Spanish to revoice the problem. She modeled mathematically (MR_S) the concept of doubling and also used an upward, increasing hand gesture (MG) to demonstrate how Alice's height doubles for each ounce of cake she eats. Immediately after Ines spoke, Jessica demonstrated a productive outcome of resustained effort (OE-2) when she realized they needed to double Alice's height, not the number of ounces of cake.

Following this discussion, the students created a new table of values that correctly modeled Alice's exponential growth (MV & MN). This new approach demonstrated the group amending their plan and making a second attempt to make sense of the All About Alice function. The equation was not yet correct, however, and did not span all of their entries (see Figure 6).

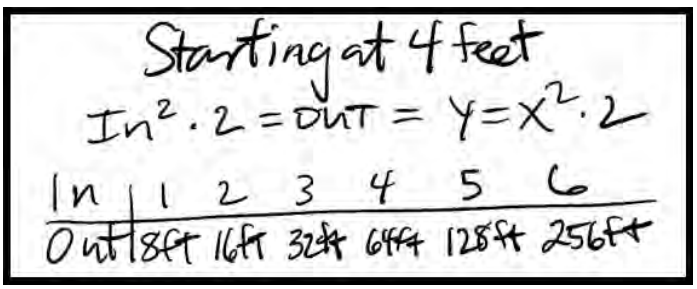


Figure 6. Revision of Student-Made In/Out Table From Case 1

Still, this vignette showcased how this group was making progress understanding the ways in which the relationship in "All About Alice" was not linear nor quadratic while engaging in translanguaging mathematical practice. This was an important opportunity for the group to recognize more mistakes and continue to persevere, which took place near the end of Day 3 (see Table 4).

Table 4
Translanguaging Practice in the Additional Attempt Phase of Perseverance for Case 1

Transcript *(English Translation)*	Evidence of Translanguaging	Evidence of Perseverance
CARINA: Mira, la pongo en la calculadora y luego pido la "TABLE" y me da otra answer de lo que nosotros tenemos aquí. *(Look, I put it in the calculator and then I push "TABLE" and it gives me a different answer from the one we have here.)* [See Figure 7]. *y* equals *x*, quedamos al *(we have)* *x* squared times two, verdad *(right)*? **ELENA:** Three es *(is)* nine… times two … is eighteen. **CARINA**: Y es lo que sale aquí *(That's what comes out here)*. Thirty-two. And out of three sale *(gives)* eighteen. **INES**: Pero ¿por qué? *(But, why?)*	**ER_S, MV, & MN:** Students compare the student-made table with the table generated by the graphing calculator.	**SE-2:** Creating another representation of the function **SE-2:** Comparing two representations of the function
JESSICA: Mhmm. Entonces lo hicimos mal…Pero esto, el OUT, tiene que ser así. *(Mhmm. Well then we did it wrong… but this, the OUT,* (referring to the output values) *has to be like this.)* **INES**: Lo que está mal es esto. *(This* (the equation) *is what is wrong.)*	**ER_S, MR_E, & MV:** Jessica uses the word "OUT" to represent output values from the hand generated table.	**OE-2:** Recognizing equation mistake
JESSICA: Porque también el zero tiene que ser el cuatro, IN tiene que ser cero y luego el out tiene que ser cuatro. Así tiene que ser. Y primero el zero cuatro uno ocho dos dieciseis. (Well, the zero has to be four, IN has to be zero and then the OUT has to be four. That's how it has to be. And, first, the zero four, one eight, two sixteen.) **ELENA**: Solo si tenemos que cambiar el formula, el equation, ¿no? *(The only thing we have to change is the formula, the equation, no?)*	**MR_S:** Jessica explains about pairs of numbers that satisfy the relationship of doubling Alice's height.	**OE-2:** Understanding how her height changes **OE-2:** Recognizing equation mistake
MS. PATRICK: Which one are you guys doing? Alice growing? **ELENA**: Yeah. We thought we had it but now we realize that we don't.	N/A	N/A
MS. PATRICK: What's the base? The base is two. Then what happens? She grows. You put the equation in the calculator. What do you do? **CARINA**: *y* equals. **MS. PATRICK:** *y* equals, so you have *y* equals two to the *x* power. *y* is equal to two to the *x* power.	**MR_E & MR_S:** Ms. Patrick tries to support their understanding of the problem using words like "base," "equation," and "*x* power," which are all part of the mathematics register for exponential functions.	N/A

Near the end of Day 3, Carina used a graphing calculator to make a table of values for $y = 2x^2$ (SE-2; MV & MN; see Figure 7) and determined that it did not match their current table (SE-2). Carina, Elena, and Ines then questioned the source of this inconsistency (SE-2) and ultimately realized and discussed, mostly in Spanish interwoven with the mathematics register in English (ER_S & MR_E), the ways in which their equation was incorrect (OE-2). The students recognized the flaws in their attempt to model the situation with the function $y = 2x^2$ (OE-2) and were ready to continue to explore ways to revise their equation and enter another Additional Attempt Phase of perseverance. However, they had ran out of time and Ms. Patrick needed to move on to a discussion of solution processes. Although this group did not yet find the proper equation to span their In/Out table in this vignette, they were agents of their own translanguaging mathematical practice and leveraged this practice to help persevere and think more deeply about exponential functions.

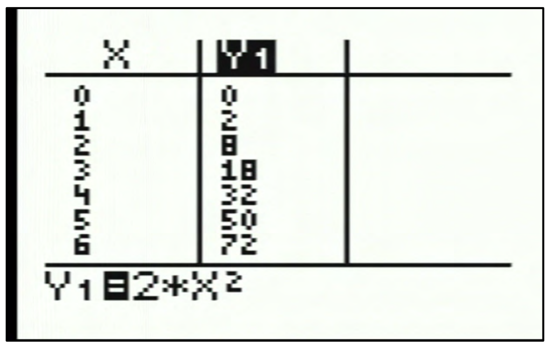


Figure 7. Calculator-Generated In/Out Table From Case 1

In summary, the findings from Case 1 demonstrated a group of secondary Latinx bilingual students leveraging their translanguaging mathematical practice to persevere in their efforts to better understand an exponential relationship. Although Carina, Jessica, Elena, and Ines did not find the proper equation in the allotted class time, they persevered over several obstacles to facilitate more deep mathematical thinking about the All About Alice function. In our view, it was within Ms. Patrick's established, productive learning environment that these students spontaneously leveraged each other's linguistic repertoires and viewed their linguistic resources as an asset to help persevere past obstacles and make progress toward a solution. Ines' strong academic discourse in Spanish was particularly celebrated and played a crucial role in the meaning-making process for the group.

Vignettes From Case 2 – Yasmin, Julia, Bernardo, and Lorenzo

This case depicts a different group of students from a different class than described in Case 1. Ms. Patrick was also the classroom teacher for Case 2 students. Ms. Patrick introduced the activity to this group on Day 1, and, similarly to Case 1, this group of students demonstrated evidence of understanding the goal of the task (C-0), but an immediate solution pathway was not known (IO-0). Our analytic process began on Day 2 when this group continued in the Entrance Phase of the 3PP (see Table 5). Interestingly, although this group of students were similarly poised as the group in Case 1 to leverage their translanguaging mathematical practice to persevere, they never progressed to enter the Initial Attempt Phase of perseverance.

Table 5
Translanguaging Practice in the Entrance Phase of Perseverance for Case 2

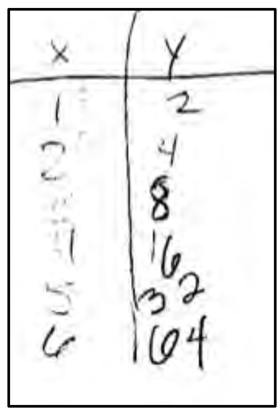
Transcript *(English Translation)*	Evidence of Translanguaging	Evidence of Perseverance
LORENZO: ¿Qué tenemos que decir? ¿Qué tenemos que decir? *(What do we have to say? What do we have to say?)* What we have to say? ¿Qué tenemos que decir? *(What do we have to say?)* **BERNARDO:** We have to explain this, man. **LORENZO:** Pero. ¿Qué hay que decir? *(But what is there to say?)*	**ER_S:** Lorenzo is confused about what he needs to explain.	**C-0:** Reviewing the task objectives **IO-0:** Aspects of solution not immediately apparent
BERNARDO*: Alice in Wonderland*, which one do we have to do? **JULIA & YASMIN***:* "All about Alice." Cuando crece come? Toma o come? (*When she grows, she eats? Drinks or eats?*) **YASMIN***:* Page 385, we have to do this [reads task]. She was telling us yesterday. **JULIA***:* Yea, aqui lo tengo ya. (*Yes, I have it here already.*)	**ER_E & ER_S:** This group moves between languages to clarify the objectives.	**C-0:** Reviewing the task objectives
MS. PATRICK: How are you guys doing? You got your equation? You got your function table? So how much is Alice's height going to grow by after one? **JULIA**: It's going to double.	**ER_E & MR_E:** Ms. Patrick attempts to support their understanding of the task. Julia says that the function is doubling.	**C-0:** Reviewing the task objectives
MS. PATRICK: After one ounce of cake? You need to write that out. *y* equals two to the…? **JULIA & BERNARDO**: Four? … Third? **MS. PATRICK**: Follow your equation and write it out. **BERNARDO & LORENZO**: Two to the… to the power… **JULIA**: But we do for every one? **MS. PATRICK**: Yes.	**MR_E:** Ms. Patrick provides some scaffolds to help the group to symbolize the equation for the function: "*y* equals two to the …?"	N/A
MS. PATRICK: Are you doing it on your calculator too? **YASMIN**: Like that Ms. Patrick? **MS. PATRICK**: No, that's not the general equation, what's the general equation, Julia? **STUDENTS**: *y* equals two…? **MS. PATRICK**: Two what? **YASMIN**: To the second. **MS. PATRICK**: No, what is this equation? **JULIA**: *x* **YASMIN & JULIA**: *y* equals two *x*. **MS. PATRICK**: No! It is not two *x! y* is equal to two to the *x* power.	**ER_E & MR_E:** Ms. Patrick provides verbal sentence frames to help the students to symbolize the equation.	N/A

These students began work on Day 2 by reminding each other that they would need to present their solution to the whole class and by rereading the All About Alice problem to determine how they might proceed (C-0). In both Spanish and English, they discussed details of the instructions and context and seemed ready to enter the First Attempt Phase of perseverance (ER_E & ER_S). At this point, Ms. Patrick intervened and asked the group about their progress obtaining an equation, a corresponding table of values, and a general understanding of the context (ER_E & MR_E). Julia confirmed that they understood the problem's context, that is, that Alice's height doubles for every ounce of cake she eats (C-0). Ms. Patrick then tried to use leading questions to help scaffold Yasmin, Julia, Bernardo, and Lorenzo toward writing an equation to model the scenario. This group seemed to understand that there was a connection between values like 2^2, 2^3, and 2^4 with Alice's changing height but were not sure how to generalize the function of height and ounces of cake. Ms. Patrick then shifted the conversation toward using a graphing calculator to help obtain the correct equation to solve the All About Alice problem. Still, the group did not seem to understand the meaning of their model and its values. Lastly, the students began to guess at appropriate equations for the scenario, and eventually Ms. Patrick stated that the correct equation was $y = 2^x$. The group spent the rest of their time on Day 2 and Day 3 monitoring the routine steps to enter the equation that Ms. Patrick provided to them into the calculator to access the table of values. See Table 6 for an excerpt of their discourse on Day 3.

Table 6
Translanguaging Practice While Monitoring Routine Calculator Steps for Case 2

Transcript *(English Translation)*	**Evidence of Translanguaging**	**Evidence of Perseverance**
YASMIN: Tienes que escribir siempre el *x* la, la equation *(you always have to write the* x*, the equation)* the equation you have to write down, you get it? And then you just give, you… in case you want to an In/Out table asi y no sabe los numeros que van el la *(so and you don't know the numbers that go)* in the x in the y nomas ponla equation aqui *(and just put the equation here)* and then you just, ah um.. **BERNARDO:** Table. **YASMIN:** Table. **BERNARDO:** Second Table.	**ER_S, ER_E, & MV:** Students describe across languages the technical steps to enter an equation and generate a table.	N/A
YASMIN*:* Second Table, Second Table y te da la answer *(and it gives you the answer)*, you get it? **BERNARDO***:* Yeah. **LORENZO**: A ver *(show me).* **YASMIN***:* Good. **JULIA**: [Copying table of values into her notes].	**ER_S, ER_E, & MV:** Students double check the technical steps to generate a table with their calculator while Julia copies the table.	N/A

During Day 3, this group spent their time reviewing the procedures by which to enter the equation into their calculator, the equation Ms. Patrick provided them. In Spanish and English, Yasmin explained how the calculator can produce a table of values for them, as long as they can enter the equation into the calculator (ER_S, ER_E, & MV). Bernardo reminded Yasmin that they needed to push the buttons "Second" and then "Table" in order to access the table of values after they entered their equation. Yasmin then showed the group the table of values on the calculator screen and confirmed that her partners understood how to execute these steps (ER_S & ER_E). While this was happening, Julia copied the table of values into her notes (MV; see Figure 8).



x	Y
1	2
2	4
3	8
4	16
5	32
6	64

Figure 8. Copied In/Out Table From Calculator's Table of Values From Case 2

In summary, the findings from Case 2 demonstrate little to no evidence of the students' translanguaging mathematical practice to persevere to better understand an exponential relationship, compared to Case 1. Although Yasmin, Julia, Bernardo, and Lorenzo were working in a similar context as Case 1—an environment with built-in supports for bilingual problem solving—the students in Case 2 did not spontaneously leverage the available linguistic resources to help persevere past obstacles and make progress toward a solution. Instead, much of the Case 2 students' engagement was in pursuit of rehearsing procedures modeled by Ms. Patrick without much meaning. These findings from Case 2 suggest the over-scaffolding of a group activity in an unproductive way (Stein et al., 1996). We find such over-scaffolding (i.e., modeling procedures without time for exploration) to be incompatible with a translanguaging stance that, ideally, should be facilitating opportunities for students to leverage their linguistic resources to make meaning and persevere. Unfortunately, in this case, the consequences of teacher over-scaffolding resulted in very few perseverance opportunities for Yasmin, Julia, Bernardo, and Lorenzo and, thus, few opportunities to make meaning of the exponential relationship.

Discussion

In this section, we interpret our findings relative to the research questions (i.e., evidence of translanguaging mathematical practice, perseverance, and resultant pedagogical insights). As we reflect on these cases and respective classroom vignettes, we suggest several pedagogical moves to substantially encourage the translanguaging and perseverance practices of Latinx bilingual students. First, we draw on Ramirez and Celedón-Pattichis' (2012) Five Guiding Principles for Teaching Mathematics to Latinx bilinguals to help reflect on the presented vignettes from both cases. These principles articulate effective ways to engage Latinx bilinguals to persevere with rigorous mathematics while engaging in translanguaging mathematical practice. Recall Ramirez and Celedón-Pattichis' (2012) guiding principles: enacting challenging mathematical tasks, leveraging cultural and linguistic differences as intellectual resources, building a linguistically sensitive social environment, offering support for learning English while learning mathematics, and offering mathematical tools and modeling as a resource. These principles help illuminate two meaningful but conflicting insights from the shared vignettes. On one hand, our inferences of the classroom culture built by Ms. Patrick suggested an intentional focus on supporting Latinx bilingual students' mathematics learning. On the other hand, despite this classroom culture, a variance of demonstrated translanguaging mathematical practice and perseverance is evident when comparing the Case 1 and Case 2 vignettes. We discuss each of these insights in the following paragraphs.

Regarding the alignment of Ms. Patrick's practice to Ramirez and Celedón-Pattichis' (2012) five guiding principles, we first note the importance of selecting and enacting the Function Analysis Task (Fendel et al., 2015b). The thoughtful selection of this challenging mathematical task designed with several supports for bilingual learners demonstrates Ms. Patrick's commitment to nurturing her bilingual students toward mathematical understanding. Enacting the Function Analysis Task was an important decision because it allowed students to access mathematical ideas through their prior work and created an environment conducive for perseverance. Ms. Patrick also used collaborative learning strategies to allow her students to communicate with each other in their groups. Such emphasis on student-to-student communication in any language evidenced Ms. Patrick's belief that cultural and linguistic differences are an intellectual resource and valuable to the meaning-making process when learning mathematics. It was evident in the way Ms. Patrick insisted her students use their native language to engage with the task that she genuinely intended to create a productive, inclusive, and linguistically sensitive learning environment. Additionally, Ms. Patrick encouraged her students to first read their textbook in English and then discuss the mathematical meaning in both English and Spanish. This supported their language and literacy development by helping make the mathematical content more comprehensible. Finally, Ms. Patrick created a classroom environment where using mathematical tools was the norm. Students' understandings were mediated by the use of the graphing calculator, In/Out tables, and other multimodal visual and symbolic representations. Access to these mathematical tools proved to be instrumental in the ways in which the students in Case 1 persevered past several mathematical obstacles.

Regarding the variance of demonstrated translanguaging mathematical practice and perseverance when comparing the vignettes from Case 1 and Case 2, we recognize that the alignment of Ms. Patrick's practice to Ramirez and Celedón-Pattichis' (2012) five guiding principles is not enough to support Latinx bilingual students' in-the-moment meaning-making. As depicted in Case 1, Ms. Patrick's acceptance of students using their first language while collaborating can encourage students to engage in mathematical meaning making using their full repertoire of linguistic, symbolic, multimodal resources. However, what we saw in Case 1 was the spontaneous actions of Carina, Jessica, Elena, and Ines using their translanguaging mathematical practice to persevere with the mathematical task. Although the classroom environment was certainly supportive, we do not see much evidence of Ms. Patrick monitoring and facilitating this group's progress to understand the function's complex symbolic representations nor do we see an explicit attempt by Ms. Patrick to make space for students to express bilingually what they did or did not understand in those moments.

As depicted in Case 2, we saw Latinx bilingual students collaborating using Spanish but only to monitor instructional and routine steps—not to navigate meaning. Although Yasmin, Julia, Bernardo, and Lorenzo seemed similarly poised (to those from Case 1) to draw on translanguaging mathematical practice to help persevere with the exponential relationship, Ms. Patrick's early interjection seemed to unintentionally derail any such progress. From a perseverance perspective, the students in Case 2 never had the opportunity to invest effort in a first attempt at solving the problem and quickly became focused on rehearsing the procedures modeled by Ms. Patrick. On Day 3, without Ms. Patrick interjecting, the group in Case 2 had another opportunity to persevere to try to make sense of the function but instead reviewed in both Spanish and English the procedures of how to enter an equation into the graphing calculator to make sure they obtained the correct answers. Contrary to Case 1, in which students spontaneously leveraged translanguaging mathematical practice to help overcome conceptual obstacles, the group in Case 2 drew on their bilingualism to discuss procedures alone. When all group members seemed to understand how to conduct these procedures that their teacher seemed to value so highly, they stopped working and waited for further instructions from Ms. Patrick about what to do next.

The comparison of these cases suggests, in part, that if Latinx bilingual students are given a limited opportunity to productively struggle with important mathematical ideas, regardless of their collective prior knowledge, then they may be reluctant to use the ample resources at their disposal, including translanguaging mathematical practice. Of course, the students in Case 2 were different from the students in Case

1, so it is possible the groups' collective content knowledge was different and might help explain why students in Case 1 were able to make more progress than students in Case 2. However, we do not think this is likely because each group chose to revisit the All About Alice function because they had experienced success with it during Year 2. This implies that the observed differences between groups is less about whether one group was mathematically stronger than the other and more about the different opportunities each group had during the Function Analysis Task to make meaning. The support conditions set forth by Ms. Patrick across Case 1 and Case 2 were different, and it is apparent that Case 2 students required more substantial support to help them make mathematical progress.

Teachers of Latinx bilinguals must develop and enact a substantial translanguaging stance in their mathematics classrooms. In reflecting on the classroom environment examined in this study, we see several ways that teachers could more substantially develop a translanguaging stance and enact a translanguaging pedagogy in their mathematics classrooms (García et al., 2017), specifically by actively supporting translanguaging mathematical practice and perseverance practice in their students. To develop a translanguaging stance, teachers must believe that bilingual students have one holistic language repertoire on which they draw. Although teachers can advocate for students' native language use, we recognize it can be difficult for teachers, especially monolingual English-speaking teachers, to enact in-the-moment teacher moves that respond to student thinking and align with a belief in one holistic language repertoire. Teachers would benefit from a mindset that goes beyond acceptance of dynamic bilingualism to being more celebratory of creating spaces that model and encourage productive struggle specific to the translanguaging practice of Latinx bilinguals (Palmer et al., 2014). Teachers could also build on bilingual students' linguistic resources and home language repertoire by revoicing, clarifying, and probing students' understandings by asking questions about the content in their first language to inform instruction (Ramirez & Celedón-Patichis, 2012). For the classroom environment examined in this study, we contend that there is room to grow to earnestly establish a translanguaging stance by finding ways to access and build on students' cultural, linguistic, and community funds of knowledge (Maldonado et al., 2018).

To further substantiate the development of a translanguaging stance and pedagogy in mathematics classrooms, we iterate on García et al.'s (2017) translanguaging strategies for teachers to employ, which could help support translanguaging mathematical practice in classrooms. First, a teacher could have offered extended reading support for students as they engaged with the English IMP textbook. Students from Cases 1 and 2 may have benefitted from rereading the problem with a teacher present, allowing students to make connections across languages and linguistic and multimodal repertoires. Second, a teacher could have provided more in-the-moment opportunities for students to develop linguistic practices for academic contexts.

Students from Cases 1 and 2 may have benefitted from a teacher prompting them to voice how drawing on both their English and Spanish mathematics registers helps them make sense of a mathematical idea as they work. Third, a teacher could do more to make space for students' bilingualism in her mathematics class. Despite encouragement for students to work in any language they prefer, the students' bilingualism existed in a peripheral space in the classroom: students used their bilingualism in small groups but rarely shared their full linguistic repertoires with the entire class.

In addition to creating the translanguaging stance above, we also suggest that teachers adjust the separative language nature of their classrooms. García (2017) posits that we cannot gauge bilinguals' mathematical proficiency if we artificially separate the students' language repertoires. Much of the work involving bilingual students is framed around an ideology and policies of language separation that encourage teachers to create separate instructional spaces (physical and other) for bilingual students to communicate in either the dominant or minority language (Palmer et al., 2014). Teachers might consider creating a more substantial space in their classrooms that privileges bilingualism and supports the development of their bilingual mathematics identities. One step toward creating such a substantial space would be for teachers to encourage their students to present their problem-solving process to the entire class in Spanish and have other students interact with their process in a bilingual way. In addition, students could help translate these presentations for monolingual English speakers (including the teacher), furthering the development of their bilingual mathematics identities. We see students from Cases 1 and 2 benefitting from such intentional identity work.

Conclusion

Through this study we addressed calls to action by Martinez et al. (2017) and NCTM (2014) by examining the ways in which Latinx bilingual students used translanguaging to persevere to make sense of challenging mathematical ideas. Such work helps rehumanize mathematics education for Latinx bilingual students in the United States by privileging their unique resources to learn mathematics with understanding and finding ways to better encourage the use of these resources (Gutiérrez, 2018). We observed and conducted member-check interviews with two groups of 12th-grade Latinx bilingual students in different classes working to make sense of an exponential function with the same monolingual, English-speaking teacher. We found that a group of students were capable of spontaneously translanguaging to persevere with a challenging problem, seemingly supported by peripheral classroom supports established by the monolingual teacher. However, we also found that a similarly poised group of students did not spontaneously collaborate in this way and required more substantial support from their classroom teacher to encourage translanguaging mathematical practice. Unfortunately, such teacher support did not happen, and these students had few opportunities to persevere to make meaning. This suggests that teachers should not solely rely on students to spontaneously engage in these practices and instead must fully commit to having a translanguaging stance. Having a translanguaging stance that manifests as native language encouragement around rich mathematical tasks is not enough; teachers must also plan for and enact in-the-moment moves to help Latinx bilingual students leverage their linguistic resources and persevere to make meaning of mathematics. Building from suggestions from Ramirez and Celedón-Patichis (2012), García et al. (2017), and Palmer et al. (2014), these teacher moves can work toward creating intentional spaces for bilingualism to be present, visible, and celebrated in mathematics classrooms.

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