

# Improvement of Learning Skills in Geometry Incorporating a Metacognitive Learning Model in Boys Compared to Girls

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Received: June 7, 2021 Accepted: July 15, 2021 Online Published: July 29, 2021

doi:10.5539/jel.v10n5p51 URL: <https://doi.org/10.5539/jel.v10n5p51>

## Abstract

“We learn by doing and by thinking about what we are doing.” (John Dewey)

In this article, we shall present findings that describe the degree to which metacognitive orientation contributes to the study of the geometry the plan in boys compared to girls in 9th grade of middle school. The geometry study process does not only involve knowledge but also high thinking abilities. Beyond the knowledge of definitions and sentences, the students are required to write a full, precise, and logically constructed proof, as well as to show the validity and its correctness. In this article, we shall present a model of metacognitive orientation aiming to develop higher-order thinking skills in geometry. We built and applied the model to 9th-grade students. Since students experience difficulties in the study of geometry, the development of a structured study process is required. Numerous studies clearly show that the study process involving metacognitive orientation improves their study ability and deepens their understanding of the topic in question. The question that we addressed was to what extent the metacognitive orientation in geometry impacted boys in comparison to girls?

In this study, we shall present data according to which metacognitive learning explicitly benefits girls more than boys. Nevertheless, as a modular model, it allowed every student of both sexes to strengthen the weak aspect and to overcome blockades inhibiting the learning process.

**Keywords:** self-regulated learning, metacognition, levels of thinking, Van Hiele theory, metacognitive orientation model

## 1. Introduction

The study of geometry is part of the mathematics curriculum and is considered by teachers, students, and parents alike as one of the most important study areas in school. In the light of the importance of the subject, scholars, mathematicians, and educators are all dealing with the question of how to improve the ability of students and their performance in this subject.

In recent years, researchers have found that the development of thinking in students improves their performance and their ability to deal with complex problems (Strauss, 1997). Thinking is an activity during which data are absorbed from the surroundings through the senses or recalled from memory and thereafter is processed and reorganized in the mind.

In the process of teaching and learning, the main emphasis is on the development of reflective thinking that encourages the student to discuss and ask questions. This type of thinking that revolves around asking questions and looking for answers enables the student to broaden the scope of acquired knowledge and to inquire about the subject that he is learning (Beyer, 2001).

Accordingly, “the student as researcher” receives validation in the subject of mathematics, given that the development of mathematical and geometric skills requires tools and skills that allow him to feel capable, curious, combine knowledge and information, develop proofing faculties, use logic and combine different fields of mathematics.

Accordingly, the educational system the student learns many subjects through which he acquires knowledge, does he or she acquire thinking skills? At what age should thinking skills be acquired?

Howard Gardner (1996) in the theory of multiple intelligences claims there are eight different types of intelligence in every person. Some are more dominated and some are less; however, what matters most is that this intelligence can be developed and improved thanks to adequate orientation already at a young age. In the realm of mathematics, two bits of intelligence can be discerned: Logical-mathematical intelligence and spatial-geometric intelligence. The questions that have preoccupied many researchers in the fields of mathematics, mathematical education, and psychology include the following:

- Which are the skills and tools that students should acquire to develop thinking, in particular, mathematical and geometric thinking?
- How can this be achieved?

The answers to these questions have brought about a shift in the world of education, as teachers and researchers gradually started to understand that knowledge does not suffice to develop thinking, which leads to the question of what is missing?

The answers to these questions lie in the acquisition of skills by the student through self-regulated learning—SRL (Ben-Eliyahu & Linnenbrink-Garcia, 2015). Self-regulated learning relates to the production of self-thoughts, feelings, and actions that are planned to achieve learning goals such as reading and processing data (Zimmerman, 1990). According to this concept, the student should be actively learning and be capable of tackling the tasks presented to him. In other words, learning is not something that happens to students, but rather something happening via the students. Moreover, this learning method applies the metacognitive thinking process that triggered a shift in the world of learning in general and in mathematics in particular.

Since students do not score high marks in the study of geometry (according to the findings of TIMSS exams 2007, 2011), there is no doubt that the necessary teaching and learning skills would develop geometric thinking and self-confidence.

This article aims to show the contribution and influence that self-regulated learning has for the study of mathematics, with an emphasis on metacognition in the study of geometry by boys compared to girls. To this end, we shall present a model of cognitive orientation that assists students in acquiring learning skills and strategies to understand geometry. On the one hand this model is uniform, and on the other hand, it provides a fitting solution for every single student according to gender.

## 2. Literature Overview

According to the publication of the U.S. National Council of Teachers of Mathematics (NCTM, 2000), the study of geometry is an integral and important part of the mathematics curriculum. The Israeli Ministry of Education attributes great importance to build broad infrastructures in the fields of mathematics and geometry already from a young age (Note 1). In the general, study, processes require thinking and the main goal is to develop higher-order thinking. The teaching of thinking while learning plays an important role (Ball, 1996). Numerous studies mentioned the importance of developing thinking early on in the parental home (Sigel, 1991). Children go to kindergarten with intuitive concepts that grow into precise concepts at a later stage (Department for Curriculum Development and Planning, Ministry of Education—The State of Israel).

In kindergarten and elementary school, the study of mathematics is based on two main areas: numerics and geometry. In geometry, beyond knowing axioms and formulas, building logical arguments and basing a proof plays an important role. Numerous studies show that the lack of understanding in the early stages of learning geometry is at the root of misunderstanding newer subjects (Biber, Tuna & Kormaz, 2013). Van Hiele characterizes understanding in general and with respect to geometry in particular as follows: The student understands the topic that is being studied, whether he can apply it in a new situation, to perform tasks deriving from the given situation correctly and appropriately, and to present this way with awareness (Van Hiele, 1999).

Van Hiele grounds his theory on five developmental stages in the study of shapes and bodies. According to this theory, every learning stage is based on the stage that preceded it. Van Hiele showed that the student will not acquire the ability of writing a geometric proof if one of the following stages is missing from the five levels of thinking:

1) Recognition—at this level the student can study a set of geometric shapes and differentiate between them. The shape is perceived as a whole (without attention being paid to its components) as it appears and the student bases his explanation on the classification of shapes according to their general shape. At this stage, the student still ignores the characteristics of the given geometric shape. If the student asks why the shape is called a rectangle, He will answer that the shape looks like a rectangle.

2) Analysis—at this level the student can identify and analyze characteristics of shapes. The student knows the characteristics of the geometric shapes that he sees, however, he does not know or understand every single characteristic. Yet he is not able to connect between the different characteristics, and he cannot explain how one characteristic is derived from the other. In other words, he still does not understand the relations between the characteristics. The explanations of the students at this level are based on an informal analysis characteristics of the geometric shape. If the student asks why the square is a rectangle, his will answer that the opposite sides are equal.

3) Ordering—the student understands the logical meaning of shape, the relation between different shapes and their characteristics, as well as the importance of precise definitions. He still doesn't grasp the meaning of the deductive structure as one entity, yet can understand how one characteristic derives from another and is still not able to prove the characteristics of the geometric shapes.

The student will be able to prove that the sum of the angles in a square amount to 360 degrees, yet he is not able to prove that the sum of the angles in a triangle amounts to 180 degrees.

4) Deduction—the student understands the meaning of deduction as a means to develop geometric theory, he understands the role of basic principles, the definitions, axioms, sentences, and proofs (as links in the chain of the deductive structure). At this stage, he can use hypotheses to prove sentences and to understand the meaning of necessary and sufficient conditions. At this level, the student can provide reasons and explanations for the proof stages; however, he still does not understand the importance of precision, neither does he understand the formal aspects of deduction.

5) Precision—the student understands the importance of precision when dealing with different structures, he can perform abstract deductions, while he understands the formal aspect of deduction. At this level, he can research the results stemming from one system of axioms being replaced by another. He knows and can compare different proof strategies. He can “discover” new sentences and proof strategies, and can think about the problem of identifying a wider context, wherein a particular sentence may be applicable.

Van Hiele theory represents a breakthrough in the world of teaching geometry. Studies show that using the five levels guides the student clearly through learning that combines collecting information with knowledge. Therefore, it can be considered to be constructed learning. The Van Hiele theory is mainly directed at the teacher who leads the learning process and it raises the question of whether the student has a share in this process? Will the student be able to perform activities that will lead him from one level of thinking to another?

To answer these questions, the current state of the teaching system in Israel should be examined, as follows:

The mathematics curriculum in middle school includes three areas:

Numerical mathematics (including statistics and probability), algebra, and geometry (Note 2). This program is based on the content studied in 1st to 6th grade, providing the student with a deeper and wider understanding of these subjects.

In these grades, a shift occurs in the levels of knowledge and analysis that are required to form the student. At these ages, students are required not only to display knowledge and understanding but also to gradually be capable of applying analysis, synthesis, and evaluation (Bloom, 1956). The levels of thinking identified by Van Hiele get a new meaning in both middle and high school as well as at university. In middle school emphasis is placed on geometric writing (Paul, 2015), such as mathematical modeling and mathematical symbol—using mathematical language to represent and solve problems, to criticize the solution provided, to develop explanations and claims, to use representations and different tools, and to develop strategies for solving problems (Curriculum Unit, Pedagogical Secretariat, Ministry of Education, State of Israel, 2012–2013).

#### A. Implementing geometric language and learning the proof structure

Reaching their first year in middle school, students have no knowledge how to write a mathematical proof, and therefore conceptualization is new to them. Students deal with patterns that they do not know and are required to write a proof based on several sentences which they have to understand, remember and know how to apply (OECD, 2004). In elementary school, students learn first and second-degree thinking according to Van Hiele, and now they are required to construct the remaining levels of thinking. The study of geometry necessitates a language that must be internalized by the students. This language is not only based on an array of sentences and definitions, but also on the ability to build logical claims and deducing conclusions (Paul, 2015).

The Ministry of Education—the State of Israel's Curriculum Development Unit—built a geometry dictionary that every student should know during his years of schooling. The difficulty that the student is confronted with

does not lie in the content but its application and writing. Writing a proof in geometry requires precision in the writing of sentences, the ability to perceive geometric shapes, and in finding the link between different shapes for example overlap, resemblance, reflection, shift, rotation, and symmetry. The student must write a proof that requires higher-order thinking, which includes complex mental activities, as opposed to lower-order thinking skills that require memorizing and recalling information (Ball, 1996). In this study, we shall attempt to decipher how the difficulty and its impact should be addressed.

#### B. The impact of thought and intelligence on learning processes in geometry

In the previous paragraph the world of content was presented as the basis of geometric language. Content is relatively easily dealt with by students, as it only requires memorizing a set of definitions. The main challenge that students face is the ability to write proofs and to develop logical geometrical proofs. To assist students in this challenge, two important concepts must be understood: Thinking and intelligence. These concepts accompany and influence the entire learning process, even more so about geometry.

Thinking—is a mental activity that deals with the information that is processed by the senses from the surroundings or the information that is recalled by memory, and then processed and reorganized by the senses. Intelligence—in parallel to thinking intelligence has also been the subject of research: the entirety of skills through which problems that require thinking may be solved.

Until 1983 there was a widespread opinion that there exists one single general intelligence, which is evaluated using an IQ test. Howard Gardner (1996) claimed that there isn't only one general intelligence, rather there are several forms of intelligence. For this study, the following are of relevance: Mathematical-logical intelligence and geometric intelligence. Quite often we notice a student who displays high abilities in calculation, deduction, use of an algorithms for problem resolution while having trouble in visual perception and abstraction.

#### C. Mathematical modeling and mathematical symbolization as pillars of geometric language development

Geometric language includes a set of signs and molds that the student has to know in addition to the sentences and definitions. Writing a geometric proof requires correct and precise writing to develop proof based on sentences and logical claims. The deductive proof required from students of geometry requires a high level of precision (level 5 in the levels of thinking according to Van Hiele). Therefore, two pillars are needed to combine between different sentences and definitions to obtain a clear geometric proof: Modelling and symbolization.

Symbolization—is the set of symbols that are used in geometry, for example,  $CB + BA$ . The student should know that the sign  $\perp$  means that there is a straight angle between two parts. Ignoring the symbols prevents the student from knowing how to analyze explicit and implicit data.

Modeling—This term refers to all proof structures that the student should know. For example, when applying rules showing two triangles to be congruent, the student should know the difference between the ASA and the SAS rule. Incorrect use of congruence rules leads to mistakes in the proof and consequently in the conclusion drawn.

The acquisition of strategies for the study of mathematics in general and statistics, in particular, is already required at an early age. Learning by understanding from a young age allows the student to acquire the capacity to deal with more complex material in advanced subjects over the years (Putnam, 1992). Students face several difficulties in the study of geometry and mainly in writing a proof that includes: Correct use of sentences and definitions, correct use of geometric symbols, and building clear proof structures according to the rules. The teaching methods should combine between proof components: Rules, definitions, symbols, and proof structures. Due to the complexity involved in writing a proof in geometric language, students should receive metacognitive guidance, which will be addressed in this article. Geometric language (axioms, sentences, symbolization, modeling, and proofs) is acquired over time along with the gradual acquisition of a wide breadth of terminology. Full mastery of the mathematical language with a special focus on geometrical language is necessary but does not suffice to understand and tackle geometric tasks (Fauzi, Dirgeyase, & Agus, 2019). Therefore, the student needs to develop skills and tools that will enable him to learn in a way that improves and develops the language as well as the ability to successfully solve exercises (Leviathan, 2012). The most important challenge posed by mathematics in general and geometry, in particular, is the ability to find new understandings and perceptions based on prior knowledge to build new knowledge.

### 3. Scientific Background of Self-Regulated Learning (S.R.L) and Its Importance in the Study of Geometry

Since the 1980's research has not only dealt with thinking and cognition but also has come up with the new concept of metacognition (Flavell, 1979) in response to a question that preoccupied the researchers regarding the path to developing thinking processes and whether the latter can be imparted to the students.

As we shall see in the course of this study, to complete an exhaustive and meaningful study process, the four following components should be integrated: Cognition, metacognition, motivation, and the studied context. The study process comprising all four elements is called “self-regulated learning”.

This study focuses on one of the elements found in self-regulated learning, i.e., metacognitive thinking. The aim is to check how self-regulated learning in study processes will promote and benefit geometric thinking in boys compared to girls, with an emphasis on the contribution of the question model presented below.

*3.1 SRL—Self-Regulated Learning Refers to the Generation of Self-Thoughts, Feelings, and Actions to Achieve Study Goals (Pintrich, 2000).*

The students who master self-regulation are capable of developing a targeted study based on adapting approaches to the study of specific subjects. This learning is characterized by the student’s mastery of four elements by which self-regulated learning is characterized: Cognition, metacognition, motivation, and the context of the learning in question.

*3.2 The Importance of Self-Regulated Learning in the Development of Mathematical Literacy*

According to the Pisa study, mathematical literacy is defined as follows: “An individual’s capacity to reason mathematically and to formulate, employ and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts, and tools to describe, explain and predict phenomena. It helps individuals know the role that mathematics plays in the world and make the well-founded judgments and decisions needed by constructive, engaged, and reflective 21st Century citizens.” This definition focuses on the mathematics that is necessary for the solution of problems in everyday life—the mathematics that human beings use regularly in a way that assists them whenever they judge or make a decision as engaged citizens who think and reflect.

Moreover, the PISA study defines mathematical literacy as a characteristic, with a permanent potential for development. As every characteristic, it can be described on the spectrum between high and low literacy skills. Those who possess high literacy skills, compared to those with low literacy skills, know how to use mathematics and mathematical tools to judge and reach informed decisions, which are necessary as an engaged and reflective citizen.

Based on this definition and the analysis of secondary data from previous years, it appears that self-regulated learning is the main key to success. This learning does not only serve a short-term purpose, but it bears potential for the future development of the independent learner.

The analysis of PISA findings shows that students with high scores in scientific literacy have a high level of self-regulated learning based on texts (OECD, 2007). Many studies in the field of teaching sciences have strengthened this claim and even applied it (Pintrich, 2000; Kramarski & Michalsky, 2009). The processes of self-regulation are not innate but may be developed and nurtured by relevant intervention and support. Thus, the challenge posed by self-regulated learning is to teach students using adequate intervention and support how to channel the different aspects of learning: Planning, applying strategies, reflective understanding while learning, motivation, joint study, etc. These processes allow the student to take an active part in learning.

#### **4. The Importance of Metacognition to Improve Skills and Thinking in Geometria Learning**

*4.1 The Integration of Metacognition into Mathematical Learning Processes*

Self-regulated learning cognition and metacognition is comprised of two major elements. Cognition—includes all the internal processes involved in processing data that mediate between stimulus and response. Cognition refers to all the knowledge and information that are at the disposal of human beings as well as to the processes whereby it was acquired, processed, and applied (Flavell, 1979). These internal processes are thinking, perception, understanding, learning, listening, memory, inferring conclusions, decision-making, and problem-solving.

Metacognition refers to higher-order thinking concerning knowledge and awareness about one’s cognitive processes and the ability to actively direct and control them (Flavell, 1979). Through metacognition, the student has better management of his cognitive skills. It is therefore essential to learning. He can define weaknesses and mend them by acquiring new cognitive skills and improve his learning achievements (Schraw, 1998).

The term metacognition denotes two different meanings:

- One’s knowledge about one’s cognitive system.
- One’s ability to regulate, criticize and adjust the cognitive processes that one uses.

Metacognitive knowledge refers to the thinking about one's own thoughts and to the processes of performing specific cognitive tasks.

Becoming aware of the thinking processes involves reflective thinking that occurs before, during, and after the performance of the task. While devising a plan to solve the task the student should define the task's objective, nature, and schedule as well as the means required to perform it. While carrying out the task, the student should check whether the objective is logical, if he progresses towards its achievement, understands what he has done, and whether any changes are required. Upon completion of the task, he should compare between the successful and the unsuccessful parts of the exercise regarding the goals he had set out to achieve to conclude as to the right way of action for the next time he will be asked to do a similar task. Studies on the subject indicate that explicit reference to metacognitive knowledge proves to be most effective for the development of the student's thinking (Garner, 1987; Nelson, 1996).

The first mathematician to address and develop mathematical-geometric thinking without using the term metacognition was George Polya in 1945.

With his four-step model, Polya became the pioneer of mathematical problem-solving. The model is based on four principles of problem-solving, while every phase consists of a strategy to be applied using directive questions. It is only in the 1980s that mathematical education and psychology researchers started to use the term metacognition and build new models based on Polya's steps. The necessary metacognitive process requires being acquainted with the student and the study material. Students become more aware of their thinking; they get a better grasp of cognitive processes in general and metacognitive processes in particular, and the more they work according to cognitive awareness, the better students they become, capable of higher-order thinking (Shabtai, 2018; Schraw, 1998).

#### *4.2 Metacognition to Promote Learning, Teaching, and Evaluation of Geometry*

Metacognitive thinking is of great importance not only for the student but also for the way of teaching and the evaluation in class (Shabtai, 2018).

Metacognitive knowledge is directly linked to the way the student will learn and carry out tasks in class. Students who are acquainted with different learning strategies, ways of thinking, and problem-solving will tend to make use of them, whereas those who are not acquainted with them will not be able to use them.

An additional and important aspect consists of the student's self-observation; in other words, his strengths and weaknesses allow him to prepare accordingly towards learning. The following example from geometrics will serve as an illustration: If the student has difficulties solving questions in geometry, he should be taught words that guide him through the question, which means disclosed, clear, data, along with implicit data that exists but that we need to infer as part of the question (Polya, 1945).

Polya was also a pioneer in rationally applying metacognitive guidance to the study of mathematics. Metacognitive guidance assists the student in analyzing given data and thereby enabling both student and teacher to devise a valid logical and formal geometric proof according to the rules of mathematical writing.

The stages of metacognitive guidance enable the student to make the most out of every stage and thereby progress in the study of geometry. The stages of metacognitive guidance assist one in moving between Van Hiele's levels of thought and thereby locating the difficulties and addressing them.

The added value is the ability to replicate and evaluate in a final stage where the student certifies whether the solution is logical and correct, or whether he can solve it in different ways. The student's strategies to find additional ways of solving the problem will eventually lead him to a deep acquaintance with the study material and to expand his knowledge. In every task in geometrics, one should pay attention to carry out the following stages:

- Devise a plan—choose adequate thinking strategies and apply resources before performing the task.
- Manage the process of the task according to the chosen plan.
- Control and monitor—being aware of thinking processes and evaluating them throughout the performance of the task.
- Identify and correct mistakes—detect mistakes and insert corrections to improve the performance of the task.
- Evaluate and check—appraise the thinking process and outcome at the end of the task's performance and find additional ways of solving the problem.

Concerning teaching—metacognitive knowledge should be explicitly taught. Therefore, schools require an explicit metacognitive plan.

Studies that address the link between metacognition and learning achievements constantly show that the study results improve the more metacognitive processes occur throughout the learning. Although metacognitive thinking develops with age and experience, its development can be accelerated and made more efficient by teaching that is specifically geared to that end. Metacognitive teaching improves metacognitive thinking itself and contributes to the student's achievements in the different fields of knowledge (Tzohar-Kramarski, 2000; Rozen & Kramarski, 2013). This type of teaching proved to be particularly efficient for students with low achievements (Zohar & Ben David, 2008), as they were guided on how to construe the whole from its parts.

The PISA study defined a model for problem-solving in mathematics and geometrics that included the three following processes: Formulating—mathematically formulating states; Employing—employing terms, facts, prior knowledge, mathematical procedures, and inferences; Interpreting and evaluating the mathematical results.

Another element featured in the PISA study were all the skills and capabilities on which mathematical processes are based (communication, mathematization, representation, inference and claim, planning strategies for problem-solving, use of language, and symbolic actions that are both formal and technical, and use mathematical tools). To assist students in thinking and problem-solving in geometry according to the aforementioned model, self-guidance should be integrated into learning. By integrating cognitive and metacognitive guidance, the student will be able to plan, reflect, interconnect and evaluate his way of action.

The metacognitive self-guidance model will serve the student as a systematic guide in the learning process. The questions arising from the model do not depend on each other; however, the progress according to the proposed order is of great importance.

#### *4.3 Gender Gaps in the Study of Geometry*

Research shows that geometric skills are lower in girls than in boys (Mainali, 2019). This piece of data requires thorough analysis first and foremost concerning learning processes. As is well known, there are differences between the attitude of boys and girls to the study of geometry as early as elementary school (Markovitz & Forgasz, 2018). The learning gaps between boys and girls in the study of mathematics or the attitude to learning mathematics tend to grow at a later stage (Markovitz & Forgasz, 2018). In order to advance learning processes in general and geometry in particular, we created a model that strengthens learning skills in girls and boys and thereby promotes their geometric thinking ability. Metacognitive guidance assists girls in building a logical process leading to complete proof, whereas in boys it contributes to a more precise reference to explicit and implicit data. The guideline we followed was to find a question model that gives metacognitive guidance to students and will enable every student to progress and bridge gaps or get a deeper understanding of the study material.

The metacognitive guidance model will systematically lead the student in the learning process. The questions appearing in the model are not interdependent; however, it is indeed important to advance according to the suggested order.

### **5. The Research**

#### *5.1 The Research Question*

a. What is the contribution of the metacognitive guidance model to improving geometric abilities in boys compared to girls?

The hypothesis: Given that metacognitive guidance maximizes geometric capability in boys and girls, we wish to examine more closely to what extent self-guidance influences the development of geometric capabilities from a gender point of view

#### *5.2 Experiment*

a. Development of a model of questions for metacognitive guidance featuring six questions and its adaptation to the study of geometrics.

b. Instructions for teachers on how to apply and implement the model throughout learning.

c. Devising a multiple-choice questionnaire for students. The questionnaire consists of eight questions. The answers ranged from 1 to 5 on the Likert scale, whereby 1—included not at all 5—very much.

d. Data segmentation is shown graphically and numerically.

e. Use of statistic exams, t exam for independent sampling.

### 5.3 Methodology

The research population consisted of 12 students—six boys and six girls. The students were learning geometry using a metacognitive guidance model throughout ten school years. Students and teachers received the model questions. The subject studied during these classes was a new topic in geometry that required prior knowledge. Teaching and learning were accompanied by guidance questions (The model).

The purpose of the research was to examine the extent to which the model contributes not only to the study of a new topic but also to bridge the learning gaps with the intention to overcome blockades. In the light of the modular character of the model, additional aspects were examined to this end such as: dealing with complex questions, writing a full proof, etc. The group of boys and the group of girls practiced the guidance questions both in class and during self-study.

As aforementioned, the model is meant to develop abilities of independent learning, enabling the student to criticize and regulate his study process.

The teacher received the guidelines before the start of the research. The teacher devised preparation cards that included the model questions as they are presented below. The student will be able to use the guidance cards in the course of the study in class and for homework.

Guidelines for the teacher:

- 1) Students should be prepared using the following questions. The teacher should distribute the guidance questions in the form of cards for practice in class and homework.
- 2) Section 6 (in Table 2)—the questions should mainly be prepared during the stages of practice. The student should be encouraged to express in writing given developing thinking skills.
- 3) Geometric language is a language that consists of rules and laws including information, prior knowledge, and logic. Students should be trained in the correct and sound use of language.

Table 1. Student questionnaire for learning geometry according to the metacognitive guidance method

Question number	Questions in the metacognitive guidance process	Question goals
1	What is the subject matter of the exercise? Is all the data clear? Mark or explain in words what is the meaning of every piece of data	Understanding
2	Is there any prior knowledge that you can use? Discover and identify the hidden data (which are not explicit in the question)? Can they assist you?	Context
3	Which ways of action will you choose to solve the problem / write the proof? Which mathematical sentence do you use? Which symbols will you use?	Strategy
4	How will you verify whether the proof is valid?	Verification
5	Are there additional ways? Compare with your classmates. Which difficulties did you meet?	Self-verification
6	Are you able to construe a question in geometry based on sentences and information from the aforementioned question? If so, let your classmate solve the question.	Activation

Table 2. Final assessment questionnaire for students

The answers to the questionnaire are from 1–5 according to the Likert scale (1—not at all; 5—very much). The reliability of the questionnaire is 0.804 in the Alpha Cronbach index.

Question number	The question
1.	Is the learning process assisted by the “Question Sheet” helpful to the understanding of the topic?
2.	Is the learning process assisted by the “Question Sheet” helpful in understanding the data in the question?
3.	Did the learning process assisted by the “Question Sheet” help you connect the study topic with other topics that you have already learned?
4.	Did the learning process assisted by the “Question Sheet” help you write a complete proof?
5.	Did the learning process assisted by the “Question Sheet” help you understand the geometrical terms?
6.	Did the learning process assisted by the “Question Sheet” help you solve complex problems in geometry?
7.	Did the learning process assisted by the “Question Sheet” help you present several solutions to the same problem?
8.	Did the learning process assisted by the “Question Sheet” help you get a better understanding of the hidden data – data that was not explicitly written in the question?

The results of the questionnaire are presented in Table 3.



#### 5.4 Data and Results

Table 3. The averages of each question in the group of boys compared with the group of girls

The assumption was that no change would be found between boys and girls as a result of using the metacognitive guidance model in geometry. The boys demonstrated better geometric abilities than the girls before the start of the experiment.

	1	2	3	4	5	6	7	8
male	3.33	4.16	3.83	3.50	2.83	3.50	3.83	3.50
female	3.50	4.00	3.83	4.16	4.16	4.00	4.50	4.50

Table 4. T-TEST for independent samples

gender	NUMBER	MEAN	STD DEVIATION	STD ERROR MEAN
male	6	3.5600	0.39684	0.14031
female	6	4.0813	0.33323	0.11781

\* The results presented in the table above were analyzed based on a questionnaire that was distributed between the students. The reliability of the questionnaire is 0.804 in the Alpha Cronbach index.

To validate the hypothesis that there is a difference in the development of geometric ability in boys compared to girls with a metacognitive orientation model, a T-test of independent samples was performed. The results show a clear difference between the average values ( $T(8) = 4.0813$ ,  $p < 0.05$ ). The average of the questionnaire that evaluated the improvement of geometric ability in girls ( $M = 4.0813$ ,  $SD = 0.33323$ ) was found to be higher than the average of geometric ability in boys ( $M = 3.560$ ,  $SD = 0.39684$ ). This result provides us with insight that the metacognitive guidance model improves learning processes and understanding in girls more than it does in boys.

As can be seen the girls' results in most of the questions in the questionnaire were higher. These results suggest the model's contribution to improving learning skills among girls is greater than in boys.

Another thing that can be learned from the results is that the standard deviation among the girls was smaller than in the boys. This figure indicates a better and more reliable prediction in the answers. In addition, the standard deviation error mean among the girls was smaller and thus the results are more reliable.

The results reflect a much greater contribution of the metacognitive guidance model for girls than among boys. Accordingly to the results, seen it is possible to improve the learning skills in geometry and improve the abilities among the girls and thus reduce the gap between boys and girls in geometry studies.

#### 6. Discussion and Conclusions

The use of a model with questions for guidance proves that although boys have an advantage in geometric abilities and tend to be more successful, the ability can also be improved in girls. The improvement of the ability of girls is made possible by a systematic learning process. As research about learning methods in girls has shown, the learning processes in girls include supervision and self-evaluation, defining objectives, and planning (Bidjerano, 2005). The model devised by this study successfully presents the ways to reinforce these learning methods.

Previous studies show that boys have an advantage in studying mathematics in general and particularly in geometry compared to girls (Markovitz & Forgasz, 2018; Mainali, 2019). The results show that use the metacognitive guidance model has been able to build and improve the skills in geometry and reduce the gap between boys and girls.

The learning process constructed using the metacognitive training model was usefully obtaining missing or unclear information. In this approach, it is possible to develop the required skills in geometry and develop a sense of competence. It is worth noting that the model also helps boys, but its contribution to girls was significant.

The study examined several parameters, each of which consists or could consist of a learning bottleneck for the students. In the study of geometry, beyond knowing sentences and definitions, students are required to write a valid mathematical proof. In writing the proof, the students are required to refer to mathematical symbols and mathematical modeling, which constitute the foundation stones of geometrical language.

Table 3, listing the averages, shows that in six out of eight questions the average of girls was higher than that of

boys.

The research findings are detailed below:

1) the First question—Was the model helpful in understanding the subject matter?

In the group of girls, there was an improvement in writing a complete proof using the model. The ability of write a complete proof strengthens the feeling of being able to understand and develops the ability to understand the material.

2) Second question—was the metacognitive guidance model helpful in understanding the data presented in the question?

As we already mentioned, the model provides a differential answer to every student. The evidence thereto is provided by the findings according to which boys would need to focus more on understanding the data, compared to girls and in comparison with other parameters that were chosen. Nevertheless, it can be seen that the question model proves to be very helpful for girls. It can be inferred with certainty that thanks to the model, blockades that delay learning processes can be removed. Moreover, it should be pointed out that the model is unique in that it provides a differential answer to every single student.

3) Third question—Did the model help you link the study topic to other topics that have already been learned?

In this question, the averages were equal and it appeared that both boys and girls used the model to connect between the topics that had been studied in the past. One of the “blockades” that the question model aims to remove is the inability to find a context during learning processes. Without context, students will find it difficult to build proof based on explicit and hidden data at once.

4) Fourth question—Did the model help you in writing a complete proof?

In girls, there was a noticeable improvement in writing a complete proof. Writing a proof is one of the most complex tasks for students. To solve a problem, students are required to examine strategies that provide a solution to what they were requested to prove. The process of proving involves both the understanding of the data and the ability to integrate it correctly and precisely into writing. The ability to write a complete proof shows a successful connection between all the parts integrated into a whole that constitutes the required proof.

5) Fifth question—Did the model help you get a clearer understanding of the geometrical terms?

The group of girls showed noticeable improvement in understanding concepts in geometry. This finding strengthens the finding mentioned in the previous section that an understanding of the concepts results in an improvement in the ability to write a complete proof.

6) Sixth question—Did the model help you solve the complex problems in geometry?

Complex questions in geometry are problems that combine data that is both explicit and hidden within the question, necessitating an intermediate proof (a proof using which we can find additional data) before the required proof. It was found that the model helped solve complex problems in girls more than it helped boys. This finding connects to the previous findings showing that the clearer and more precise the basic levels are, the higher are the levels of thinking required by the learning processes.

7) Seventh question—Did the model help you present several solution for the same question?

One of the challenges in the study of mathematics in general and geometry, in particular, is to enable students to present several ways to solve a problem. A presentation of several ways to reach the solution reflects a wide range of abilities that are expected from the student. The “model” contributed more to the construction of the subject studied by the girls vs boys. Thus, contributed to the development of their ability to represented several solutions.

8) Eighth questions—Did the model help you get a better understanding of the hidden data?

A hidden piece of data is an element that does not explicitly appear in the question. The student is expected to identify which one is the hidden piece of data and how it contributes to the writing of the proof. The “model’s” contribution to the identification of hidden data strengthens the findings that have been presented so far, showing that in the absence of an understanding of hidden data it is not possible to write proof.

From all that has been stated above, it appears that the model significantly contributes to the learning processes and the development of geometric thinking faculties in girls more than in boys. In the adaptation of the questions, the guideline was to enable comprehensive learning without missing details. Comprehensive learning was made possible thanks to the eventuality that the students had to go back to the previous question to complete the

missing information and to move on to the next question.

Metacognitive guidance questions gradually help students to progress, enabling independent learning that encourages not only thinking but also thinking about thinking and performance evaluation – that is, the verification of the result. It can be seen that the model did not only benefit the group of girls, but also the group of boys. It can thus be inferred that the “model” suits everyone, albeit not equally.

## 7. Recommendations

- 1) A horizontal study with a greater number of participants should be carried out to examine to what extent the “model” contributes to the increase of the number of students taking 5 units in mathematics for the matriculation certificate from high school, and to examine the increase in the number of girls learning mathematics on the level of 5 units (highest level of difficulty towards obtaining the Israeli high school matriculation certificate).
- 2) It should be further inquired whether students can use the model in additional areas of study of mathematics.
- 3) Mathematics study programs should be created that integrate metacognition.
- 4) Personal study programs should be created based on the model, and thereby reduce learning gaps and improve the students’ achievements in geometry.
- 5) Research should be carried out to examine the extent of the model contribution to decrease students’ anxiety relating to the study of mathematics in general and geometry in particular.

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## Notes

Note 1. The Ministry of Education – the article relates to geometry studies in the State of Israel.

Note 2. According to the 9th-grade curriculum of middle schools in Israel.

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