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DETERMINATION OF 5TH-GRADE STUDENTS STRATEGIES IN COMPARING FRACTIONS

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Abstract: The aim of this study is revealing the strategies used by 5th-grade students in comparing fractions. In line with this purpose the study is based on qualitative research. Participants of the study conducted with convenient sampling method are seventy 5th-grade students. The students have been asked six questions which necessitate comparison of fraction pairs, considering the sample questions included in mathematics textbook within the limitations of 5th-grade gains on fractions. In addition, for the purpose of revealing the strategies of students in comparing fractions in more detail, face-to-face interviews are held with ten students. The students interviewed have been selected from among those who use different strategies, apply the rules and make faulty reasoning. In order to ensure the validity and reliability of the research, research data are checked by another researcher and similar themes and categories to the ones found by the researcher are reached. Collected data are analyzed by using content analysis and the strategies that the students use in comparing fractions are tried to be determined. As a result of the analyses, it is seen that students mostly prefer "*Applying the Rules*" where they use procedural information. It is determined that the number of students who prefer "*Strategy Development and Discovery*" which require conceptual comprehension is less than others. The findings show that consideration of conceptual and procedural comprehension in comparing fractions and not imposing rules on students and ensuring the students reach the rule is important.

Key words: Mathematics Teaching, Fractions, Comparison in Fractions

1. Introduction

In order for individuals to maintain an effective life in society, they need to perform basic calculations such as counting, addition, and multiplication (Yıldırım, 2014). At this point, natural numbers as one of the basic subjects of mathematics play an important role in supplying individuals with basic skills. However, natural numbers remain insufficient to solve some problems in our daily life. For example, natural numbers do not help if we need to distribute 4 apples among 5 children. That is because the answer is 4/5. Here, we need to do a different mathematical process like fractions. It is possible to count the quantities around you and express them with a natural number. However, some cases (e.g., one and a half kilos of tomato, half a liter of milk) cannot be expressed in natural numbers. The ancient Egyptians thought these centuries ago and found fractional numbers (Baykul, 2014). Therefore, fractions are a subject that is taught immediately after the natural numbers in the curriculum of the Ministry of National Education. One of the main reasons for this is that we use fractions in our daily life frequently (Bingölbali & Özmantar, 2015). In order to express fractions, concepts such as division and measurement are required. The main difference between fractions and natural numbers is that while natural numbers respond to the question "how many", fractions respond to the question "how much" (Doğan-Temur, 2011). In this direction, it is possible to define the fraction as a common feature of sets consisting of parts of the integral with the same quantity (Altun, 2008).

The concept of fraction, which refers to the parts of an integral, can be among the difficult concepts to comprehend for children. Despite the difficulty of comprehension, children face with fractional concepts from the early years and develop various schemes related to the subject. Children begin to

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learn fractional concepts such as whole, half, and quarter at a very early age, and assign meanings to these concepts as different sizes. In other words, when doing comparisons between the two parts, they use expressions such as big and small about the parts (Nures and Brgant, 2008). For example, when using the word 'half', a child uses it to express sometimes a small piece and sometimes a part close to a whole. Such fractional expressions that students face in everyday life form the basis of a systematic fraction teaching (Olkun & Toluk Uçar, 2014). Being able to comprehend fractions and make deductions about them will enable children to perform comparisons and ordering on fractions, and consequently make calculations related to fractions (Kılıç & Özdaş, 2010). And, this seems to be possible only with an effective and efficient fraction teaching.

Comparative strategies are used to determine whether two objects are the same or different according to a particular feature (Erdoğan, 2009). In order to make a comparison in fractions, students should be able to internalize the concepts of number sense and equivalence (Gersten & Chard, 1999; Kılıç & Özdaş, 2010). Although the concept of fraction forms the basis of teaching many subjects in mathematics, it is seen that students make many mistakes in ordering and comparison processes in fractions (Kılıç & Özdaş, 2010). The main reasons why students make these mistakes are shown as failing to understand the part-whole relationship in fractions, doing wrong operations, misconceptions with regard to ordering and comparisons of fractions, and not comprehending the sense of fraction (Haser & Ubuz, 2002; Stafylidou & Vosniadou, 2004; Soylu & Soylu, 2005; Biber, Tuna & Aktaş, 2013; Okur & Çakmak-Gürel, 2016). As a fractional number is a number indicating the relationship between a whole and its parts, the part-whole relationship should be emphasized in the studies about fractional numbers (Pesen, 2003). Especially when the mistakes made by the students in the primary and secondary school are examined, it is seen that students cannot understand the part-whole relationship in fractions (Kocaoğlu & Yenilmez, 2010; Kılıç & Özdaş, 2010; Karaağaç & Köse, 2015). The reason why students experience difficulties in comparing fractions is that students make comparisons like in natural numbers and they are made to memorize the rules of fractions rather than learning them (Haser & Ubuz, 2002; Vamvakoussi & Vosniadou, 2004; Olkun & Toluk Uçar, 2014).

The first concept that students should learn when comparing fractions is that the fractional numbers with the greater denominator are the smaller. This concept is very difficult to acquire. Because the students have been using the natural numbers in the comparisons they have made so far. The relationship of greatness and smallness in fractional numbers is different from natural numbers. So, 4 is smaller than 5, but 1/4 is greater than 1/5. The subject becomes more complicated when it comes to numbers that are closer to one another. The path to be followed here by teachers is to help students find their own solution strategies. While comparing fractional numbers, instead of giving the rules and asking the problems that require applying them, teachers should allow students to grasp the relationship between the fractional numbers and guide students to deduce the rules (Baykul, 2014).

In their study, Clarke & Roche (2009) asked 6th Grade students to explain which of the fraction pairs was greater. It was found that students had difficulty in fractions especially other than those with the same numerator and denominator. Students had the most difficulty in fractions 3/4 and 7/9. More than half of the students who answered this fraction correctly reached the correct answer by using the common denominator. It was found that the students who reached the answer using a different strategy were quite a few. Clarke, Roche & Mitchell (2008) gave students pairs of fractions and asked them to choose the greater fraction in their study called ten tips of building fractions based on logic and cognizance. They also asked the students to explain why they chose that fraction and develop strategies. These tasks were given to 323 students from 6th grade at the end of the school year. Their performance of comparing various pairs was 77.1% correct. While the most appropriate explanation was (3/8, 7/8), the most difficult fraction pair was (3/4, 7/9), and the students achieved only 10.8% success in this question. The most successful students used two strategies that were not taught at school. The first one is the method we call comparison, in which students compare the size of fractions according to 0, ½, or 1. For (3/7, 5/8) in matching pairs, students noted that 3/7 was smaller than a half, and compared that 5/8 was greater than a half. Another innovative strategy was the thought strategy in which we think about the remainder. Students looked at the amount needed to reach the whole, and comparing 5/6 and 7/8, they reached the conclusion that 1/6 was needed to make the first fraction a whole, and 1/8 was needed for the second.

Many studies conducted on students' comparison of fraction sizes revealed that students thought that fractions with greater numbers were the greater one between the two fractions given (Bingölbali & Özmantar, 2015). A strong sense of fractional numbers acquired in primary school years forms the cornerstone of rational numbers that students will encounter in the future education levels. This is because students can use their reasoning when making comparisons between fractions and better comprehend the sense of fraction (Way, 2011).

1. 2. Purpose of the Study

This study is important to ensure the students to learn to compare and sequence fractions by discovery instead of rote learning. Students should be allowed to develop strategies to compare fraction pairs and reach the rule by using the strategies they develop because it will contribute to the realization of permanent learning. In this respect, it is obvious that the results of the study will guide teachers when they teach comparing fractions. In this direction, the aim of the study is determined as finding out the solutions used by students in comparing fraction pairs. In accordance with this purpose, the answer is sought to the question of "What type of solutions the students use in comparing fraction pairs?"

2. Method

2. 1. Research Design

This study which is conducted to determine the strategies of students in comparing fractions, examine them thoroughly and analyze why they use the said solution is carried out with the qualitative research method. Qualitative researchers try to understand how people evaluate and interpret their experiences, how they relate to their experiences, and how they build their world (Merriam, 2015). Qualitative research was preferred while conducting this study as it is a research type which is widely used in social sciences and aims to elaborate on social phenomena, the human behavior and the reasons of these behaviors in detail (Güler, Halıcıoğlu & Taşkın, 2015).

2. 2. Study Group

The workgroup of this research is determined after two phases. In the first phase, 70 (33 males & 37 females) 5th grade students have been selected by using the convenient sampling method to perform the "Achievement Test to Determine Strategies in Comparing Fractional Numbers". Convenient sampling method can be utilized to minimize the loss of time, effort and cost, and in order to add speed and practicality to the research (Patton, 2015; Yıldırım & Şimşek, 2016). In the second stage, after the achievement test was applied to seventy students, ten students in the study group were selected for interview based on some criteria. Ten students who have composed the study group were selected by using the criterion sampling method which is a purposeful sampling type. Criterion sampling is a study of all situations that meet a pre-determined set of criteria (Yıldırım & Şimşek, 2016). Participants who will be selected by the criterion sampling method in the research should have some criteria according to the purpose of the research. Accordingly, the second phase of participants is determined by considering criteria such as students who answer the questions in the achievement test by using different strategies, who answer them by applying the rule and who answer them by making faulty reasoning.

2. 3. Data Collection

To determine the participants as a data collection tool, "Achievement Test to Determine Strategies in Comparing Fractional Numbers" is used in this study. 6 questions are used in this form with regard to comparing and sequencing fractions. The textbook of the Ministry of National Education that is taught in schools was taken into consideration while preparing the questions. No fractional expression that is not included in the book was used in the test. The achievement test was applied to seventy students and ten students who used different strategies, applied rules and conducted faulty reasoning were selected for the study group. These students we selected in order to provide a clearer understanding of

the answers they gave in the achievement test. During the research conducted with ten students, data were obtained with the interview technique frequently used in qualitative research. It is possible to classify qualitative interviews as structured, semi-structured and unstructured. The semi-structured interview form is the interview forms where the predetermined questions are asked by the researcher to the participants and also new questions are asked during the interview based on the main questions (Güler, Halıcıoğlu & Taşkın, 2015). In this respect, the interview is necessary to learn about directly observable behaviors, situations, emotions or how people express the world around them (Merriam, 2015; Patton, 2015).

The achievement indicators for the comparison of fraction pairs are included in the primary school mathematics education program. The purpose of choosing the fifth-grade students for the research is because they know the fractions subject from the fourth grade and they can make fractional comparisons. In order to conduct the study, it was assumed that the students who participated in the research had learned the fractional comparison in the fourth grade of primary school. While the students in the current program are expected to make comparisons in the fourth grade, the updated program includes the following statements about fractions. "*M.5.1.3.5. He/She is able to sequence fractions with equal numerator and denominator. Examples of sequencing fractions whose denominator is a multiple number of the other are also included* (MEB, 2018, p. 53)". In this context, an achievement test was applied to a total of seventy 5th grade students (33 male & 37 female) studying in public schools in Istanbul. The participants were given the Achievement Test to Determine Comparison Strategies in Fractional Numbers developed by the researchers. Before the achievement test was applied, expert opinion was taken from a specialist who had completed his doctorate in mathematics education and the achievement test was finalized. A content analysis was applied to the answers of seventy 5th grade students to the questions via a document analysis. As a result of the achievement test, a total of ten students (six female, four male) who used different strategies, applied rules and found to have faulty reasoning were selected for the interview. In order to reveal the student strategies in more detail, face-to-face interviews were conducted with the students by using semi-structured interview forms prepared by the researchers and the answers of the students to the achievement test were examined in depth.

2. 4. Data Analysis

In line with the purpose of this research; the data obtained from the achievement test to determine the Comparison Strategies in Fractions were subjected to content analysis by document analysis and interviews were made with ten students using different strategies, rules and faulty reasoning based on random selection and volunteering. The data obtained through the semi-structured form were analyzed by the content analysis method. Content analysis is a qualitative data analysis method used to identify and quantify the existence of words, concepts, experiences, characters or sentences within one or more texts (Seggie & Bayyurt, 2015). In addition, the names of the students participating in the research were coded according to the ethical rules and included in the study findings.

2. 5 Validity and Reliability

The credibility of the results obtained by qualitative research is considered as one of the most important subjects of scientific research (Yıldırım & Şimşek, 2016). Validity in qualitative research is related to whether the findings of the research are correct or not. In this respect, the two most important aspects are that the research is valid and reliable. In order to ensure validity and reliability in the research, the following procedures were performed.

- Students were accepted in the study on a voluntary basis.

- Since the validity in qualitative research is generally related to the meanings and results extracted from the data (Güler, Halıcıoğlu & Taşkın, 2015), the research data was checked by a different researcher and similar themes and categories were obtained.

• Comprehensibility and relevance of the questions used in the achievement test which requested to compare the fractional pairs were also checked by two researchers who are experts in their field.

• In order to understand the suitability of the questions in the semi-structured interview form, firstly 2 students (different from 10 students who formed the participants of the research) were piloted and both the comprehensibility of the questions were measured and prior experience of the actual application was acquired.

2. 6. The Role of the Research

Qualitative research is based on interpretation, and researchers often have intense experience and continuity with participants (Creswell, 2017). In this study, the researcher was directly involved in the data collection process and in the data analysis, and served as a tool. In the process of collecting data, the researcher tried to be as objective as possible.

3. Findings

In this part of the research, semi-structured interview notes with students and the data obtained by the achievement test which requires comparison of fraction pairs were analyzed and supported by quoting the answers of the students to the questions. In this direction, firstly the achievement test performances of the students were given in Table 1.

Table 1. Students' Answers

Question	The Number of Students Who Gave a Correct Answer	The Number of Students Who Gave an Incorrect Answer	The Number of Students Who Left Blank
1	62	7	1
2	59	9	2
3	44	21	5
4	48	20	2
5	51	15	4
6	34	33	3

Table 1 presents the distribution of the correct, incorrect and blank answers given by the students to the achievement test for determining the comparison strategies in fractions. The students generally answered the questions correctly. A total of 70 5th grade students participated in the study. The most correctly answered question by the students was the first question (n = 62) and the least correctly answered one was the third question (n = 34). The strategies used by the students for the solution of related questions were obtained through interviews. The themes obtained in this study are given below.

Themes Related to Students' Comparison Strategies in Fractions

In the framework of the findings obtained from the study, it is seen that students use two basic strategies when comparing fractions. These are defined as *rule learning* and *strategy development - discovery*, and explained below by presenting evidence.

Theme 1: Rule learning

It was found that students generally used the information they learned from their teacher or written materials (explanatory sources such as books, journals) while comparing the fractional pairs. They particularly used certain pattern expressions which we call rule learning (such as in fractions with the equal denominator, the fraction with the greater numerator is greater) when comparing fractional pairs.

When comparing simple fractions, the rule that the smaller denominator is the greater was the most commonly used solution strategy by students. For example, the Student-M saying that *"If the denominators are equal, then the one with the greater numerator is the greatest. This is how I solve it.*

Also because eight is greater" showed that he/she applied rules while comparing in simple fractions. Again the rule that the fraction with the smaller denominator is greater in fractions with the same numerator was another strategy frequently used by the students. For example, Student-Z saying that *"8/9 fraction is greater; in fractions with the same numerator, the one with the greater denominator is smaller*" shows that he/she uses this rule as a strategy. In addition, the strategy used in the fractions whose numerator and denominator is not the same was usually as such stated by the Student-K *"If the numerator and the denominators are not matching in fractions, then the ones with the smallest numbers are the smallest"*.

During the interviews with the students, it was observed that the students learned the rules from their teachers and had a tendency to apply these rules. In this respect, students use the rules as a solution strategy more often in the case of fractional problems. However, it is seen that some students have developed different strategies by going out of the rule. This situation is explained under the heading of theme 2.

Theme 2: Strategy Development, Discovery

In comparing fractional pairs, some students reached the right answers with logical reasoning outside the rule application. However, this situation was used by very few students considering the number of students who took the achievement test. Especially *completing to one* and *dividing by expanding* were the mostly used strategies by students. As a result of the interviews with the students, it was understood that these rules were not taught in schools and that students did them as a result of their own logical reasoning. Because, while the majority of students reached the correct answer through the rule application, some students were seen to reach the answer with a perspective based on discovery as follows. For example;

Student-C; *4/9 + 5/9 = 9/9* *smaller*

8/9 + 1/9 = 9/9 *greater*

Above, the student completed the given fractions to 1 and reached the correct answer. After the interview with the student, the student's strategy was tried to be understood by asking why the student did so. The student explained "*Simple fractions are fractions smaller than one. If we bring them closer to one, the one we add the least is greater*". Smith (1995) stated in his study that the students can use the well-known numbers such as 0 and 1 to compare the fractions. Therefore, we can easily say that the student can understand the basic concepts of fractions for comparison in fractions. In addition, when the student was given the rule and asked why he had not done it in that way, the student answered, "*if it a multiple answer exam, I would do it that and not lose time, but here it wanted us to write our strategy to compare fractions"*.

Some students primarily developed a strategy for themselves, and then reached the solution by using the existing rules. For example; the statement of Student-S *"7/9 is greater. Because if model them both, 7/9 is greater as it is closer to one. If we divide them both, we see that 7/9 is greater. And since the numerator and denominator of this fraction are greater and also since more parts are taken, it is greater."* shows that student first solved the question by using strategy and then stated the rule. In line with this example, it can be said that some students tend to combine their own strategies with the rule.

It is observed that the students who make up the study group use two basic strategies in comparing the fractional pairs. Findings on how students use these two basic strategies in the questions given to them are stated below.

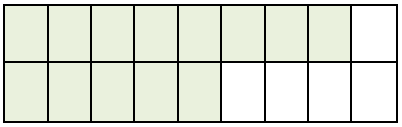
Findings from the Fraction Comparison Strategies Test

Findings Related to the Question 5/9, 8/9 which is Greater

This question was the most correctly answered question by the students (62 students). In the question, it was seen that generally no strategy was produced and the problem was solved by rule application. The rule applied to this question is that "*The fraction with the greater numerator is greater in fractions whose denominator is the same*". The example below supports this situation.

Student-M says *"8/9>5/9, if the denominators are the same, then the one with the greater numerator is the greatest. This is how I solve it. At the same time because eight is greater"*. When the answer of the student is examined, it can be said that the student chose the rule application in order to solve the question. When the student was asked why he/she did not choose a different way, he/she answered as

follows: *"There is no other way, our teacher has taught us this way, if there was another solution way our teacher would have told us"*.



In another example, it was seen that Student-L solved the question in the following way: *"When we count the fractions with the same denominators normally, the numbers are the same size"*. The symbolic representation of the shape used by the student in solving the problem is given on the side. The student showed which fraction was greater by drawing two shapes of the same length. It can be said that the student has understood that the fractions should have wholes with the same length in formal representation.

As a result of the interviews with the students, it is seen that the students generally tend to apply the rules taught by their teachers. Instead of trying different strategies, students tend to apply the rules given by their teachers. Indeed, the statement of Student-M as *"Our teacher told so, that's why I'm doing it this way because he/she knows everything"*. is an indication of this. In this respect, teachers should not give the rules in advance and they should enable students to develop strategies. In short, the teacher should enable the students to learn by discovering the rule.

Some students did not apply rules when comparing fractions and compared the fractions with a strategy (by determining a reference point). The students completed the fractions to one and determined that the one which was added more was the smaller fraction. Petit, Laird & Marsden, (2010) says that the strategy of completing to one can be used in comparing fractions. This strategy was used by two students. The following example illustrates this situation. For example;

Student-C; *4/9 + 5/9 = 9/9* *smaller*

8/9 + 1/9 = 9/9 *greater*

In the interview with the student using the above strategy, it was tried to determine why the student solved the question in this way. The student answered, *"8/9 is greater because we added less to complete it to one"*.

Findings Related to the Question 8/9, 8/7 which is Greater

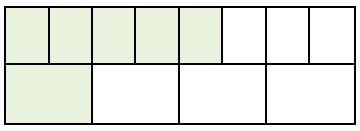
Many of the students who answered this question (59 students) reached the correct answer by applying the rule. The rule they applied was generally that "*in fractions with the same numerator, the one with the smaller denominator is greater*". Some examples given below show that students tend to apply rules.

For example, Student-S says *"8/7 fraction is greater because in fractions with the same numerator the one with the smaller denominator is greater. If we divide the cake into eight pieces and share it with seven people, they would have more"*. During the interview with the student, the student was asked why he/she gave this answer. The student said, "*I did so because that's the rule"*. In this respect, it is possible to say that the student applies rules when solving the question. It is thought that the student's failure to develop a strategy is due to the fact that he has not learned different approaches to the question.

When the answer of the student who answered the question incorrectly is examined, for example in terms of Student-K who said *"8/9 fraction is greater. In fractions with the same numerator, the one with the greater denominator is greater*", it is seen that the student confused the rules with one another. Since the student memorized the rule, he/she confused the rules and answered incorrectly. During the interview with the student, he/she was asked why he/she had solved the question in this way, and the student gave the correct answer. However, when we told the students that he/she answered the question incorrectly, he/she said *"I must have been confused"*. Thus, it is seen that comparing fractions based on rote learning triggers students to make mistakes.

Findings Related to the Question 5/8, 3/4 which is Greater

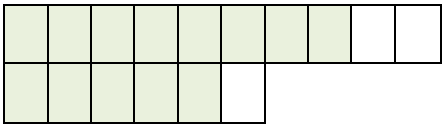
It is seen that more than half of the students (44 students) answered this question correctly. The explanations of how students solved the questions are given below.



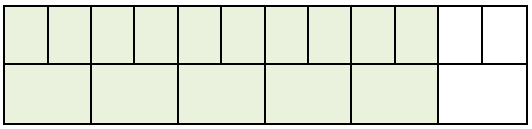
The majority of the students who answered correctly reached the correct answer by expanding the fractions. For example, the symbolic representation used by the Student-S in solving the problem which is shown on the side can be an example of this

situation. The Student-S said *"3/4 fraction is greater. Because when we expand the fractions it becomes 6/8 and in the fractions with the same denominator the one with the greater numerator is greater"*. As a result of the interview, it was seen that the student knew the concepts of expansion and simplification in fractions and applied them to the questions. The Student-L, on the other hand, tried to reach to the answer of the question by stating that *"5/8>3/4, 5/8 fraction is greater"*. As a result of the interview with the question, when the student was asked why this fraction was greater, he/she answered: "*I looked at the numbers and saw that 5/8 fraction was greater which had greater numbers*". It is seen that the student conducts faulty reasoning in solving this problem. The student thought in the same way as in natural numbers and pointed out that 5/8 fraction was greater thinking that it had greater numbers.
The Student-Z: *"If neither is the same in fractions, then the one with the smaller numerator and denominator is greater. Therefore, 3/4 is greater"*. When the answer of the student is examined, it is seen that he/she answered correctly to the question by applying the rule. However, it can be said that there is a misconception here. For instance, when the fractions 7/5 and 4/5 are given, the correct answer of the student would be 4/5 according to this rule. However, while 7/5 fraction is a compound fraction, 4/5 fraction is a simple fraction and 7/5 is greater.

Findings Related to the Question 10/12, 5/6 which is Greater
It is seen that more than half of the students (48 students) answered this question correctly. Most of the students who answered correctly applied expansion in fractions and a few of them reached the correct answer by simplification. It was seen that the students who gave the wrong answer did not know the concepts of simplifying and expanding the fractions.



The symbolic representation of the shape used by the Student-P in solving the problem is given on the side. Student-P *"10/12 is greater because it has greater numbers. He/she gets more cake"*. This expression of the student shows that he/she thinks that they would get more cake because the numbers are greater. In addition, it is seen that student has not comprehended that he/she needs to draw shapes of the same size when comparing pairs of fractions. He/she drew the 10/12 fraction bigger than 5/6 fraction and said that 10/12 was greater taking its amount into consideration.



The symbolic representation of the shape used by the Student-L in solving the problem is given on the side. Student-L revealed the reasoning he/she applied while solving the question by saying *"Here again we make the denominators same and see that they are equal"*. According to Bray & Sanchez (2010), the majority of students prefers to equalize the denominator while comparing fractions. When the student was asked how he/she would solve the question without equalizing the denominators during the interview, the student answered: "*Then I would divide both the numerator and denominator by two and see that I get 5/6=5/6 which shows that they are both equal"*. It is seen that the student knows the concepts of simplifying and expanding the fractions.

Findings Related to the Question 7/6, 9/11 which is Greater
In this question, it is seen that the students who answered correctly applied equalization in the denominator. It is determined that very few of the students answered this question correctly by using the concepts of simple and compound fractions.
The statement of Student-D saying *"I think 7/6 is smaller it has smaller numbers"* shows that the student thinks as in natural numbers. When the question was asked if he/she could show the answer to the question by drawing, he/she drew 7/6 fraction bigger and the other smaller and once again said that 7/6 is smaller.

With the statement of Student-A saying *"If we subtract 11 from 9 we get 2, but if we subtract 6 from 7 we get 1, so 9/11 is greater"* it is seen that student has a misconception. During the interview we have made with the student, the student makes the mistake of subtracting the bigger number from the smaller one and tends to apply the operations used in natural numbers to fractions.

Student-P saying *"7/6 is greater, I don't know why"* has revealed that he/she did not know its reason although he/she answered correctly. The student said that *"when we divide 7 with 6, the result is*

greater than the other that's why it's greater". When we asked why he/she had not written that, he/she responded: "*that would not be correct, our teacher showed us another way"*. When the mentioned discourses are examined, it is seen that the students cannot go beyond what is taught by their teachers.

With the statement of Student-K saying *"7/6 is greater because when we divide 7 with 6 we get 1/6, but the other is not a whole, and compound fractions are greater."* it is seen that student has comprehended simple and compound fractions and approaches questions in this way. The student also says "*When we equalize the denominators, 77/66 is greater than 54/66. As we have learned in class, in fractions with the same denominator, the one with the greater numerator is greater."*

Findings Related to the Question 3/5, 7/9 which is Greater

It was determined that the students made the most mistakes in this question. Considering the response and correctness rates in fractions with the same numerator and denominator, it is observed that the students had difficulty in this question. From this point of view, it can be said that the rules do not facilitate students' job, but only work to solve the questions where the rules can be applied. For example, the statement of Student-K saying *"If the numerator and denominator are not matching in fractions, then the one with the smaller numbers is the smaller"* shows this situation.

The students who answered the question correctly generally preferred the way to equalize the numerators and denominators. For example, Student-F says *"If we equalize the denominators in 3/5 and 7/9 fractions, the fraction 7/9 will be greater, so, 7/9 is greater"* As a result of the interview with Student-F, the student was asked why he/she was doing such an operation. The student answered *either the numerator or the denominator should be the same in order to make comparisons in fractions*. The student actually applied rules, but he/she learned the rule in fractions with the same numerator or denominator. Therefore, he/she chose to reach the result by equalizing the denominators of the fractions. The Student-A saying *"7/9 because when we expand it we get a bigger number"* followed a similar solution strategy.

It is seen that some students used a different strategy to solve this problem. For example; Student-S answered *"7/9 is greater. Because if model them both, 7/9 is greater as it is closer to one. If we divide them both, we see that 7/9 is greater"*. Similarly, Student-H *"Equalized the denominators and you get more cake per person from a 35-pieces cake than a 27 -pieces cake. I think fractions with the same denominator is like money. Ali has got 8 liras and he divides it to 9 people whereas Ayşe divides her 8 liras to 7 people. In this case, the people who took money from Ayşe has more money. Therefore 8/7 is greater".*

4. Conclusion

! In this study conducted to determine the strategies of fifth-grade students in comparing fractions, it is determined that students use some out of the box strategies which are not taught by teachers. However, it is determined that a large part of students have applied the rules but not a different strategy in comparing fractions even though they have answered the questions correctly. When the literature was examined (Aksu, 1997; Haser & Ubuz, 2003; Soylu & Soylu, 2005; Gould, 2005; Sowder & Wearne, 2006; Kılıç & Özdaş, 2010), it was determined that students used the applied the rules they memorized and had difficulty in comparing and sequencing the fractions in many studies conducted. This is in line with the findings of our study.

When another finding of the study was examined, it was determined that the students generally applied rules when comparing fractions. The majority of the students who solved the questions by applying rules answered the comparison questions in the fractions with the same numerator and denominator. This result also clearly shows that students tend to apply rules when comparing fractions. This result is similar to the study results of Reys, Kim & Bay (1999) and Şengül (2013). When comparing fractions, it was observed that students had difficulty in fractions other than those with the same numerator and denominator. This finding of the study is in line with the research carried out by Clarke, Roche & Mitchell (2008). Researchers determined that the majority of students were prescriptive and did not develop a strategy although they correctly answered the fractions with the same numerator and denominator.

Some of the students reached the correct answer as a result of faulty reasoning. In the first question, the student said that 8/9 was greater. However, during the one-on-one interview with the student, he/she said that it was greater because it had bigger numbers. And, this shows that the student reached the correct answer with faulty reasoning. In a study conducted in the field of sequencing and comparison in fractions in the literature, Gould (2005) also reached a similar conclusion. In his research "Year 6 Students' Methods of Comparing the Size of Fractions", Gould concluded that students reached the correct answer although they had faulty reasoning. This result is in line with the result of our study.

In conclusion, it can be said that allowing students to develop strategies without giving the rules may facilitate students' job when comparing fractions. Students should use their own strategies to solve the question and discover the rule. In a way, they should learn by discovering the rule. This kind of teacher approach will also help to improve the cognitive structure of students.

5. Suggestions and Limitations

There are some limitations to the research. It is possible to list these limitations as carrying out the second stage of the study with 10 students and carrying out the research with the six question pair which requires comparison of the fractions used in the study. When findings of the research are taken into consideration, it is considered that providing an opportunity to students to determine their own comparison strategies in comparing fractions instead of imposing rules on them and guiding them to apply the rule will be effective for permanent learning. Moreover, it should be remembered that the conceptual mistakes of students will be minimized in this way. Considering the findings of the study, researchers can carry out similar studies in different subjects of mathematics.

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