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Probabilistic Thinking Profile of Mathematics Teacher Candidates in Problem Solving based on Self-Regulated Learning

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Abstract: Many problems in real life are full of uncertainty. Probabilistic thinking is one way to solve this uncertainty problem. The personality abilities that are interesting to examine include how independent the students are in facing probabilistic solutions. For this reason, it is necessary to study the probabilistic thinking profile of prospective mathematics teachers in solving probabilistic problems in terms of learning the self-regulation. The research method used in this research is qualitative research. In this study, the researcher conducted an analysis of students' probabilistic thinking with the scope of the probability material related to the sample space, the probability of events, and the probability of the random variable space. The profile of the level of probabilistic thinking is influenced by cognitive factors in the form of probabilistic knowledge obtained by students and personality in the form of self-regulated learning. Student learning self-regulated influences the mental activity in decision making. By studying the profile of probabilistic thinking, students are better prepare to face uncertainty with various aspects that influence it.

Keywords: *Probabilistic thinking, problem-solving, self-regulated learning.*

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Introduction

Thinking is one aspect that is always associated with human cognitive and thinking is one indicator that humans are normal or abnormal (Hodiyanto & Oktaviana, 2018). Thinking is manipulating or managing and transforming information in memory (Santrock, 2009). From this definition thinking means analyzing and processing data received into memory or the brain is thinking for something that humans do all the time; of course, thinking influenced by people's experiences and the theoretical frameworks they acquire in school and elsewhere.

The word probability is found in mathematics, in other fields of science, and also in everyday life. Other words that are expressed with the same meaning as probability, such as the word possibility, hope, prediction, or opportunity is often used in everyday problems that are probabilistic in nature. Probabilistic problems are problems that contain an element of uncertainty. A problem that contains an element of uncertainty is a problem that refers to a random activity or experiment that can get various possible results, but the exact result cannot be determined in advance with precision. The term probabilistic thinking will use to describe students' thinking in responding to various probabilistic problems (Groth et al., 2019; Malaspina & Malaspina, 2020; Mooney et al., 2014).

Probabilistic thinking is an individual cognitive process in responding to the problem of uncertainty through understanding the concepts of probability, intuition, the use of strategies, and making decisions. This shows that probabilistic problem solving is a higher-order thinking skill or High Order Thinking Skill, because it characterizes higher thinking. Three categories indicate the ability of higher-order thinking, namely: (1) initiating the transfer of one concept to another, (2) critical examining ideas and information, (3) using the information to solve problems (Brookhart, 2010).

Probabilistic thinking is used to describe a person’s thinking in response to various kinds of probabilistic problems (Groth et al., 2019; Mooney et al., 2014). Based on the results of research related to probabilistic thinking, it shows that the majority of students do not have a clear construction picture (Sharma, 2016). Students' probabilistic thinking

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cannot be seen from the age factor only. Some students indicated that their level of probabilistic thinking was below their age (Groth et al., 2019; Mahyudi, 2017). This means that even though a person is an adult, their probability of thinking ability is still low.

There are six main constructs of probabilistic thinking, namely sample space construction, experimental probability of an event, probability theory of an event, comparison of odds, conditional odds, and free odds involving one-stage and two-stage probabilistic events. For example, tossing a coin or two coins. This is to reveal the various levels of probabilistic thinking and their relationship (Jones et al., 1999).

The results of research on probabilistic thinking show that most students do not have a clear picture of the probability construct (Sharma, 2012). Students' probabilistic thinking process cannot be seen from the age factor only (Mahyudi, 2017). Some students indicated that their level of probabilistic thinking was below their age. Regarding probabilistic thinking, students who have a field-dependent learning style are at level II while the independent field is at level 4 (Taram, 2016). Research related to the probabilistic level has four levels of probabilistic thinking (Jones et al., 1999), and there are five levels of probabilistic thinking (Sujadi, 2010).

There are several opinions about the stages or categories of probabilistic thinking. Developing probabilistic thinking has four phases, namely: (1) classical concept phase, (2) frequency or empirical concept phase, (3) subjectivist concept phase, (4) axiomatic or formal concept phase (Nacarato & Grando, 2014). Develop probabilistic reasoning with four categories, namely: (1) types of strategies, (b) representations, (c) use of probabilistic language, (4) nature of cognitive barriers (Jan & Amit, 2009).

In solving problems, one of them is needed self-regulated learning (Shodiqin, Sukestiyarno et al., 2020). Learning is a cognitive process that is influenced by several factors such as individual circumstances, initial knowledge, attitudes, individual views, content, and presentation methods (Kerlin, 1992). The learning process also needs to pay attention to students' cognitive styles (Shodiqin, Waluya et al. 2020). One thing that affects learning is self-regulated learning. Self-regulated learning as a process of designing and careful self-monitoring of cognitive and affective processes in completing an academic task (Hargis, 2000; Kerlin, 1992). The characteristics of learning independence describe a high individual personality state and contain metacognitive processes in which individuals consciously design, implement, and evaluate their learning and themselves carefully.

Self-regulated learning is a broad construct that involves interactions between various control systems (cognition, attention, metacognition, emotion, motivation, and will), (Boekaerts, 1996; Corte et al., 2011; Lim & Yeo, 2021; Zimmerman, 2008; Zimmerman & Schunk, 2011). Cognitive and constructive in the learning process emphasizes more on self-regulated learning (Çetin, 2017). Self-regulation can be defined as feelings, thoughts, and behaviors that are developed by themselves to achieve certain goals (Gorgoz & Tican, 2020). The structural model describes the main components involved in students' self-regulation in their learning (Panadero, 2017): 1) content domain, 2) cognitive strategies, 3) cognitive regulatory strategies, 4) metacognitive knowledge and motivational beliefs, 5) motivational strategies, and 6) motivation regulation strategy (Boekaerts, 1996).

Based on the above background, the purpose of this study is to obtain a profile of probabilistic thinking of prospective mathematics teachers in solving probabilistic problems in terms of self-regulated learning.

Methodology

Research Gold

The research method used in this research is qualitative research. The form of research used is case study research. In this study, the researchers conducted a leveling analysis of the student test result data related to the probability material, namely: the sample space, the event and probability of an event, the random variables, and the probability of the random variables.

The research consists of two variables, namely: probabilistic thinking and independent learning. Probabilistic thinking is a cognitive as well as mental activity in processing or solving probabilistic problems, transforming them into memory, and making a conclusion. Learning independence is an individual attribute that leads to a person's mental influence in decision making.

Sample and Data Collection

The sampling technique in this study was using the purposive sampling technique. The purposive sampling technique is a sampling technique with certain considerations (Creswell, 2014; Sukestiyarno, 2020). Through the purposive sampling technique, the sample subjects taken were students of semester VII of the Mathematics Education Study Program at the PGRI University of Semarang.

Analysis of Data

The stages of data analysis in this study were data reduction, data presentation, and verification (Sukestiyarno, 2020). Reducing data means summarizing, selecting the main points, focusing on the important things, looking for themes and patterns, and eliminating unnecessary (Creswell, 2014). Presentation of data can be done in the form of brief descriptions, charts, relationships between categories, and the like by displaying the data so that it will be easier to understand what happened, plan the next work based on what has been understood. verification or conclusion of this research is a description or description of an object that was previously dim or unclear.

In this study, the procedure for analyzing data to be obtained from the results of written tests and the results of interviews to conclude was carried out by following the concept of the following processes: (1) reduction data, (2) presentation data, and (3) conclusion and verification (Miles & Huberman, 1994).

Data Validity

The instruments used in this study were written tests and guided interviews based on the problems posed by the task. Before using the instrument, the instrument was first validated by three validates who are experts in mathematics education and declared eligible to be used to obtain a profile of prospective mathematics teacher candidates' thinking in solving probabilistic problems in terms of learning independence.

Research Procedure

In this descriptive qualitative research, the researcher himself becomes the research instrument assisted by the I assist instrument, namely the written test, the II auxiliary instrument, the interview, and the third assist instrument, the self-regulated learning test. Data analysis was carried out before the researcher went into the field, analysing in the field, and reporting the results of the research.

Based on a study of various theories about self-regulated learning, six indicators of self-regulated learning were determined, namely: (1) Independence from others or self-reliance, (2) Self-confidence, (3) Disciplined behavior, (4) Having a sense of responsibility, (5) Behave based on own initiative, and (6) Conduct self-evaluation (Hidayati & Listyani, 2010). The lattice of the self-regulated learning instrument in this study was formulated in six indicators which in total consisted of 20 statements. Based on the research results, the self-regulated learning instrument that has been produced can be said to be valid and reliable. The self-regulated learning instrument was obtained from a modification from (Hidayati & Listyani, 2010). The filling out of the questionnaire is written by checking the column with the choice of strongly agree, disagree, agree, and strongly disagree. The questionnaire statement can be seen in Appendix I.

There are several leveling indicators of probabilistic thinking of revealing students' level of probabilistic thinking (Hodiyanto & Oktaviana, 2018; Jones et al., 1999; Taram, 2016). The instrument is adapted to the sample space material, the experimental probability of an event, and the random variables and their probabilities.

Table 1: Guidelines for Leveling Probabilistic Thinking

Sub Topics	**Level 1 (Subjective)**	**Level 2 (Transitional)**	**Level 3 (Quantitative Informal)**	**Level 4 (Numerical)**
Sample Space	1. List an incomplete set of experimental results at one level 2. Predict the most likely or least likely events based on subjective opinions	1. List a complete set of experimental results for one-level and sometimes, for a two-stage but not systematic experiment	1. Construction of the sample space members with a certain pattern. 2. List all sample points in a set 3. Consistently list the results of two-tier experiments using a portion of the generative strategy	1. Implement and employ a generative strategy that allows a complete listing of experimental results at two or three variables
Events and their probabilities	1. Predict the most likely or least likely events based on subjective opinions. 2. Construct members an event by not following a certain pattern	1. Defines an occurrence from the sample space 2. Predict the most likely event or the least likely to be based on quantitative opinions but return to subjective opinions	1. List all members of the incident according to the definition made 2. Begin to realize that broader sampling is needed to determine the most likely or least likely events.	1. Gathers appropriate data to determine numerical values for experimental odds. 2. Can identify situations where the likelihood of an event can be determined only experimentally.

Table 1: Continued

Sub Topics	Level 1 (Subjective)	Level 2 (Transitional)	Level 3 (Quantitative Informal)	Level 4 (Numerical)
Random Variables and their Probabilities	1. Write down the notation for all sample points 2. Determine all possible values for X (a) not complete	1. Write the notation of the random variable X (a) 2. Determine all possible values for X (a) completely and correctly	1. Make an arrow chart for all the values of X (a) 2. Draw a probability distribution graph	1. Determine probability values for all random variable prices 2. Construct a random variable probability distribution table 3. Verify the sum of the probability values is equal to one

Results

In this study, data collection begins with taking subjects based on the self-regulated learning of 27 students. Two experts asked for validation before using the self-regulated learning instrument. The instrument was declared fit for use. Furthermore, three subjects were taken based on their self-regulated learning abilities. Next, the table is shown as follows:

Table 2. Subjects in the study based on self-regulated learning

No	Subject All Students	Subject Selection	Category
1	AL, RT, CH, H, ML, MA	AL	High
2	NN, IM, VM, ES, TO, AM, LB, SA, DW, AN, SV, MD, DP, FW, RV, FP, DR, TP	IM	Medium
3	SH, ZI, DS	SH	Low

The data analysis was carried out by researchers based on the criteria for the leveling of probabilistic thinking developed by Jones et al, with the subject having obtained probability material in the mathematics education study program.

To get a profile of probabilistic thinking based on the results of a probability test with the following materials: (1) sample space, (2) events and their probabilities, and (3) random variables and their probabilities. The written test given to the subject can be seen in appendix II. The following is a discussion of the profile of probabilistic thinking based on the material subject All Students:

1) Sample Space

1.a) Subject AL with highly self-regulated learning

The AL subject is able to construct sample space members with a certain pattern with tree diagrams and is able to write in the form of a set, namely S = {JJJ, JJB, JBJ, JBB, BJJ, BJB, BBJ, BBB}. So that the AL subject has been able to apply and use a generative strategy that allows a complete listing of the sample space results. In the sample space material, the AL subject is able to think probabilistically at the numerical level or level 4 because it is able to express decisions on the number of members of the sample space based on numerical data. This can be seen in Figure 1.

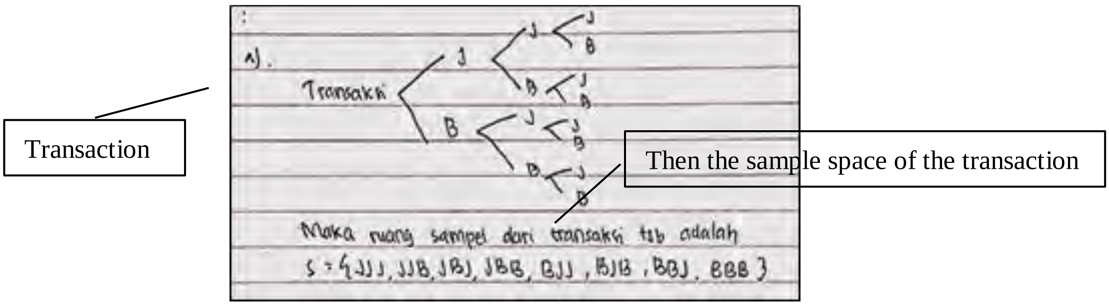


Figure 1. Result answer no. 1.a from subject AL

This is supported by interviews:

R : When trading stocks 3 times every day, is it possible, impossible, or certain to make buying transactions twice? Why?

S1-Al : Maybe sir.

1.b) Subject IM with medium self-regulated learning

IM subject is able to construct sample space members with a certain pattern with tree diagrams and also be able to write in the form of a set, namely S = {JJJ, JJB, JBJ, JBB, BJJ, BJB, BBJ, BBB}. IM subject can already find the sample space from the experiment with one random variable. In the sample space material, the IM subject in probabilistic thinking is at the Informal quantitative level or level 3. This is shown in Figure 2.

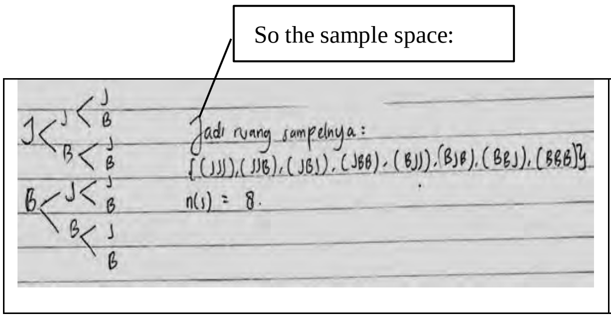


Figure 2. Answer no. 1.a from subject IM

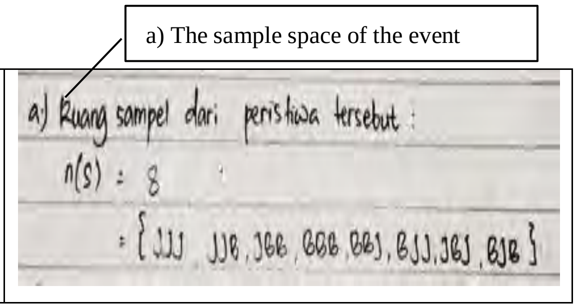


Figure 3. Answers no. 1.a from subject SH

The following is an interview with IM Subjects:

R : When trading stocks three times every day, is it possible, impossible, or certain to make buying transactions twice? Why

S2- IM : Maybe, because there could be buying transactions twice, namely BBJ, BJB, and JBB

R : There are 8, namely the possibilities obtained by JJJ, JJB, JBB, BBB, BBJ, BJJ, JBJ, BJB

S2 IM : There are 8, namely the possibilities obtained by JJJ, JJB, JBB, BBB, BBJ, BJJ, JBJ, BJB

1.c) Subject SH with low self-regulated learning

The SH subject has Figure 3 Answers to the SH Subject been able to construct members of the sample space with a certain pattern, so that in probabilistic thinking in an informal quantitative level or at level 3.

2) Events and their probability

2.a) Subjects AL with highly self-regulated learning

In this event material, the AL subject can collect data precisely to determine the numerical value of the experimental probability. This is shown by the results of the event U with one purchase transaction, getting U = {JJB, JBJ, BJJ}, while for event V doing the buying transaction twice, getting V = {JBB, BJB, BBJ}, while event W Make a purchase transaction at most once, where you get W = {JJJ, JJB, JBJ, BJJ}. Based on the written results in Figure 4 and the results of interviews with AL subjects in the incident and probability material, they have entered the numerical stage or at level 4.

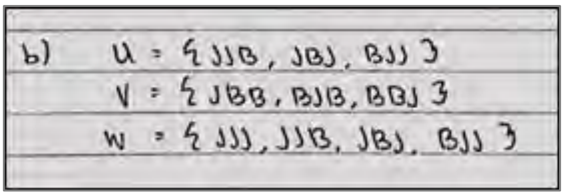


Figure 4. Answer results 1.b subject AL

Researchers interviewing the S1 AL subject can be seen as follows:

R: In the share sale and purchase transaction carried out, it is known those events are in the form of a set. For example, U is an event that makes a one-time purchase transaction; V is an event to make a purchase transaction twice; W is an event to make a purchase transaction at most one, how do you show this event?

S1-Al: In showing the event, I first used my help by making a tree diagram. Then after that, I wrote down the sample space based on the tree diagram that I had made, pack. Next, I identify which includes an event U, event V and event W from the sample space.

In looking for the probability of experimental opportunities, AL subjects were able to collect the right data to determine the probability value numerically. It begins with the determination of the space for the random variable X, namely $\mathcal{A}_X$ = {0,1,2,3}. This is shown in the picture of the distribution of opportunities as follows:

1. When x = 0

   Suppose A is an event where there is no purchase transaction

   $A = \{JJJ\}$

   $$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

   When x = 1

   Suppose B is an event where there is a one-time purchase transaction

   B={JJB, JBJ,BJJ}

   $$P(B) = \frac{n(B)}{n(S)} = \frac{3}{8}$$

   When x = 2

   For example, C is an event where there is a buy transaction twice

   C={JBB, BJB, BBJ}

   $$P(C) = \frac{n(C)}{n(S)} = \frac{3}{8}$$

   When x = 3

   Suppose D is an event where there is a purchase transaction three times

   D={BBB}

   $$P(D) = \frac{n(D)}{n(S)} = \frac{3}{8}$$

(Translated from Indonesian)

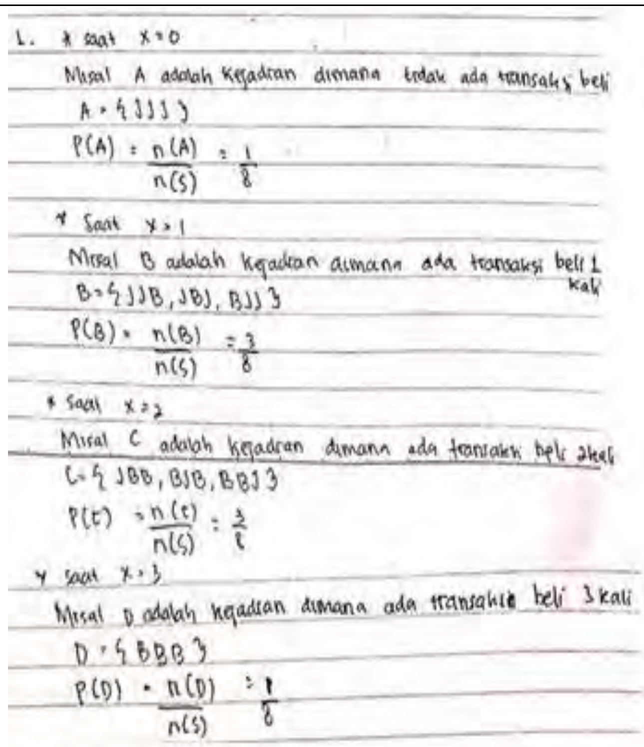

1. * saat x = 0
Misal A adalah kejadian dimana tidak ada transaksi beli
A = {JJJ}
P(A) = n(A)/n(S) = 1/8
* saat x = 1
Misal B adalah kejadian dimana ada transaksi beli 1 kali
B = {JJB, JBJ, BJJ}
P(B) = n(B)/n(S) = 3/8
* saat x = 2
Misal C adalah kejadian dimana ada transaksi beli 2kali
C = {JBB, BJB, BBJ}
P(C) = n(C)/n(S) = 3/8
* saat x = 3
Misal D adalah kejadian dimana ada transaksi beli 3 kali
D = {BBB}
P(D) = n(D)/n(S) = 1/8

Figure 5. Answer results 1.c subject AL

From the Figure 5, the odds for each x in $\mathcal{A}_X$ or $P(X = x)$ are obtained

Table 3. The odds of each X

X = x	0	1	2	3
P(X = x)	1/8	3/8	3/8	1/8

This Table 3 shows that the AL subject can determine the probability value numerically against the experimental odds. This is also shown in calculating the opportunity for the buying transaction to be at most 2 (two) chances of getting 7/8. This can be seen in Figure 6.

2. Suppose E is the occurrence of the buy transaction opportunity at most, then:

E = {$JJJ, JJB, JBJ, JBB, BJJ, BJB, BBJ$} and $P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}$.

(Translated from Indonesian)

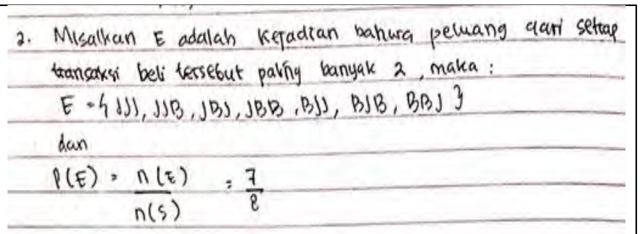

2. Misalkan E adalah kejadian bahwa peluang dari setiap transaksi beli tersebut paling banyak 2, maka:
E = {JJJ, JJB, JBJ, JBB, BJJ, BJB, BBJ}
dan
P(E) = n(E)/n(S) = 7/8

Figure 6. Answer results 1.c2 subject AL

2.b) Subjects MI with medium self-regulated learning

Subjects MI have difficulty defining an event from the sample space so that to determine the members of the event is not complete. Thing shows Figure 7.

Answer:

$\frac{n(M)}{n(S)} = \frac{3}{8}$ (One-time purchase transaction)

$\frac{n(V)}{n(S)} = \frac{3}{8}$ (Buy transaction twice)

$\frac{n(W)}{n(S)} = \frac{3}{8}$ (Buy at most once)

(Translated from Indonesian)

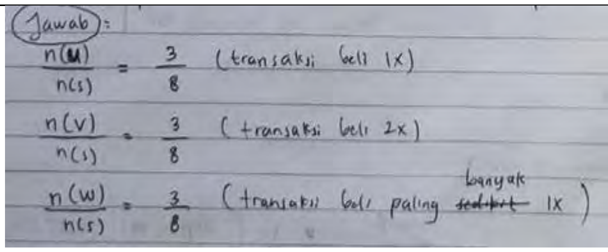


Figure 7. Answer 1.b from subject MI

2.c) Subjects with low self-regulated learning

The subject of SH can write down all members of the incident as defined. This is indicated by U as an event to make a purchase transaction once, get U = {JJB, JBJ, BJJ}, while for the event to make a purchase transaction 2 times it is represented by V, obtained V = {JBB, BJB, BBJ}, while the event make a purchase transaction at most 1 time symbolized W, where you get W = {JJJ, JJB, JBJ, BJJ}. This can be seen in Figure 8.

b) U= Buy once

= $\{JBB, JBJ, BJJ\}$

V= Buy twice

= $\{JBB, BBJ, BJB\}$

W= Buy at most once

=$\{JJJ, JJB, JBJ, BJJ\}$

(Translated from Indonesian)

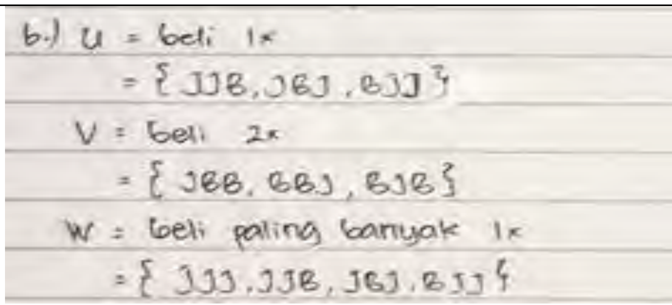


Figure 8 Results answer no.1.b from subject SH

The subject SH has tried to get members of a random variable, with members $X = \{0, 1, 2, 3\}$. The writing of the opportunity notation is still in accurate and there is an error, because it is not symbolized by the event or the event variable, for example $P(X = x)$. However, in determining the odds, it is correct to get the odds correctly.

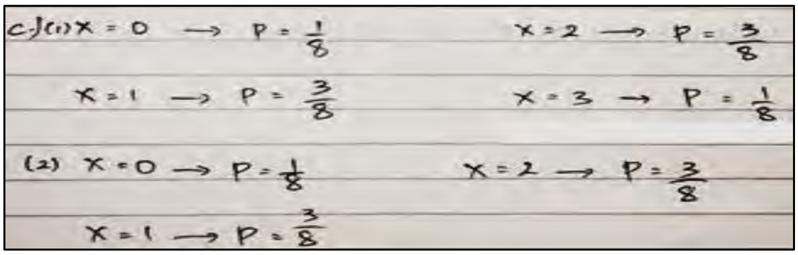


Figure 9 Result of answer 1.c. of the subject SH

This Figure 9 is also shown in calculating the probability of buying transaction occurrences at most two, the SH subject is not able to show the opportunity.

3) *Random variables and their probabilities*

3.a) Analysis of Subject AL with high self-regulated learning

- For one variable random variable

From Table 4 the AL subject can determine the space of one random variable written $\mathcal{A}_X = \{0,1,2,3\}$, and the AL subject can prove that the random variable is a function of probability density. This can be seen from the probability value in each member of the random variable space has a positive value and the sum of all odds of the members of the random variable are equal to 1. So that the AL subject for one random variable, can show his probabilistic thinking at the numerical stage or level 4.

Table 4. Answer 1.d. the subject AL

Answer 1.d Subject AL: Suppose $\mathcal{A}$ is the space of the random variable X, where X represents the number of purchase transactions, then: $f: \mathcal{A} \to R$, where $\mathcal{A} = \{0,1,2,3\}$ and the function pairs $\left\{\left(0,\frac{1}{8}\right),\left(1,\frac{3}{8}\right),\left(2,\frac{3}{8}\right),(3,\frac{1}{8})\right\}$ Obviously $f(x) = P(X = x) = \begin{cases} \frac{1}{8};\ x = 0,3 \\ \frac{3}{8}; x = 1,2 \\ 0; for\ other\ x \end{cases}$ For i) $x = 0 \to f(x) = \frac{1}{8}; x = 1 \to f(x) = \frac{3}{8}; x = 2 \to f(x) = \frac{3}{8}; x = 3 \to f(x) = \frac{1}{8}$ ii) $\sum_{x\ di\ \mathcal{A}} f(x) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1$ Since it meets the conditions i) and ii), the occurrence of X is a function of probability density.

- For Two-Variable Randomized Variables

The AL subject has been able to write the random variable notation X, by determining the space value of the random variable from one random variable. This can be seen from the answer no. 2a is shown in Table 5. The notation for random variable space X has not been written correctly. The random variable space X is correct, namely $\mathcal{A}_X = \{0,1,2,3\}$, but the Y variable space has a slight error, because it raises the member -1 which should be $\mathcal{A}_Y = \{0,1,2\}$, not $\mathcal{A}_Y = \{-1, 0,1,2\}$.

Table 5. Answer no. 2.a. from the subject AL

2a. Transaction $S = \{JJJ, JJB, JBJ, JBB, BJJ, BJB, BBJ, BBB\}$ Space of X = {0,1,2,3} Space of Y = {−1,0,1,2} Shared Space X and Y= $\{(x, y)\ \vert x = 0,1,2,3\ and\ y = -1,0,1,2\}$

Next, suppose X = states the number of buying transactions, Y = states the number of selling transactions that begin with a buy transaction. Furthermore, for the probability of random variables X and Y, the probability obtained from $P\ (X = x, Y = y)$, it was found that the AL subject was not yet right in calculating the odds, it was still difficult to translate the odds in $X = x$ and $Y = y$. It is seen that as follows:

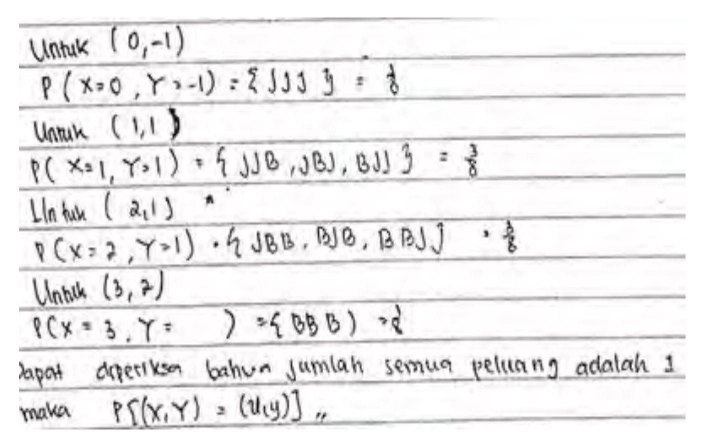
Answer No. 2b. from the subject AL
For (0,-1)
$P(X = 0, Y = -1) = \{JJJ\} = \frac{1}{8}$
For (1,1)
$P(X = 1, Y = 1) = \{JJB, JBJ, BJJ\} = \frac{3}{8}$
For (2,1)
$P(X = 2, Y = 1) = \{JBB, BJB, BBJ\} = \frac{3}{8}$
For (3,2)
$P(X = 3, Y =..) = \{BBB\} = \frac{1}{8}$
It can be checked that the sum of all odds is one, then $P[(X, Y) = (x, y)]$.
(Translated from Indonesian)
	
	Untuk (0,-1)
	P(X=0, Y=-1) = {JJJ} = 1/8
	Untuk (1,1)
	P(X=1, Y=1) = {JJB, JBJ, BJJ} = 3/8
	Untuk (2,1)
	P(X=2, Y=1) = {JBB, BJB, BBJ} = 3/8
	Untuk (3,2)
	P(X=3, Y=) = {BBB} = 1/8
	Dapat diperiksa bahwa jumlah semua peluang adalah 1
	maka P[(X,Y) = (x,y)]

Figure 10. Answer No. 2.b from the subject AL

Whereas the opportunities obtained are not in accordance with the two defined random variables, that X represents the number of buying transactions and Y states the number of selling transactions that begin with buying transactions. Even though it should be written as a whole the odds of random variables X and Y are as follows.

$$P(X=x, Y=y) = f(x,y) = \begin{cases} \frac{1}{8}\ ;\ (x,y) = (0,0),(1,0),(1,1),(1,2),(2,0),(3,0) \\ \frac{2}{8}\ ;\ (x,y) = (2,1) \\ 0\ ;\ for\ another\ x,y \end{cases}$$

Based on Figures 10, and the analysis above, the AL subject is at the post-subjective level of probabilistic thinking leading to the transitional stage for the material probability of the two-variable random variable.

3.b) Subjects with medium self-regulated learning

Subject IM has not been able to determine the possible values for random variable space X, subject IM only writes the sample space S = {JJJ, JJB, JBJ, JBB, BJJ, BJB, BBJ, BBB}.

From Figure 11, the subject of IM has not shown that they can confidently answer every time a purchase transaction occurs. So that for the random variable material and its probability, for either one or two random variables, the subject shows it at the subjective level or level one. This is because we cannot decide on the opportunity if it is based on random variables.

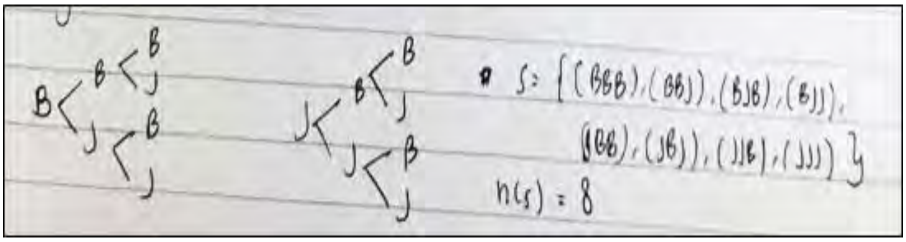


Figure 11. Result of answer 2.a subject IM

3.c) Subjects with low self-regulated learning

For random variables and their probabilities, the SH subject has written the random variable notation, determined the value of the random variable space of one random variable, but there is a little error of representation in the random variable space Y, the correct one should be $\mathcal{A}_Y$ = {0,1,2}. Subject SH made a mistake in the shared space material of the random variable XY, because of an error from his search for space in the random variable Y.

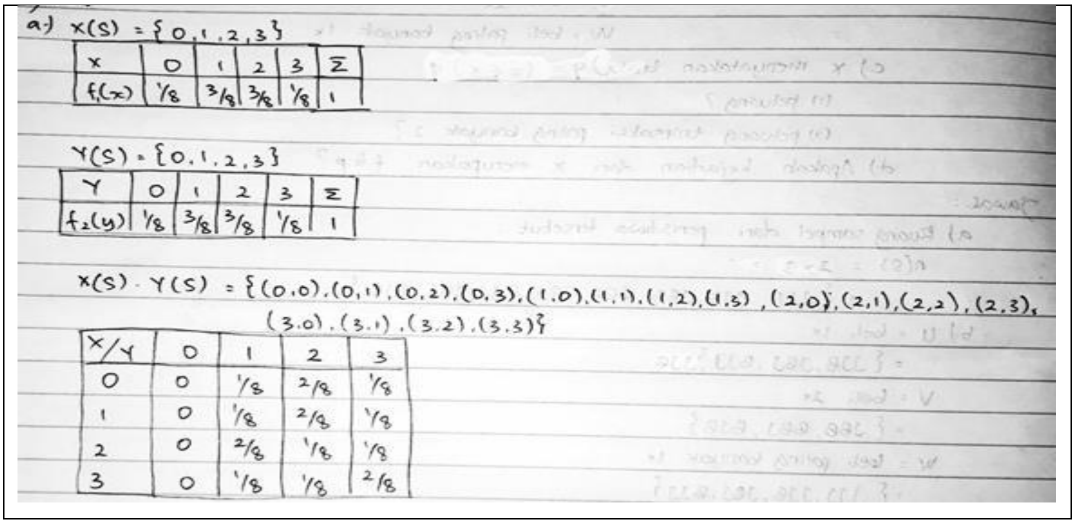


Figure 12. Result of answer 2.a subject SH

The SH subject has not been able to describe the opportunity function properly and has not been able to show that X is a function of opportunity density so that the SH subject based on empirical data is at the Transitional. This can be seen in Figure 12.

Discussion

Three factors affect independent learning, namely a) personal factors; b) behavioral factors; c) environmental factors (Zimmerman & Schunk, 2011). While a person's personality is influenced by the knowledge that is in him. The knowledge possessed by a person is influenced by cognition. Probabilistic thinking is a mental and cognitive activity that involves informal and formal knowledge of probability (Amir & Williams, 1999). So that probabilistic thinking is influenced by a person's personal factors, how to obtain this knowledge. Someone's personal probabilistic thinking is influenced by a person's culture, language, beliefs, and experiences (Amir & Williams, 1999; Malaspina & Malaspina, 2020; Sharma, 2012). From the results obtained an analysis based on probabilistic thinking as follows:

Table 6. Recap of Subject Probabilistic Thinking Based on Self-Regulated Learning

Self-Regulated Learning	Theory		
	Sample space	**Events and Probabilities**	**Random Variables and their Probabilities**
High Self-Regulated Learning	The subject expresses the decision on the number of members of the sample space based on numerical data	Able to collect the right data to determine the numerical value of experimental odds so that it can be in the numerical stage	▪ Already able to make arrow diagrams to get all values from random variable space or $X(a)$, ▪ Be able to describe the value of the probability function p.a. for one random variable, but two random variables but not perfect. ▪ So the subject tends to be at the informal quantitative level
Medium Self-Regulated Learning	In the sample space material, the IM subject in probabilistic thinking is at the quantitative Informal level or level 3	Be able to define an event from sample space, but difficulty determining an event, so it tends to be transitional or level 2	Determine all possible values for the random variable space X, but not completely so that they are on the subjective level
Low Self-Regulated Learning	The SH subject has been able to construct members of the sample space with a certain pattern, so that in probabilistic thinking at an informal quantitative level or level 3	Able to write all members of the event according to the definition made, but not yet perfect in getting the chance, so they are at the quantitative Informal level or level 3	▪ The subject writes the random variable notation, determining the space value of the random variable from one random variable. ▪ However, the subject has not been able to describe the opportunity function properly. The subject is on the transitional level

Table 6 shows data analysis and a recap of probabilistic thinking. There are several cognitive factors in the form of material knowledge of the problem and the presence of personality factors in the study. A good self-regulated learning attitude affects decisions making so that it affects the profile of the level of probabilistic thinking. Decision making in probabilistic thinking is influenced by aspects of culture and personality (Sharma, 2012, 2016). Subjects with good self-regulated and an understanding of cognition related to sample space material, events, and the probability that the subject can be able to be on the numerical level. Subjects with good independence for random variable material and the probability of a probabilistic thought process at the informal quantitative level.

The subject of self-regulated learning was in the midst of his probability thinking for sample space material at an informal quantitative level. Whereas in the matter of events and probability, the subject's probabilistic thinking is at the transitional level, and while in the material of random variables and its probability, the subject's probabilistic thinking is at the subjective level or level one.

Whereas subjects who have self-regulated learn less in their probabilistic thinking for the sample space material on the informal quantitative or the level 3, for the event probability material on the quantitative Informal level or level 3 and the random variable material and the probability is on the transitional level.

Cognition factors affect probabilistic thinking, this can happen because of the knowledge it receives and understanding the opportunity material that is understood. This is very influential in making the decision in probabilistic thinking. Solving probability problems, many students use strategies based on beliefs, previous experiences (daily and school), and intuitive strategies (Sharma, 2012, 2016). Probabilistic thinking factors include cultural factors (Sharma, 2012); cognition (Taram, 2016); intuition, heuristics, beliefs, experiences, and individual social setting (Amir & Williams, 1999); and the ability of students to represent mathematics (Maher & Ahluwalia, 2014). Based on the descriptions of research subjects, good self-regulated learning will affect cognitive processes that involve understanding the concept of probability, intuition, strategy use, and decision making, so that it also affects the ability to think probabilistically.

Probabilistic thinking in solving problems can be seen from the strategies and representations of students in solving probability problems. Strategies in thinking include beliefs or reasons, subjective estimation, experimental estimation, and selection of theoretical probability calculations (Jan & Amit, 2009). Probabilistic thinking has begun with the investigation into intuition and learning difficulties to which he has different knowledge (Malaspina & Malaspina, 2020). Knowledge of mastery of the material affects the depth of content for students (Lestari et al., 2019). Knowledge of probability and reasoning is needed in everyday life for decision-making (Batanero et al., 2016). So it confirms that the profile of probabilistic thinking is influenced by various aspects.

Conclusion

Many everyday uncertainty problems cannot be separated from probabilistic thinking. The level profile of probabilistic thinking is influenced by cognitive factors. The students' probabilistic knowledge, as well as a personality in the form of self-regulated learning, are factors that influence probabilistic thinking. Student self-regulated learning has a mental impact on decision making. The knowledge factor of probabilistic thinking is divided into the sample space material, the chance of the event, the event, and its chance of occurrence, as well as the probability of random variables. Subjects with good self-regulated learning and an understanding of cognition related to sample space material, events, and the probability that the subject can be able to be on the numerical level. Subjects with good self-regulated learning for random variable material and the probability of a probabilistic thought process at the informal quantitative level.

Meanwhile, subjects who have self-regulated learning are in the middle of seeing their probabilistic thinking for the sample space material on the informal quantitative level, and the events and probabilistic are only capable on the transitional level. Meanwhile, subjects who have self-regulated learning are less likely to think about their probabilistic thinking for the event sample space and their probability at the informal quantitative level, and the probability material from random variables thinks the probability is at the transitional level. Probabilistic thinking for prospective Mathematics teachers who are still lacking, there needs to be an increase in understanding of material knowledge well. This is done so that prospective Mathematics teachers in probabilistic thinking can be on the highest level, namely on the numerical level.

Recommendation

Many problems that exist in everyday life are full of uncertainty. It takes attitude and probabilistic thinking skills to solve this uncertainty. At present, probabilistic thinking is still rarely applied in classroom learning, so the need for teachers and education policy makers to strengthen the curriculum on probabilistic material, even though they unconsciously carry out a probabilistic thinking. In addition to the factors of knowledge and personality, self-regulated learning affects probabilistic thinking. It needs further studies related to the factors that affect probabilistic thinking and the implementation process. As well as the need for studies to implement probabilistic thinking in classroom learning and problems of everyday life which are full of uncertainties.

Limitation

The limitations of research related to probabilistic thinking are as follows. (1) this study takes the subject of prospective mathematics teachers based on independent learning, even though there are several other factors and different levels of learning, so that knowledge cognition and levels in making decisions are different; (2) During the written test and interview there is a difference in time that is quite long, so that there are subjects who explain less fluently; (3) This study explores the profile of probabilistic thinking based on four levels, then other researchers can study further with different points of view and different school levels of subjects.

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Authorship Contribution Statement

Shodiqin: Conceptualization, design, data acquisition, data analysis, writing, securing funding, admin. Sukestiyarno: Supervision, critical revision of manuscript, technical or material support, final approval. Wardono: Reviewing, supervision. Isnarto: reviewing, supervision.

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Appendices

Appendix I

Table A1. The items of the Student Self-Regulated Learning Instrument statement

No	Statement
A.	**Dependence on others or self-efficacy**
1	I study under the control of others (-)
2	I improve learning achievement because of encouragement from others (-)
3	I choose my learning strategy (+)
4	I complete my lecture assignments according to my ability (+)
B.	**Have self-confidence**
5	I have confidence that I can achieve my learning goals (+)
6	I don't have the confidence that I can overcome the problems or obstacles I face in my learning activities (-)
7	I dare to convey an opinion that is different from the opinion of others (+)
C.	**Behave Discipline**
8	I always make plans for my study activities (+)
9	I'm not trying to be on time (-)
10	I always collect lecture assignments on time (+)
D.	**Have a sense of responsibility**
11	I encourage myself to continue to be passionate about learning (+)
12	I don't try to plan my learning activities as best I can (-)
13	I was able to focus attention on lecture activities (+)
E.	**Thinking on your initiative**
14	I argue consciously of my own will (+)
15	I act consciously of my own will (+)
16	I don't plan my study activities (-)
17	I did practice questions, although not as a lecture assignment. (+)
F.	**Conducting Self Evaluation**
18	I believe that my learning activities ultimately have an impact on myself (+)
19	I don't evaluate my study result (-)
20	I look at the increase and decrease in learning outcomes that I get (+)

Appendix II

PROBABILISTIC PROBLEMS

Instructions:

• Read carefully each question below.

• Work on folio paper and write down the answers accurately and clearly.

• Time to work on the questions 120 minutes.

Question:

1. In trading precious metal commodities on international exchanges, Budi conducts business transactions for profit with the option of Sell (J) or Buy (B), if Budi makes three transactions a day, Question:
   a. The sample space of the transaction?
   b. Define the following events in the form of a set, U = the event of making a one-time purchase, V is defined as the event of making a purchase transaction twice, W is defined as the event of making a purchase transaction at most once
   c. For example, X states the number of purchase transactions, determine: (1) the probability of each transaction ?, (2) the probability of each buying transaction is at most 2?
   d. Is the occurrence of X a function of probability density (f.k.p)? explain.

Figure 1: Commodity prices with buy and sell options

2. In trading precious metal commodities on the stock exchange, Doni conducts his share business transactions to gain profit with the option to sell (J) or buy (B), if Doni makes three transactions a day, the question is:
   a. For example, it is known that the variable X represents the number of buying transactions, Y represents the number of selling transactions that begin with a buy transaction, determine the space of X, the space of Y, and the common space of X and Y.
   b. The probability of each of these transactions or P [(X, Y) = (x, y)] for each (x, y) in the sample space?
   c. Are the occurrences of X and Y a function of probability density (f.k.p)? explain

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