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Understanding the Meaning of the Equal Sign: A Case Study of Middle School Students in the United Arab Emirates

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Abstract: The equal symbol has been used in diverse mathematical frameworks, such as arithmetic, algebra, trigonometry, set theory, and so on. In mathematical terms, the equal sign has been used in fixed command of standings. The study reports on the students meaning and interpretations of the equal sign. The study involved Grade 6, 7, and 8 students in a secondary school in Alain, United Arab Emirates (UAE). Much of the earlier research done on the equal sign has focused on the primary school level, but this one focuses on middle school students. The study shows that the maximum foremost understanding of the equal sign amongst Grade 6, 7, and 8 students is a do-something, unidirectional symbol. Students realize the equal sign as an instrument for marking the response moderately than as an interpersonal symbol to associate extents.

Keywords: *Equal sign, unidirectional, United Arab Emirates, middle school.*

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Introduction

One simple interpretation of the equal sign is to recite that what appears on the left-hand side of a mathematical expression is equal to its right-hand side. The literature on this subject shows that college, high school, middle school, and even primary learners are not fully aware of the meaning of the equal sign (Jones et al., 2013). There are many problems with connotations and words/wording - not everything can be put on a child. This study fills the gap on the above-mentioned research in middle school United Arab Emirates (UAE) students. We start with the theoretical framework, purpose, research questions, findings, and conclusions. Finally, we discuss the limitations vs. expectations of the study findings.

The Study Purpose

The main aims of the study are to examine the sympathetic connotation of the equal sign for intermediate students among United Arab Emirate 6, 7, and 8 graders. This investigation cast off a qualitative approach to examine alterations in student perceptive of the equal sign: what do school students think about the connotation of the equal sign? Does it matter if students' progress a cultured understanding of the equal sign?

The Study Questions

The purpose of this study is an attempt to respond to the following study question, which represents the primary endpoint:

1. To what extent do the UAE students in grades 6 to 8 understand the meaning of the equal sign?

Misapprehension of Equal Signs

Misapprehension emerges after a misunderstanding of some statement. It emerged from the notion of knowing something before and made it correct again after using reasoning, culture, tradition, and other factors (Ningrum et al., 2018). This misunderstanding will eventually cause difficulties in students' logical understanding of things - if not

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determined at early stages; for instance, students who describe the equal sign as a function have trouble answering related algebraic problems. Students were required to answer the problem $5 + 6 = ___ + 2$. Some students added $5 + 6$ and introduced the answer on the dotted line. The correct way is, of course, to anticipate for the equivalence: $5 + 6$ equals 11, so the right-hand side has to be 11 as well. 9 achieves the difference: $5 + 6 = 9 + 2$. This is one of the five factors that turn out as mistakes in students, teachers, schoolbooks, background, and way of teaching (Sherman & Bisanz, 2009).

Mathematics procedures teach us to solve algebraic calculations by concentrating on associations rather than calculating the answer (Carpenter et al., 2005). There exists a serious curriculum disjointedness between the algebra taught in upper grades in the UAE and the arithmetic taught in elementary school. The primary school arithmetic syllabus emphasizes a computational presentation, deprived of appearing to relations and arithmetic essential characteristics (Abdallah & Wardat, 2021).

Consequently, students grasp the misunderstanding around mathematical items and experience trouble when studying algebra in higher grades. Also, it affects the student's discovering a conclusion without considering the notion of the equation and simply finding it by a computational technique. The outcome is that students get stuck in a repetitious system of discerning and incompetent ways to mature their imagination. Originality is a significant skill obligatory for 21st-century education and critical thoughtfulness, communication, and teamwork.

The History of the Equal Symbol

Cajori's (1928) mentioned in his book (*The Whetstone of Witte*), the concept and the equivalent "=" symbol have been used countless times. People used the same symbol in multiple ways throughout the centuries, with specific symbols to convey equality.

Education System in the United Arab Emirates

The education system of the United Arab Emirates (UAE) has been divided into three groups, known as primary, secondary, and high secondary, respectively. The academic curriculum of public schools is Arabic, while private schools follow 17 different curriculums. The national curriculum followed by schools is from the India, U.S., U.K., and the Ministry of Education (MoE) supervises up to 90 percent of the student population in private schools. The remainder of the curriculum comprises International Baccalaureate (IB), German, Iranian, Canadian, Pakistani, French, Philippines, Russian, Japanese, and Iranian. The MoE secondary and primary education is provided for all United Arab Emirates (UAE) citizens and obligatory until the ninth grade (Alotaibi et al., 2021); the current educational construction, which was recognized in the early 1970s, is a four-tier system casing education for 14 years, as follows:

Playgroup – 4 to 5 years old (1-2 years program).

Primary – 6 to 12 years old (6 years program).

Introductory – 12 to 15 years old (3 years program).

Secondary – 15 to 18 years old (3 years program).

Dubai and Abu Dhabi represent more than 67% of U.A.E.'s inhabitants, out of which Emiratis characterize 16% out of the entire population. Foreign emigrants characterize 91% in Dubai compared to 55% in Abu Dhabi of the whole population. In the academic year 2016/2017, United Arab Emirates (UAE) had 580 private schools, a maximum of which were situated in Abu Dhabi (122) and Dubai (185). There were roughly 241,493 students who joined private schools in Abu Dhabi and 273,599 in Dubai, compared to 238,632 and 265,299 correspondingly in 2015/2016. Regardless of students' age or which curriculum is followed, the Ministry of Education has adopted a vision that promotes creative talents and places a greater emphasis on students and self-learning abilities (Jarrah et al., 2020).

Literature Review

Conceptual analysis

The equal sign is utilized to signify correspondence amongst two-equation margins (Blanton, 2011). It is a cipher used in arithmetic to characterize an association. In the teaching and grasping of arithmetic, the equivalent symbol is ubiquitous and must be one of the maximum utilized symbols in arithmetic (Knuth et al., 2008).

Previous Studies

An early related study on this subject is Falkner et al. (1999), who studied at Elementary Lapham School, Madison. It focuses on the concept of equality and the corresponding symbol. The concept of equality has been worked out and examined by the 5th and 6th graders, and the increasing misunderstandings among young students about the concept of equality have been mentioned. A few classroom activities are proposed after the study to help students address the concept of equality and the same symbol throughout their understanding.

Bennett (2015) conducted a study at the University of Louisville under the title “Comprehending the equivalent symbol's connotation: examining primary students and teachers.” The study aimed to discover the consequence of the two different approaches for new arithmetic values in two schools, intended for grades two to five, concerning student thoughtfulness of the equal sign. Additionally, the study also intended to scrutinize teachers' information and comprehension of the equal sign. The design aimed at finding out how values were applied and two contiguous conditions were utilized. Two different conclusion-making methodologies were applied under the same conditions, taking into consideration the implementation standards adopted. People who would benefit most from a dissertation of this nature would be teachers, education administrators, teacher educators, and various other stakeholders. Findings revealed that the different timelines for the employment of the arithmetic values in the two conditions appeared to influence scholars' understanding of the equal sign and designated that teacher's information was not the greatest momentous standard for student comprehension equal sign'. Nevertheless, two features, specifically state employment timeline and student grade level, showed momentous features in forecasting student thoughtfulness of the equal sign.

Duncan (2015) performed a study at Louisiana State University under the title “A Study of Mathematical Equality: The Equal Sign significance.” The study aimed to examine education students' consideration of the equivalent sign, discovering approaches to resolving corresponding calculations and the relationship amongst the two in arithmetic'. The techniques are cast-off to gather 167 first-year students of arithmetic education in the University of Muhammadiyah, Malang, East Java, Indonesia. Data composed concluded the allocated errands, was descriptively characterized employing Chi-square statistics. Outcomes of the investigation exposed that students' working comprehension of the equal sign was more leading than their interpersonal commencement. When it came to solving corresponding equations, one shared inadequacy: schoolchildren were inclined to accept working measures by making solutions, contrasts, and replacements, despite paying consideration to the active relationships in correspondence, called strategy to identify equality. Additionally, it was exposed that no meaningful connection happens amid students' consideration of the equal sign and approaches cast-off in resolving equal equations. Afterward, gender and defendants' topographical origin was further discussed as considerations for the study's resolutions.

Powell (2014) conducted the study with the title “The Influence of Symbols and Equations on Understanding Arithmetical Correspondence.” The primary determination of the study was to evaluate eight basic prospectuses for the quantity of student experience to different equation types. Prospectuses were oblique crossways six basic grade levels, for so many standard and non-standard equation types seeming in student's textbooks. With the exclusion of 1 out of the eight prospectuses, students fundamentally do not obtain experience to non-standard equation types that endorse an interpersonal empathetic of the equal sign. The textbook proposes that scholars obtain negligible education on the equal sign's interpersonal descriptions by examining the mathematics teacher's physical. However, the mainstream of teaching should be happening in grades K–2, and negligible teachings should be offered in grades 3–5.

Methodology

Method

The study was undertaken in a given high school in UAE in which the language of education is English. Students in the school are proficient in English even though English is their second language. The primary endpoint of the study is to discover what conceptions do grade 6, 7, and 8 students hold about the equal sign? The study involved 18 students, 6 students in each grade. Information letters that invited students to contribute to the study and consensus procedures were given to all the grades 6, 7, and 8, and those who took part were those who completed and returned signed consent forms. Data were collected by a written test and individual interviews. Students' thoughtfulness of the equal sign is not easily recognizable in verbal expression. Part of the confusion associated with the equal sign stems from this verbal-written dichotomy (Luneta, 2015; Pirie, 1998). Although mistakes in using the equal sign may not be effortlessly noticeable in verbal scientific expression (for example, see question 2 of the research instruments below), they become prominent in the written form. Therefore, we used the written test as one of the methods of data collection. Moreover, an inscribed confirmation of how students utilized the equal sign was critical. Below are the test items:

Item 1

1) Write the missing number in each square box below:

- a) $14 \times 3 = \square - 3$
- b) $9 - 5 = \square - 9$
- c) $24 + \square = 27 + 31$
- d) $100 \div 5 = \square + 5$
- e) $169 \div 13 = 13 - \square$

Element 2

Is the arithmetical declaration below precise? If yes, say why; if no, say why.

$$5 + 9 = 14 \div 2 = 7 \times 3 = 21$$

Element 3

I have 5 apples, and I add one more. Now I have 6. Then, I add two more apples again. How many apples do I have now?
Show ALL your workings (show all the given information in your solution).

Element 4

Solve for x in the following equation. SHOW each step of your solution clearly and STATE what you did in each step.

$$X + 1 = 5 \text{ (Grade 6)}$$

$$4x + 7 = 15 \text{ (Grade 8)}$$

$$4x + 4 = 4x + 1 \text{ (Grade 9) (Essien \& Setati, 2006).}$$

Elements 3 and 4 were nearly indistinguishable in structure, and these tasks were performed to measure students' consciousness of the correct arithmetical grammar. These items weathered the written-verbal dichotomy, which has been accredited to nearly all students' misunderstandings, with equal signs. Interviews provided a tool to ensure the accuracy of interpretations (Silverman, 2000). This is more so in a study such as this. Interviews "produce located understandings beached in detailed interactional episodes" (Amis, 2010). In addition to giving a written test, we conducted individual interviews with selected students to provide those students with opportunities to explain their reasoning in the test items. Of the 18 students who participated in the written test, four were selected for interviews. The selection was based on students' solutions in the written test. Each of the four interviewees had to answer questions based on their test answers. As the interviews concluded, students were requested to provide solutions to the question: "How do you understand the equal sign? Give an example to illustrate what you mean."

Data Analysis

In the analysis of the research data, Data were collected by a written test and individual interviews. The response papers of the students were coded as 6S1, 6S2, ..., 6S6 for 6th graders; 7S1, 7S2,..., 7S6 for 7th graders, and 8S1, 8S2,...,8S6 for 8th graders and the solutions of the students in each question were examined in three categories as correct, incorrect and no response. However, for each question, the solutions in the incorrect category were analyzed in detail, and the errors that students made are coded according to their types. The reasons for each type of error are emphasized. According to the explanations made by the students, it was investigated whether these mistakes are caused by misunderstandings or not.

To ensure the reliability of the study, in the analysis of data, two expert researchers from Mathematics independently coded the data. By comparing the analysis of these two field experts, the number of overlapping and non-overlapping codes has been determined. By using the formula, (Reliability = number of overlaps / (number of overlaps + number of non-overlaps)) 89% reliability level was found. In addition, while interpreting the obtained data, direct citations from students' answers were included to ensure reliability and validity.

Findings

Following data examining, the consequences are separated into two units. The segments are seen from scholars' test results and students' interview facts consequences. Researchers have given three questions to students to identify them.

The written test examination shows that (5 students out of 18) 28% of the students in the study obtained the correct answer to sub-question 1(a). Also (13 students out of 18), 72% of students wrote 42 as the answer as an incorrect answer. For these students, the equal sign was an invitation to find the operation's answer on the left. The equal sign was seen as a *do-something symbol* (Behr et al., 1980). This was also the case in sub-question (b), where most of the students answered 4 as an incorrect answer (13 students out of 18), so only (5 students out of 18) 28% answered sub-question (b) correctly. 72% of students gave the correct answers to sub-questions (c), (d), and (e), respectively. The questions that record students' most deficient performance are ones in which the placeholder comes just after the equal sign (see questions (a), (b), and (d)). The students who wrote incorrect answers to questions (a) and (b) were consistent in seeing the equivalent symbol as a "find-the-answer" symbol. Conversely, 95% of those who correctly answered (a) or (b) also answered all or most of the other sub-questions in question 1 correctly. One grade 7 students and one grade 8 students correctly answered all the sub-questions in question 1.

Tables 1, 2, and 3 below show that the number of grades 6, 7, and 8 students who correctly solved the questions, the number of students who gave incorrect answers, and those who did not respond to the questions.

Table 1: Grade 6 test results

| Question | Incorrect answers | Correct answers | No response | Emerging conception |
|----------|-------------------|-----------------|-------------|---|
| 1 (a) | 5(83.3%) | 1 (16.7%) | 0 | An equal sign as a 'do something symbol' is reflected in students' answers to the three questions—overgeneralizing the commutative law in question (b). |
| 1 (b) | 5 (83.3%) | 1 (16.7%) | 0 | |
| 1 (c) | 2(33.3%) | 4(66.7%) | 0 | |
| 1 (d) | 2 (33.3%) | 4 (66.7%) | 0 | |
| 1 (e) | 2 (33.3%) | 4 (66.7%) | 0 | |
| 2 | 5(83.3%) | 1 (16.7%) | 0 | An equal sign as a unidirectional symbol. (Lack of syntax awareness) |
| 3 | 3(50%) | 3 (50%) | 0 | An equal sign as a unidirectional symbol. (Lack of syntax awareness) |
| 4 | 0 (0.0%) | 0 (0.0%) | 6 | No error. Lack of knowledge of solving equations |

Table 2: Grade 7 test results

| Question | Incorrect answers | Correct answers | No response | Emerging conception |
|----------|-------------------|-----------------|-------------|---|
| 1 (a) | 4(66.7%) | 2 (33.3%) | 0 | An equal sign as a 'do something symbol' is reflected in students' answers to the three questions—overgeneralizing the commutative law in question (b). |
| 1 (b) | 4(66.7%) | 2 (33.3%) | 0 | |
| 1 (c) | 2 (33.3%) | 4(66.7%) | 0 | |
| 1 (d) | 2 (33.3%) | 4(66.7%) | 0 | |
| 1 (e) | 2 (33.3%) | 4(66.7%) | 0 | |
| 2 | 5(83.3%) | 1 (16.7%) | 0 | An equal sign as a unidirectional symbol. (Lack of syntax awareness) |
| 3 | 3(50%) | 3(50%) | 0 | An equal sign as a unidirectional symbol. (Lack of syntax awareness) |
| 4 | 1(16.7%) | 1(16.7%) | 4 | No error. Lack of knowledge of solving equations |

Table 3: Grade 8 test results

| Question | Incorrect answers | Correct answers | No response | Emerging conception |
|----------|-------------------|-----------------|-------------|---|
| 1 (a) | 4 (66.7%) | 2 (33.3%) | 0 | An equal sign as a 'do something symbol' is reflected in students' answers to the three questions—overgeneralizing the commutative law in question (b). |
| 1 (b) | 4 (66.7%) | 2 (33.3%) | 0 | |
| 1 (c) | 1 (16.7%) | 5 (83.3%) | 0 | |
| 1 (d) | 1 (16.7%) | 5 (83.3%) | 0 | |
| 1 (e) | 1 (16.7%) | 5 (83.3%) | 0 | |
| 2 | 5 (83.3%) | 1 (16.7%) | 0 | An equal sign as a unidirectional symbol. (Lack of syntax awareness) |
| 3 | 3 (50%) | 3 (50%) | 0 | An equal sign as a unidirectional symbol. (Lack of syntax awareness) |
| 4 | 1(16.7%) | 1(16.7%) | 4 | No error. Lack of knowledge of solving equations |

As the tables above show, the grade 6 students' performance level is lower than that of the grade 7 and 8 students. In question 1, (14 students out of 30), 47 % of students in Grade 6 produced the correct answers as opposed to 53% in grades 7 and 63% in grade 8. Carpenter et al. (2005) made a similar observation in their study, which involved Grades 1 to 6. They noted that performance did not improve with age in the question: $8 + 4 = \square + 5$.

The tables above also show that most of the students failed to answer question 2 correctly. 83.3% thought that the mathematical sentence $5 + 9 = 14 \div 2 = 7 \times 3 = 21$ is correct. Even those who correctly answered the sub-questions in question 1 could not see the symbolization error in question 2. In question 3, however, 50% of the students were talented at interpreting the term problem into mathematically correct sentences, while 50% were consistent in their lack of syntax awareness. Most of the grade 6 students did not attempt question 4. All but one grade 7 and 8 students attempted the question. 88.8 % of the students who attempted it indicated that they moved the like terms together and then solved for x . one student first subtracted four from both sides and then subtracted one from both sides.

The previous discussion highlights students' performance in the test and some of the emerging interpretations. While the analysis helps show broad categories, it does not show the students' details and why. In the discussion section, we provide an analysis of the interviews with students, which details how the students used the equal sign and why. The interviews focused more on the students' justification of their use of the equal sign than their answers' correctness. Students 1 and 2 were from grade 6, students 3 and 4 from grade 7, and students 5 and 6 from grade 8.

Section 2

Equivalent symbol as a 'Do-something' symbol

The excerpt below from Student 1's answers to the interviews and written test is typical of how the students (grades 6, 7, and 8) who displayed the do-something conception responded to question 1. Student 1's answers in the test suggested a thoughtfulness of the equal symbol as a representation in which on the left you have the operation and just after the equal sign you have the answer. For example, for problem number (d), he answered $100 \div 5 = 20 - 5$. The interview extract below shows his explanation. Student 1: 100 divided by 5, right? So, you have 5, 10, 15, 20, 25, 30, 35, 40, 45, 50 (Counting fingers). Its 20, right? Then you plus the 5 ... You do nothing to this other 5. This same notion was evident in the response by Student 4:

Researcher: How did you get an answer of 20 in sub-question (d)?

Student 4: 100 divided by 5 is equal to 20.

Researcher: And sub-question (c)? How did you get 3?

Student 4: 24 plus 3 gives 27.

When were students asked: "What do you understand by the equal sign? Tell me in your own words. What does "=" mean? Give an example to illustrate what you mean", their responses below emphasized the notion of the equal symbol as a do-something or a find-the-answer symbol.

Student 1: When you add or subtract something, elm, that particular number that you get, the number you get after you have done the addition or subtraction is the equal number.

Student 2: It means what you get from the things you are combining or adding or subtracting. It gives you the answer to something, equals the answer to something.

Student 3: for an equal sign, wouldn't it mean like the quantity you have at the end of your math equation. What your answer to the math equation, but it's like a quantity. It tells you how much of that equation you get at the end.

Student 4: If you have to add two things, what would the end answer be, or if you were to subtract something, what would the end answer be. But, like also as in this (referring to $5 + 1 = 6 + 2$). What's on the right must equal what's on the left. All students understood the equal sign as necessitating that the response is written afterward instead of an interpersonal symbol associating.

The equal sign as a unidirectional symbol

The equal sign's conception as a unidirectional symbol is closely related to the equivalent symbol's thoughtfulness as a do-something symbol. Both conceptions call for computing the quantity on the left of the equivalent sign. Besides, the equal sign is seen as one-way traffic. For instance, Behr et al. (1980) found that students refused to accept that the mathematical sentence $\square = 2 + 4$ is correct because it has the equal sign "in the wrong place." For these students, the arithmetical sentence is only correct if written in form $2 + 4 = \square$, where the undertaking is from leftward to the right. Such unidirectional use would view the arithmetical sentence $5 + 9 = 14 \div 2 = 7 \times 3 = 21$ as $5 + 9 \Rightarrow 14 \div 2 \Rightarrow 7 \times 3 = 21$. The reverse movement would make no sense, and neither would the double movement as in $5 + 9 \Rightarrow 14 \div 2 \Rightarrow 7 \times 3 \Leftrightarrow 21$. Consistent with this, Student 1 explained that the mathematical sentence is correct "because $5 + 9$ is equal to 14. You divide that by 2, it gives 7, then you times it by 3, it's 21. So, it is right." he interpreted the equal sign as a unidirectional symbol. This explains why he made a syntax error in question 2. This conception of the equal sign as a unidirectional sign was consistent across Grade 7 and 8 students. Below is an excerpt from an interview with Student 4. Student 4: You said, I have 5 apples, and I add one more. 5 plus 1 equals 6. Now I have 6.

Then I add 2 more apples again. 6 plus 2 equals 8. How many apples do I have? Eight. That's how I got eight. (This student had written $5 + 1 = 6 + 2 = 8$ in the test). This erroneous symbolization stems from translating verbal expression into a mathematical sentence (Luneta, 2015). Student 4's verbal explanation above is correct, but her translation into a written mathematical sentence is erroneous.

There is a wealth of evidence that students' interpretation of the equivalent symbol as a do-something symbol or as a unidirectional symbol stems from the way they are taught in earlier grades (Behr et al., 1980; Carpenter et al., 2005; Frequental, 1983; Kieran, 1981; MacGregor & Price, 1999; Stacey & MacGregor, 1997). In higher grades, this notion persists and is even reinforced by instrumental teaching strategies in algebra: teaching students' rules (without reason) for manipulating algebraic expressions can lead to a limited understanding. For example, Student 3: Ok, solving for x, right? Okay, we must get all the like terms on one side, right, so we get all the numbers to one side and all the x's to the other side to get the like terms together. So that we have $4x + 4 = 4x + 1$ (original question). So, we've got this 4x this side. To bring this 4x to this side, we have to change it because it was positive on this site; it will change to a negative, right? (writes $4x - 4x$). Then this 4 needs to go that side so it can come with 1. So, if we move it to this site, then it becomes negative. The response suggests that the student's comprehension of the equal sign is contributory. Neither of

the Grade 7 students explained their solution approach to highlight the serious role of the equal sign in the equation. Neither could elucidate how altering the sign when an amount is occupied to the other side or calculating the same amount to both sides of the equation was essential to keeping the quantities equal. They only knew the 'rules without reason' (Jones et al., 2013).

This, however, does not necessarily mean that instrumental understanding is a result of instrumental teaching. As Olivier argues, "Incorrect new learning is mostly the result of previous correct learning" (p. 17). There are many instances in which the equal sign's interpretation as a do-something, unidirectional signs produce the correct answer. The problem here is that that kind of interpretation of the equal sign does not always denote equivalence. Jones et al. (2013) argue that such an understanding of mathematical concepts is insufficient if students adapt their knowledge to new situations. They understand the equal sign as a do-something symbol. A unidirectional symbol is most likely to pose problems when dealing with algebraic equations where students have to work with equivalent statements and equivalent equations.

Another source of the students' errors was due to the overgeneralization of the commutative law. As Student 1 explains below, in either $-=1313169 \square$ or $=13169 \square - 13$, zero should go into the box, just as 4 should go into the box in $4 + 3 = 3 + \square$ and $4 + 3 = \square + 3$.

Researcher: ... Let's go back to sub-question (e)... If the placeholder, the square box, were here (pointing to the placeholder in the question: $13169 = \square - 13$), what would be the answer?

Student 1: I'm not sure. I think it would be naught again.

Researcher: Why would it be zero?

Student 1: You just swap them around, that's all.

This same error of the overgeneralization of commutativity was evident in Student 2's responses.

Researcher: ... in sub-question (b), you got 5. What did you do?

Student 2: Because it says 9 minus 5 equals something minus 9. So, I said, it's the same thing because it says it's equal, it says it's equal, so the sum should be equal. That's why I wrote 9 minus 5 equals 5 minus 9.

These two students in Grade 7 were not the only ones who over-generalized the commutative law. 37.5% of Grade 7 students who wrote the test made the same mistake in sub-question (b) of question 1. None of the students in Grade 8 made this error.

Discussion

Data analysis shows that amongst these grades 6, 7, and 8, the equal sign's dominant meaning is a signal for do-somewhat or find-the-answer symbol. The equal sign is an instrument to inscribe the answer. The interpretation of the equal sign as a unidirectional symbol is another concept that originated in this study. The students did not show this in questions 2 and 3 due to a lack of syntax awareness. This education's conclusions are dependable with the conclusions of preceding investigation at the primary school level (Behr et al., 1980). One of the students' conceptions of the equal sign as a do-something or a unidirectional symbol stems from the verbal language mathematical language tension. Another source of the limited conceptions is due to overgeneralization of the law of commutativity.

The fact that the findings for Grade 6, 7, and 8 students are the same as those for primary school students is, in our view, a significant finding and indicates that we cannot take lightly the importance of teaching the concept of equivalence when students are introduced to algebra at Grade 7 level. One of the main hesitant chunks in the knowledge of algebra is an incomplete sympathetic of the equivalent sign since practically all managements in algebraic reckonings necessitate a grip of the equivalent symbol as suggesting a relation (Carpenter et al., 2005).

A student with a "do-somewhat" or "find-the-answer" understanding of the equal sign may have difficulties understanding a simple algebraic equation like $a + b = ?$ because there is nothing to be *done* (unlike the equation $4 + 5 = ?$) (Lubinski & Otto, 2002). Algebra essentially deals with equivalence; not equality is the marked difference between arithmetic and high school algebra. $2a + a$ is not the same as $3a$! They are different calculation procedures, yet they are equivalent since they produce the same output values for all input values. Worse still, how would students understand the equation/ relation $\pi = 3,1415$ the same applies to the equation $4x + 4 = 4x + 1$. Only understanding the equivalent symbol as signifying a correspondence relative would allow a student to comprehend that it makes sense to subtract 4, for example, from both sides (Molina, & Ambrose, 2008).

Our finding that interpretations of the equal sign in Grades 6,7 and 8 in this school are similar and similar also to published findings in primary schools suggests that, as Falkner et al. (1999) endorse, it is significant that teachers challenge students' mistaken misapprehension of the equal sign beforehand they develop resolutely engrained. Students face challenges when it comes to solving nonstandard equations. At the moment, curricula have a lot of inconsistencies with their definition across all grade levels. For example, the equal sign instruction incorporated in the student textbooks, and the teacher manuals, is relatively not consistent with each other. It is important to consider that

most curricula do not present opportunities for students to solve nonstandard equations using their student textbooks. According to (McNeil et al., 2006) brings to light how nonstandard equations such as operations-both-sides equations may assist in bringing an understanding of the equal sign. Therefore, it is essential to feature some nonstandard equations in the teacher manuals to foster an understanding of the equal sign role in its relationship with two sides of an equation (Demonty & Vlassis, 1999).

Giving students the definition of the equal sign and its instructions through the teacher manuals within elementary mathematics curricula will significantly understand the equal sign. This will help students interact with nonstandard equations more often and from an early stage. Though incorporating these nonstandard equations in the teacher manuals may be time-consuming while teaching, most importantly is that it will help the student familiarize him/herself with these equations. Therefore, it will help students understand the equal sign. In addition, the curricula should offer a consistent definition of the equal sign across all grade levels and provide an opportunity for students to interact with nonstandard equations more often, with similar proportions in all grade levels. In a situation where the curriculum developers do not want to expand the curriculum lesson to fit in lessons on the equal sign, teachers will have to deviate and take up the responsibility themselves and provide their own instructions.

Conclusion

The study reports on the students meaning and interpretations of the equal sign. The study involved Grade 6, 7, and 8 students in a secondary school in Alain, UAE. Much of the earlier research done on the equal sign has focused on the primary school level, but this one focuses on middle school students. The study shows that the maximum foremost understanding of the equal sign amongst Grade 6, 7, and 8 students is a do-something, unidirectional symbol. Students realize the equal sign as an instrument for marking the response moderately than as an interpersonal symbol to associate extents.

Recommendations

Current studies show that most students receive minimally or no exposure to nonstandard equations in the student textbooks and classwork. One significant fact that cannot be ignored is that further research needs to be done to grasp the correlation between opportunities and exposure to handle or solve nonstandard equation types in the student textbooks and equal sign understanding. Curriculum developers will be more inclined to use a combination of nonstandard equations and standard equations in the curriculum if a correlation between the exposure to various types of equations and the ability to work with the equal sign is established. Also, it is important to acknowledge that further research needs to be done on methods that are favorable for teaching the relation of the equal sign, such as the vocabulary to give instructions, or if a student needs more exposure to various equations, including nonstandard equations.

In elementary mathematics, the equal sign is the most used symbol. In addition, subtraction, division, and multiplication student have to use the equal sign. Therefore, for this fact, it is important for a student to understand what the symbol means. This will also be essential in helping a student to solve other equations, such as algebraic equations. The purpose of the equal sign means to balance both sides. Helping students understand the equal sign from elementary grade will help prevent further misconceptions concerning the equal sign from building up.

Limitation

A limitation of the current study includes Grade 6, 7, and 8 students in a secondary school in Alain, UAE. Much of the earlier research done on the equal sign has focused on the primary school level, but this one focuses on middle school students; the study was undertaken in a given high school in UAE. The language of education is English. Students in the school are proficient in English even though English is their second language. The primary endpoint of the study is to discover what conceptions do grade 6, 7, and 8 students hold about the equal sign? The study involved 18 students, 6 students in each grade. Information letters that invited students to contribute to the study and consensus procedures were given to all the grades 6, 7, and 8, and those who took part were those who completed and returned signed consent forms. Data were collected by a written test and individual interviews. Students' thoughtfulness of the equal sign is not easily recognizable in verbal expression. Part of the confusion associated with the equal sign stems from this verbal-written dichotomy (Luneta, 2015; Pirie, 1998).

Authorship Contribution Statement

Wardat: Concept design for the study, data collection and analysis. Jarrah: Data analysis, interpretation, discussion, final approval of manuscript, critical revision of manuscript. Stoica: Literature review, Theoretical framework, critical revision of manuscript.

References

- Abdallah, R., & Wardat, Y. (2021). Teachers' perceptions on the effectiveness of professional development programs in improving the curriculum implementation at Jordanian schools. *Elementary Education Online*, 20(5), 4438-4449. <https://doi.org/10.17051/ilkonline.2021.01.126>
- Alotaibi, A., Khalil, I., & Wardat, Y. (2021). Teaching practices of the mathematics male and female teachers according to the PISA framework and its relation to their beliefs towards their students. *Elementary Education Online*, 20(1), 1247-1265. <https://doi.org/10.17051/ilkonline.2021.01.126>
- Amis, J. (2010). Book Review: Amis: Denzin, N. K., & Lincoln, Y. S. (Eds.). (2008). The landscape of qualitative research. *Organizational Research Methods*, 14(1), 239-242. <https://doi.org/10.1177/1094428109332198>
- Behr, M., Erlwanger, S., & Nichols, E. (1980). How children view the equal's sign. *Mathematics teaching*, 92(1), 13-15.
- Bennett, V. M. (2015). *Understanding the meaning of the equal sign: an investigation of elementary students and teachers* [Doctoral dissertation, University of Louisville]. ThinkIR: The University of Louisville's Institutional Repository. <https://doi.org/10.18297/etd/2303>
- Blanton, M. L. (2011). *Developing essential understanding of algebraic thinking for teaching mathematics in grades 3-5*. National Council of Teachers of Mathematics.
- Cajori, F. (1928). *A History of Mathematical Notation: Notations in Elementary Mathematics (Vol. 1)*. Open Court.
- Carpenter, T. P., Levi, L., Franke, M. L., & Zeringue, J. K. (2005). Algebra in elementary school: Developing relational thinking. *central sheet for didactics of mathematics*, 37(1), 53-59. <https://doi.org/10.1007/bf02655897>
- Demonty, I., & Vlassis, J. (1999). Teaching practices at CP [Structures and effects on student learning]. *French Journal of Pedagogy* 93(1), 5-15. <https://doi.org/10.3406/rfp.1990.1369>
- Duncan, C. D. (2015). *A study of mathematical equivalence: The importance of the equal sign* [Master's thesis, Southern University]. LSU Digital Commons. https://digitalcommons.lsu.edu/gradschool_theses/398
- Essien, A., & Setati, M. (2006). Revisiting the equal sign: Some Grade 8 and 9 learners' interpretations. *African Journal of Research in Mathematics, Science and Technology Education*, 10(1), 47-58. <https://doi.org/10.1080/10288457.2006.10740593>
- Falkner, K., Levi, L., & Carpenter, T. (1999). Early childhood corner: Children's understanding of equality: A foundation for algebra. *Teaching Children Mathematics*, 6(4), 232-236. <https://doi.org/10.5951/tcm.6.4.0232>
- Jarrah, A. M., Khasawneh, O. M., & Wardat, Y. (2020). Implementing pragmatism and John Dewey's educational philosophy in Emirati elementary schools: case of mathematics and science teachers. *International Journal of Education Economics and Development*, 11(1), 58. <https://doi.org/10.1504/ijeed.2020.104287>
- Jones, I., Inglis, M., Gilmore, C., & Evans, R. (2013). Teaching the substitutive conception of the equals sign. *Research in Mathematics Education*, 15(1), 34-49. <https://doi.org/10.1080/14794802.2012.756635>
- Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics*, 12(3), 317-326. <https://doi.org/10.1007/bf00311062>
- Knuth, E. J., Alibali, M. W., Hattikudur, S., McNeil, N. M., & Stephens, A. C. (2008). The importance of equal sign understanding in the middle grades. *Mathematics Teaching in the Middle School*, 13(9), 514-519. <https://doi.org/10.5951/mtms.13.9.0514>
- Lubinski, C. A., & Otto, A. D. (2002). Meaningful Mathematical Representations and Early Algebraic Reasoning. *Teaching Children Mathematics*, 9(2), 76-80. <https://doi.org/10.5951/tcm.9.2.0076>
- Luneta, K. (2015). Understanding students' misconceptions: An analysis of final Grade 12 examination questions in geometry. *Pythagoras*, 36(1), 1-11. <https://doi.org/10.4102/pythagoras.v36i1.261>
- MacGregor, M., & Price, E. (1999). An exploration of aspects of language proficiency and algebra learning. *Journal for Research in Mathematics Education*, 30(4), 449-467. <https://doi.org/10.2307/749709>
- McNeil, N. M., Grandau, L., Knuth, E. J., Alibali, M. W., Stephens, A. C., Hattikudur, S., & Krill, D. E. (2006). Middle-school students' understanding of the equal sign: The books they read can't help. *Cognition and Instruction*, 24(3), 367-385. https://doi.org/10.1207/s1532690xci2403_3
- Molina, M., & Ambrose, R. (2008). From an operational to a relational conception of the equal sign. Thirds graders' developing algebraic thinking. *Focus on Learning Problems in Mathematics*, 30(1), 61-80.
- Ningrum, R. W., Yulianti, M., Helingo, D. D. Z., & Budiarto, M. T. (2018). Students' misconceptions on properties of rectangles. *Journal of Physics: Conference Series*, 947, 1-7. <https://doi.org/10.1088/1742-6596/947/1/012018>

- Pirie, S. (1998). Crossing the gulf between thought and symbol: Language as (slippery) stepping-Stones. In H. Steinbring, B. Bussi, & A. Sierpienska, (Eds.) *Language and communication in the mathematics classroom* (pp. 7-29). National Council of Teachers of Mathematics.
- Powell, S. R. (2014). The influence of symbols and equations on understanding mathematical equivalence. *Intervention in School and Clinic, 50*(5), 266–272. <https://doi.org/10.1177/1053451214560891>
- Sherman, J., & Bisanz, J. (2009). Equivalence in symbolic and nonsymbolic contexts: Benefits of solving problems with manipulatives. *Journal of Educational Psychology, 101*(1), 88–100. <https://doi.org/10.1037/a0013156>
- Silverman, D. (2000). Analyzing talk and text. In N. K. Denzin & Y. Lincoln (Eds.). *Handbook of qualitative research* (2nd ed., pp. 821-834). Sage Publications.
- Stacey, K., & MacGregor, M. (1997). Building foundations for algebra. *Mathematics Teaching in the Middle School, 2*(4), 252–260. <https://doi.org/10.5951/mtms.2.4.0252>