Effects of Three Neo Piagetian Constructs on Students' Portrayed Representations of the Atomic Structure: A Latent Class Analysis

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Abstract

The present study investigates the effect of three neo-Piagetian constructs, namely Formal Reasoning (FR), Field Dependence/ Independence (FDI) and Divergence (DIV), as well as the effect of age and gender, on students' portrayed representations of the atomic structure, considering their degree of coherence. For this purpose students' representations were examined for their consistency across three task contexts, using Latent Class Analysis (LCA), in order to identify distinct latent classes of participants providing specific consistent representations. Participants (n=421) were students of the grades 8th, 10th and 12th of secondary education. LCA led to three clusters, in each of which, students' responses demonstrated a consistency across tasks, with Bohr's model, Nuclear model and Particle model, respectively. LCA with the covariates provided evidence for the association of the three cluster-memberships with the neo-Piagetian variables and age, while no effect of gender was found. Implications for science education are also discussed.

Keywords: Atomic structure, Representations, Latent Class Analysis, Cognitive variables.

INTRODUCTION

Research on the nature of student mental models for the atom and the representations of the atomic structure is quite extended (e.g. Papaphotis & Tsaparlis, 2008; Park & Light, 2009; Papageorgiou, Markos & Zarkadis, 2016a; Zarkadis, Papageorgiou & Stamovlasis, 2017). Relevant studies indicate that students within a wide range of ages/grades often have difficulties in understanding a more sophisticated and abstract quantum view of the atom holding naïve, deterministic or hybrid mental models (e.g. Papaphotis & Tsaparlis, 2008; Dangur et al., 2014). From another point of view, a number of studies show that, when students use such models in order to explain particular situations, there is not always a consistent way in their use (e.g. Wang & Barrow, 2013; Zarkadis et al., 2017). Zarkadis et al. (2017) for instance, found that there is an extended inconsistency both between and within the models, when characteristics of the atomic structure are used in order to explain familiar phenomena.

On the other hand, research focusing on the developmental and cognitive factors' effects on the way students represent the atomic structure is rather limited (Papageorgiou et al., 2016a). Taking also into account that such factors have been found to play an important role on the understanding of topics relevant to the structure of matter (e.g. Tsitsipis et al., 2010; Stamovlasis & Papageorgiou, 2012; Tsitsipis et al., 2012), we investigate in the present study, the effect of cognitive and developmental (age) factors on students' representations of the atomic structure, taking also into account their degree of coherence.

THEORETICAL BACKGROUND

Students' Models of the Atomic Structure and Their Coherence

For many decades, studies relevant to students' models of the atomic structure were elaborating their data in a way accepting that they are stable and coherent. Consequently, these student models were categorized in distinct categories, which can be briefly described, from the simplest and concrete to the most sophisticated and abstract, as follows (e.g. Park & Light, 2009; Papageorgiou et al., 2016a; Zarkadis et al., 2017):

A. The 'Atom–cell model', which considers the atom as a living organism.

B. The 'Particle model', where the atom is perceived as a particle without reference to sub-atomic characteristics.

C. The 'Nuclear model', where the atom consists of sub-atomic particles (electrons around the nucleus consisting of protons and neutrons).

D. The 'Bohr's model', where the atom comprises electrons moving in certain paths around the nucleus.

E. The 'Quantum model', where the atom is perceived through a probabilistic view, comprising orbitals and electron clouds.

However, during recent years, the aspect that there is not always coherence in the above mental models (e.g. Park & Light, 2009; Zarkadis et al. 2017; Zarkadis & Papageorgiou, 2020) tends to connect these models to the fragmented knowledge perspective (e.g. di Sessa, 1993; diSessa et al., 2004). For instance, there are cases where, although students represent the atomic structure within the context of the quantum model, some characteristics of the Bohr's model are also present, or cases where the opposite happens (e.g. Taber, 2002; Nakiboglu, 2003; Taber, 2005; Papaphotis & Tsaparlis, 2008; Dangur et al., 2014). According to Zarkadis et al. (2017) this is a 'within' inconsistency. On the other hand, there are cases, where students use more than one atomic model interchangeably when trying to explain particular situations - a 'between' inconsistency (Zarkadis et al., 2017). In addition, particular pieces of knowledge relevant to the atomic structure have been identified and possible routes, through which they are activated by the students when constructing relevant explanations, are also explored (Zarkadis & Papageorgiou, 2020). Thus, the research seeking for answers to the question of coherence/incoherence of the mental models regarding the atomic structure is ongoing, stimulating the interest of science researchers and educators.

Neo-Piagetian Constructs / Cognitive Variables

The neo-Piagetian premises consider that cognitive processes are driven by mental resources, which can explain the observed differences in performance across various tasks. The representative Pascual Leone's theory of Constructive Operators (Pascual-Leone, 1970) suggests that cognitive performance is associated with a number of constructive operators, each of which performs a specific function. For instance, the M-operator deals with information processing capacity, the L-operator is associated with the Formal Reasoning (FR), e.g. formal logic or conservation, the F-operator deals with Field Dependence or Independence (FDI), etc. The importance of the neo-Piagetian framework is that the constructive operators in question, which are activated during particular cognitive tasks, can be operationalized by psychometric variables. Research evidence has supported the role of those cognitive variables, such as *Formal Reasoning* (FR), *Field Dependence/ Independence* (FDI) and *Divergence* (DIV), which have been found to be predictive of students' performance in science and especially, in understanding both the particulate nature of matter (e.g. Tsitsipis et al., 2010; Stamovlasis & Papageorgiou, 2012; Tsitsipis et al., 2012) and the atom and atomic characteristics (e.g. Papageorgiou et al., 2016a,b). These neo-Piagetian cognitive variables are briefly presented below.

- (FR): *Formal Reasoning* is a cognitive factor associated with the ability of students to use concrete and formal operational reasoning (Lawson, 1978).
- (FDI): *Field Dependence/ Independence* is associated with the ability of the students to identify the significant information from a complex and confusing context (Witkin et al., 1971).
- (DIV): *Divergence* is a cognitive factor associated with the ability of the students to find several equally acceptable solutions to a problem (e.g. Bahar, 1999).

Despite the interesting findings concerning the effects of these cognitive factors/variables on the student understanding of the atom, there is not any relevant study yet, investigating their effects on students' representations of the atomic structure in dependence to their degree of coherence. In order to do this, the application of a Latent Class Analysis (LCA) would be extremely useful.

LCA

Latent Class Analysis (LCA) is an analysis designed to identify Latent Classes (LCs), i.e., groups of students sharing similar response patterns. It is considered that these similar responses (observable variables) originate from the same latent variable, which is the common causal-cause of the responses (McCutcheon, 1987; Clogg, 1995). LCA is in fact, a model-based cluster analysis and a psychometric modeling procedure, which implements the *conditional probabilities* (CP) in order to assign class-memberships to students. CP is defined as the probability of providing a certain student's response to an item, under the condition that the student belongs to a specific LC. The procedure of an

LC classification provides a number of cluster solutions. Thus, the researcher has to choose the most fit and interpretable solution. This choice is based on a number of indicators, namely: the number of parameters, the Bayesian Information Criterion (BIC), the likelihood ratio statistic (L^2), the entropy-R^2, the Akaike's Information Criterion (AIC), the degrees of freedom and the bootstrapped *p*-value. Additionally, it is important that an analysis of covariates could be also included in LCA, determining the effects of the relevant class memberships on the external variables (Bakk et al., 2013).

Rationale and Research Questions

The validity of neo-Piagetian framework has been established in education through an extensive research demonstrating the determinant role of above cognitive factors in understanding science (e.g. Tsitsipis et al., 2010; Stamovlasis & Papageorgiou, 2012; Papageorgiou et al., 2016a,b). Among the most popular topics, the structure of matter and the relevant concepts to the atom and the atomic structure have attracted a special interest of science researchers (e.g. Cokelez & Dumon, 2005; Papaphotis & Tsaparlis, 2008; Park & Light, 2009; Cokelez, 2012; Dangur et al., 2014; Papageorgiou et al., 2016a,b; Zarkadis et al., 2017; Allred & Bretz, 2019; Zarkadis & Papageorgiou, 2020). From another angle, students' knowledge and representations on these topics, which are changing across ages, has been studied within the framework of coherence mental model hypothesis (e.g. Papageorgiou et al., 2016a; Zarkadis et al., 2017). Applications of LCA to empirical data in this area have shown that often distinct clusters of students are revealed possessing similar mental representations manifested by consistent responses under varying conditions. Thus, a connection between such cluster-memberships resulting for students' portrayed representations of the atomic structure, with neo-Piagetian constructs could provide an explicative interpretation of them, as emerged via processes where the corresponding constructive operators are actively participating.

Based on the above, two research questions are posited:

- Are the cluster-memberships, emerged from an LCA, concerning students' portrayed representations of the atomic structure, associated with neo-Piagetian constructs, such as formal reasoning (FR), field dependence/ independence (FDI) and divergent thinking (DIV)?
- Are the above ensued class-memberships associated with age and gender?

METHODOLOGY

Sample and Procedure

Participants were 421 students (189 male and 232 female) comprised of four age-cohorts of 8th, 10th and 12th grades attending classes of secondary schools in Northern Greece. The participation in the study was voluntary. Table 1 shows the four-student age-cohorts.

Table 1. *The Four Age-Cohorts*

Cohorts	Description
1	[Age 13] 127 students (30.2%) of grade 8
2	[Age 15] 167 students (39.7%) of grade 10
3	[Age 17] 82 students (19.5%) of grade 12, 'technological direction'
4	[Age 17] 45 students (10.7%) of grade 12, 'science and math direction'

Students were from mixed socio-economic backgrounds and they attended mixed ability classes in regular public schools according to the National Science Curriculum for Greece (Greek Pedagogical Institute, 2003). In every particular cohort, all students were using the same textbook. However, the textbooks of the 3rd and 4th cohorts were different since the former was focused on the Bohr's model approach, whereas the latter was focused on the quantum mechanical approach. Data were collected through four paper-and-pencil tests (one test designed to assess students' representations for the atomic structure and three for the corresponding cognitive variables). Data collection took place during the second semester of the school year.

Instruments

Atomic Structure Representations Test

An instrument was developed in the context of a project aiming to assess students' ideas, misconceptions and representations of the atomic structure. For the needs of the present study, three tasks were developed targeting to students' portrayed representations of the atomic structure. The instrument was the same for all student cohorts. Students were asked to draw how they imagine the 'atom' if they could observe it through a 'powerful microscope' and to describe in detail what they have drawn, as follows:

i) Independently of any context (Task 1)

ii) Considering the electron moving in specific orbits, i.e., within the context of the Bohr's atomic model (Task 2)

iii) Considering the electron as an electron cloud, i.e., within the context of the probabilistic quantum model (Task 3).

Cognitive Variables Tests

Students' abilities concerning the three cognitive factors, i.e., Formal Reasoning (FR), Field Dependence/ Independence (FDI) and Divergence (DIV), were measured as independent variables based on the tests presented in Table 2. More details for the content of these tests can be found elsewhere (e.g. Tsitsipis et al., 2010, 2012; Papageorgiou et al., 2016a,b). In Table 2, the content of the tests, the duration of their completion and the value of the Cronbach's alpha reliability coefficient, were presented in brief.

Table 2. *Basic Characteristics Of The Cognitive Variables' Tests*

	FR	FDI	DIV
Test	The Lawson paper-and-pencil test (Lawson, 1978)	The Group Embedded Figures Test (Witkin et al., 1971)	A test designed by Bahar (Bahar, 1999)
Content	A fifteen-item test dealing with conservation of mass, displaced volume, control of variable, proportional reasoning, combinational reasoning, and probabilistic reasoning	A twenty-item test, in which students had to dissembled simple figures incorporated in twenty complex ones	A six-item test, in which students were asked to generate words similar to given, sentences using given words, sketches relevant to a given idea, things having a common trait, words beginning with one specific letter and ending with another, ideas about a given topic
Duration	45 min	20 min	20 min
Cronbach's alpha	0.77	0.84	0.69

Data Analyses

Taking into account students' drawings and relevant descriptions, their responses were categorized in the five categories A, B, C, D and E, reported in the theoretical background, i.e., '*Atom-cell model*', '*Particle model*', '*Nuclear model*', '*Bohr's model*', '*Quantum model*'. The categorization scheme was validated by two of the authors and any discrepancy was discussed in detail until a total agreement was reached. Thus, the *Atomic structure representations Test,* included three variables at the nominal scale, identifying categories corresponding to the distinct five specific representation models A to E. These variables were used as input for the LCA.

RESULTS

In the LCA analysis the set of tasks 1, 2 and 3 was used. As Table 3 shows, among a number of solutions (from 1-cluster to 5-cluster), the LCA lead to a 3-cluster solution as the most parsimonious and best fitting model. This fitting of a latent-class model is assessed by the co-evaluation of a number of indicators already reported earlier, such as the Npar, the entropy-R^2, the bootstrapped p-values and especially the lower BIC values (Vermunt and Magidson, 2000).

Table 3. *LCA Solutions And The Model Fit Indexes*

	LL	BIC(LL)	Npar	L^2	df	p-value	Class. Err.	R2
1-Cluster	-1047.72	2164.84	12	2095.43	313	0.00	0	-
2-Cluster	-806.695	1786.90	30	1613.39	295	0.00	0.001	0.98
3-Cluster*	-744.164	1765.95	48	1488.33	277	0.10	0.0027	0.97
4-Cluster	-716.358	1814.45	66	1432.72	259	0.06	0.0354	0.88
5-Cluster	-702.615	1891.07	84	1405.23	241	0.03	0.078	0.84

*Note: * The most parsimonious and best fitting model*

As a result, in our dataset three distinct clusters were identified, namely, Cluster 1, which has been found accounting 69.09% of the sample, Cluster 2, accounting 23.68% of the sample and Cluster 3, which has been found accounting 7.24% of the sample. In each one of these three clusters, there is a particular conditional probability (CP) of a student to provide one of the five categories of representations of the atomic structure A, B, C, D and E, i.e., '*Atom-cell model*', '*Particle model*', '*Nuclear model*', '*Bohr's model*', '*Quantum model*', under the condition that the student belongs to one particular cluster (out of these three clusters). Figure 1 shows these conditional probabilities (CPs) of students' representations in each one of the three clusters. For instance, in task 1, when a student belongs to Cluster 1, '*Atom-cell model*' has a probability 0.00 to be provided by the student, '*Particle model*' has a probability 0.07 to be provided by the student, '*Nuclear model*' has a probability 0.30 to be provided by the student, '*Bohr's model*' has a probability 0.59 to be provided by the student and '*Quantum model*' has a probability 0.04 to be provided by the student.

Thus, as Figure 1 presents, in Cluster 1 (69.09% of the sample) the *Bohr's* model seems to dominate with the *Quantum* model also appearing with high CP in task 3. In Cluster 2 (23.68% of the sample) the *Nuclear* model dominates, while in Cluster 3 (7.24% of the sample) the *Particle* model seems to be prevailing. Within these ensued clusters, there are prevailing representations, that is, students in each cluster demonstrate a consistency in their drawings across tasks. This observation leads to hypothesize a coherent latent variable or a coherent mental model (Stamovlasis, Papageorgiou & Tsitsipis, 2013; Zarkadis et al., 2017).

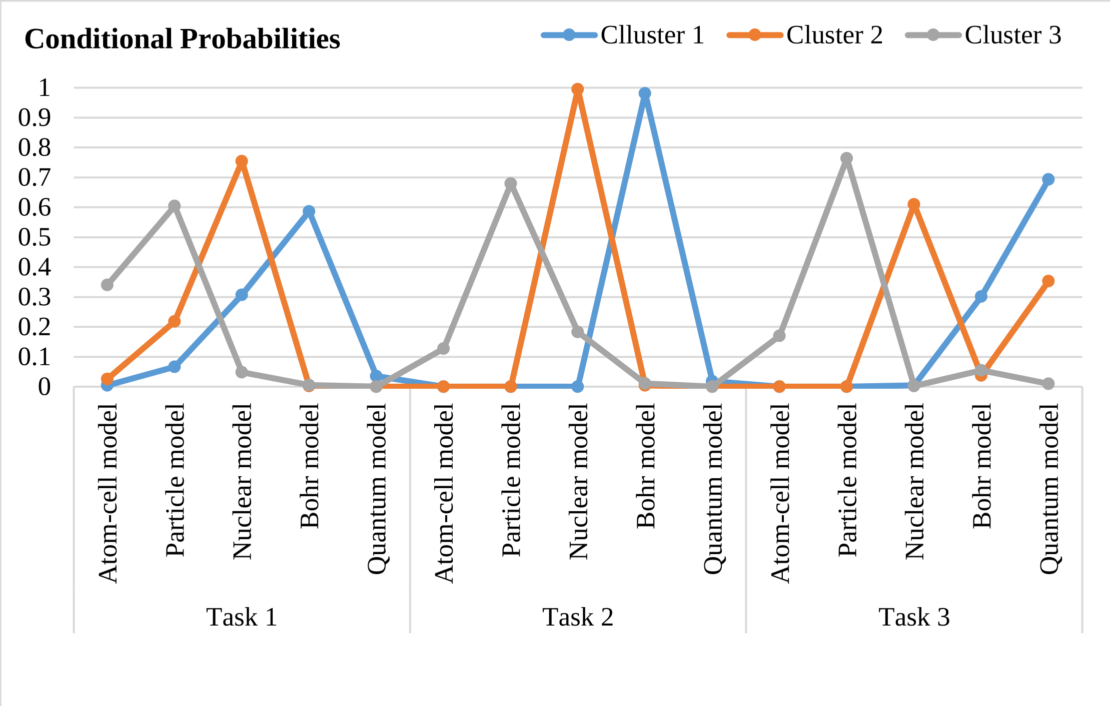

Figure 1. *The Conditional Probabilities (CPs) Of Each Representation/Model In Each Of The Three Clusters*

Table 4. *Effects (B's) Of The Three Cognitive Variables, Age And Gender On Clusters Membership*

Covariates		**FR**	**DIV**	**FDI**	**Age**	**Male**	**Female**
Cluster 1	b	0.0769	0.0539	0.1215	0.5077	0.1866	-0.1866
	s.e.	0.0106	0.0116	0.0234	0.0625	0.0963	0.0963
	z-value	7.229	4.634	5.1839	8.1274	1.9381	-1.9381
Cluster 2	b	-0.0084	-0.023	-0.0881	-0.275	-0.0264	0.0264
	s.e.	0.0111	0.0124	0.028	0.0793	0.1123	0.1123
	z-value	-0.7506	-1.8551	-3.1439	-3.467	-0.2347	0.2347
Cluster 3	b	-0.0686	-0.0309	-0.0333	-0.2327	-0.1602	0.1602
	s.e.	0.0165	0.0179	0.0353	0.0917	0.1583	0.1583
	z-value	-4.1465	-1.7301	-0.9449	-2.5393	-1.0123	1.0123
	Wald	55.8158	27.0939	34.6465	70.1724	4.0328	
	p-value	<0.0001	<0.0001	<0.0001	<0.0001	0.13	

Subsequently the effects of the three neo-*Piagetian* constructs on the portrayed representations of the atomic structure were investigated by applying LCA with covariates the three cognitive variables, FR, FDI and DIV. The results are depicted in Table 4. It is observed that Cluster 1, in which *Bohr's* model seems to dominate (with

Quantum model) associates positively with FR (*b*=0.0769, p<0.0001), DIV (*b*=0.0539, p<0.0001) and FDI (*b*=0.1215, p<0.0001). That is, the three neo- *Piagetian* constructs have significant positive effects on attaining *Bohr's* and *Quantum* model. Cluster 2, in which the *Nuclear* model dominates, does not associate significantly with FR, and DIV, while it associates negatively with FDI (*b*= - 0.0881, p<0.001). Cluster 3, in which the *Particle* model dominates, does not associate significantly with DIV and FDI, while it associates negatively with FR (*b*= - 0.0686, p<0.0001). In addition, age and gender were used as independent variables on cluster memberships. Age affect positively Cluster 1 (*b*=0.5077, p<0.0001) and negatively Cluster 2 (*b*= -0.275, p<0.0001) and Cluster 3 (*b*= - 0.2327, p<0.0001). No effect of gender was observed.

DISCUSSION AND CONCLUSIONS

Based on the resulted clusters' conditional probabilities, it is quite clear that students' portrayed representations of the atomic structure show a high degree of consistency with specific model. The majority of them (69.09%, Cluster 1) are moving mainly within the context of the Bohr's model, indicating the dominant role of this model in secondary education (e.g. Papaphotis & Tsaparlis, 2008, Tsaparlis & Papaphotis, 2009). However, when the task context is appropriate (task 3), a remarkable probability of the Quantum model appears. Please note that, this probability to get the Quantum model when the context is appropriate is also supported for the teaching context by a number of science researchers (e.g. Kalkanis et al., 2003; McKagan et al., 2008). As a result, any effect of an independent factor (i.e., any of the neo-Piagetian constructs in the present study) on the students of cluster 1, actually, reveals the effect of this factor on their representations that are dominated by the Bohr's model, with a probability for the Quantum model to be present as well. Respectively, cluster 2 is dominated by the Nuclear model with a rather small probability for the Quantum model (when the task context is appropriate, like task 3). For the students of this cluster (23.68% of the sample) the effect of an independent factor rather targets their representations within the Nuclear model. As far as the small group of students belonging to cluster 3 (7.24% of the sample), it is a case indicating (more intensive than the case of cluster 2) that there is a students' trend to approach atomic structure through simple and concrete models (e.g. Harrison & Treagust, 1996; Cokelez & Dumon, 2005). In this cluster, any effect of an independent factor mainly concerns the Particle model.

Taking into account the above, it seems that students with an increased FR have also increased probabilities to represent the atomic structure within the context of the Bohr's model, whereas an appropriate teaching context (compatible with the quantum theory) could lead them to adopt the corresponding Quantum model (since cluster 1 associates positively with FR). On the contrary, students with a relatively low FR have increased probabilities to represent the atomic structure within the context of the Particle model (since cluster 3 associates negatively with FR). In between, students' representations of the atomic structure within the context of the Nuclear model are not associated with their FR. Nevertheless, FR appears one more time to be a significant factor (e.g. Tsitsipis et al., 2010, 2012; Papageorgiou et al., 2016a,b). In the present study FR appears to significantly affect students' portrayed representations of the atomic structure, since it is connected to the degree of sophistication of the models adopted by the student.

In accordance with other relevant studies (e.g. Tsitsipis et al., 2010, 2012; Papageorgiou et al., 2016a,b), FDI appears to be also a significant factor. The fact, that it associates positively with Cluster 1 and negatively with Cluster 2, provides evidence that students with an increased FDI have an advantage in representing the atomic structure within the Bohr's model and especially within the Quantum one. Since these models have a significant degree of portrayed representations' complexity compared to the others, an increased ability of the students to separate the significant information from the 'noise' could help them in representing the appropriate characteristics. On the contrary, FDI does not associate with the Particle model (cluster 3), probably because this model does not appear any complexity and thus, a students with an increase FDI has not any particular advantage over others with a low FDI.

The finding that divergence (DIV) associates positively only with Cluster 1, without effects on the other two clusters, indicates its significance, as well. Students with an increased divergent thinking have the ability to find several equally acceptable solutions to a problem, something that is related to the creativity (e.g. Danili & Reid, 2006; Tsitsipis et al., 2012). Thus, they can better find ways to manage difficult concepts, like those presupposing the probabilistic view of the atom (e.g., orbital or electron cloud) or can go beyond the simple Nuclear model (e.g., orbit or energy level). On the contrary the lack of any effect of DIV on clusters 2 and 3, advocates the simplicity of Nuclear and Particle models, where such a students' ability does not serve any advantage.

Looking from the developmental point of view, and in line with relevant research (e.g. Papageorgiou et al., 2016a; Zarkadis et al., 2017), age seems to be a significant factor affecting positively the acquisition of Bohr's and especially Quantum models (cluster 1). This finding, together with the fact that age associates negatively with Cluster 2 and Cluster 3 (Nuclear and Particle models, respectively), advocates the aspect that students' representations of the atomic structure becomes more sophisticated along with the age. Although expected to a certain degree (e.g. Papageorgiou et al., 2016a; Allred & Bretz, 2019), this aspect contributes to the gradually increase of the ability of an individual to cope with concepts and situations of a corresponding increased difficulty and complexity, something also compatible with the Piagetian and neo-Piagetian principles (e.g. Tsitsipis et al., 2010; Stamovlasis & Papageorgiou, 2012; Tsitsipis et al., 2012; Papageorgiou et al., 2016a,b). This appears to be common for both genders since there is not any association of gender with any of the clusters.

IMPLICATIONS FOR SCIENCE EDUCATION

Since findings of the present study indicate the significant positive effect of FR and age on students' portrayed representations of the atomic structure, relevant implications for the teaching and learning procedure emerges. It is quite clear that the acquisition of the

Bohr's model presupposes a degree of FR and elder ages, both of which are rather connected to higher grades, whereas the Quantum model needs additionally an appropriate teaching context (as conditional probabilities for task 3 in cluster 1 show). On the contrary, in younger ages and lower FR, Particle and Nuclear models are dominant. Taking these into account, curricula designers have to gradually introduce the Bohr's model along with the grades and to create an appropriate context for the introduction of the quantum model. This appropriateness is also associated with the findings concerning DIV. The positive association of DIV with cluster 1, practically means that multiple representations of the atomic structure have, respectively, positive effects on the acquisition of the Bohr's and especially the Quantum model (e.g. Harrison & Treagust 2000; Rau 2015), since a high degree of DIV is associated with an increased ability to manage several equally acceptable solutions to a problem (e.g. Bahar, 1999). Furthermore, the positive association of FDI with cluster 1 gives an advantage to students, who have the ability to identify the significant information hiding within a more abstract and sophisticated context of representation of the atomic structure, i.e., that of Bohr's and especially Quantum models (Witkin et al., 1971). However, an appropriate teaching context for these models, which anticipate simple ways in representing the characteristics of the atomic structure, could give the opportunity to a wider range of students (even having a moderate or low FDI) to 'see' the important parts of the representations and understand the Bohr's and Quantum models to a better degree.

Conclusively, the findings concerning the effects of the above cognitive and developmental factors on students' representations and understanding of the atomic structure are interpreted in the present endeavor as crucial points, where teaching interventions should focus on, in order to overcome the relevant conceptual obstacles. Nevertheless, this challenge appeals to pedagogical content knowledge.

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