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ETHNOMATHEMATICS: *PRANATAMANGSA* SYSTEM AND THE BIRTH-DEATH CEREMONIAL IN YOGYAKARTA

Rully Charitas Indra Prahmana[1], Wahid Yunianto[2], Milton Rosa[3], Daniel Clark Orey[3]

[1]Universitas Ahmad Dahlan, Jl. Pramuka No. 42, Pandeyan, Umbulharjo, Yogyakarta, Indonesia
[2]SEAMEO QITEP in Mathematics, Jl. Kaliurang, Condongcatur, Sleman, Yogyakarta, Indonesia
[3]Universidade Federal de Ouro Preto, Ouro Preto, State of Minas Gerais, Brazil
Email: rully.indra@mpmat.uad.ac.id

Abstract

Mathematics exists as rooted in human ideas, ways, and techniques to respond to their environments. Along with its development, mathematics has been taught through formal education or schooling in the flat- not flexible ways and far from students' social and cultural lives. Indonesia is rich with culture, including the Special Region of Yogyakarta (DIY). It is called a city of culture. Educators are not yet aware of this richness in the learning process to integrate it as the starting point. It is hard to find in mathematics textbooks in Indonesia, which put cultural context as starting points. Therefore, this study aims to explore Yogyakarta's culture in terms of contexts used in mathematics learning. It is an ethnography study. Furthermore, the data was collected through literature artifacts, field observation, and interviews with resource persons who understand the seasons, system, and calculation of birth and death days. It is to clarify the researcher's understanding of the literature. This study showed that Yogyakarta's people use mathematical modeling to determine the seasons' system and funerary dates. These models have the potential to be used as a starting point in learning mathematics.

Keywords: mathematical modelling, ethnomodeling, Yogyakarta culture, seasons system, birth and death dates

Abstrak

Keberadaan matematika bersumber dari gagasan, cara, dan teknik manusia untuk merespons lingkungan mereka. Seiring dengan perkembangannya, matematika telah diajarkan melalui pendidikan formal atau sekolah dengan cara yang tidak fleksibel dan jauh dari kehidupan sosial dan budaya siswa. Indonesia kaya dengan budayanya, termasuk Daerah Istimewa Yogyakarta (DIY). Itu disebut kota budaya. Pendidik belum menyadari kekayaan ini dalam proses pembelajaran untuk mengintegrasikannya sebagai titik awal. Oleh karena itu, penelitian ini bertujuan untuk mengeksplorasi budaya Yogyakarta dalam konteks yang dapat digunakan dalam pembelajaran matematika. Penelitian ini merupakan studi etnografi dengan *ethnomodelling*. Selanjutnya, data dikumpulkan melalui studi literatur, observasi lapangan, dan wawancara dengan narasumber yang memahami musim, sistem, dan perhitungan hari kelahiran dan kematian. Ini untuk memperjelas pemahaman peneliti tentang literatur. Hasil penelitian ini menunjukkan bahwa masyarakat Yogyakarta menggunakan pemodelan matematika untuk menentukan sistem musim dan tanggal kelahiran dan kematian. Hal ini berpotensi untuk digunakan sebagai titik awal dalam belajar matematika.

Kata kunci: pemodelan matematika, *ethnomodeling*, budaya Yogyakarta, sistem musim, hari kelahiran dan kematian

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Mathematics stems from ways, styles, and techniques developed by humans to respond to environments such as seeking explanations, understanding, experiences, and solutions to surrounding phenomena (Freudhental, 2006; D'Ambrosio, 2007; Rosa & Orey, 2016; D'Ambrosio, 2016). Mathematics does not stand alone but is influenced by historical aspects, environment, social, and geography, or we refer it as a culture where humans evolve in life processes (D'Ambrosio, 2016; Risdiyanti & Prahmana, 2018; Utami, Sayuti, & Jailani, 2020). However, mathematics becomes formal when it comes to formal education or schooling in routine-not flexible ways and far from cultures in which it developed and is taught (Alangui, 2010; Muhtadi, Sukirwan, Warsito & Prahmana, 2017). This issue is related to western colonization, which tried to hegemon the knowledge in the world (D'Ambrosio, 2007; Joseph 2010; Rosa & Orey, 2016). So, mathematics learning becomes transferring knowledge, and students receive it without any reflective thinking and not knowing the use of mathematics in daily life.

Based on how mathematics has been taught at schools, reflecting on how mathematics developed, D'Ambrosio (1985) initiated Ethnomathematics as a solution. Ethnomathematics is a way to learn and combine ideas, ways, techniques that have been used and developed by socio-culture or members of different cultures (D'Ambrosio, 2016; Rosa & Orey, 2016). Ethnomathematics tries to reposition mathematics to be rooted in different cultures, accommodating different ideas so that students become critical reasoners, democratic, and tolerant (D'Ambrosio, 2016; Zevenbergen, 2001). Therefore, Ethnomathematics, as pedagogical innovation in mathematics teaching and learning aims to make students love mathematics, get motivated and improve creativity in doing mathematics.

There are several ways to integrate Ethnomathematics in teaching and learning (Rosa & Orey, 2017). One of the approaches is ethno-modeling which was firstly introduced by Bassanezi (2002). In learning mathematics, the use of Ethnomathematics and mathematical modeling towards ideas, ways, and techniques from what societies have developed is expected to be an alternative for introducing surrounding lives to students (Rosa & Orey, 2016). Rosa and Orey (2013) stated that ethnomodelling would allow us to see mathematics perform holistically. In this way, it is possible to critically explore local mathematics by appreciating the diverse cultural values of different societies (Abdullah, 2017). Therefore, mathematics learning can be initiated by critically exploring phenomena around students' neighborhood and model them mathematically, to develop awareness and critical reasoning, and get motivated.

Indonesia is a country with abundant cultures embedding mathematical ideas, ways, and techniques using mathematical modeling. This creates opportunities in mathematics education to use local contexts and to boost students' critical reasoning and interest by reinventing mathematics rooted in students’ culture existing in their surroundings to get the benefit from it. One of the mathematical phenomena in Yogyakarta is the seasons' system repeating every year or called *pranatamangsa*. Besides, Yogyakarta's people still organize birth and death Memoriam ceremonies and determine when it should be done by using a mathematical calculation. Exploring mathematical modeling in *pranatamangsa* and determining the dates for birth-death ceremonial can be contexts for learning mathematics.

Exploring mathematical modeling in Indonesia has been documented by some researchers. Abdullah (2017) studied mathematical modeling on the clock and measuring unit in Tasikmalaya, the Sunda district. Furthermore, there are several ethnics which implemented of Ethnomathematics in their life in Cipatujah, West Java, for example, they apply the traditional mathematical concept in the way they determine the time to sail for fishing, and the way they construct their houses (Kusuma, Dewanto, Ruchjana, & Abdullah, 2017). The motifs of batik are an Ethnomathematics context related to lines, angles, triangles, quadrilaterals, circles, and transformation geometry, which exist in several cultures in Indonesia (Risdiyanti & Prahmana, 2018; Lestari, Irawan, Rahayu, & Parwati, 2018; Pramudita & Rosnawati, 2019; Irawan, Lestari, Rahayu, & Wulan, 2019). However, few researchers have explored mathematical modeling in Yogyakarta's literature. Therefore, the researchers aimed to study Yogyakarta's culture in mathematical modeling, which has the potential to be used as a starting point to learn mathematics.

The next sections explain the methodology, how the data was collected. Furthermore, results and discussion of the seasons' system and a birth-death ceremonial will be described. The results of this study then were compared to previous related studies in different areas. In conclusion, this study revealed that Yogyakarta's culture includes mathematical ideas, ways, and techniques that can be explored through mathematical modeling.

METHOD

The method used in this research is an ethnographic method, which is a method that describes the culture of a community (Spardley & McCurdy, 1989). Ethnography was chosen as the method in this study because it is in line with the aims of Ethnomathematics which study ideas, methods, and techniques in a particular culture from the original view of members of that culture (Ascher & D'Ambrosio, 1994; Shirley & Palhares, 2016). Ethnographic methods involve learning about cultures that see, hear, speak, and act in different ways and in ways that they find themselves (Spradley & McCurdy, 1989). Data collection was carried out by field studies and interviews with Mr. Riyadi and Mr. Gasiman, farmers and fishermen on the coast of Bugel, Panjatan, Kulon Progo. From them, we learn, explore, and clarify comprehensively about one of the bases for catching fish at sea such as predicting catch fish, tools, weather conditions, and risks, as well as a basis to farm in the fields, predict the kind of plant to plant and predict the times to plant, harvest, and others.

In this study, three boundaries of the coverage area are used which are the basis for determining the research subject; that is community unity consisting of education that speaks one language or accent in the same language, community unity which is limited by the boundaries of an administrative political area and community unity has similar experiences. By using the same history, as the limits that have been set in ethnographic research to show the authenticity of culture under study, there is no mixture with other cultures (Koentjaraningrat, 2015). Therefore, it was determined that the community unit to be studied was the original Yogyakarta people who used the same accent (Javanese language), limited by the same administrative area, namely the Special Region of Yogyakarta and experienced the same historical experience, that is history when living, growing and developing in Yogyakarta.

Koentjaraningrat (2015) explains that in ethnographic research there were seven main descriptions produced by ethnographers, that is language, technology systems, economic systems, social organizations, knowledge systems, arts, and religion. In this study, the researcher will focus on one main description, that is the knowledge system because the researcher must observe and dive into the knowledge and technology systems to find the knowledge base used in the process of catching fish and farming activities. Even so, it does not rule out that other cultural elements will also be studied because they are related to one another.

In conducting ethnomathematical exploration, researchers begin with four general questions that are the essence of ethnographic principles, that is "where to start looking?", "how to look?", "how to recognize that you have found something significant?", "how to understand what it is?". The results of data collection were collected in the form of pictures, videos, and field notes, then were analyzed to see the relationship between the mathematical knowledge system and culture and to see the mathematical conceptions that exist in the catching fish and farming activity. Then the findings are described in the results of this study. Based on these four general questions, the research stages are organized in Table 1.

Table 1. Design of Ethnography Research

General Questions	Initial Answers	Starting Point	Specific Activity
Where to start looking?	In the activities of catching fish and farming carried out by the people of Yogyakarta where there are mathematical practices in it.	Culture	Conducting interviews with people who have knowledge of Javanese culture in the Yogyakarta community or those who catch fish and farming.
How to look?	Investigating aspects of catching fish and farming of the people of Yogyakarta related to mathematics practice.	Alternative thinking, technology and knowledge system	Determine what ideas are contained in catching fish and farming activities of the people of Yogyakarta related to mathematics practice.
What is it?	Evidence (Results of alternative thinking in the previous process)	Philosophy of mathematics	Identifying characteristics in the activity of catching fish and farming Yogyakarta society related to mathematics practice. It shows that the activity of catching fish and farming activities for the people of Yogyakarta does have a mathematical character seen from the elements of knowledge and art systems used in everyday life.
What does it mean?	Valued important for culture and important value patterns for mathematics	Anthropologist	Describes the relationship between the two systems of mathematical knowledge and culture.

	Describe mathematical conceptions that exist in the activity of catching fish and farming for the people of Yogyakarta.

This is an ethnography that studies the description of the life of people in a culture community (Spradley & McCurdy, 1989). This method fits the Ethnomathematics study as it is to understand perspectives, ideas, ways techniques of the members from members' views (Spradley & McCurdy, 1989; Ascher & D'Ambrosio, 1994; Koentjaraningrat, 2015; Shirley & Palhares, 2016). This is in line with the research aim; to explore mathematical ideas, ways, and techniques of Yogyakarta culture concerning mathematical modeling. Koentjaraningrat (2015) described seven cultural aspects that can be the focus on ethnography; languages, systems of technology, the system of economics, social organization, a system of knowledge, arts, and religion. This study only observes and describes the system of knowledge of Yogyakarta's culture.

In this study, data were collected through field observation, literature review, documentation, and interview with Mr. Gasiman and Mr. Slamet Riyadi. They are a farmer and fisherman respectively in Bugel, Panjatan, Kulon Progo district to discuss *pranatamangsa*. Another conversation was with Mrs. Suminah and Mrs. Pariyem in Karangsari village, Pengasih sub-district, Kulon Progo district, to review the determination of dates for birth-death ceremonial in Yogyakarta. These respondents were purposively selected to gain more information based on their experiences. The interviews were conducted with semi-structured interviews. The data from interviews were analyzed through content analysis to find the general ideas of *pranatamangsa* being used and helpful for them. The data from the documents were analyzed through content analysis and showed useful diagrams depicting seasons division.

RESULTS AND DISCUSSION

The results showed that Yogyakarta's culture has mathematical modeling called *pranatamangsa*. This is a unique seasons calculator. Using this calculator, fishermen can tell which fish to catch, and which tools to use. Farmers use this calculator to decide which crop to plant and when to harvest it. It is also found that mathematical modeling is used in determining dates for birth-death ceremonial.

Mathematical Modeling of Pranatamangsa in Yogyakarta

Pranatamangsa is a season system dividing periods in a year into smaller units aligned with cropping seasons. It divides 365 days into four seasons, aligned with seasons for farming. These four periods are also called *Mangsa*, such as *Mangsa Ketiga* for dry season, *Mangsa Labuh* for pre-rainy season, *Mangsa Rendheng* for rainy season, and *Mangsa Mareng* for the transition season (Kridalaksana, 2001; Gasiman, 2017).

In addition, *pranatamangsa* divides a year into 12 units of time, in accordance with the Solar calendar. Each unit has a different number of days. It indicates natural occurrences to determine the seasons. *Pranatamangsa* is still used and believed by Javanese to do farming and fishing as well as the tools they need. The seasons' divisions are as follows (Partosuwiryo, 2013; Riyadi, 2017; Gasiman, 2017):

1. *Mangsa Kasa* (First season)

   This is the first season, lasting for 22 days and starting from June 22nd to August 1st. In this season, natural occurrences on land are characterized by cool and fluctuating temperatures, fallen leaves and no rain, while natural occurrences in the sea are marked by the west currents and east winds. This condition is usually used by farmers to plant crops, while for fishermen, this condition is best to catch tuna, yellowfin, skipjack, stingray, and sailfish.
2. *Mangsa Karo* (Second season)

   It is the second season, lasting for 23 days, and starting from August 2nd to August 24th. In this season, natural occurrences on land are characterized by trees beginning to flower, cool temperatures, and dry air, while natural occurrences in the sea are characterized by cool sea surface, strong east winds and strong west currents. For farmers, it is to grow *palawija*. Farmers are usually clean weeds and wild plants that grow around the crops. As for fishermen, they catch yellowfin tuna and skipjack sharks.
3. *Mangsa Katelu* (Third season)

   It is the third season, lasting for 24 days, and starting from August 25th to September 17th. In this season, natural occurrences on land are characterized by strong east winds, fallen flowers, tubers begin to sprout and cool temperatures, while natural occurrences in the sea are characterized by cool sea surface, cloudy seawater, and jellyfish appearance. Farmers start harvesting their crops, while for fishermen, they catch yellowfin, tuna, and milk shark.
4. *Mangsa Kapat* (Fourth season)

   It is the fourth season, lasting for 27 days, starting from September 18th to October 12th. The natural occurrences on land are characterized by cold temperatures, fallen flowers and moderate winds, while natural occurrences at sea are characterized by changes in sea water currents and changes in wind direction, cloudy sea water, moderate west winds and calm waves. Farmers are still harvesting their crops, while for fishermen, they catch *layur* fish, mackerel, tuna, skipjack tuna, black pomfret, and white pomfret.
5. *Mangsa Kalima* (Fifth season)

   It is the fifth season, lasting for 27 days, starting from October 13th to November 8th. The natural occurrences on land are characterized by starting to rain, tree branches begin to sprout and the wind is blowing moderately, while the natural occurrences in the sea are characterized by warm sea surface, the emergence of tiny shrimp and cloudy seawater. Farmers begin to plant rice seeds in paddy fields, while for fishermen, they catch *layur* fish, mackerel, mackerel, tuna, white pomfret, black pomfret, anchovies, and lobster.

6. *Mangsa Kanem* (Sixth season)

   It is the sixth season, lasting for 43 days, starting from November 9th to December 21st. Natural occurrences on land in this season are characterized by moderate rainfall, trees that are beginning to bear fruit and moderate winds, while natural conditions at sea are characterized by warm sea surface, winds blowing westward, water currents moving east, and cloudy seawater. Farmers plant rice in the fields, while for fishermen, they catch black pomfret, white pomfret, mackerel, anchovies, and lobster.

7. *Mangsa Kapitu* (Seventh season)

   It is the seventh season, lasting for 43 days, starting from December 22nd to February 2nd. In this season, the natural occurrences land is marked by pouring rain and the river starts to flood, while the natural occurrences in the sea are characterized by turbid sea water, and wind blows to the west. Farmers plant rice in the fields while fishermen catch *layur* fish, snapper, small rays, *cucut*, *mayung*, and lobster.

8. *Mangsa Kawolu* (Eighth season)

   It is the eight-season, lasting for 27 days, starting from February 3rd to February 29th. In this season, the natural occurrences on land are characterized by strong west winds and heavy rain, while natural occurrences at sea are characterized by west winds blowing hard, eastern currents getting slower and the seawater becoming cloudy. Farmers are yet to harvest rice, while fishermen catch mackerel, pomfret, *layur*, *mayung*, stingray, *jerbung*, and prawns.

9. *Mangsa Kasanga* (Ninth season)

   It is the ninth season, lasting for 25 days, starting from March 1st to March 25th. In this season, the natural occurrences on land are marked by erratic wind direction, less rain though rivers are still flooded and flowers start to fall, while natural occurrences at sea are marked by the presence of seagulls, cloudy seawater, slow eastern current and small waves. Farmers can start preparing for rice harvest, but yet to harvest it. As for fishermen, they catch *mayung*, *layur*, pomfret, *jerbung*, and shrimp.

10. *Mangsa Kasepuluh* (Tenth season)

    It is the tenth season, lasting for 24 days, starting from March 26th to April 18th. In this season, natural occurrences on land are characterized by moderate winds and birds start to hatch, while natural occurrences at sea are marked by changes in water currents. This is the time farmers harvest rice, while fishermen catch pomfret, snapper, *mayung*, *jerbung*, and shrimp.

11. *Mangsa Dhesta* (Eleventh season)

    It is the eleventh season, lasting for 23 days, starting from April 19th to May 11th. In this season, the natural occurrences on land are marked by no rain falling and the flowers are falling, while the natural occurrences at sea are marked by the presence of seagulls, western currents and lit up seawater at night. Farmers are still harvesting their rice, while fishermen catch tuna, marlin, sailfish, and black pomfret.

12. *Mangsa Sadha* (Twelfth season)

    It is the twelfth season, lasting for 41 days, starting from May 12^{th} to June 21^{st}. In this season, the natural occurrences on land are marked by no rain and fallen leaves. Farmers are still harvesting their rice, while fishermen catch marlin, tuna, and sailfish.

For Javanese who work as farmers and fishermen, *pranatamangsa* plays essential roles in their lives. Not only as a way to understand the nature but also to determine tools for them to catch fish, to predict the possible bad weather, to predict sea current directions, and to discover time for seeding, growing, and harvesting. This *pranatamangsa* is related to mathematical modeling. For simplicity, the *pranatamangsa* is formulated in Table 2.

Table 2. Calculation in Determining *Mangsa* on *Pranatamangsa*

No	**Month**	**Calculation**				***Mangsa***
		Month	**Value**	**Calculation**		
1	January-June	January	1		1 + 6 = 7	*Mangsa Kapitu*
		February	2		2 + 6 = 8	*Mangsa Kawolu*
		March	3		3 + 6 = 9	*Mangsa Kasanga*
		April	4	+ 6	4 + 6 = 10	*Mangsa Kasepuluh*
		May	5		5 + 6 = 11	*Mangsa Dhestha*
		June	6		6 + 6 = 12	*Mangsa Sadha*
		July	7		7-6 = 1	*Mangsa Kasa*
2	July-December	August	8		8-6 = 2	*Mangsa Karo*
		September	9		9-6 = 3	*Mangsa Katelu*
		October	10	- 6	10-6 = 4	*Mangsa Kapat*
		November	11		11-6 = 5	*Mangsa Kalima*
		December	12		12-6 = 6	*Mangsa Kanem*

Table 2 determine the mathematical modeling on calculating *pranatamangsa* is as follows:

1. For the seasons from January to June, the formula used is as follows:

$$Mangsa = \text{Value of the month (January– June)} + 6$$

2. For the seasons from July to December, the formula used is as follows:

$$Mangsa = \text{Value of the month (July - December)} - 6$$

The detailed explanation of the *pranatamangsa* is depicted on in Figure 1. It depicts *pranatamangsa* as a primary reference for farmers and fishers, and includes the formulas to determine the seasons and its units. This can be a reference for students and teachers in Yogyakarta.

Figure 1. Wheel of *Pranatamangsa*

Mathematical Modelling for Determining the Day of Birth-Death Ceremonial

People of Yogyakarta still use this mathematical model to determine the days for birth-death ceremonies. In Javanese culture, especially in Yogyakarta, the death is mourned in day 3, 7, 40, 100, and 100 after one's death. It is to remember and to pray for deceased one (Suminah, 2017). Besides, Javanese also determine the proper days for organizing the funerary ceremony (Pariyem, 2017).

1. Mathematical modeling for determining one's *Neptu* day and value using day of birth

   In Javanese culture, people use a combination of national days (7 days) and Javanese (*Pasaran*) days (5 days) in their daily activities, such as Monday *Pahing*, Tuesday *Kliwon*, Wednesday *Pon*, Thursday *Wage*, Friday *Legi*, and others. In Javanese culture, each national and *pasaran* days have their own values, shown in Table 3.

Table 3. The Sequence of National and *Pasaran* Days and Its Value

National Days	Value
Monday	4
Tuesday	3
Wednesday	7
Thursday	8
Friday	6
Saturday	9
Sunday	5

Pasaran Day	Value
Legi	5
Pahing	9
Pon	7
Wage	4
Kliwon	8

In Javanese society, the term *neptu* or *weton* is also defined as the day of one's birth, which is described as the combination of national and *pasaran* days. The calculation of *neptu* value in Javanese society consists of the sum of the value of the national day and the *pasaran* value. For example, if someone was born on October 28, 1996, which is on Monday *Legi*, then the calculation of his *neptu* value is as follows:

Neptu value	= Weighted value of national day	+ Weighted value of *pasaran* day
	= Weighted value of Monday	+ Weighted value of *Legi*
	= 4	+ 5
	= 9	

This *neptu* calculation is usually used by Javanese people to predict human characteristics, a good day for making events, a good day for traveling, a good day for marriage, predicting compatibility of a prospective spouse, job suitability, and so on. The results of this prediction are written in a book called the *Primbon* Book, which is a book of ancestral heritage that contains predictions and teachings that come from the relationship between life and the universe (Utami, Sayuti, & Jailani, 2019). A number of these predictions are still used in Javanese society. Apart from reasoning from the human ratio of the relationship between life and the universe, some predictions in the *Primbon* book and/or warning predictions that have been used in people's daily lives can be explained rationally using mathematical modeling, such as prediction of birthdays, anniversaries day of death, and the prediction of the day of the particular year.

This study reveals that in the culture of Javanese, people determine and calculate precisely their *pasaran* day for a specific year. This is used to determine the birth date to make memoriam date for praying him/her on the localized wisdom birthday. This mathematical modeling involves modulo seven and modulo five. Given any year n, modulo seven is used to determine the national day, and modulo five determine its *pasaran* (Robiyanto & Puryandani, 2015). Modulo seven is used to determine days, as in the solar calendar; Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday. While, in Javanese, people call it *pancawara* (5 numbers), or *pasaran*. There are 5 days; *Pahing*, *Pon*, *Wage*, *Kliwon*, and *Legi*. Before the modern calendar, Javanese used *pasaran* to name days.

In determining the *neptu* in a specified year, people need to know one's day of birth. It is possible to meet a leap year of 366 days. Such a year is divisible by four, that is how this year is called leap year. Therefore, we need to calculate how many leap years from the year of birth to the current year. For every leap year, we add one to the addition of day. The detailed description is in Table 4.

Table 4. Calculating the Addition to Day

Days in a Year	Addition to Day
365	365 (mod 7) = 1 means +1

Based on the explanation from the mathematical modeling in Table 4, it concludes that:

Addition to day = Days in a year (mod 7)

Day in year-n = day of birth - [{(Year-n – Year today) x Addition to day} + Number of leap years] mod 7

or it can be written as:

$$q = t \pmod 7$$

$$HN = m - [\{(n-s) \times q\} + k] \bmod 7$$

Information:

HN = Day in the year-n

m = Day of birth

N = Year-n

s = Year today

q = Addition to day

k = Number of leap years

t = Days in a year (365 days)

Meanwhile, the *pasaran* day is described on Table 5.

Table 5. Calculating the Addition of Day

Days in a Year	**Addition to Day**
365	365 (mod 5) = 0 means +5

Based on the explanation from the mathematical modeling in Table 5, it concludes that:

Addition to day = Days in a year (mod 5)

Pasaran day in year-n = *Pasaran* day of birth- [{(Year-n – Year Today x Addition to day} + number of leap years] mod 5

or it can be written as:

$$p = t \text{ (mod 5)}$$
$$PN = u - [\{n\text{-}s) - p\} + (k + 1)] \text{ mod } 5$$

Information:

PN = *Pasaran* day in year-n
u = Today's *pasaran* day
n = Year-n
s = Year today
p = Addition to day
k = Number of leap years
t = Days in a year (365 days)

An example of determining the date of birth ceremonial and *pasaran* day in year-n is the following. A person was born on Thursday, 17 August 1945. His birth ceremonial in the year 2017 is on:

$$\begin{aligned} Q &= t \text{ (mod 7)} \\ &= 365 \text{ (mod 7)} \\ &= 1 \end{aligned}$$

$$\begin{aligned} HN &= m - [\{n\text{-}s) \times q\} + k] \text{ mod } 7 \\ &= \text{Thursday} - [\{(2017\text{-}1945) \times 1\} + 18] \text{ mod } 7 \\ &= \text{Thursday} - [\{72 \times 1\} + 18] \text{ mod } 7 \\ &= \text{Thursday} - [72 + 18] \text{ mod } 7 \\ &= \text{Thursday} - [90] \text{ mod } 7 \\ &= \text{Thursday} - 6 \text{ days} \\ &= \text{Friday} \end{aligned}$$

Meanwhile, his *pasaran* day is:

P = t (mod 5) = 365 (mod 5) = 0

PN = u - [{(n-s) x p} + k] mod 5
= Wage - [{(2017-1945) x 0} + 18] mod 5
= Wage - [{72 x 0} + 18] mod 5
= Wage - [18] mod 5
= Wage - 3 days
= Legi

Therefore, from the calculation above, it can be concluded that the *neptu* day date for a person who was born on 17 August 1945 would be on Friday *Legi*.

2. Mathematical modelling in determining the day of funerary ceremony using the death day of the deceased one

This study reveals that in the culture of Yogyakarta, families mourn the deceased 7, 40, 100, and 100 after the person died (Suminah, 2017; Pariyem, 2017). This is to memorize the family of one family members' death, so they can pray for the deceased one. Determining the day of funerary ceremony uses mathematical modeling involving modulo 7 for the 7-day-of-the-week and modulo 5 for the *pasaran* day of the deceased one. The details are as follows.

a. Mathematical modeling on determining the day of funerary ceremony

On calculating the day of the funerary ceremony, it involves mathematical modeling using Modulo 7. It is due to the number of days in a week being seven, and so the days in year-n are divided by seven and resulted in the remainder. If it is divisible by 7, and then it has the remainder seven. Then the remainder is subtracted by one, in which we don't take deceased day into account. It resulted in the number of days needed to be added to the day of death in the year-n. The detailed description is in Table 6.

Table 6. Model of Funerary Day Calculation

The Mourn Day	Calculation of Its Value	Value of the Day	Addition to Day
3	3 (mod 7) = 3	3	3 -1 = 2 means +2 days
7	7 (mod 7) = 0	7	7 -1 = 6 means +6 days
40	40 (mod 7) = 5	5	5 -1 = 4 means +4 days
100	100 (mod 7) = 2	2	2 -1 = 1 means +1 day
1000	1000 (mod 7) = 6	6	6 -1 = 5 means +5 days

Based on the explanation from the mathematical modeling in Table 6, it concludes that:

Addition to day = Mourn day (mod 7)-1

Funerary day ceremony = Day of death + Addition to day or it can be written as:

$$a = b \text{ (mod 7)-1}$$
$$H = c + a$$

Information:

H = Funerary day ceremony

a = Addition to day

b = Mourn day in n by n = 3, 7, 40, 100, and 1000

c = Day of death

b. Mathematical modeling in determining *pasaran* day of a death person

In predicting the *pasaran* day of the deceased one, we apply mathematical modeling integrating modulo 5. Therefore, for those days in a year n which is divisible by five, and it has remainder five (Utami, Sayuti, & Jailani, 2019). The remaining is then subtracted by one because we don't take the deceased day into account. Then, it results in the number of days to be added to the *pasaran* day of the deceased one. The detailed explanation is in Table 7.

Table 7. Model for Calculating the *Pasaran* Day of the Deceased One

The Mourn Day	The Value of the *Pasaran* Day	Value of the Day	Number of Addition to *Pasaran* Days
3	3 (mod 5) = 3	3	3 - 1 = 2 means, +2 *pasaran* day
7	7 (mod 5) = 7	7	7 - 1 = 6 means, +7 *pasaran* day
40	40 (mod 5) = 0	5	5 - 1 = 4 means, +4 *pasaran* day
100	100 (mod 5) = 0	5	5 - 1 = 4 means, +4 *pasaran* day
1000	1000 (mod 5) = 0	5	5 - 1 = 4 means, +4 *pasaran* day

Based on the explanation from the mathematical modeling in Table 7, it concludes that:

Addition to the *pasaran* day = The Mourn day (mod 5) -1

The ceremonial *pasaran* day = Deceased day + addition to *pasaran* day

or it can be written as:

$$D = b \text{ (mod 7)-1}$$
$$P = c + d$$

Information:

P = The *pasaran* day of the deceased one in year-n

d = Addition to *pasaran* days

b = The-n mourn day n = 3, 7, 40, 100, and 1000

c = The actual *pasaran* day of the deceased one

It is an example to calculate the mourning celebration and its *pasaran* day for a person who died on Friday *Legi*. It is detailed in Table 8.

Table 8. An Example of the Mourn Day of the Death Day

The-n Mourn Day	The Death Day	Addition to Day	The Mourn Day
3	Friday	+ 2 days	Sunday
7	Friday	+ 6 days	Thursday
40	Friday	+ 4 days	Tuesday
100	Friday	+ 1 days	Saturday
1000	Friday	+ 5 days	Wednesday

Meanwhile, the day for the ceremonial is detailed in Table 9.

Table 9. An Example of the Mourn Day of the *Pasaran* Day

The-n Mourn Day	The *Pasaran* day	Addition to Day	The *Pasaran* Day for the Funerary Ceremony
3	*Legi*	+ 2 days	*Pon*
7	*Legi*	+ 1 days	*Pahing*
40	*Legi*	+ 4 days	*Kliwon*
100	*Legi*	+ 4 days	*Kliwon*
1000	*Legi*	+ 4 days	*Kliwon*

Therefore, a person who died in Friday *Legi* will have funerary ceremony day in Table 10.

Table 10. Result for *Pasaran* Day of Funerary Ceremony

The-n Mourn Day	The Day of Funerary Ceremony	The *Pasaran* Day for the Funerary Ceremony
3	Friday	*Pon*
7	Thursday	*Pahing*
40	Tuesday	*Kliwon*
100	Saturday	*Kliwon*
1000	Wednesday	*Kliwon*

The results of the mathematical modeling exploration of Yogyakarta's culture *pranatamangsa* have added references and knowledge concerning the use of cultural contexts which are potential as starting points in learning mathematics. In Hawaii, there is a lunar calendar system namely Hawaiian moon calendar similar to the lunar calendar-*pranatamangsa* in Yogyakarta, Indonesia. It turns out to be used well for learning mathematics based on past, present, and future mathematics to prepare leaders who have wisdom about their ancestors' culture in using mathematical modeling (Kaomea, 2019). Furthermore, Utami, Sayuti, and Jailani (2019) investigated the Javanese *pasaran*-fortune day (primbon), Sugianto, Abdullah, and Widodo (2019) explored number patterns, 2D figures, and number operations in Reog Ponorogo culture, and Kemaro island legend can be used to teach statistics (Lestariningsih, Putri, & Darmawijoyo, 2012). In addition, several games have been explored as a starting point in learning number operations using *bermain satu rumah* traditional game (Nasrullah & Zulkardi, 2011), social arithmetic using *kubuk manuk* games (Risdiyanti, Prahmana, & Shahrill, 2019), and Gundu for learning linear measurement (Wijaya, Doorman, & Keijzer, 2011).

Integrating Ethnomathematics can be helpful to make mathematics relevant, meaningful to students, and foster their performances. If we look into the case of the low performance on critical thinking and reasoning from the Program for International Student Assessment (PISA), it might be affected by teachers who have not integrated students' social and cultural life in learning mathematics (Muhtadi, Sukirwan, Warsito, & Prahmana, 2017). Teachers deliver what is written on the textbooks without igniting interactive dialogue to enhance students' critical reasoning and communicate different ideas (Stacey, 2011; Arisetyawan, Suryadi, Herman, & Rahmat, 2014). This led students to memorize formulas without knowing its meaning and not being reflective of mathematics they learn at schools (Nurhasanah, Kusuma & Sabandar, 2017; Risdiyanti & Prahmana, 2020). Therefore, to solve this problem, we need to relate mathematics and social-culture contexts close to students by exploring cultural backgrounds to be used in mathematics learning.

Ethnomathematics studies have helped teachers and students to understand mathematics in contexts of ideas, ways, techniques used in real life to enhance students' interest, understanding, and creativity (D'Ambrosio, 1999; Freudenthal, 2006; D'Ambrosio, 2007). This study is in line with the aim of mathematics education in Indonesia, such as to make students understand mathematical concepts and their relationship. The reason is to make mathematics generalization-proof-ideas, explain mathematical ideas, and solve real-life problems mathematically. Mathematics exists because of the need for humans to respond to the problems and or environment and solve problems, as it is crucial to infuse social values through ethnomathematics so that students can reflect on it for their lives (D'Ambrosio & D'Ambrosio, 2013). Lastly, mathematics education in Indonesia needs to contextualize mathematics in a social environment and culture.

CONCLUSION

Yogyakarta's culture includes mathematical modeling to determine seasons in the *pranatamangsa* system and the birth-death ceremonial. In determining the seasons, people of Yogyakarta use a one-to-ten season system, which is essential for them to, especially for farmers and fishermen. They predict which fish they will catch and tools to be used by studying natural phenomena.

Meanwhile, the farmers use *pranatamangsa* to determine the crops they will plant, seeding time, and crop time.

Besides, in Yogyakarta's culture, people mourn the death on days 3, day 7, day 40, day 100, and day 1000 to pray for the deceased one. They also celebrate their birthday by using *neptu* days. The determination of the date of birth-death ceremonial uses mathematical modeling integrating modulo 5 and modulo 7. Lastly, these rituals still exist in people of Yogyakarta who live in villages. This study has shown a rich culture containing mathematical modelling which are potential to be used in learning mathematics topics such as patterns, modulo, and number sense. It is expanded to be a reference for educators in Yogyakarta to improve students' understanding and relation of mathematics and their culture and lives.

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