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Incorporating the Dragging Feature of Dynamic Geometry Environments in Teaching and Learning College Geometry

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Background

Proof and theorems form part of the core content of secondary geometry curriculum and should be well grasped by secondary math teachers and their students (NCTM, 2000, 2003, 2012). Studies show that both secondary teachers and students have encountered challenges in teaching and learning proofs (Cirillo, 2009; Knuth, 2002; McCrone & Martin, 2004; National Center for Education Statistics (NCES), 1998; Senk, 1985). In order to develop knowledge about pre- service secondary mathematics teachers' (PSMTs) conceptions of theorems and provide mathematics educators and researchers with a possible means to unpack their conceptions, the researcher investigated the essential elements of four PSMTs' conceptions of the nature of theorems (NoT) through research-informed task-based interviews in 2016-2017. Findings of the study provided interpretations of PSMTs' conceptions of the NoT, in terms of the ways they claimed the truth of mathematical statements, examined validity of the given proof, disproved the given statement, as well as the role of the task-based interviews in understanding their conceptions (An, 2017).

During the above study, the researcher noticed that it was challenging for the PSMTs to draw and visualize counterexamples using paper and pencil to disprove the given geometrical statements. Both the PSMTs and the researcher felt the need to modify some of the geometry tasks that the researcher developed for the study by incorporating the dragging feature of dynamic geometry environments (DGEs), such as The Geometer's Sketchpad and GeoGebra. Especially, under the background of the rapid development of mobile devices and touch technology, the use of the dragging feature of DGEs in teaching and learning proof and reasoning has been discussed more and more widely and has become an ongoing research trend in mathematics education (Mariotti, 2014; Sinclair et al., 2016). Furthermore, introducing the dragging feature of some easily accessible DGEs to my secondary geometry content classes can provide PSMTs with a handy tool to explore the meaning and applications of counterexamples in writing proofs. In this paper, the researcher and her colleague propose a research design intending to answer the research question: *What are PSMTs' conceptions of counterexamples in geometrical reasoning when using the dragging feature of DGEs?*

Literature Review

Challenges in Learning and Teaching Proof and Theorems

The relationship between proof and theorems can be viewed as the relationship between a process and products in the world of mathematics (Farrell & Farmer, 1980). Being able to understand and apply theorems is considered as a relatively high level of proof and reasoning ability. For example, understanding of the axiomatic system of Euclidean geometry is ranked as higher level geometric thinking by the van Hiele levels (van Hiele, 1959). Despite the important role of proof and theorems in school mathematics, many secondary students have difficulty in writing valid geometry proofs (McCrone & Martin, 2004; NCES, 1998; Senk, 1985). Their difficulties may relate to incomplete conceptions or confusion about proof and theorems, such as accepting empirical evidence as formal proofs, questioning the generalizability of deductive reasoning, not accepting counterexamples as refutation, and overemphasizing the forms without logical coherence in proofs (Chazan, 1993; McCrone & Martin, 2004; Schoenfeld, 1994; Weber, 2001). Studies also show that teachers' conceptions,

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knowledge, and prior learning experiences of proof and theorems can have significant impact on their teaching of proof and theorems and thus affect their students' understanding and achievement in learning proof and theorems (Cirillo, 2009; Knuth, 2002; Lacourly & Varas, 2009; Oehrtman & Lawson, 2008; Rozner, Noblet, & Soto-Johnson, 2010). The extensive focus on what teachers do not know for teaching proof and theorems has driven the researchers' interest in examining what they do learn about proof and theorems in their undergraduate programs.

Conceptual Framework for Task Design

As mentioned earlier in this paper, the proposed study is an extension of an earlier study in which the researcher focused on unpacking PSMTs' conceptions of geometric theorems. The researcher created a set of principles of theorems which served as the conceptual framework for the development of the data collection instrument: task-based interviews. The principles were developed by incorporating Cirillo's (2014) conceptual model of mathematical proof tools, Dreyfus and Hadas' (1987) six logical principles of geometry theorems and proofs with additional revisions by McCrone and Martin (2004), and Duval's (2007) indicators of misunderstandings in proof writing. The principles of theorems included three aspects: (a) nature of theorems (NoT), (b) logic of theorems (LoT), and (c) application of theorems (AoT). The current study only focuses on the two tasks built on two subcategories (NoT 1 and NoT 3) of the principles of the NoT (Table 1), as these tasks reflected PSMTs' need of utilizing draggable figures to explore possibilities of counterexamples.

Table 1. Principles of the NoT

Nature of Theorems (NoT)	1. A mathematical statement is not a theorem until it has been proved (using axioms, definitions, postulates, previously proved theorems, lemmas, and propositions) (NoT 1) 2. A theorem has no exceptions a. A general statement is true implies that the statement is true for all specific instances. Therefore, a proof must be general to be valid for proving a statement (i.e., non-exhaustive proofs, empirical evidence, and checking a few specific cases are invalid proofs) (NoT 2) b. A general statement is not true in one specific instance, proves that the statement is false (i.e., one counter example is sufficient to disprove a statement) (NoT 3)

Use of Dragging Feature of DGEs in Geometry Education

Dynamic geometry environments (DGEs) refer to geometry software that supports the "continuous real-time transformation often called 'dragging'" (Goldenberg & Cuoco, 1998, p. 351). The dragging feature allows the user to change certain elements (e.g., a point) in a constructed geometric figure and observe the change of the corresponding geometric relationships in the figure. The constructed figures are referred as "draggable" or "moving" figures, which can provide the user with opportunities to experience "motion dependency" and further explore "logical consequence between properties within the geometrical context" (Mariotti, 2014, p. 159).

Research has shown that using the dragging feature of DGEs can promote student understanding and reasoning ability in learning geometry. By letting junior high students work on a series of DGE tasks designed for constructing draggable quadrilaterals, Jones (2000) found that students were able to make a transition from DGE-based arguments to formal mathematical arguments, which indicated their development of "formal-geometric conceptualizations" (p. 877). By studying the use of a particular dragging modality in dynamic geometry with pairs of high school students, maintaining dragging (MD) – "dragging a base- point of the dynamic figure on the screen trying to maintain some geometrical property of the figure" (p. 110), Baccaglini-Frank's (2011) study implied the potential of MD to foster "a greater cognitive unity" (p. 117) – generating of conjectures that can lead to proofs, when internalized as a psychological tool by the learners.

As Battista (2007) pointed out as one of the future research issues, both qualitative (to understand the nature) and quantitative research (to determine the effectiveness) is much needed on the use of DGEs in teaching and learning geometry, and it serves the best to integrate the two types of studies. Therefore, the proposed study adopts a qualitative case study design to develop knowledge about PSMTs' conceptions of counterexamples when using the dragging feature of DGEs. The result of study will be compared with the result of the previous study in which non-DGE tasks were used. If the comparison indicates the dragging feature of DGEs as an effective instructional tool, a quantitative study will be conducted to examine its effectiveness.

Methodology

Overall Design

Since the purpose of the study is to provide descriptive accounts targeted at understanding PSMTs' conceptions of geometric counterexamples, the nature of the study is determined as a basic interpretive case study. Basic interpretive studies aim at "understanding a phenomenon, a process, or a particular point of view from the perspective of those involved" (Ary, Jacobs, & Sorensen, 2010, p. 453). Case studies are appropriate when the objective is an "in-depth data collection involving multiple sources of information rich in context" (Creswell, 1998, p. 61). In this study, a case is defined as each individual PSMT's work on the two tasks designed for the task-based interview session. Task-based interviews are used as the data collection method, because they provide a structured and somewhat controlled mathematical environment for the researcher to focus directly on the subjects' processes of addressing the tasks, rather than just on the correctness of the answers (Goldin, 2000).

Site and Participants

Four to six participants will be recruited from the department of mathematical science at a research-oriented public university in the South. The recruitment criteria include: (a) candidates committed themselves to secondary mathematics teaching in the future, namely, they are PSMTs; and (b) candidates have taken a university-level geometry course to ensure sufficient prior knowledge of geometric theorems and proofs. A recruitment letter will be sent to the students through department email lists, introducing the goal of the study, recruitment criteria, and involvement in the study. Students' participation is completely voluntary and irrelevant to their course grades.

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Task-based Interviews

Each PSMT will take part in an individual task-based interview session, lasting approximately 60 minutes. The interview session focuses on unpacking PSMTs' conceptions of the NoT, including the subcategories *theorem has to be proved* (NoT 1) and *one counterexample is sufficient to disprove* (NoT 3) (see Table 1). The researchers will demonstrate how to use the dragging feature of DGEs and let the participant practice on a few geometric constructions in the first 20 minutes of each session. During the interview sessions, PSMTs will be asked to think aloud while working on the tasks. The researcher will ask probing questions following a pre-designed interview protocol in order to understand their thinking process through the tasks. Because the goal of the study is not to assess participants' memorization skills, a list of Euclidean geometry definitions, postulates, and theorems will be provided to the PSMTs. Interviews will be video and audio recorded. The researchers will collect interview notes and PSMTs' worksheets as supplementary materials. A pilot interview will be conducted before the official data collection to test out the tasks and the interview protocol.

Goldin (2000) summarized the exploration process of task-based interviews as a four-stage process: (1) posing the question (free problem solving), (2) minimal heuristic suggestions (if no spontaneous responses), (3) the guided use of heuristic suggestions (if again no expected spontaneous responses), and (4) exploratory, metacognitive questions. This sequence of interview questions for each task is consistent with the "hard to easy" order suggested by Tzur (2007) in terms of minimizing the influence of prompts on students' conception development during the task exploration process. Since the main goal of this study is to unpack PSMTs' current conceptions rather than foster new conceptions, minimal heuristic suggestions will be provided when a PSMT is not able to provide any responses to the task questions.

Tasks

The two tasks used in the interview session are designed based on the Principles of the Nature of Theorems in Geometry framework (see Table 1), and targeted PSMTs' conceptions of NoT 1, and NoT 3, respectively. Task 1 (NoT 1) shown in Figure 1 is designed to unpack the ways in which PSMTs can show that a mathematical statement is true or not true, which indicates their conceptions that theorems are true mathematical statements proved through deductive reasoning using axioms, definitions, postulates, etc.

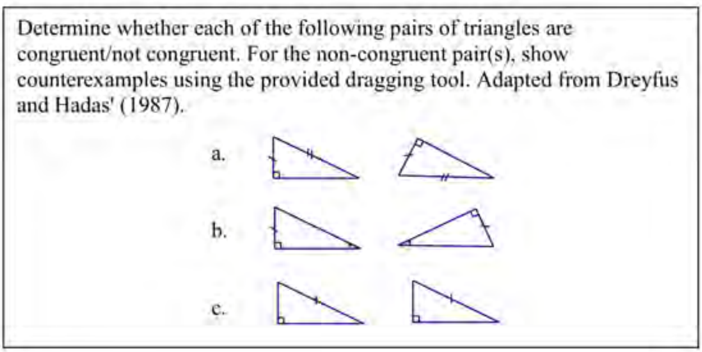

Figure 1. Task 1 (NoT 1) – Theorem must be proved

Task 2 (NoT 3) shown in Figure 2 task is designed to see in what ways the PSMTs can disprove a statement, which indicates their conceptions about the meaning and use of counterexamples.

> Disprove the following statement using the provided dragging tool.
>
> In ΔABC, if D and E are points on AC and BC respectively, BC=2BE and DE= 1/2 AB, then AC=2AD, DE∥AB.

Figure 2. Task 2 (NoT 3) - One counterexample is sufficient to disprove

The dragging tool (GeoGebra) will be provided to the participant using an iPad during the interview. Figure 3 below shows a possible counterexample of Task 2 that can be created with the dragging feature of GeoGebra. As mentioned earlier, each interview will start with a 20-minute tutorial session about the use of the tool.

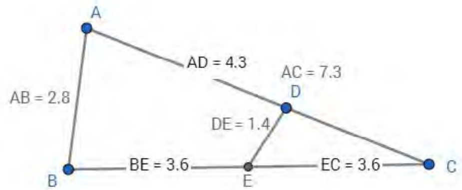

Figure 3. A counterexample of Task 2 created by GeoGebra

Data Analysis

Since the data are collected through task-based interviews with a semi-structured interview protocol, the data analysis process will follow steps suggested by the typological analysis method that "data analysis starts by dividing the overall data set into categories or groups based on predetermined typologies" (Hatch, 2002, p. 152). The typologies of the study are PSMTs' *goals* of the given task, their available *goal-directed activities* (*GA*), and what they believe are the *effects* of their goal-directed activities, which are the essential components of their conceptions (Simon et al., 2004). PSMTs' goal of a task refers to their desired status of problem solving, namely, what they want to or intend to do. The goal structures what they can notice, compare, and abstract, but it is not necessarily conscious (Simon et al., 2004; Tzur & Simon, 2004). PSMTs' GA of a task refers to their mental activity that is related to their goal of the task. GAs include the observable actions and the corresponding mental process that generated these actions (Simon et al., 2004; Tzur & Simon, 2004). An effect refers to what the PSMT identifies as the outcome of his/her GA (Simon et al., 2004; Tzur, 2007; Tzur & Simon, 2004), which, in this study, includes his/her conception of whether the goal of the GA is met (or about to be met) and the decision about the next step of actions. PSMTs' goals, GAs, and effects of GA are structured and governed by their current conceptions. In other words, their conceptions are embodied by their goals, GAs, and effects of GAs. Taking a close look at these elements can help the researchers gain a deeper understanding of PSMTs' current conceptions of theorems.

Timeline and Potential Impact

In terms of the timeline of the research, a pilot study will be conducted in Spring 2018 to collect feedback on the tasks and interview protocol. The institutional review board (IRB) application will be submitted by the end of Spring 2018. The participant

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recruitment and official data collection will be completed in Fall 2018. Data analysis and dissemination will start in late Fall 2018 and will continue until publications are generated.

This research project could potentially benefit all PSMTs at the university in which the researchers are teaching. Once the research results indicate the effectiveness of incorporating the dragging feature of DGEs in geometry learning, the researchers plan to introduce this tool to students in her college geometry classes. If the students can gain the knowledge of how to use the dragging feature to better understand the concept of counterexample, they will have a tool for more effective learning of geometric proof and reasoning. In turn, their new knowledge will be able to be applied in their future teaching roles and increase the quality of secondary geometry education in Georgia.

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