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Understanding functional dependency on Dynamic Geometry Systems (DGS): Napoleon and von Aubel Theorems on Geogebra

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SUMMARY

Functional dependency is a term coming out of mathematical software terminology. It shows the dependency of one representation (either diagrammatic or algebraic) to one another in the form of a formula, a conceptual relation or an algorithm. In mathematics, it is important since it shows students' understanding of those conceptual relationships if used properly. In this study, we have used representation of two mathematical theorems on DGS specialty of Geogebra: Napoleon and von Aubel Theorems. Both theorems are related with the geometrical relations of simple shapes as in the forms of squares and triangles. A class of 33 students divided into two lab groups were given randomly these two theorems without the knowledge of the names of the theorems to the students of Material Development course students. In each lab hour, students' assignment to the theorems were related to their seat numbers in order hence no one could see the same theorem person seating next to a student. The main study was a teaching experiment. Results are analyzed according to correct reasoning, inconsistencies, lacking diagrams etc. Data demonstrate that these kind of theorem use in mathematics classroom may be possible by Geogebra use and may empower students to think mathematically. Functional dependency is not connected to the task, hence can be given in all types of tasks by Geogebra. It was interesting to see some students' conjectures on the results of these two theorems.

Keywords: Dynamic geometry, understanding math, functional dependency, Geogebra

INTRODUCTION

Functional dependency is a term dedicated to the relation of one activity with other in computer software use. In other words, change in one representation results in a change of a related representation. One parameter can be changed in relation with another parameter. Both of these changes can be seen simultaneously. In some way, the representation acts as a student mind, hence it can be a real showcase for the conceptual relations. In this study, it is studied whether functional dependency is connected to the task or not. This study aimed to search for the role of functional dependency on dynamic geometry (DGS) with two specific tasks. Geogebra is an open-coded software incorporating DGS, CAS, and spreadsheet facilities. Here, the study only focused on the DGS facility of the software. Geogebra enables dynamism in diagrammatic reasoning. The structure of the diagrams is specified by theorems.

Mariotti & Falcade (2003) point to the lack of experience of functional dependence in a qualitative way. Hence, somehow not only movement and change but also knowledge of why those changes happens is a requirement for students. They name this as experience of variation in the form of motion. And this in turn results in functional dependency. In a DGS, dynamism is constructed through as sequence of operations linked by functional dependencies. Important research question is how it affect students' understanding and meaning making. In their paper, they name functionalities as: drag mode (a dependent point cannot be dragged by itself), delete function (deletion of an object leads to all related objects' deletion), redefinition of an object (changing its deleted dependency relayions with other objects-they name CABRI for this feature). In their study, students state that: "They all are shapes that you can move them without deforming", "initial object is the one that others depend on", "just because it depends on it, it moves". Hence, they are invariance, reference point/object, and movement.

Carreira, Clark-Wilson & Faggiano (2017) point to drag and loci as functional dependencies or name it as what remains constant and what changes? They stress that dragging takes a new form than before. İt is like leaving a trace and generating a new object- a locus. And this could lead to new mathematical investigations.

Hollebrands, Laborde & Straber (2008) stress the difference of three kinds of points in a DGS: a free point, a point on an object and a constructed point (a point which cannot move by its own but by another points' movement). When an element of a diagram is moved or dragged, modifications is according to the geometrical constructions not with respect to the wished of the user. In other words, practically rule bound movements are enabled.

Von Aubel and Napeleon Theorems worked as tasks. Hence, functional dependency could be seen in the diagrams of both of these theorems on Geogebra DGS (dynamic geometry system). Napoleon Theorem states that if equilateral triangles are constructed on the sides of any triangle; either all outward or all inward, lines connecting the centers of those equilateral triangles themselves form an equilateral triangle (Figure 1). Von Aubel Theorem states that *given* an *arbitrary quadrilateral*, if we place a square outwardly on each side, and connect the centers of opposite Squares, then the two lines are of equal length and cross at a right angle (Figure 2).

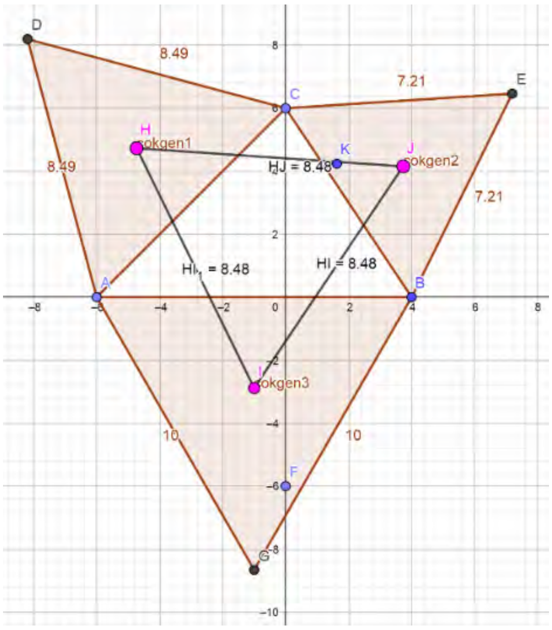

Figure 1 Napoleon Theorem

Good thing is both of these theorems are enabled by geogebra. Students can demonstrate what is stated by the theorem and can name out deductions possible just by observing them.

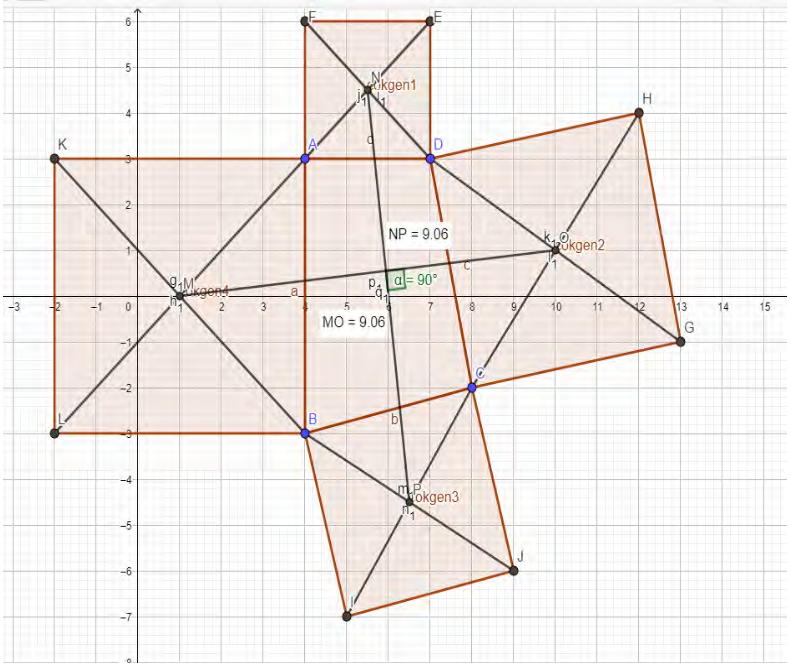

Figure 2 Von Aubel Theorem

Functional dependency on these two tasks

Some points are fixed, while some others are changed with respect to functional dependency it carries. Napoleon and von Abel theorems are perfect for studying functional dependency since one should be able to change the starting point: corner points of an arbitrary triangle in Napoleon Theorem and corner points of an arbitrary quadrilateral in von Aubel theorem (Mariotti & Falcade, 2003; Holdbrands, Laborde & Straber, 2008). In other words, initial points were free points where all others depended upon. Constructed points were centers of triangles and centers of squares. Somehow, both these theorems are parallel as tasks but also, since they are related with different concepts, they are structurally different as well.

At first, student needs to draw a scalene triangle and quadrilateral as given in the theorem statements. But then, one can show special examples agreeing with the theorems as with equilateral triangle in Napoleon and square in von Aubel Theorem (Yağcı, 2004). Geogebra enables changing the corner points of the starting triangle and consequently starting quadrilateral and special points are arranged according to these changes. If starting point is right, following functional dependencies carry relations as stated in the theorems. In the Napoleon Theorem, one understands the center of an equilateral triangle and in the Von Aubel Theorem, diagonals of square and the idea of perpendicular lines. By drawing of these theorems by Geogebra, students also understand the affordances of the software by reinforcement. Points are in conjunction with each other within the task. When one point is moved, related points also replaced properly. This is a results of functional dependence.

Functional dependency is not only a software term, here one can teach students related concepts and conceptual relationships by functional dependency. Hence, by solving the dependency of points in Geogebra, they somehow solved the theorems and their conceptual arbitrariness as well. Functional dependence shows the student related concepts and points in a problem, hence, related to the understanding of the structure. Both of these theorems ask for very specific structure. This study aimed to search for the role of functional dependency on dynamic geometry (DGS) with two specific tasks.

METHOD

Students seated in the lab as no two students next to each other would take the same task. These two theorems were somehow parallel regarding what they ask for and what they give, hence two groups started equal in the beginning. After the exam, they have print screened the output and sent author their file via e-mail. They did have experience with this sort of theorems on Geogebra. No diagrams but verbal explanations of the theorems were given without theorem names. By using Geogebra, they drew the dynamic diagrams and showed three examples illustrating the same functional dependency. It was a teaching assessment experiment of Geogebra use.

33 preservice mathematics teachers of the Material Development and Technology course were taught Geogebra in a computer lab for one semester. Then they were required to draw these two theorems (von Aubel & Napoleon) in the exam without any knowledge of the names of the theorems or even if they were any theorems or not. It was their first acquaintanceship with these theorems since they are not part of the general curriculum not only of high school but even of university. These two theorems are not in the curriculum; hence they would not have any pre knowledge of these before the exam. Besides, they were not given the names of the theorems.

The data collection instrument was acted as a midterm exam. Verbal representations of the two theorems were given without the theorem names or any affiliations. Students were assigned first and second theorems in order of their sitting in the lab. Then there was a table to gather data on the three sets of examples, and their conclusions of what changed from starting point to the end.

ÖTBT Vize Sınavı –Geogebra

1) Bir ABC üçgeninin kenarları üzerine oluşturulan eşkenar üçgenlerin ağırlık merkezleri birleştirildiğinde ortaya bir eşkenar üçgen çıkar. Gösteriniz.
2) Bir ABCD dörtgeninin kenarları üzerine oluşturulan karelerin ağırlık merkezlerini karşılıklı olarak birleştiren doğru parçaları hem eşit uzunluktadır, hem de birbirini dik keser. Gösteriniz.

Üstteki soruda özel üç durumu inceleyerek verileri tabloya kaydediniz.

	Başlangıçtaki şekil	Sonuç şekil
1		
2		
3		
Ne öğrendiniz?		

Figure 3. Data collection instrument

Teaching experiment

We could think the case as a random assignment to two groups. Two groups could be thought as experiment and control groups respectively. Though, each was experimental in itself. A teaching experiment consists sequence of teaching episodes (Steffe, 1984). A teaching episode includes a teaching agent, one or more students, a witness of the teaching episodes, and a method of recording what transpires during the episode. These records can be used in preparing subsequent episodes as well as in conducting a retrospective conceptual analysis of the teaching experiment. These elements are very important to all teaching experiments (Steffe, 1984).

Here, teaching episode was only one episode, the theorem. Teaching agent was author; as teacher of the material development course and the witness was the author as well and screen outputs of the student work were the recording method. Subsequent episodes may be developed, but here since it was a part of the exam, it turned out to be the last activity of the course. Conceptual analysis is done afterwards as part of this study.

Reliability and validity

Reliability is consistency of the data collection tools and validity is measuring what it supposed to measure. To achieve these, at first students needed to draw the diagram by their sole understanding of the verbal representation of the theorem. It somehow asked for understanding of representational registers and treatments and conversions between them. Student answers to the pre-constructed form were analyzed for existence of examples, correct dynamic diagram of the theorem in the paper form and in the Geogebra file, and the consistency in the representations involved in the answer. Form structure would help in foundation of reliability of the instrument. No explanations were given to the students. For validity, exact verbal explanation of the theory was stated. Since they were not given the theorem names, no memorization fell into the place. Even though, students have seen these kind of mathematics theorems in the course template, they were not acquainted with these theorems specifically.

FINDINGS

5 students came out with their conjectures. For example, one student came out stating that, the side length of the last equilateral triangle is equal to the arithmetic mean of the sides of the first original triangle. This was looking right for the example they chose for the Napoleon Theorem. And for the von Aubel Theorem, one student stated that at least for the special cases, total length of those two perpendicular lines equals to the perimeter of the quadrilateral at the beginning. From 33 students, only 5 of them (%15) came out with the deductions but as can be seen, those deductions looked theoretical as much (Table 1). Results are in parallel with literature stating the value of content knowledge supporting instrumental genesis. In other words, the degree of the content knowledge needed can be in conjunction with the specifics of the instrument in our case Geogebra.

Table1. Descriptive data from the experiment

Problem	Napoleon Theorem (16)	Von Aubel Theorem(17)
Mean grade	87	86,65
# Lacking examples	10	8
# inconsistency	4	2
#Lacking Drawing in the paper	0	1
#Lacking Drawing by geogebra	0	0
#Lacking drawings	2	14
Top points	3 100 points	4 100 points
Least points	65 points	70 points

As expected, ones with a correct starting point, ended with the correct functional dependency. In other words, if triangle was scalene, and if trapezoid was really a four unequal sided figure, then correct deductions could be presumed. Otherwise, they thought that only specific triangles and quadrilaterals would end in these two theorems. Hence, in theorem understanding, analyzing what is given, or in some way the structure of the theorem should be drawn just in parallel to the original figure, not less not more. As can be seen from Table 1, inconsistency was seen in less than % 25. This was due to the problems with part whole relationships. Students should understand the parts of the structure of the theorem, to be able to draw the figure flawless. Since, they are theorems no numbers are given in the beginning. Students decided upon the numbers in examples. But they were also supposed to appreciate the complete structure of the theorem and relationships of all parts related. Even though there were some differences in numbers between those theorems, more or less results could be counted as same. Results showed similarities with respect to two theorems. Tasks (theorems) did not prohibit the understanding of relations hence we can conclude that functional dependency is not related with task used.

Some did come out with interesting deductions not given in the theorems. Interestingly, some of these deductions were correct. Can we say that this kind of activity may make students feel like a mathematician? Even though they did not know it was a theorem? Can learning by discovery should be replaced by learning by drill? We hope not.

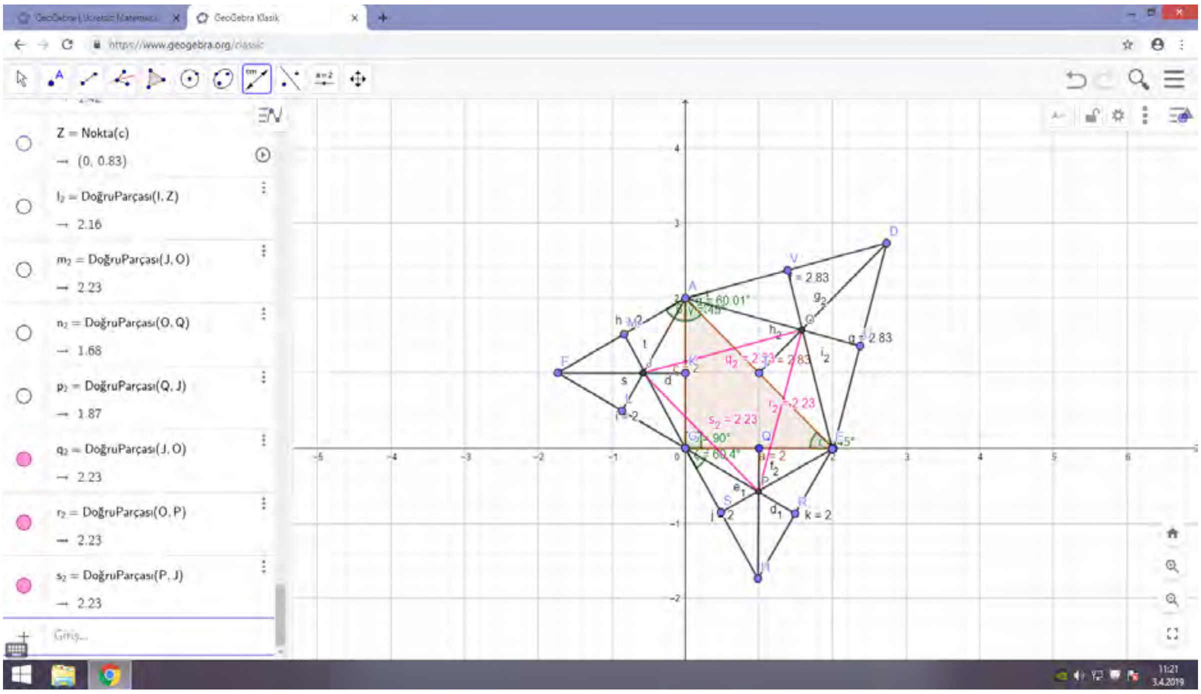

Figure 4 Student answer to Napeleon Theorem

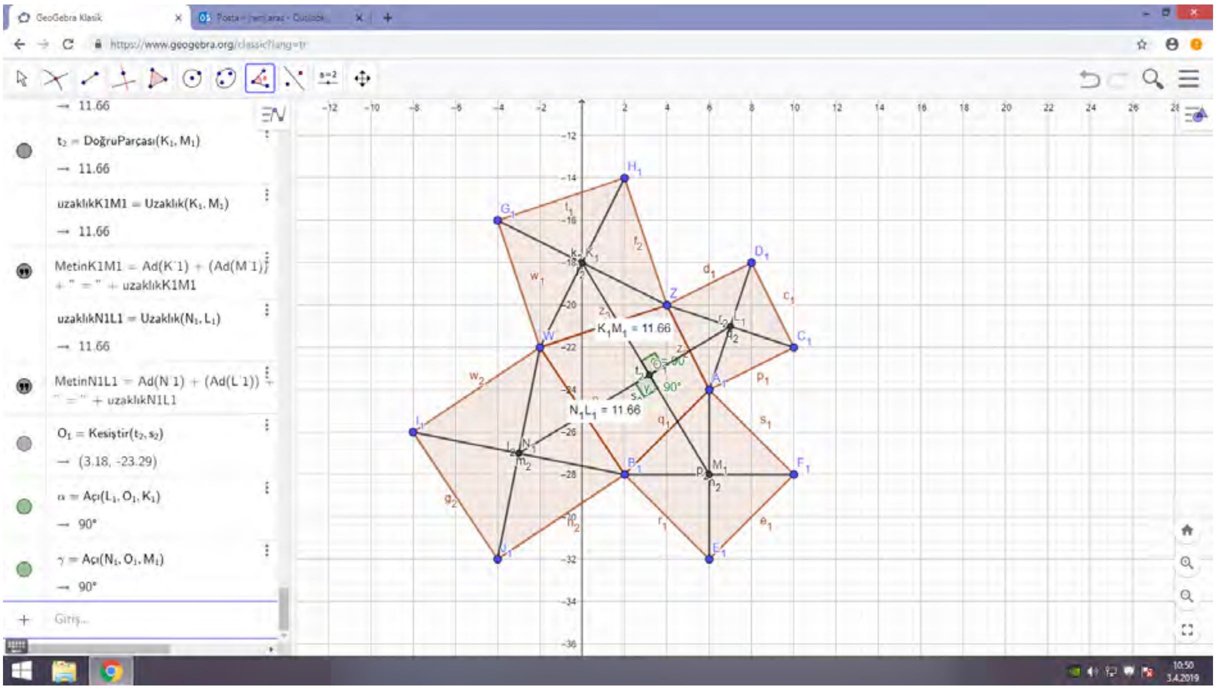

Figure 5 Student answer to von Aubel Theorem

Figure 4 and figure 5 are example answers from two students. Hence, they were able to see the correct functional dependencies and were able to conclude the exact diagrams required from them. Students who came with specific shapes but not arbitrary shapes concluded that the theorems were right for only the shapes they found. Hence, their generalized judgments were injured.

CONCLUSION AND DISCUSSION

In the literature, there are unfortunately not so much experimental research on functional dependency. Hence, we could only be able to find research on the software facilities as in Kabaca et.al. (2010), Akyüz (2015) and Baki (2000), Mariotti & Falcade (2003), Carreira, Clark-Wilson & Faggiano(2017), and finally Holdbrands, Laborde & Straber (2008). Functional dependency is mostly given as a specialty of the software program but not in relation to mathematics topics. However, students can find reasonable connections with the correct use of functional dependency. Geogebra enables these connections since its multi representational facility.

We concluded that functional dependency is not dependent on the task even though it may be related to in a manner. We have seen some students acted like mathematicians even though these theorems were from advanced curriculum. They could deduce some hypotheses from the theorem representations. Very few inconsistencies found from student answers show that Geogebra is a good tool to study these kind of functional dependencies. Simple geometry should be a start point, and from that point on, students may be forced to develop more relationships if possible. In the end, students and pre service teachers may feel empowered mathematically as well.

The tasks (theorems) were only different on the number of free corner points of the starting shape (name as triangle in Napeleon and name as a quadrilateral in von Aubel theorem. This should be an important difference for the idea of functional dependency since, those points define the starting object, or the loci. If we can say that 4 is more than 3, hence von Aubel was more complex than Napeleon Theorem, then following results come out. For less complex; inconsistencies are more, examples were less and for more complex; number of drawings are less. Hence, students run away from drawings even in the presence of the Geogebra program. Just by dragging they could generate many examples for two of the theorems as much, but they could not come out with required examples as much as intended.

We could not see any differences regarding the understanding of the initial drawing of the theorem just be reading its verbal representation. They may need to be taught regarding the functional dependencies of Geogebra in detail so that they would not be challenged by different or complex problems or theorems.

RECOMMENDATIONS

It may be a good idea to search mathematics literature for theorems like these sort. Teachers may try to incorporate advanced mathematics more in natural K-12 curriculum. To study functional dependency, students may be required to search for the conceptual relations occurring inside the tasks, and especially in terms of theorems if possible. Material development courses, as well as mathematics courses may be a good place to apply for these tasks. Student teachers may be precluded to find more tasks like these for that course or for mathematics.

Napoleon Theorem is interesting since, students may see that with 120 degree angles, the resulting hexagons (although not regular) tessellate the plane with Geogebra. And with von Aubel Theorem, students may be directed to create n-gons with n squares around and see the relationships of the diagonals hopefully in the form of new theorems and deductions.

A teaching experiment can be a good method for these kind of investigations. It gives room to the researchers and it gives freedom to the students to go their own ways. Data collection instrument could be revised to gather data on what changed and what did not to understand invariance and also it could include some empty space for identifying the dependencies as what depends upon what.

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