Learning Disabilities: A Contemporary Journal 18(2), 213-240, 2020

Working Memory Sensitive Math Intervention for Primary School Students: A Multiple Baseline Design Study

Sarah Schulze*
Technical University of Dortmund

Timo Lüke
University of Graz

Jan Kuhl
Technical University of Dortmund

Interventions to support children with mathematical learning difficulties typically address deficits in domain-specific knowledge. However, not all students benefit from these instructional programs. In this case, some authors suggest an even more intensive instructional program combined with other factors assumed to be relevant for learning. Previous research has demonstrated that working memory is such a factor. It underpins a range of cognitive abilities, including arithmetic and mathematical development. For this reason, we developed an integrated approach, a working memory sensitive math intervention. This new approach aims at domain-specific knowledge, while a) taking poor working memory into account and b) stimulating learners to invest cognitive resources in mental confrontation with the learning content. The present multiple-case study was designed to investigate its effects. On the basis of their low performance in a standardized test of mathematical precursors, we identified 13 first graders (mean age 6.8 years) to take part in our study. Over a period of four weeks, the children participated in 12 half-hour sessions of our program. Results show small to large positive training effects on competencies that are near to the training, though not for all participants. We discuss why the intervention works well for some children, only moderately for others, or fails to work.

Keywords: **Working Memory, Mathematical Learning Difficulties, Training, Intervention, Multiple Baseline Design, Single-Case Research**

Introduction

Many children struggle with basic mathematical skills, and their challenges may go undetected until they fall behind at school. The reason for their mathematical difficulties, however, often lies in their early development (e.g., Aunola, Leskinen, Lerkkanen & Nurmi, 2004; Krajewski & Schneider, 2009a; Passolunghi, Vercelloni & Schadee, 2007). Precursors such as counting abilities and quantity-number competencies (connecting numbers with quantities) are important predictors of later mathematical achievement (Jordan, Glutting & Ramineni, 2010; Krajewski & Schneider, 2009a; Locuniak & Jordan, 2008). There is an effective approach in supporting these domain-specific skills. Several studies have shown that support

*Please send correspondence to: Sarah Schulze, Department of Differential Didactics, Technische Universität Dortmund, Otto-Hahn-Straße 6, 44227 Dortmund, Germany, Phone: (+49)231 755-5201, Email: sarah.schulze@tu-dortmund.de.

of precursors improves later mathematical skills (Ennemoser, Sinner & Krajewski, 2015; Kroesbergen & van Luit, 2003; Griffin, 2004). Despite these promising findings, not all children benefit from such interventions, and some show little or even no improvement. This phenomenon, known as nonresponsiveness, can occur in all learning areas (Fuchs, Compton, Fuchs, Paulsen, Bryant, & Hamlett, 2005; Fuchs & Fuchs, 2006).

Another capacity whose influence on learning has been discussed widely is working memory. In their *model of good information processing*, Pressley, Borkowski, and Schneider (1989) present factors that are considered to be individual requirements for successful learning. Working memory is one essential characteristic that differentiates between good and weak information processors. To date, several studies have demonstrated a close relationship between working memory and mathematical learning (Alloway & Alloway, 2010; Alloway & Passolunghi, 2011). Regarding domain-unspecific factors, working memory explains more variance in mathematical performance than other known predictors such as intelligence (Cragg & Gilmore, 2014). Findings indicate that children with mathematical learning difficulties show weaker working memory functioning (Klesczewski et al., 2018; Raghubar, Barnes & Hecht, 2010; Vukovic & Siegel, 2010).

It is obvious that an appropriate intervention should address as many of these problems as possible. We argue that a greater consideration of working memory might be a useful approach for math intervention. To close this instructional gap, we have developed an intervention that aims at mathematical precursors as domain-specific skills and includes working memory as an additional factor. In the present study we investigate the effects of the newly developed intervention in single cases.

Working memory and mathematical learning

The construct of working memory refers to a limited-capacity storage system in which information is maintained and manipulated over a short period of time (Baddeley, 1986, 2012). Thus, it plays an important role in every mental task as well as mathematical learning and math performance. There are various models of working memory which are still discussed (Baddeley, 2012; Wilhelm, Hildebrandt & Oberauer, 2013). In this work, we refer to the domain-general model proposed by Baddeley (1986, 1996, 2012). This multicomponent model with its components phonological loop, visuospatial sketchpad and the central executive is a widely accepted theoretical framework. The phonological loop stores acoustic and verbal information, whereas the visuospatial sketchpad temporarily stores static visual and dynamic spatial information. The central executive is a composite of mechanisms that coordinate and control information processing (Miyake, Friedman, Emerson, Witzki, Howerter & Wager, 2000). Miyake et al. (2000) found that three executive functions – shifting, inhibition, and updating – are correlated with each other, but still clearly separable (e.g., in factor analysis). Shifting, also referred to as task switching, concerns shifting between multiple tasks or operations (Monsell, 2003). The updating function requires simultaneous storing and processing of information relevant to a specific task. Thereby irrelevant items held in working memory are replaced by new relevant items. The ability to suppress dominant response tendencies and attention to irrelevant stimuli is labeled as inhibition. In many studies, the results of Miyake et al. (2000) are used to specify the construct of the central executive.

The functional working memory capacity limits the amount of information held in consciousness (Pressley et al., 1989). If only a small amount of information can be held in working memory, unfavorable conditions arise that are reflected in limitations in learning (Kirschner, Sweller & Clark, 2006). Here, working memory influences math performance in two ways: 1) There is a direct impact of working memory during the mental arithmetic process called *online impact* (Raghubar, Barnes & Hecht, 2010). Students confronted with complex mental arithmetic problems have to deal with many subtasks, for example retaining partial results while calculating the next step. 2) Efficient working memory functions are required for mathematical learning, e.g., for the acquisition of precursors that in turn have an influence on later mathematical learning (*offline impact*; Bull, Espy & Wiebe, 2008; Grube & Seitz-Stein, 2012). The functional working memory capacity also depends on strategy use and prior knowledge (e.g., schema construction and automation). Here, the use of automated skills can free up working memory resources. For example, the retrieval of basic addition facts requires less resources than effortful calculation over several steps. As a result, the children cannot meet the working memory demands of the learning situations and therefore, cannot benefit from typical math interventions. As a further consequence, they cannot take advantage of automation to free up their working memory resources, which again causes them to be unable meet the working memory requirements of the learning situation and so on.

All working memory functions are relevant for mathematical learning (Clearman, Klinger & Szucs, 2017; Friso-van den Bos, van der Ven, Kroesbergen & van Luit, 2003). However, the component involved depends on age as well as on the specific mathematical skill (Friso-van den Bos et al., 2013; Alloway & Passolunghi, 2011). The phonological loop seems to be especially relevant for early mathematical precursors such as acquiring the correct number word sequence (Preßler, Krajewski & Hasselhorn, 2013; Viesel-Nordmeyer, Ritterfeld & Bos, 2020). Some studies also indicate an indirect influence of the phonological loop via language skills on mathematical precursor skills (e. g. Nys et al., 2013, Röhm et al., 2017; Viesel-Nordmeyer et al., 2020). The visuospatial sketchpad seems to be relevant for linking number words to quantities (Ansari et al., 2003; Preßler et al., 2013).

Simanowski and Krajewski (2017) investigate the predictive value of executive functions on various aspects of mathematical skills. The results reveal a relationship between updating and mathematical precursors at a very early stage (number word sequence). Only an indirect influence was seen on higher precursors (e.g., the quantity-number concept), mediated by previous skills. However, we suspect that the small influence observed is a result of the research design. The authors measured quantity-number competencies that were already automated rather than complex arithmetic problem solving. Furthermore, they showed that a combined factor of inhibition and shifting has a significant influence on the acquisition of contents that require a conceptual understanding of numbers.

Weak working memory functioning "[…] therefore places a child at a high risk of slow rates of academic progress […]" (Gathercole & Alloway, 2008, p. 53). They are not able to cope with the working memory demands of many of the interventions that are designed to help them learn (Gathercole & Alloway, 2008). Swanson (2014, 2015) showed that this applies to the effects of strategy training as well. His findings suggest an interaction between the type of strategy instruction and working memory. Children with relatively better working memory were more likely to benefit from strategy instruction than children with a weaker working memory. On the contrary, cognitive strategies decreased problem-solving accuracy in children with lower working memory (Swanson, 2014).

Consideration of working memory

Theoretically, there are two options in dealing with limited working memory: (1) direct training and (2) preservation of available resources through more efficient usage. Whether working memory capacity can be increased directly by training is a long-discussed question. There is no clear answer, but the results from training studies are mixed and not very promising (Sala & Gobet, 2017). Training effects tend to be short-term and often occur in learners with weak working memory starting conditions (Melby-Lervåg & Hulme, 2013; Sala & Gobet, 2017). Furthermore, as far as the benefits of math performance are concerned, the "curse of specificity" (Strobach & Karbach, 2016, p. 2) has to be considered. That means, if children already have deficits in mathematical competencies, we cannot expect them to eliminate these gaps through a pure working memory training (Schulze & Kuhl, 2019), because it doesn't compensate for the instruction and practice they have already missed. Therefore, an intervention for children with persistent difficulties should always include mathematical content.

Research on the second possibility (more efficient usage) refers to *cognitive load theory* (CLT; Sweller, 1988, 1989; Sweller & Chandler, 1991). According to the CLT, all tasks contain a high number of elements that have to be processed to induce learning (Paas & Van Gog, 2006). This cognitive load occurs at different instances during the learning process. Commonly, we differentiate between load that is caused by the complexity of the learning task (*intrinsic load*) and cognitive load that is caused by the instructional design (*extraneous load;* Paas & Sweller, 2014). The third type of load, called *germane load*, refers to the development of cognitive schemata (Paas & Sweller, 2014). As many resources as possible should be free for this process, which we also call learning. Thus, the learning environment and learning material should fit the limited functioning of working memory, especially in children with learning difficulties. Some authors have explored principles that aimed at preserving cognitive resources during learning (e.g., Hecht, 2014; Magner, Schwonke, Aleven, Popescu & Renkl, 2014; Wiley, Sanchez & Jaeger, 2014) or provided guidelines for instructional design (Gathercole & Alloway, 2008; Krajewski & Ennemoser, 2010). Exemplary principles are the omission of seductive details (Harp & Mayer, 1997), spatially close and integrated presentation of associated pieces of information, and unambiguous representations (Hecht, 2014). Gathercole and Alloway (2008) integrated several recommendations that refer to classroom support for children with poor working memory functioning. (1) Using familiar and meaningful content, (2) reducing the amount of material used, (3) using repetitions, and (4) restructuring complex tasks are only some of them. Many of these principles relate to the learning environment and material, but there are also principles related to individual competencies, for example the development of basic skills or the automation of sub-competencies. Single elements can be subsumed into a schema that can be treated as a single element in working memory, thus decreasing the number of interacting elements (Paas & Van Gog, 2006). We assume that children with weak working memory functioning cannot use the latter principle because the offline working memory impact confounds the development of knowledge in long-term storage.

Some authors point out the limitation of these principles, emphasizing that vacant resources are not supposed to automatically lead to germane load (Paas, Tuovinen, Tabbers & van Gerven, 2003; Paas & van Gog, 2006). The surplus working memory resources can be devoted to activities that further contribute to learning. But, such activities are unlikely to be spontaneous in children with learning difficulties, because they tend to work less systematically and reflect on their solutions less often (Smith, 2004). Learners must be encouraged to actively engage with the content (Paas et al., 2003; Paas & van Gog, 2006). We suggest that this is necessary at both behavioral and cognitive dimensions, in particular for students with learning difficulties, as they are hindered from learning by several factors (Jacobs & Petermann, 2007; Smith, 2004). Therefore, we assume that the existing evidence should be combined into one approach.

Idea of a working memory sensitive math intervention

Based on empirical findings and theoretical models, it is repeatedly discussed that working memory should be integrated more strongly into conventional learning or be given greater attention (Passolunghi & Costa, 2019; Röhm, 2020; Viesel-Nordmeyer et al., 2020).

We developed a *working memory sensitive math intervention* that embeds mathematical content in classical working memory tasks to address relevant working memory functions directly. Thus, it is a combination of relieving and challenging methods that extend the conventional learning material. From this point of view, the approach aims at the improvement of mathematical precursors while a) weak working memory is considered and b) learners are stimulated to invest cognitive resources in mental confrontation with the learning content. Consequently, the freed-up working memory capacity can be used directly for mathematical learning. With regard to the latter, the approach can also be situated in the larger construct of self-regulation. In the paradigm of information processing, self-regulating activities can take place in the storage and integration of information, which are essential points of the intervention.

In order to implement these aspects, we have to consider which working memory function is essential for which mathematical task (Schulze & Kuhl, 2019). It is therefore not a matter of including an additional task with an additional requirement, but rather of finding a format in which the mathematical content and the relevant working memory function can be combined. When trying to master the number word sequence, we should integrate the phonological loop and tasks that directly activate it. Such a task is the digit span task (forward or reverse), which is typical for measuring working memory. There is also evidence for a relationship between updating and building the number word sequence (Simanowski & Krajewski, 2017), so it should also include tasks that require this function. If working memory is burdened as less as possible by the use of suitable learning materials and a small step-by-step procedure that is adapted to the learners previous knowledge, working memory functions should be used as well as possible during the learning process.

Our training aims at mathematical precursors. In this piloting study we aim at evaluating the effects of our newly designed intervention. We expected a gradual improvement during the training. The children's performance should improve in both level and slope. However, mathematical learning does not occur immediately. We are investigating a rather new intervention; hence, the first indications of effects should be reflected, so that further evaluation steps with more comprehensive research designs can follow.

Method

Participants

The study took place in seven primary school classrooms of a metropolitan area in Germany. At mid-term, we contacted class teachers and asked them to name students with low arithmetic skills who do not respond to the current classroom instruction. We obtained parental consent for all nominated students and assessed their basic mathematical competencies using a German standardized quantity-number competencies test ("Test mathematischer Basiskompetenzen ab Schuleintritt"; MBK 1+ by Ennemoser, Krajewski, & Sinner, 2017). Students below a percentile rank of 16 were included in our study.

Thirteen first grade students (five male and eight female; see Table 1 for detailed information on participants) from these seven schools participated in our study. They all took part throughout the entire study. Their average age was 6.8 years (SD = 0.60) at the start of our study. Given the available time slots, it was not possible to assess the participants' language skills systematically. Instead, we collected judgments of the respective classroom teachers. Emma (we changed all names to ensure anonymity) is the only second-language learner in this sample. Teachers evaluated all participants as having sufficient German language skills to understand the instructions used in our study. This was later confirmed by the unproblematic course of the sessions. One of the children, Mason, was already receiving special educational services, due to learning difficulties ("Förderschwerpunkt Lernen"). The mean score in the test of basic mathematical competencies (MBK 1+) was 21.19 (SD = 4.04), corresponding with a mean percentile of 11.23 (SD = 4.90) and a percentile range from Min = 2 to Max = 18. Our sample consists of students with clear mathematical learning difficulties, even based on a very strict cutoff.

Table 1. ***Participant Overview***

				MBK 1+ Raw Score		MBK 1+ T Scores			
	Sex	**Age**	**Grade**	**Pre**	**Post**	**Pre**	**Post**	***nA***	***nB***
Ava	female	6	1	21.50	40.50	31.00	51.00	6	12
Benjamin	male	7	1	24.00	43.50	34.00	55.00	5	12
Charlotte	female	7	1	22.50	35.50	24.00	38.00	5	11
Destiny	female	7	1	18.50	22.50	18.00	20.00	5	11
Emma	female	8	1	21.50	35.00	23.00	39.00	5	11
Faith	female	8	1	12.00	20.50	10.00	16.00	5	11
Gabriel	male	7	1	26.00	28.50	30.00	28.00	5	12
Harper	female	7	1	22.50	24.00	24.00	22.00	5	10
Isaac	male	7	1	24.50	37.50	28.00	42.00	5	12
Julia	female	6	1	19.50	35.50	21.00	39.00	5	12
Kevin	male	6	1	25.00	44.50	29.00	53.00	5	11
Lily	female	6	1	15.00	40.50	14.00	46.00	5	11
Mason	male	8	2	23.00	33.50	25.00	36.00	5	10

Notes. MBK 1+ = German standardized quantity-number competencies test; *nA* = Number of observations in phase A; *nB* = Number of observations in phase B.

Research design and procedure

We implemented a single-case intervention study, using an AB multiple-baseline design across participants (Gast, Lloyd, & Ledford, 2018) to evaluate the effects of this new intervention approach. All sessions took place as one-on-one situations in a quiet room on the premises of the respective primary school. Participants received 12 30-minute intervention sessions over a period of three to seven weeks depending on their presence in school. Every measurement session took about 15 minutes. We ensured that the baseline (A phase) had a minimum of five measurements. During the intervention (phase B), measurements followed the intervention session. We made sure that every participant worked with the same instructor from our team (first author and five pre-service special education teachers) for the entire study. To ensure that the intervention was carried out as intended, the instructors were trained by the first author in at least four meetings. The meetings were also held during the intervention process to review the progress of the intervention and discuss any problems.

Intervention

On the basis of an integrated approach (Schulze & Kuhl, 2019), we developed an intervention for children with persistent difficulties in acquiring mathematical precursors that considers working memory as an additional factor. We focus on working memory while supporting the development of mathematical precursors, and, as mentioned above, aim at combining two essential aspects: (a) extraneous and intrinsic load should be reduced to a minimum, and (b) free capacities should be reallocated to the mental examination of the learning object.

On the domain-specific part (mathematical precursors), we focus on Krajewski's developmental model of quantity-number competencies (QNC; Michalczyk, Krajewski, Preßler & Hasselhorn, 2013; Krajewski, 2008; Krajewski & Schneider, 2009a, 2009b) and address the development of counting and quantity knowledge. The QNC model postulates three levels of development through which children acquire a deeper understanding of quantity to number-word linkages. On the first level, children acquire the awareness of numbers and the correct number word sequence. The milestone on the second level is when children become aware that number words are linked to quantities. At the end of the second level, children understand that the number-word sequence represents an order of strictly increasing quantities (*5 is exactly one more than 4*). They develop a precise quantity to number-word linkage. On the third level, children understand the relations between numbers. Their understanding of the principles of composition and decomposition is now applicable on numerical relations (Krajewski & Schneider, 2009a; Kuhl, Sinner & Ennemoser, 2012).

On the mathematical part, our concept consists of two modules, which in turn consist of different blocks. The blocks are further divided into units. The first module refers to the numbers up to 20, whereby the first block in this module aims for the acquisition of the exact and flexible number word sequence (module 1, block 1: unit *number word sequence to 20*). This competence requires the repetition of single number words but also depends on the updating of the relevant segment of the number word sequence in working memory for the correct reproduction. In accordance with Simanowski and Krajewski (2017), we assume that the demands of updating are reduced if exercises include strictly defined content that is worked on and deepened until it is finally automated. This assumption leads us to a highly structured concept.

To comply with point a), we have ensured that the principles above have been considered (restructuring complex tasks, reducing the amount of material used etc.). In this way, working memory should not be unnecessarily loaded.

In order to comply with point b), we have embedded the mathematical content into specific working memory tasks; for example, we challenge updating and phonological loop specifically when the number word sequence is learned. In this way, the resources available should be used as efficiently as possible for the learning process.

In the first block of module 1, the phonological loop is especially required, therefore, we create tasks where the students need to update the phonological information. To illustrate how we realized this, we would like to give an example of a very simple exercise: In one task, the children were verbally given a short sequence of unsorted numbers (e.g. 3-1-2). Depending on their previous knowledge, the sequences of numbers were of different lengths. The children were asked to remember the numbers while putting them in the right order (3-1-2 ➞ 1-2-3). During this task, the phonological loop should be activated directly, simply because they work on it. In this way we address mathematical concepts and working memory at the same time. If the children become more confident with the number word sequence, the task can also lead to automation.

The second block of the first module focuses on insight into the link between quantity and number word representations. When the children learn to deal with quantities, we use tasks that require the visuospatial sketchpad. For example, changes in quantities can be trained with tasks similar to the classical matrix span. Here, the children are asked to recognize changes of structured points (like a matrix pattern), to store them or to make changes themselves mentally. The working memory load can be adjusted via the size of the quantities. Finally, there is a third block in which the quasi-simultaneous recording of quantities is trained.

The second module targets the relational understanding of numbers. In particular it aims at the automation of number bindings, which means that children instantly know that 10 can be split into 5 and 5 or into 7 and 3 and so on. These number-related part-whole relationships represent an essential milestone on the way to a deep understanding of numbers and are crucial for later arithmetic (Krajewski, 2005). In this module, we structured three blocks based on the number space. First, we work on all bonds of 5, followed by all bonds of 10, and finally, we focus on all bonds of the remaining numbers.

A special feature of our concept is that all units are divided into one of three areas: introduction, deepening, or automation. This allows for flexible use of these units according to individual learning progress.

Measures

Quantity-number competencies

Based on Krajewski's developmental model of quantity-number competencies (QNC; Krajewski, 2003, Krajewski & Schneider 2005; 2009a, 2009b), this test is used for the early detection of mathematical development risks in first grade. Furthermore, MBK 1+ is a test for short- and long-term evaluation of interventions in a variety of longitudinal and training studies (Ennemoser et al., 2015, Sinner, 2011). The norm sample consists of 6084 students; of these, 5604 students were available for the first grade (Ennemoser et al., 2017). For the first quarter of the first grade, the MBK 1+ has a wide range of item difficulties ($p = .20$ to $p = .96$). Retest reliability varies between sub-scales ($.66 < r < .78$; Ennemoser et al., 2017); internal consistencies are good ($.88 < \alpha < .95$). Previous studies have shown that it is correlated with later math performance: MBK1+ scores in first grade are a good predictor of math performance in fourth grade ($\beta = .65$, $p < .01$; Ennemoser et al., 2017).

The test consists of several subtests that measure competencies at three levels of the developmental model (for the description of the three levels, see page 11). In addition to its screening function, we used the MBK 1+ as a first diagnostic to plan our intervention sessions. After the intervention phase, the children performed the test a second time, similar to a post-test measurement.

Working memory

All working memory subtests were administered using a span procedure in which the difficulty level increases across blocks of trials, using two errors as a stop criterion. We used the digit-span-forward and backward tasks (German version of the *Wechsler Intelligence Scale for Children;* WISC-V, Petermann, 2017) to assess students' phonological loop and updating of phonological information. In order to measure the visual-spatial sketchpad, we implemented the Corsi block paradigm (Corsi, 1972) and designed a Corsi block task forward and backward. Here, the experimenter points to a sequence of nine unsystematically located squares that are arranged randomly on a board. The child must reproduce the sequence presented in the same or reverse order. In the latter case, demands are also placed on the central executive. To test the stability of the working memory performance, we constructed parallel test forms.

Relational understanding of numbers

To operationalize the understanding and automation of relations between numbers, we developed a *number-bonds task*. The task uses number bonds in the form of the so-called pyramid notation. In this notation form, the whole stands on top and the two parts are noted to the right and left below the initial number (Figure 1). Thus, the task refers only to the bond of numbers without using the plus, minus, or equal sign.

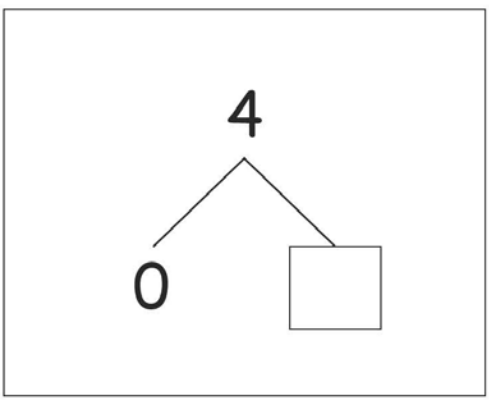

Figure 1. **Instruction number-bonds task.**
Note. Children are instructed to fill in the missing number in the square.

Each bond lacks one part that needs to be supplemented and requires the children to fill in the missing part. The test is available for two levels: The first level refers to all bonds of all numbers up to 5 (*number-bonds level 1*), whereby the second level includes the bonds from 6 to 10 (*number-bonds level 2*).

Arithmetic

Numeracy skills were measured by a simple arithmetic test where children had to solve typical *addition* problems (e.g., 3+2; 7+3). We measured the number of errors (out of 6 tasks) as a dependent variable.

Data Analysis

In addition to visual judgment and descriptive statistics, we report a set of overlap and correlation-based effect sizes: We calculated the percentage of all non-overlapping data (PAND; Parker, Hagan-Burke, & Vannest, 2007), standardized mean differences (SMD; Glass, 1976), and the baseline corrected Tau (Tarlow, 2017). The PAND is the percentage of original data remaining after removal of the fewest data points leading to non-overlap between phases. It ranges between 50 (chance level) and 100 (no removal). We report the simple standardized mean difference suggested by Glass (1976), which uses phase means and the overall variance. Baseline corrected Tau (Tarlow, 2017) conceptualizes the homogeneity of phases as effect size after correcting for monotonic baseline trends using Theil-Sen regression. It ranges between -1 and 1 and is easily interpreted as a rank correlation coefficient. All analyses and graphs were created using the package scan (Wilbert & Lüke, 2019) for R (R Core Team 2013).

Results

Our main goal was to examine the potential effect of a newly designed working memory sensitive intervention. All participants received at least 10 training sessions ($n_B \geq 10$) following the baseline phase (nA ≥ 5), resulting in 15-18 observations per individual (see Table 1 for details).

Number bonds (errors)

Visual analysis of the *number bonds* (errors) results over all participants of the multiple baseline design indicate an intervention effect. Not, however, for all participants: Destiny, Gabriel, and Harper show no change in their error rates agreeable to the phase changes. Emma reduced her error rates within the first sessions of the baseline phase, leaving no further room for improvement in this specific variable. The remaining majority of the students showed small (e.g., Faith) to large reductions of their error rates. With few exceptions the average number of errors was reduced in the intervention phase (M_A–M_B: M = -3.68, *Min–Max* = -11.35–0.40), and the trend remained or improved toward a reduction of errors (T_A–T_B: M = -0.10, *Min–Max* = -0.70–1.17). Correspondingly standardized mean differences indicate moderate to strong intervention effects for all participants except for Destiny, Faith, and Harper. Tau values indicate a strong intervention effect for five participants (Ava, Emma, Kevin, Lily, Mason), a moderate intervention effect for another five (Benjamin, Charlotte, Gabriel, Isaac, Julia), but no relevant intervention effect for Destiny, Faith, and Harper.

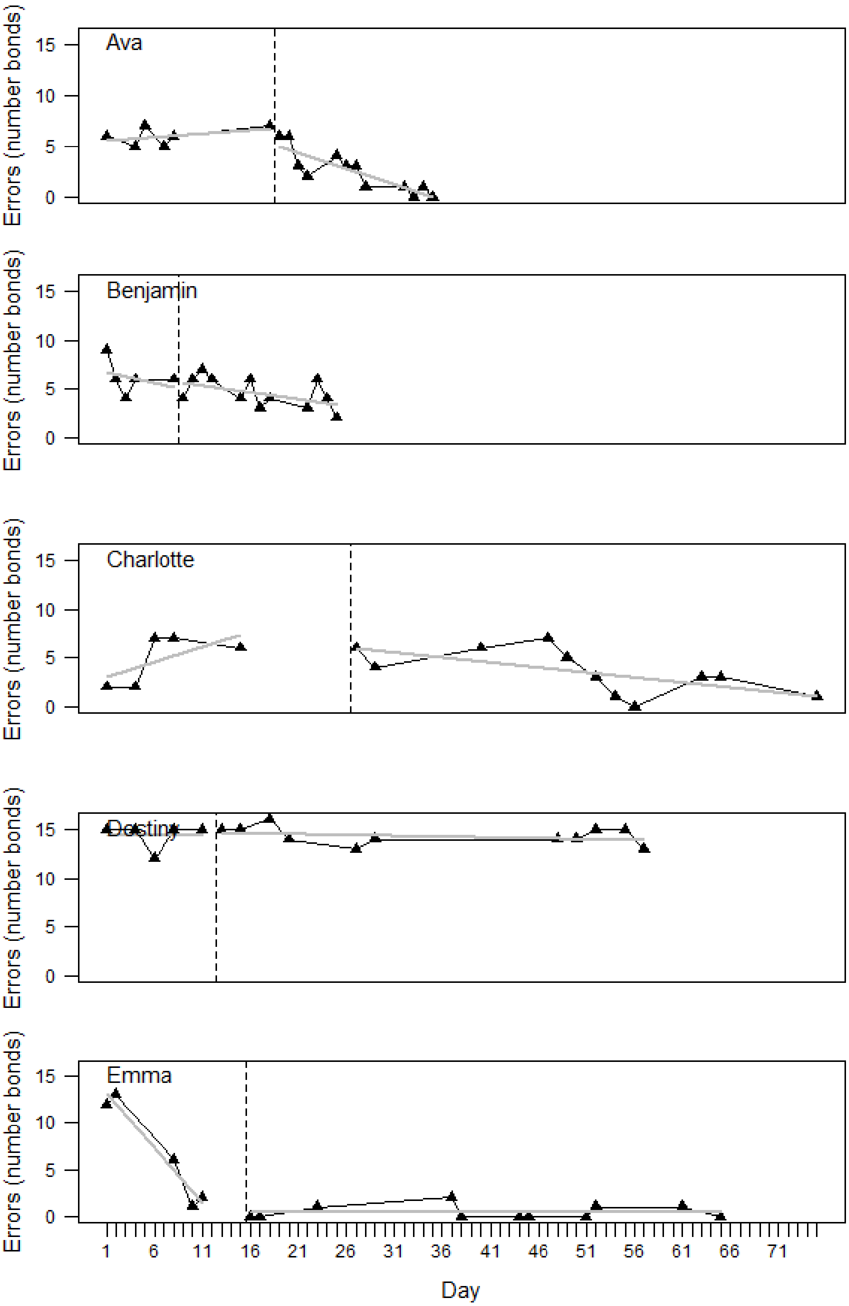

Figure 2. **Number bonds errors for all participants in baseline and intervention phase.**

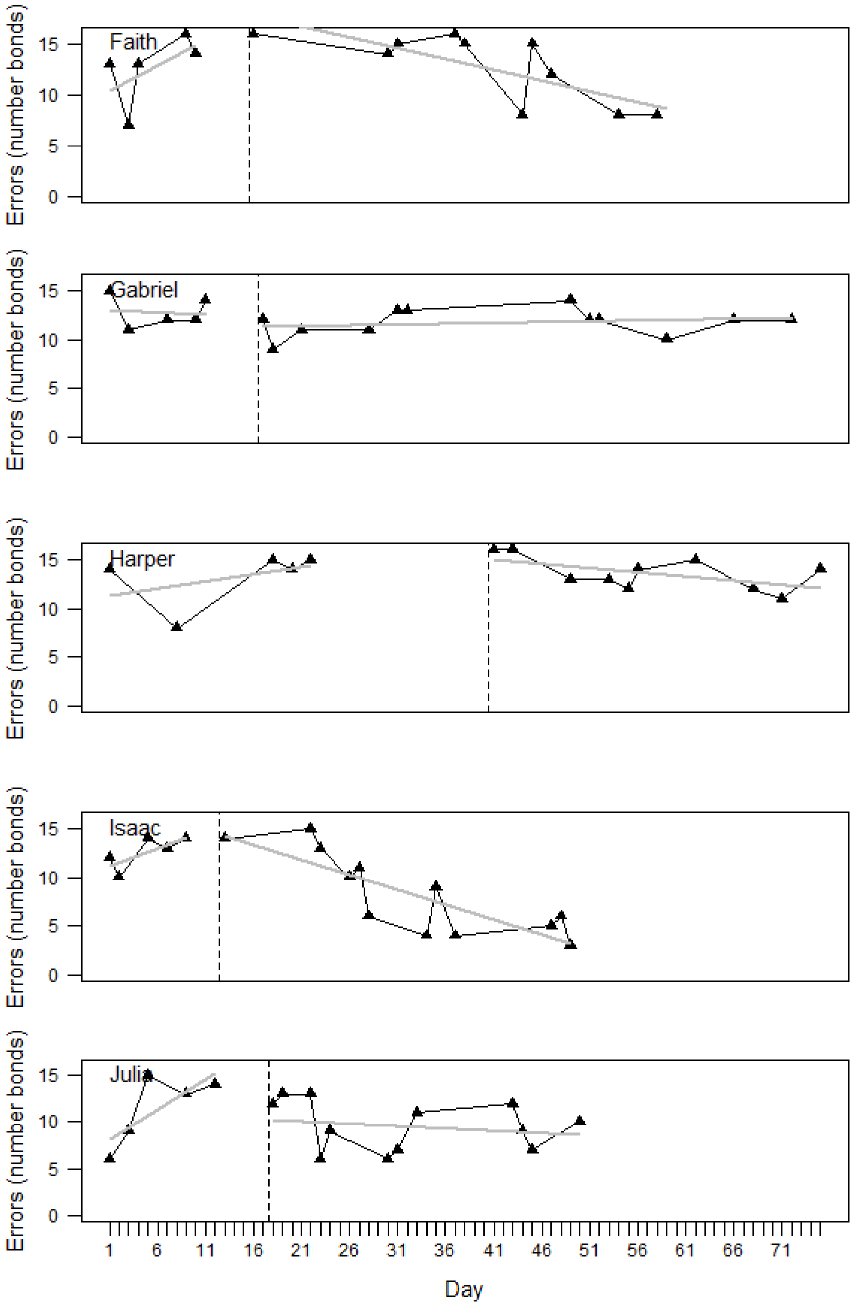

Figure 2 **(continued)**

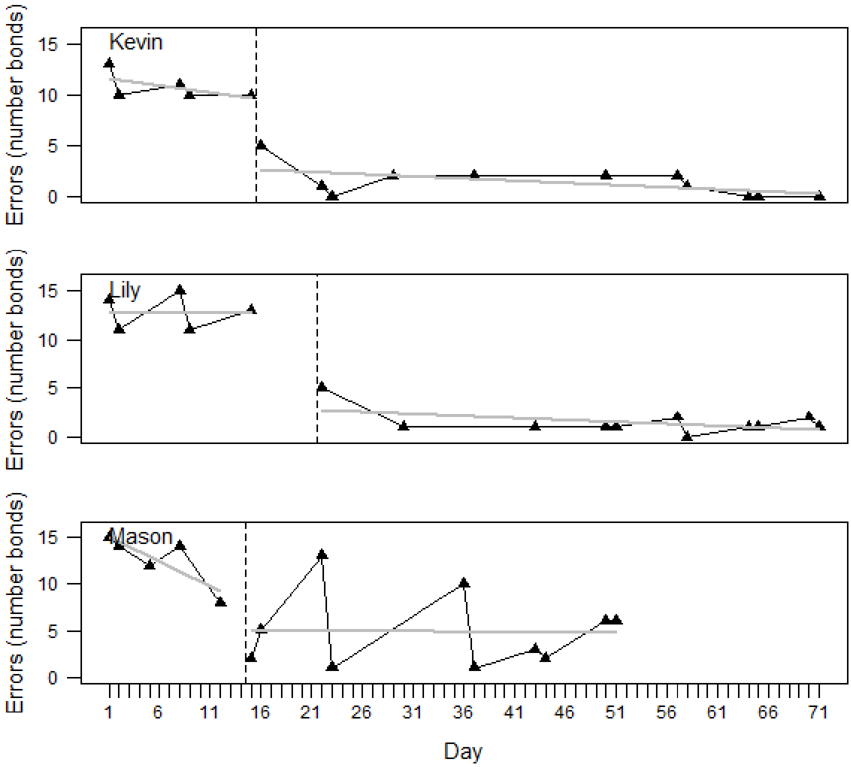

***Figure 2* (continued)**

Addition (errors)

Visual analysis of the *addition* (errors) outcomes indicates an intervention effect for Lily, unclear results for Ava, Benjamin, and Charlotte, and no intervention effect for the remaining majority of the students. The average number of errors was slightly reduced or remained the same in the intervention phase for all but three students ($M_A – M_B$: M = -0.35, *Min–Max* = -2.98–1.05). The majority of graphs show negligible trends for this outcome, corresponding with no improvements in students' error rates over time ($T_A – T_B$: M = -0.07, *Min–Max* = -0.49–0.19). The standardized mean differences indicate increases in errors comparing phases A and B for five participants (Benjamin, Destiny, Emma, Julia, Mason), and small to large positive effects (lower error rates) for the majority of participants. Baseline-corrected Tau values indicate strong intervention effects for Destiny and Lily, moderate intervention effects for six participants (Ava, Benjamin, Charlotte, Faith, Isaac, Kevin), but no effect for Emma, Gabriel, Harper, and Mason.

Table 2. ***Descriptive Statistics and Effect Sizes for Number Bonds (Errors)***

	A			B			A vs. B				
	M (SD)	***Md (MAD)***	**Trend**	***M (SD)***	***Md (MAD)***	**Trend**	M_A-M_B	T_A-T_B	***SMD***	***PAND***	Tau
Ava	6.00 (0.89)	6.00 (1.48)	0.07	2.50 (2.07)	2.50 (2.22)	-0.31	-3.50	-0.38	-3.91	77.78	-0.60
Benjamin	6.20 (1.79)	6.00 (0.00)	-0.23	4.58 (1.56)	4.00 (2.22)	-0.15	-1.62	0.08	-0.90	88.24	-0.35
Charlotte	4.80 (2.59)	6.00 (1.48)	0.31	3.55 (2.30)	3.00 (2.97)	-0.10	-1.25	-0.41	-0.48	75.00	-0.22
Destiny	14.40 (1.34)	15.00 (0.00)	0.00	14.36 (0.92)	14.00 (1.48)	-0.02	-0.04	-0.02	-0.03	87.50	-0.13
Emma	6.80 (5.54)	6.00 (7.41)	-1.17	0.45 (0.69)	0.00 (0.00)	0.00	-6.35	1.17	-1.15	87.50	-0.69
Faith	12.60 (3.36)	13.00 (1.48)	0.49	12.70 (3.43)	14.50 (2.22)	-0.21	0.10	-0.70	0.03	46.67	0.10
Gabriel	12.80 (1.64)	12.00 (1.48)	-0.05	11.75 (1.36)	12.00 (1.48)	0.02	-1.05	0.06	-0.64	64.71	-0.23
Harper	13.20 (2.95)	14.00 (1.48)	0.15	13.60 (1.71)	13.50 (2.22)	-0.08	0.40	-0.23	0.14	60.00	-0.06
Isaac	12.60 (1.67)	13.00 (1.48)	0.37	8.33 (4.23)	7.50 (5.19)	-0.31	-4.27	-0.68	-2.55	76.47	-0.39
Julia	11.40 (3.78)	13.00 (2.97)	0.64	9.58 (2.64)	9.50 (3.71)	-0.05	-1.82	-0.69	-0.48	76.47	-0.25
Kevin	10.80 (1.30)	10.00 (0.00)	-0.13	1.36 (1.50)	1.00 (1.48)	-0.04	-9.44	0.09	-7.24	100.00	-0.73
Lily	12.80 (1.79)	13.00 (2.97)	0.02	1.45 (1.29)	1.00 (0.00)	-0.04	-11.35	-0.06	-6.34	100.00	-0.75
Mason	12.60 (2.79)	14.00 (1.48)	-0.51	4.90 (4.01)	4.00 (2.97)	-0.01	-7.70	0.51	-2.76	86.67	-0.62

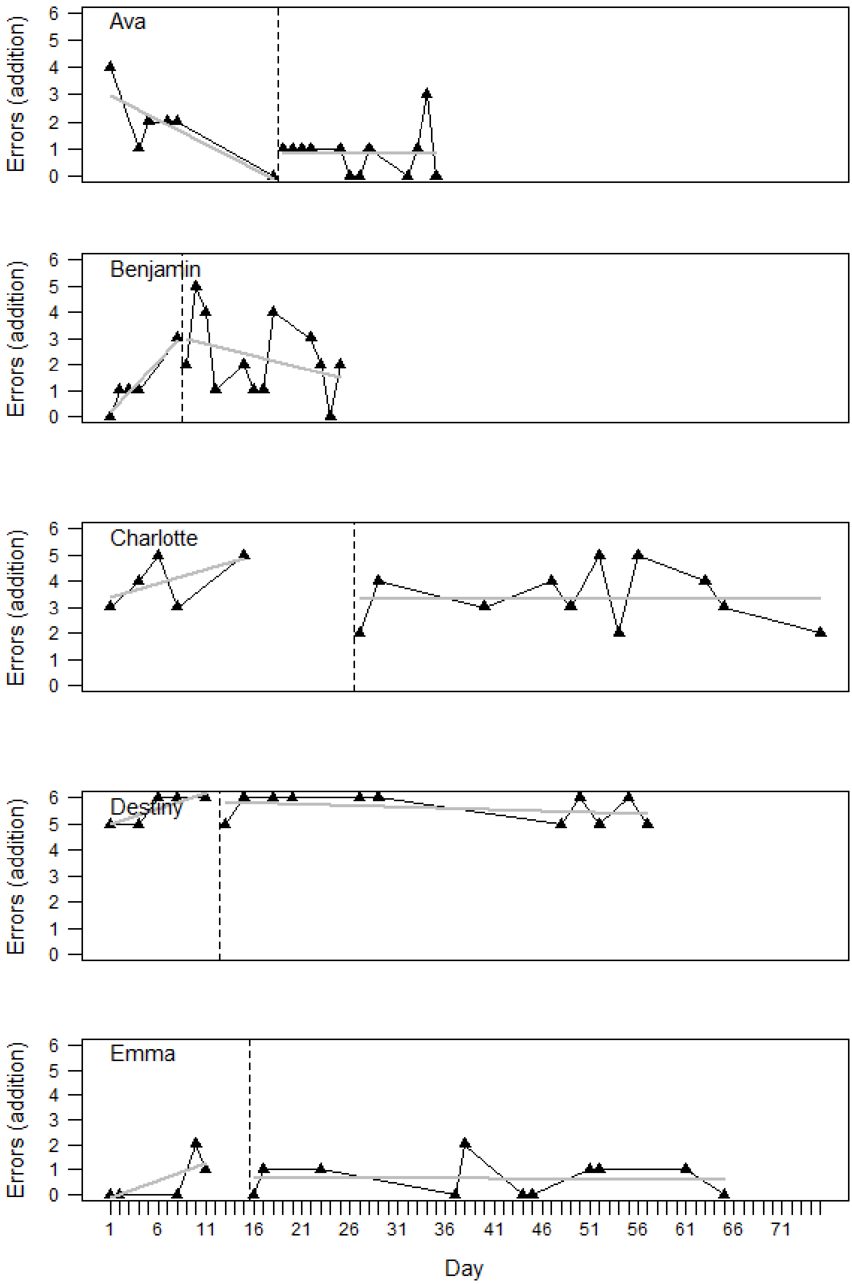

Figure 3. **Addition errors for all participants in baseline and intervention phase.**

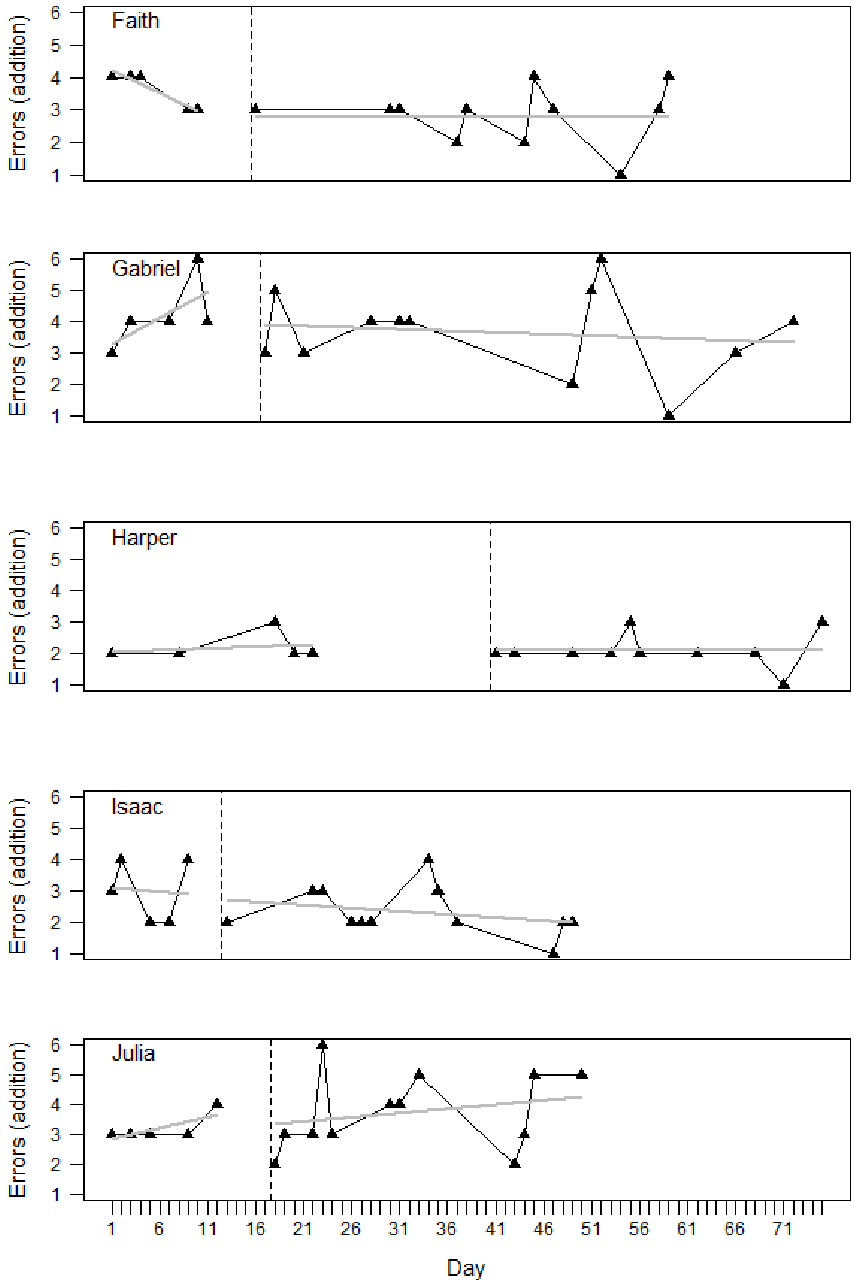

Figure 3 **(continued)**

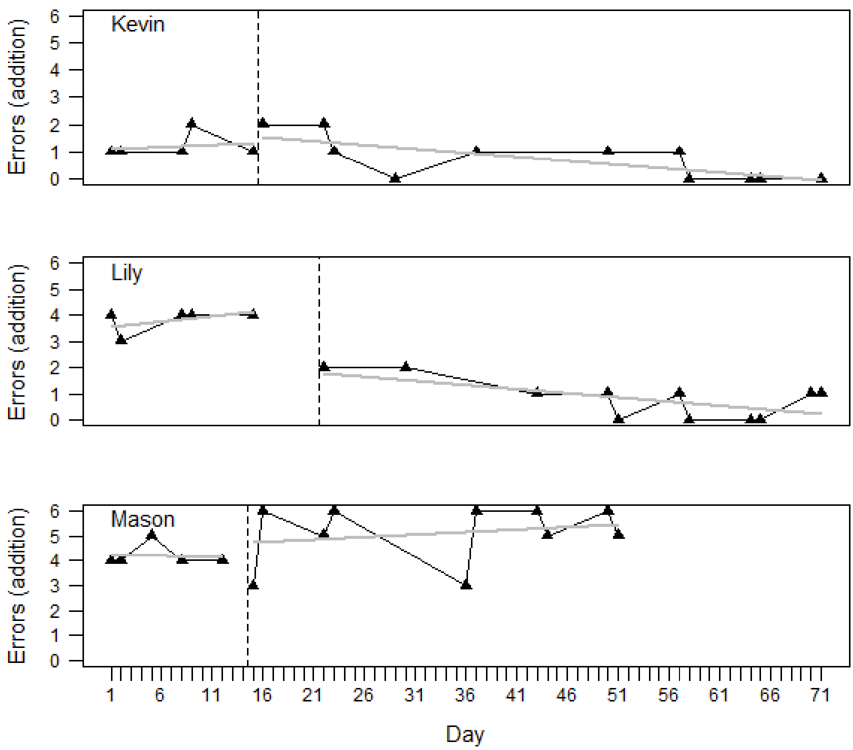

***Figure 3* (continued)**

Table 3. ***Descriptive Statistics and Effect Sizes for Addition (Errors)***

	A			B			A vs. B				
	***M* (*SD*)**	***Md* (*MAD*)**	**Trend**	***M* (*SD*)**	***Md* (*MAD*)**	**Trend**	M_A-M_B	T_A-T_B	***SMD***	***PAND***	***Tau***
Ava	1.83 (1.33)	2.00 (0.74)	-0.18	0.83 (0.83)	1.00 (0.00)	0.00	-1.00	0.19	-0.75	88.89	-0.41
Benjamin	1.20 (1.10)	1.00 (0.00)	0.39	2.25 (1.48)	2.00 (1.48)	-0.09	1.05	-0.49	0.96	52.94	-0.37
Charlotte	4.00 (1.00)	4.00 (1.48)	0.11	3.36 (1.12)	3.00 (1.48)	0.00	-0.64	-0.11	-0.64	75.00	-0.25
Destiny	5.60 (0.55)	6.00 (0.00)	0.12	5.64 (0.50)	6.00 (0.00)	-0.01	0.04	-0.13	0.07	59.38	-0.68
Emma	0.60 (0.89)	0.00 (0.00)	0.14	0.64 (0.67)	1.00 (1.48)	0.00	0.04	-0.14	0.04	62.50	0.06
Faith	3.60 (0.55)	4.00 (0.00)	-0.13	2.82 (0.87)	3.00 (0.00)	0.00	-0.78	0.13	-1.43	75.00	-0.43
Gabriel	4.20 (1.10)	4.00 (0.00)	0.17	3.67 (1.37)	4.00 (1.48)	-0.01	-0.53	-0.18	-0.49	64.71	-0.15
Harper	2.20 (0.45)	2.00 (0.00)	0.01	2.10 (0.57)	2.00 (0.00)	0.00	-0.10	-0.01	-0.22	73.33	-0.08
Isaac	3.00 (1.00)	3.00 (1.48)	-0.02	2.33 (0.78)	2.00 (0.00)	-0.02	-0.67	0.00	-0.67	76.47	-0.31
Julia	3.20 (0.45)	3.00 (0.00)	0.08	3.75 (1.29)	3.50 (1.48)	0.03	0.55	-0.05	1.23	52.94	0.18
Kevin	1.20 (0.45)	1.00 (0.00)	0.02	0.73 (0.79)	1.00 (1.48)	-0.03	-0.47	-0.04	-1.06	75.00	-0.32
Lily	3.80 (0.45)	4.00 (0.00)	0.04	0.82 (0.75)	1.00 (1.48)	-0.03	-2.98	-0.07	-6.67	100	-0.75
Mason	4.20 (0.45)	4.00 (0.00)	-0.01	5.10 (1.20)	5.50 (0.74)	0.02	0.90	0.03	2.01	33.33	0.42

Quantity-number competencies (MBK 1+)

It is atypical for a single-case design, but we did the MBK 1+ a second time as a post-test measure. The children showed some surprising improvements after the intervention. Their mean score is 34.00 (SD = 7.90), and the mean percentile rank is 43.70 (SD = 23.82). Only three of the children have a percentile rank less than 18.

Discussion

In this study we examined the effects of a new working memory sensitive math intervention on mathematical precursors, in particular the relational understanding of numbers, of 13 primary school students. The results of our multiple-baseline design demonstrate that this treatment can potentially lead to enhancement in children's performance.

Participants for which we conclude effects show a gradual improvement in the number-bonds task. This task measures the understanding and automation of relations between numbers and is especially appropriate to show near training effects. As expected, improvements did not occur immediately, but rather after a few sessions. Only two cases showed an immediate change (Kevin and Lily), and were both taught by the same instructor. We could think of at least two explanations for this result: (1) Maybe the instructor always starts with the same task, which is particularly effective. It would therefore be an effect of the task. For example, in a recent paper Hawes and Ansari (2020) assume that tasks addressing spatial thinking are more appropriate for stimulating mathematical learning. It is possible that the greatest potential for mathematical learning lies in the combination and integration of spatial and numerical instruction. We have examined the possibility of a task effect for Kevin and Lily and can exclude it based on our material. (2) The second possibility refers to the classical experimenter effect. Characteristics of the instructor can also be associated with improvement as well as its absence. This could also be the case with Gabriel and Harper, who both did not improve their error rates. The implementation of the treatment – like any teaching situation – depends on the expertise of the instructor. Despite previous training of the instructors, their content knowledge and their pedagogical content knowledge (Shulman, 1987) may affect the participants' results. If the implementation of the approach turns out to be a major challenge, this would also limit its ecological validity. A practical approach should also prove its worth outside a laboratory situation.

The performance of Mason shows a very large variability. He received special educational services, due to learning difficulties, and showed some other traits that are unfavorably associated with learning: He showed impulsive behavior and attention problems and was easily frustrated, which might have hindered his learning progress. This assumption is in line with models that summarize influencing factors of impaired mathematics learning processes, e.g., the model of Jacobs and Petermann (2007). The treatment, or the mental confrontation with the learning object, requires a certain amount of attention. For some students it is possible to combine the treatment with an additional behavioral intervention in future studies.

The results of the addition test are less positive. Not surprisingly, the children who show no improvement in the number-bonds task also show no improvement in their numeracy skills. Only Lily reduced her error rates. Although the results of the other indices are much more positive, there was no effect at all for Emma, Gabriel, Harper and Mason. Nevertheless, this does not speak against the effectiveness of our intervention, since arithmetic was not explicitly addressed. The treatment focused on the understanding of numbers, so the solution of typical addition problems could already be seen as a far transfer effect. In connection with appropriate classroom-based instruction, such transfer effects are quite conceivable; however, we can't expect that to happen over a relatively short intervention period.

Although the results show an improvement in mathematical precursors measured by the MBK 1+, it is not clear whether this improvement is valid. For one thing, pre-test / post-test measurement is not the adequate analysis for a single-case design, and we interpret the pre-test / post-test comparison conservatively as a trend. Except for two students (Gabriel and Harper), the participants show an improvement in their MBK 1+ t-scores after the short time of our intervention. Within three weeks, the children enhanced their scores by 12.81 on average. Unfortunately, the retest reliability of the test varies between $r = .67$ and $r = .77$; we must consider the possibility that this is an effect of retesting. Despite all limitations, the improvement in MBK 1+ is compatible with the other results. Gabriel and Harper are not improving in any component during the intervention. It is equally consistent that Destiny and Faith show the smallest improvement. The remaining children increased their t-scores by at least 11 points. After the intervention, 6 out of 13 children were no longer at risk for mathematical learning difficulties (all above a percentile rank of 16). Considering the measurement error, only four children are still at risk.

Destiny was reported to be unfocused in conventional lessons. This problem was also noticed during the intervention sessions. Even if the tasks are very limited, they require a certain amount of attention.

We also measured a few working memory functions throughout the study. For the interpretation of the mathematical performance, we can only see one remarkable aspect here. The evidence suggests that Gabriel and Harper benefited least from the intervention. In terms of their working memory, they are by far the weakest in the Corsi block backward. At t1 both students were not able to remember a single item, so their memory span for this function is zero. The mean memory span at t1 is 2.31 ($SD = 1.32$). In the forward Corsi block task they do not show these difficulties. Here Gabriel showed a memory span of five, marking the upper end of the values. Harper has achieved a memory span of two, which indicates that the testing has generally worked. It seems that especially the combination of storing and processing dynamic visual-spatial information is critical. In some ways this fits with research suggesting that visual-spatial working memory is strongly related to numerical reasoning and mathematical skills in general (e.g., Raghubar, Barnes, & Hecht, 2010), even though it is not clear to what extent visual or rather spatial processes are involved (e.g., Hawes & Ansari, 2020; Passolunghi & Mammarella, 2012). Holmes, Adams and Hamilton (2008) provide evidence that the spatial subcomponent is particularly relevant in younger (seven- to eight-year-old) children; a block recall task predicted mathematics performance in this age group. Further evidence comes from Passolunghi and Mammarella (2012), who showed that children with poor mathematical skills exhibit particular deficits in dynamic spatial working memory measured by a Corsi block task. In our results, however, it is remarkable that both Corsi block tasks relate to spatial processes. Perhaps the critical point is the processing or rather manipulating component. This could indicate the particular importance of combining central executive and dynamic visual-spatial functioning.

Unfortunately, to this point the relationship between working memory and mathematics is scarcely understood, and we can only describe the phenomenon in our data. Working memory is operationalized very differently, which presents a general problem for comparing and interpreting the findings. Furthermore, there are hardly any studies that have used a backward condition of the Corsi block. Our purpose with this intervention was to offer a new approach especially to students like Gabriel and Harper, and we still assume that many nonresponders have limited cognitive resources. However, we assume that these children especially need a larger number of intervention sessions.

Limitations

There are, however, some limitations to this study. (1) This multiple-case design allows for no generalization beyond the context of this experiment, neither to the population with mathematical learning difficulties nor to the population of nonresponders. The validity of our findings needs to be verified through their replication and through more comprehensive research (Kazdin, 2011).

(2) Furthermore, we were able to show that the program has potential effects. But our intervention is composed of two aspects: working memory and mathematics. Since the two are integrated, it is difficult to show which parts of the intervention lead to an effect. Is it working memory, or the strongly structured mathematical content, or a combination? At the moment we cannot make any statements about the "construct validity" (Kazdin, 2011, p. 36). In order to clarify this, future studies should examine these questions, perhaps by varying crucial aspects of the experimental design. More complex single-case designs are also conceivable, e.g., an A-B-C design.

(3) Since the children have already been tested a lot, we did not capture other interesting variables such as intelligence or attention. These might help us to understand causal relationships in future studies. Current hypotheses about missing effects can also be tested in this manner. Relationships to working memory performance should also be focused further.

(4) Unfortunately, we did not collect any follow-up data. Upcoming school vacation prevented us from continuing with the measurement. The practical significance of the intervention also results from the stability of the effects (Prentice & Miller, 1992). Have the children overcome their knowledge gaps? Can these children now participate in conventional lessons without supplemental instruction? Or do they continuously need such intensive support? On the other hand, through follow-up studies we can see whether delayed effects occur in the children who have not improved yet.

(5) We couldn't check whether the germane cognitive load was actually increased by our intervention, because there are no standard, reliable, and valid measures for the different types of cognitive load (Moreno, 2010).

(6) Finally, no observational data could be collected to ensure treatment fidelity; and we rely on regular meetings with the instructors and their statements regarding treatment fidelity. In future studies the implementation of the intervention

should be monitored more closely.

Conclusion

Despite its limitations, this study provides first empirical support for the newly designed working memory sensitive intervention. Most of the participants showed small to large positive training effects, mainly with regard to the number-bonds task, which is near to the treatment. Further research is now needed to understand why the intervention works well for some children, works only moderately for others, or – just as important – fails to work. At this point we can only make cautious assumptions. However, considering the relatively short intervention period, our results are encouraging. Pressley et al. (1989) emphasize that good information processing cannot be achieved by time-limited interventions but is rather a product of many years of good teaching practice. Perhaps this also applies to our approach, and the working memory sensitive math intervention and its principles should be linked more closely to conventional classroom teaching.

Working memory can be situated in the broader construct of self-regulation, and in this sense, our intervention can be seen as a way of influencing the regulation of learning. Here, we refer to the phase during learning, but in this context, it is possible to include further aspects, perhaps those that are more conscious (e.g. student-centered strategies, which cause children to become more autonomous). The intervention itself, but also research, can be extended in this way. In the future, a better understanding of mathematical learning processes and their relationship to working memory should help to further develop interventions such as ours.

References

Alloway, T. P., & Alloway, R. G. (2010). Investigating the predictive roles of working memory and IQ in academic attainment. *Journal of Experimental Child Psychology*, *106*, 20–29. https://doi.org/10.1016/j.jecp.2009.11.003

Alloway, T. P., & Passolunghi, M. C. (2011). The relationship between working memory, IQ, and mathematical skills in children. *Learning and Individual Differences*, *21*, 133–137. https://doi.org/10.1016/j.lindif.2010.09.013

Ansari, D., Donlan, C., Thomas, M. S. C., Ewing, S. A., Peen, T., & Karmiloff-Smith, A. (2003). What makes counting count? Verbal and visuo-spatial contributions to typical and atypical number development. *Journal of Experimental Child Psychology, 85*, 50–62. https://doi.org/10.1016/S0022-0965(03)00026-2

Aunola, K., Leskinen, E., Lerkkanen, M.-K., & Nurmi, J.-E. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology, 96*, 699–713. https://doi.org/10.1037/0022-0663.96.4.699

Baddeley, A. D. (1986). *Working memory*. Oxford, UK: Oxford University Press.

Baddeley, A. D. (1996). Exploring the central executive. *Quarterly Journal of Experimental Psychology*, *49*, 5–28. https://doi.org/10.1080/713755608

Baddeley, A. D. (2012). Working memory: Theories, models, and controversies. *Annual Review of Psychology*, *63*, 1–29. https://doi.org/10.1146/annurev-psych-120710-100422

Bull, R., Espy, K. A., & Wiebe, S. A. (2008). Short-term memory, working memory, and executive functioning in preschoolers: Longitudinal predictors of mathematical achievement at 7 years. *Developmental Neuropsychology*, *33*, 205–228. https://doi.org/10.1080/87565640801982312

Clearman, J., Klinger, V., & Szucs, D. (2017). Visuospatial and verbal memory in mental arithmetic. *Quarterly Journal of Experimental Psychology, 70*, 1837–1855. https://doi.org/10.1080/17470218.2016.1209534

Corsi, P. M. (1972). Human memory and the medial temporal region of the brain. *Dissertation Abstracts International, 34*, 819. McGill University, Montreal, Canada.

Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. Trends in Neuroscience and Education, 3, 63–68. https://doi.org/10.1016/j.tine.2013.12.001

Ennemoser, M., Krajewski, K., & Sinner, D. (2017). *Test mathematischer Basiskompetenzen ab Schuleintritt (MBK 1+)* [Test of basic mathematical competencies in 1st grade]. Göttingen, Germany: Hogrefe.

Ennemoser, M., Sinner, D., & Krajewski, K. (2015). Kurz- und langfristige Effekte einer entwicklungsorientierten Mathematikförderung bei Erstklässlern mit drohender Rechenschwäche [Short and long-term effects of quantity-number training with first grade at-risk children]. *Lernen und Lernstörungen*, *4*, 43–59. https://doi.org/10.1024/2235-0977/a000091

Friso-van den Bos, I., van der Ven, S. H. G., Kroesbergen, E. H., & van Luit, J. E. H. (2013). Working memory and mathematics in primary school children: A meta-analysis. *Educational Research Review, 10*, 29–44. https://doi.org/10.1016/j.edurev.2013.05.003

Fuchs, L. S., Compton, D. L., Fuchs, D., Paulsen, K., Bryant, J. D., & Hamlett, C. L. (2005). The Prevention, Identification, and Cognitive Determinants of Math Difficulty. *Journal of Educational Psychology*, *97*, 493–513. https://doi.org/10.1037/0022-0663.97.3.493

Fuchs, D., & Fuchs, L. S. (2006). Introduction to Response to Intervention: What, why, and how valid is it? *Reading Research Quarterly*, *41*, 93–99. https://doi.org/10.1598/RRQ.41.1.4

Gast, D. L., Lloyd, B. P., & Ledford, J. R. (2018). Multiple baseline and multiple probe designs. In J. R. Ledford, & D. L. Gast (Eds.) *Single case research methodology* (pp. 239–281). New York, NY: Routledge. https://doi.org/10.4324/9781315150666-10

Gathercole, S. E., & Alloway, T. P. (2008). *Working memory and learning: A practical guide for teachers.* Los Angeles, Calif., London, New Delhi: SAGE.

Glass, G. V. (1976). Primary, secondary, and meta-analysis of research. *Educational researcher, 5*, 3–8. https://doi.org/10.3102/0013189X005010003

Griffin, S. (2004). Building number sense with Number Worlds: a mathematics program for young children. *Early Childhood Research Quarterly*, *19*, 173–180. https://doi.org/10.1016/j.ecresq.2004.01.012

Grube, D., & Seitz-Stein, K. (2012). Arbeitsgedächtnis und Rechnen [Working memory and arithmetic]. In M. Hasselhorn & C. Zoelch (Eds.), *Tests und Trends: N.F., 10. Funktionsdiagnostik des Arbeitsgedächtnisses* (pp. 145–157). Göttingen, Germany: Hogrefe.

Hecht, A. T. (2014). Ressourcenorientierte Lernförderung in der Grundschule: Der Einfluss des Aufgabendesigns auf die Übungsleistungen von Zweitklässlern in Rechtschreiben und Mathematik [Resource-oriented learning support in primary school: The influence of task design on the performance of second graders in writing and arithmetic]. Dissertation. Justus-Liebig-Universität Gießen. http://geb.uni-giessen.de/geb/volltexte/2014/10981/pdf/HechtAgnesTeresa_2014_07_08.pdf. Accessed 4 April 2018.

Harp, S. F., & Mayer, R. E. (1997). The role of interest in learning from scientific text and illustrations: On the distinction between emotional interest and cognitive interest. *Journal of Educational Psychology*, *89*, 92–102. https://doi.org/10.1037/0022-0663.89.1.92

Hawes, Z., & Ansari, D. (2020). What explains the relationship between spatial and mathematical skills? A review of evidence from brain and behavior. *Psychonomic Bulletin & Review*, *27*, 465–482. https://doi.org/10.3758/s13423-019-01694-7

Jacobs, C., & Petermann, F. (2007). *Rechenstörungen* [Dyscalculia]. Göttingen, Germany: Hogrefe.

Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences*, *20*, 82–88. https://doi.org/10.1016/j.lindif.2009.07.004

Kazdin, A. E. (2011). Single-case research designs: Methods for clinical and applied settings (2nd ed.). New York, NY: Oxford University Press.

Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist*, *41*, 75–86. https://doi.org/10.1207/s15326985ep4102_1

Klesczewski, J., Brandenburg, J., Fischbach, A., Schuchardt, K., Grube, D., Hasselhorn, M., & Büttner, G. (2018). Development of working memory from grade 3 to 5: Differences between children with and without mathematical learning difficulties. *International Journal of Disability, Development and Education*, *65*, 509–525. https://doi.org/10.1080/1034912X.2017.1419555

Krajewski, K. (2003). Vorhersage von Rechenschwache in der Grundschule [Prediction of mathematical learning difficulties in primary school]. In *Schriftenreihe Studien zur Kindheits- und Jugendforschung*: Bd. 29. Hamburg, Germany: Kovac.

Krajewski, K. (2005). Vorschulische Mengenbewusstheit von Zahlen und ihre Bedeutung für die Früherkennung von Rechenschwache [Pre-school quantity-number awareness and their importance for the early recognition of mathematical learning difficulties]. In M. Hasselhorn, H. Marx & W. Schneider (Eds.), *Diagnostik von Mathematikleistungen* (pp. 49–70). Göttingen, Germany: Hogrefe.

Krajewski, K. (2008). Prävention der Rechenschwäche. [The early prevention of math problems]. In W. Schneider, & M. Hasselhorn (Eds.), *Handbuch der Pädagogischen Psychologie* (pp. 360-370). Göttingen, Germany: Hogrefe.

Krajewski, K., & Ennemoser, M. (2010). Die Berücksichtigung begrenzter Arbeitsgedächtnisressourcen in Unterricht und Lernförderung [The use of limited working memory resources in teaching and learning support]. In H.-P. Trolldenier, W. Lenhard & P. Marx (Eds.), *Brennpunkte der Gedächtnisforschung* (pp. 337–365). Göttingen, Germany: Hogrefe.

Krajewski, K., &. Schneider, W. (2009a). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. *Learning and Instruction*, *19*, 513–526. https://doi.org/10.1016/j.learninstruc.2008.10.002

Krajewski, K., & Schneider, W. (2009b). Exploring the impact of phonological awareness, visual–spatial working memory, and preschool quantity–number competencies on mathematics achievement in elementary school: Findings from a 3-year longitudinal study. *Journal of Experimental Child Psychology, 103*, 516–531. https://doi.org/10.1016/j.jecp.2009.03.009

Kroesbergen, E. H., & van Luit, J. E. H. (2003). Mathematics interventions for children with special educational needs: A meta-analysis. *Remedial and Special Education*, *24*, 97–114. https://doi.org/10.1177/07419325030240020501

Kuhl, J., Sinner, D., & Ennemoser, M. (2012). Training quantity-number competencies in students with intellectual disabilities. *Journal of Cognitive Education and Psychology, 11*, 128–142. https://doi.org/10.1891/1945-8959.11.2.128

Locuniak, M. N., & Jordan, N. C. (2008). Using kindergarten early math to predict calculation fluency in second grade. *Journal of Learning Disabilities, 41*, 451–459. https://doi.org/10.1177/0022219408321126

Magner, U. I. E., Schwonke, R., Aleven, V., Popescu, O., & Renkl, A. (2014). Triggering situational interest by decorative illustrations both fosters and hinders learning in computer-based learning environments. *Learning and Instruction*, *29*, 141–152. https://doi.org/10.1016/j.learninstruc.2012.07.002

Melby-Lervåg, M., & Hulme, C. (2013). Is working memory training effective? A meta-analytic review. *Developmental Psychology*, *49*, 270–291. https://doi.org/10.1037/a0028228

Michalczyk, K., Krajewski, K., Preßler, A. L., & Hasselhorn, M. (2013). The relationships between quantity-number competencies, working memory, and phonological awareness in 5- and 6-year-olds. *British Journal of Developmental Psychology, 31*, 408–424. https://doi.org/10.1111/bjdp.12016

Miyake, A, Friedman, N. P, Emerson, M. J, Witzki, A. H, Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex "Frontal Lobe" tasks: A latent variable analysis. *Cognitive Psychology*, *41*, 49–100. https://doi.org/10.1006/cogp.1999.0734

Monsell, S. (2003). Task switching. *Trends in Cognitive Sciences, 7*, 134–140. https://doi.org/10.1016/S1364-6613(03)00028-7

Moreno, R. (2010). Cognitive load theory: more food for thought. *Instructional Science*, *38*, 135–141. https://doi.org/10.1007/s11251-009-9122-9

Nys, J., Content, A., & Leybaert, J. (2013). Impact of language abilities on exact and approximate number skills development: evidence from children with specific language impairment. *Journal of Speech, Language, and Hearing Research*, *56*, 956–970. https://doi.org/10.1044/1092-4388(2012/10-0229)

Paas, F., & Sweller, J. (2014). Implications of cognitive load theory for multimedia learning. In R. E. Mayer (Ed.), *Cambridge handbooks in psychology. The Cambridge handbook of multimedia learning* (p. 27–42). New York, NY: Cambridge University Press. https://doi.org/10.1017/CBO9781139547369.004

Paas, F., & van Gog, T. (2006). Optimising worked example instruction: Different ways to increase germane cognitive load. *Learning and Instruction*, *16*, 87–91. https://doi.org/10.1016/j.learninstruc.2006.02.004

Paas F., Touvinen, J., Tabbers, H., & van Gerven, P. W. M. (2003). Cognitive load measurement as a means to advance Cognitive Load Theory. *Educational Psychologist*, *38*, 63–71. https://doi.org/10.1207/S15326985EP3801_8

Parker, R., Hagan-Burke, S., & Vannest, K. J. (2007). Percentage of All Non-Overlapping Data (PAND): An Alternative to PND. *Journal of Special Education*, *40*, 194–204. https://doi.org/10.1177/00224669070400040101

Passolunghi, M. C., & Costa, H. M. (2019). Working Memory and Mathematical Learning. In A. Fritz, V. G. Haase, & P. Räsänen (Eds.), *International Handbook of Mathematical Learning Difficulties: From the Laboratory to the Classroom* (pp. 407–421). Cham, Switzerland: Springer. https://doi.org/10.1007/978-3-319-97148-3_25

Passolunghi, M. C., & Mammarella, I. C. (2012). Selective spatial working memory impairment in a group of children with mathematics learning disabilities and poor problemsolving skills. *Journal of Learning Disabilities, 45*, 341–350. https://doi.org/10.1177/0022219411400746

Passolunghi, M. C., Vercelloni, B., & Schadee, H. (2007). The precursors of mathematics learning: working memory, phonological ability and numerical competence. *Cognitive Development, 22*, 165–184. https://doi.org/10.1016/j.cogdev.2006.09.001

Petermann, F. (2017). Wechsler intelligence scale for children - Fifth edition. WISC-V. Frankfurt a. M., Germany: Pearson.

Prentice, D. A., & Miller, D. T. (1992). When small effects are impressive. *Psychological Bulletin, 112*, 160–164. https://doi.org/10.1037/0033-2909.112.1.160

Preßler, A.-L., Krajewski, K., & Hasselhorn, M. (2013). Working memory capacity in preschool children contributes to the acquisition of school relevant precursor skills. *Learning and Individual Differences, 23*, 138–144. https://doi.org/10.1016/j.lindif.2012.10.005

Pressley, M., Borkowski, J. G., & Schneider, W. (1989). Good information processing: What it is and how education can promote it. *International Journal of Educational Research*, *8*, 857–867. https://doi.org/10.1016/0883-0355(89)90069-4

R Core Team (2013). *R: A Language and Environment for Statistical Computing.* Vienna, Austria. https://www.r-project.org

Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and Individual Differences*, *20*, 110–122. https://doi.org/10.1016/j.lindif.2009.10.005

Röhm, A. (2020). Sprache, Arbeitsgedächtnis und mathematische Kompetenz von Schulkindern mit SES [Language, working memory, and mathematical skills of schoolchildren with DLD]. *Lernen und Lernstörungen, 9*, 85–96. https://doi.org/10.1024/2235-0977/a000292

Röhm, A., Starke, A., & Ritterfeld, U. (2017). Die Rolle von Arbeitsgedächtnis und Sprachkompetenz für den Erwerb mathematischer Basiskompetenzen im Vorschulalter [The role of working memory and language skills in the acquisition of basic mathematical skills at pre-school age]. *Psychologie in Erziehung und Unterricht*, *64*, 81–93. https://doi.org/10.2378/peu2016.art26d

Sala, G., & Gobet, F. (2017). Working memory training in typically developing children: A meta-analysis of the available evidence. *Developmental Psychology, 53*, 671–685. https://doi.org/10.1037/dev0000265

Schulze, S., & Kuhl, J. (2019). Integration von Arbeitsgedächtnistrainings in die mathematische Lernförderung [Mathematical learning support and integrated working memory training]. *Lernen und Lernstörungen*, *7*, 47–59. https://doi.org/10.1024/2235-0977/a000229

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*, 1–22. https://doi.org/10.17763/haer.57.1.j463w79r56455411

Sinner, D. (2011). *Prävention von Rechenschwäche durch ein Training mathematischer Basiskompetenzen in der ersten Klasse* [Prevention of mathematical learning difficulties by training basic mathematical skills in first grade]. Inaugural-Disseration. Justus-Liebig-Universität Gießen. http://geb.uni-giessen.de/geb/volltexte/2011/8198/pdf/SinnerDaniel_2011_05_25.pdf. Accessed 4 April 2018.

Simanowski, S., & Krajewski, K. (2017). Specific preschool executive functions predict unique aspects of mathematics development: A 3-year longitudinal study. *Child Development*, *90*, 544–561. https://doi.org/10.1111/cdev.12909

Smith, C. R. (2004). *Learning diasbilities: The interaction of learner, task, and setting* (5^{th} ed.). Boston, MA.: Allyn and Bacon.

Strobach, T., & Karbach, J. (2016). *Cognitive Training: An Overview of Features and Applications.* Cham, Switzerland: Springer International Publishing. https://doi.org/10.1007/978-3-319-42662-4

Swanson, H. L. (2014). Does cognitive strategy training on word problems compensate for working memory capacity in children with math difficulties? *Journal of Educational Psychology, 106,* 831–848. https://doi.org/10.1037/a0035838

Swanson, H. L. (2015). Cognitive strategy interventions improve word problem solving and working memory in children with math disabilities. *Frontiers in Psychology, 6*, 1099. https://doi.org/10.3389/fpsyg.2015.01099

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science*, *12*, 257–285. https://doi.org/10.1207/s15516709cog1202_4

Sweller, J. (1989). Cognitive technology: Some procedures for facilitating learning and problem solving in mathematics and science. *Journal of Educational Psychology, 81*, 457–466. https://doi.org/10.1037/0022-0663.81.4.457

Sweller, J., & Chandler, P. (1991). Evidence for cognitive load theory. *Cognition and Instruction*, *8*, 351–362. https://doi.org/10.1207/s1532690xci0804_5

Tarlow, K. R. (2017). An improved rank correlation effect size statistic for single-case designs: baseline corrected Tau. *Behavior Modification*, *41*, 427–467. https://doi.org/10.1177/0145445516676750

Viesel-Nordmeyer, N., Ritterfeld, U., & Bos, W. (2020). Die Rolle von Sprache und Arbeitsgedächtnis für die Entwicklung mathematischen Lernens vom Vorschul- bis ins Grundschulalter [The role of language and working memory in the development of mathematical learning from preschool into primary school age – Longitudinal and cross-sectional path analyses of data from the national educational panel study (NEPS)]. *Lernen und Lernstörungen*, *9*, 97–110. https://doi.org/10.1024/2235-0977/a000291

Vukovic, R. K., & Siegel, L. S. (2010). Academic and cognitive characteristics of persistent mathematics difficulty from first through fourth grade. *Learning Disabilities Research & Practice*, *25*, 25–38. https://doi.org/10.1111/j.1540-5826.2009.00298.x

Wilbert, J., & Lüke, T. (2019). scan: Single-Case Data Analyses for Single and Multiple Baseline Designs. Version: 0.40. https://cran.r-project.org/package=scan

Wiley, J., Sanchez, C. A., & Jaeger, A. L. (2014). The individual differences in working memory capacity principle in multimedia learning. In R. E. Mayer (Eds.), *Cambridge handbooks in psychology. The Cambridge handbook of multimedia learning* (pp. 598–619). New York, NY: Cambridge University Press. https://doi.org/10.1017/CBO9781139547369.029

Wilhelm, O., Hildebrandt, A., & Oberauer, K. (2013). What is working memory, and how can we measure it? *Frontiers in Psychology, 4*, Article 433. https://doi.org/10.3389/fpsyg.2013.00433