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DIDACTICAL ENGINEERING OF THE SECOND GENERATION: A PROPOSAL OF THE DESIGN AND A TEACHING RESOURCE WITH THE SUPPORT OF THE GEOGEBRA SOFTWARE IN BRAZIL

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Abstract: This article presents a partial excerpt from an ongoing master's research on the methodological theoretical aspects of problems related to the content of volumes, selected from the evaluations of the National High School Exam (ENEM). The Exam consists of a test proposed by the Brazilian federal government, which aims to assess the performance of students at the end of high school, in addition to being used as a selection criterion to enter higher education. In this way, a didactic situation is proposed in the context of ENEM structured in the Theory of Didactic Situations (TSD) and subsidized by the use of the GeoGebra software as a technological resource for the construction and resolution of the problem situation on the volume of a cylinder. The didactic session was consolidated in Didactic Training Engineering (EDF) and developed to design a didactic resource for teaching and training mathematics teachers. GeoGebra was used to carry out the didactic transposition of the problem, as it benefits the student to break the difficulties inherent in the understanding and appropriation of geometric concepts. This structure facilitates didactic mediation, in addition to giving a new meaning to the study and teaching of volumes in the context of ENEM assessments.

Key words: Didactical Engineering of Second Generation, Didactic Situation, Didactic Resource, GeoGebra Software, Teacher training in Brazil.

1. Introduction

In the last decades, there has been a marked contribution of methodological and investigative resources from the Didactics of Mathematics, of French origin, related to the training of Mathematics teachers and the design and proposition of didactic and methodological resources that can support the teacher to improve planning and the execution of its didactic sessions for teaching mathematics. In Brazil, for example, it is undeniable the existence of difficulties and obstacles in the study and teaching of mathematics by teachers during their respective academic training and the exercise of the profession, because according to Damaceno (2018, p. 43), the problem "consists in the initial training of teachers, who work in the teaching of mathematics, who for the most part finish their courses without the essential knowledge for the exercise of teaching, with regard to concepts, procedures and mathematical language ".

From a concern with teaching and learning the content of volumes in the Brazilian scenario, the problem that guided our investigation was constituted: How to conceive a didactic resource that allows to improve the teaching of volumes, in order to develop both the student learning regarding the teacher education?

In this sense, the notion of Didactic Training Engineering (EDF) or Didactic Engineering of 2nd Generation is embodied as a methodology or design that relates research and teaching in order to develop resources for teaching Mathematics, as well as for the formation of math teachers. In Tempier (2016, p. 263), he describes this methodology as an instance of the basic design of the research with some peculiarities, among them, being "supported by an explicit basic theory, of intermediate level,

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the Theory of Didactic Situations (TSD). Such a relationship will allow the construction of a situation through an epistemological study of a mathematical concept, in order to provide a theoretical model that serves as a basis for designing situations for a resource that guarantees the verification of the study and promotes student leadership.

In view of this scenario, the article presents a didactic situation proposal for the teaching of Geometry, specifically for the teaching of the mathematical concept of volume of a cylinder, which is capable of promoting the development of skills that assist in problem solving, intuitive reasoning and communication, supported by the manipulation of the GeoGebra software, because according to Alves (2019, p. 115), the potential of the GeoGebra software provides the teacher with a way to encourage students to participate in the "dynamic exploration of numerical and geometric properties , so that visualization, perception and intuition play an essential role in the evolution of learning for everyone involved in each didactic situation.

In this case, we sought to address a problem of teaching a specific content to questions selected from the tests of the National High School Exam (ENEM), relating them to the initial teacher training and the conditions of applicability in regular education. Given the above, it is important to open a parenthesis to highlight the relevance of such constant assessments in the Brazilian educational system.

The National High School Exam (ENEM) is a oficial test of a program of the Brazilian federal government created in 1998, with the objective of evaluating the student's performance at the end of high school. After 22 years, these assessments have provided students with the opportunity to enter higher education, in addition to verifying the knowledge acquired during regular basic education. Such assessments are different from the traditional vestibular models, as it intends to verify if the students have acquired conditions to apply the concepts learned in the classroom in everyday situations, that is, if it is necessary to realize if the acquired knowledge is continuous or if it simply it comes down to decorated formulas and calculations.

It also highlights the importance of using educational software in mathematics classes, especially in Spatial Geometry. According to Kirikçilar and Yildiz (2018), dynamic math software is considered more influential in students' learning than other classroom tools, as its manipulation allows students to visualize (2D / 3D) the geometric relationships between points, the lines, polygons, circles and other geometric concepts facilitating the structuring of concepts and the development of mathematical thinking.

Given this context, we sought to conduct an investigation of didactic elements in an aspect of the evolution of the problem situations on volumes collected from the National High School Exam - ENEM evaluations, for two reasons: the first concerns the learning difficulties of such content , because according to Van Der Mer (2017, p. 5), despite the theme being present in several everyday situations, it is still found that students have difficulties in solving simple problems involving volumes, "in many cases, students mechanically use formulas without demonstrating a real understanding of the concept ".

The second is due to the fact that this concept is present in most of the National High School Exam ENEM tests, presenting predominant characteristics of the "relationship between mathematical models and models of school geometric solids (prisms, pyramid, cylinder, cone and sphere) and the use of formulas "(Morais, 2013, p.15). To support the situation, the Didactic Situations Theory (TSD), developed by Brousseau (1986), was used. It is a theory that makes it possible to conceive didactic teaching situations that lead to greater interaction between teacher, student and mathematical knowledge. Artigue (2015) draws attention to the fact that a situation must be planned in order to guarantee the occurrence of an adidactic interaction with the environment, in addition to stressing the importance of collective discussions mediated by teachers.

Given the above, in the following sessions, the notion of a problem situation on the volume content of a cylinder is introduced. Thus, the situation will be validated and substantiated through the experience of the four phases of the TSD (action, formulation, validation and instiutitionalization), using the GeoGebra software as an educational technological aid. This theory allows us to model a didactic resource that serves as input for mathematics teachers in initial or continuous training, based on EDF, thus establishing an environment appropriate to the understanding of essential concepts in the elaboration of strategies and problem solving schemes, in addition to enriching geometric thinking and special visualization of the volume content.

2. Didactical Engineering of the Second Generation

When thinking about the formation of the mathematics teacher, the need to implement the production of teaching didactic situations is evaluated, of adapting methodologies and tools that can contribute to the planning, execution and the corresponding transposition of didactic situations that allows a more significant teaching in the exercise of the teaching profession. So, Tempier (2013) highlights the need for Training and Development Engineering, when reporting and indicating the following aspects:

> Thus, Didactical Engineering (development) appears as an "interface" between research and teaching [...]. This type of research aims to develop resources for teachers, while increasing knowledge for basic research on the didactic transposition of the studied mathematical concept (Tempier, 2013, p. 188).

The Didactical Engineering of Training, also known as Didactic Engineering of 2nd generation, presents, according to Perrin-Glorian (2019) the study of its adaptation to the conditions of regular education aiming at the dissemination of these situations in basic education, through the production of resources and the need for teacher training. This can cause different levels of construction, including classroom situations in which the teacher / researcher is led to describe the actions selected to intervene in teaching. Thus, the role of the teacher as a learning mediator is evidenced, therefore needing to plan and anticipate the obstacles that may arise in the development of situations, therefore, facilitating the way of conducting the process of construction and appropriation of mathematical knowledge by the student. For Perrin-Glorian (2019, p. 117), in an educational engineering of development and training “research questions come directly from a teaching problem, from a specific content and include issues of teacher training, as well as, the cyclical nature of the methodology, which requires numerous achievements and a cooperation of responsibility and theory between researchers, professors and even trainers in the development of engineering, as illustrated in the diagram in Figure 1.

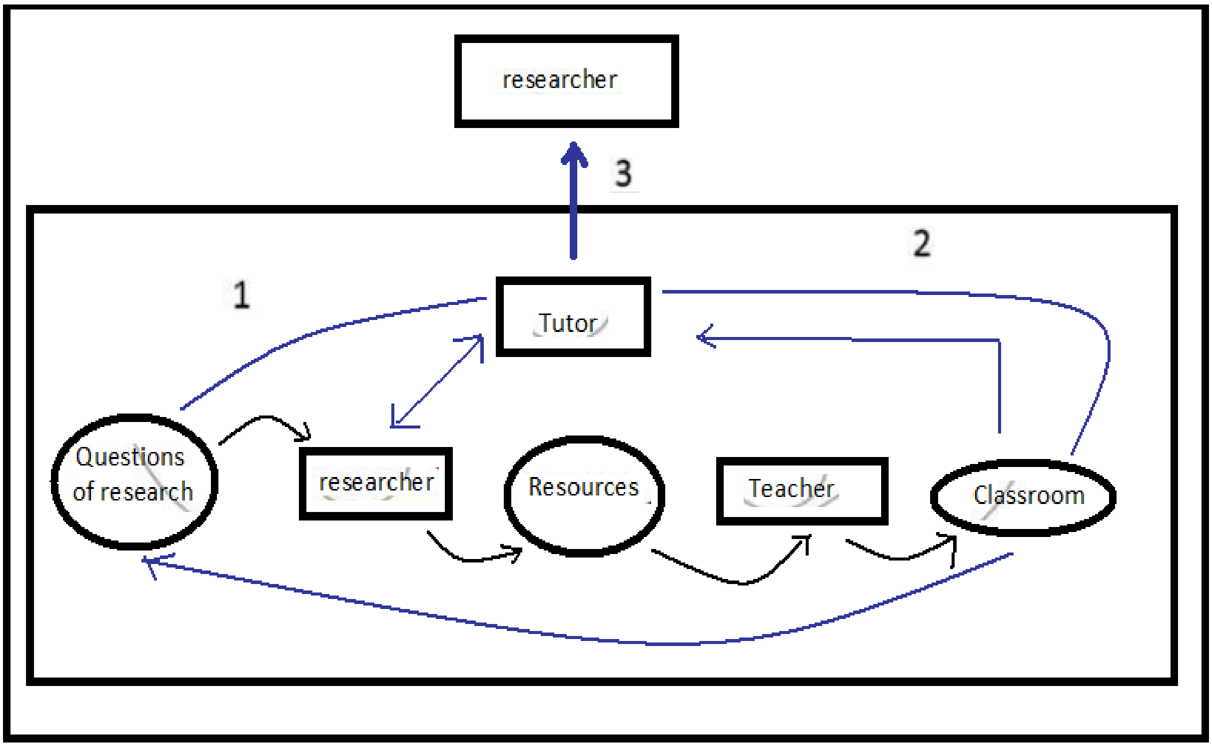

Figure 1: *Perrin-Glorian & Bellemain (2019) describe the organization of work among researchers, coaches and teachers at IDD. (Modified by the authors)*

The diagram in Figure 1 above describes the levels of cooperation between the actors involved in the development of EDF: the first sphere clearly shows the collaboration of the smaller group, in which the researcher and the trainer design the different versions of the resources, worked on during the analysis a priori of the situations. The presence of researchers, trainers and teachers play a major role in converting research into teaching, creating a place where the research hypotheses (as illustrated by arrow 1 in the diagram in Figure 1), are transformed into working hypotheses, and thus evolving for regular education (inner arrows of the large rectangle). Such transformations can specify or modify the research hypotheses, as can be seen in arrows 2 and 3. The team of researchers and trainers conduct the device and develop temporary variants of the resource that will be tested by the coaches, before being presented to the teachers who will apply classroom to be assessed (as shown in the large rectangle in Figure 1). Thus, the design of resources is organized according to the circuit of the arrows, in order to produce teaching sequences that may be suitable for use and disseminated in regular education (Perrin-Glorian & Bellemain, 2019).

According to Tempier (2012), EDF is also presented by a teaching methodology that can be used to design educational resources for teachers, allowing both student learning and teacher training. This type of research requires at least two engineering levels with different objectives, as shown in the diagram in Figure 2.

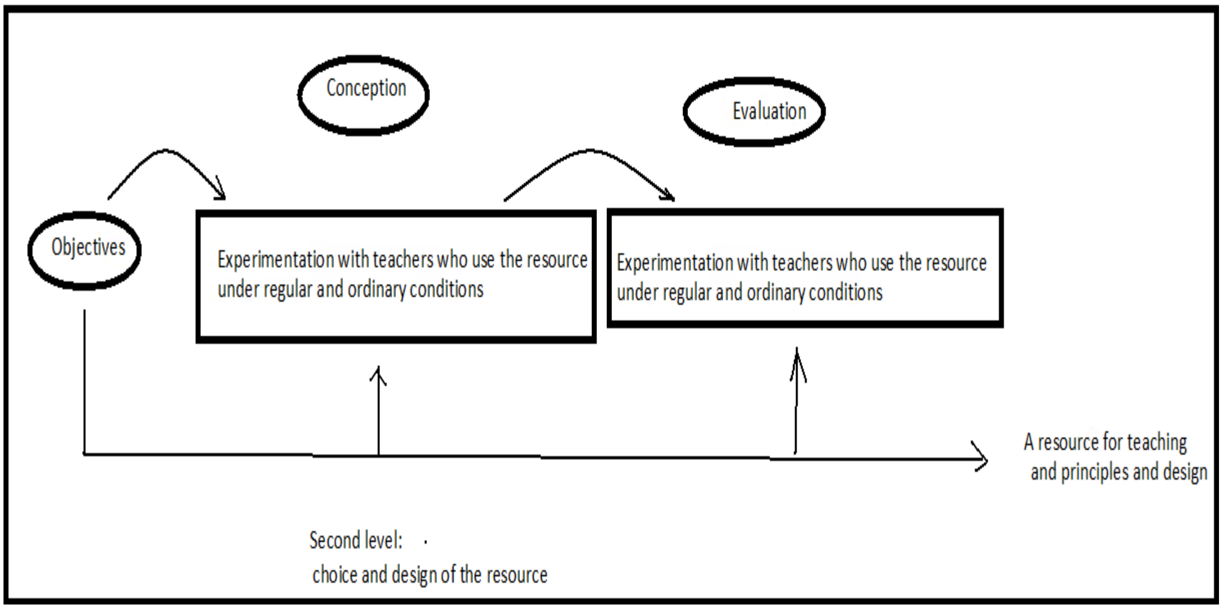

Figure 2: *Tempier (2012) describes the EDF methodology for designing a teaching resource and continuing teacher training for mathematics teachers in France. (Modified by the authors)*

In the scheme in figure 2, it is clear that after setting the objectives, there is a moment of design conception on two levels: on the first level, there are specific fundamental choices to test the theoretical validity of didactic situations, which may be an epistemological analysis of the mathematical content of the research, as well as a study of the possible options for teaching, enabling the construction of learning situations to be worked on in the classroom. At the second level, there are the choices of the type of resources, which will allow the study of the adaptability, predictability and reproducibility of the situations of regular education, the difference of an implementation and the transformations carried out as an object of study of the repercussions of the engineering itself.

In a third moment, during experimentation, teachers use resources under normal conditions, that is, they apply them in the classroom to validate learning situations. And, finally, they formalize the conception of a resource for teaching the researched mathematical object, consolidated in the design principles presented in the diagram of figure 2 (Tempier, 2012). In the case of this research, it is believed that both levels are necessary, as it presents elements that include both segments in its proposal, since it proposes an option for teaching, as well as the choice of a resource that will allow studying adapting a didactic situation during class.

The following section presents some assumptions capable of impacting the teaching process and the formation and improvement of the mathematics teacher's role, as well as some essential elements for a volume teaching model, in the context of the National High School Exam (ENEM).

3. Theory of Didactical Situation (TSD)

A problem situation structured in the Theory of Didactic Situations (TSD) was used to model a didactic teaching situation capable of assisting the teacher about volume learning, as it provides the means to investigate all the problems related to mathematical learning and reveal aspects that occurred during problem solving and the elaboration of concepts by students. According to Alves (2016, p. 59), TSD “allows the theorized discussion of a series of actions closely related to the trinomial teacher - students - mathematical knowledge”, therefore, it can be considered as an important ally in the elaboration and structuring of didactic situations that promotes the development of mathematical knowledge.

The Theory of Didactic Situations is a theoretical model developed by a French researcher, who considers teaching as a social action project capable of transforming a student by adapting it to a certain knowledge that is in the formation stage or already constituted. According to Freitas (2015):

> A didactic situation is a set of relationships established explicitly and or implicitly between a student or a group of students, in a certain environment, eventually comprising instruments and objects, and an educational system (the teacher) with the purpose of enabling these students to learn constituted or about to be constituted (…) the student's work should, at least in part, reproduce characteristics of the scientific work itself, as a guarantee of an effective construction of pertinent knowledge (Freitas, 2015, p. 80).

According to Brousseau (2008, p. 21), a situation is “a model of interaction between a subject and a determined environment”, allowing the apprentice to use all the knowledge and resources available to him to achieve and preserve relations with the environment (*milieu*). For the author, the mileu is an autonomous system, antagonistic to the subject.

In the trajectory for the construction of knowledge, some variables may arise, which are presented in two ways: didactic situation and adidactic situation. The first corresponds to an interaction of the student in a game situation, which happens whenever there is an intention of the teacher to develop the student's learning. In the second, the student evolves on his own merit in the attempt to acquire knowledge, following the rules of the game, so the teacher must program the means that enable students to create their own strategies to solve the issue at stake. When analyzing the relationship between the teaching and learning activities of mathematical knowledge Brousseau (1986) developed four stages for didactic situations, be they of action, formulation, validation and institutionalization. Defined by Sousa, Alves and Fontenele (2020), as:

1. Action Situation: “Students facing a didactic situation, relate to the class, create procedures, plan and formulate hypotheses for problem solving” (Sousa; Alves & Fontenele, 2020, p. 13).

Thus, in the diagram of Figure 3, a conventional and simplified model about knowledge in interaction with a medium in the context of a situation is presented. In the scheme of Figure 3, it can be seen that there is no mediation by the teacher and the procedures used make the learner actively participate in the elaboration of cognition, thus developing new knowledge, based on personal experiences and promoting interaction with the environment in which he lives.

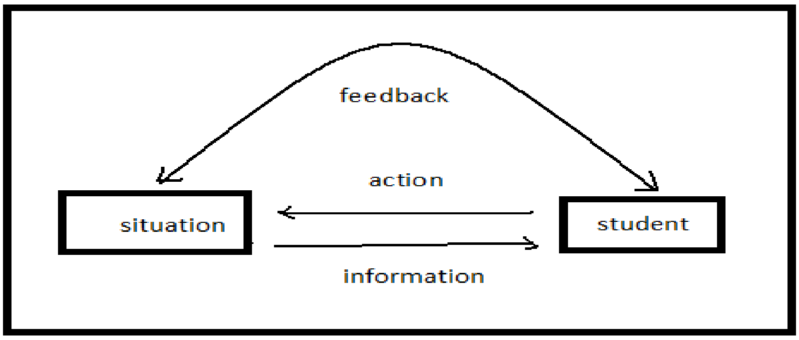

Figura 3: *General scheme of the dialectical situation of action. (Brousseau, 1997) (Modified by the authors)*

2. Formulation situation: "In this stage there are exchanges of knowledge between students, in which they try to modify the mathematical language, contextualizing it to meet their goals that were previously planned" (Sousa, Alves & Fontenele, 2020, p. 13)

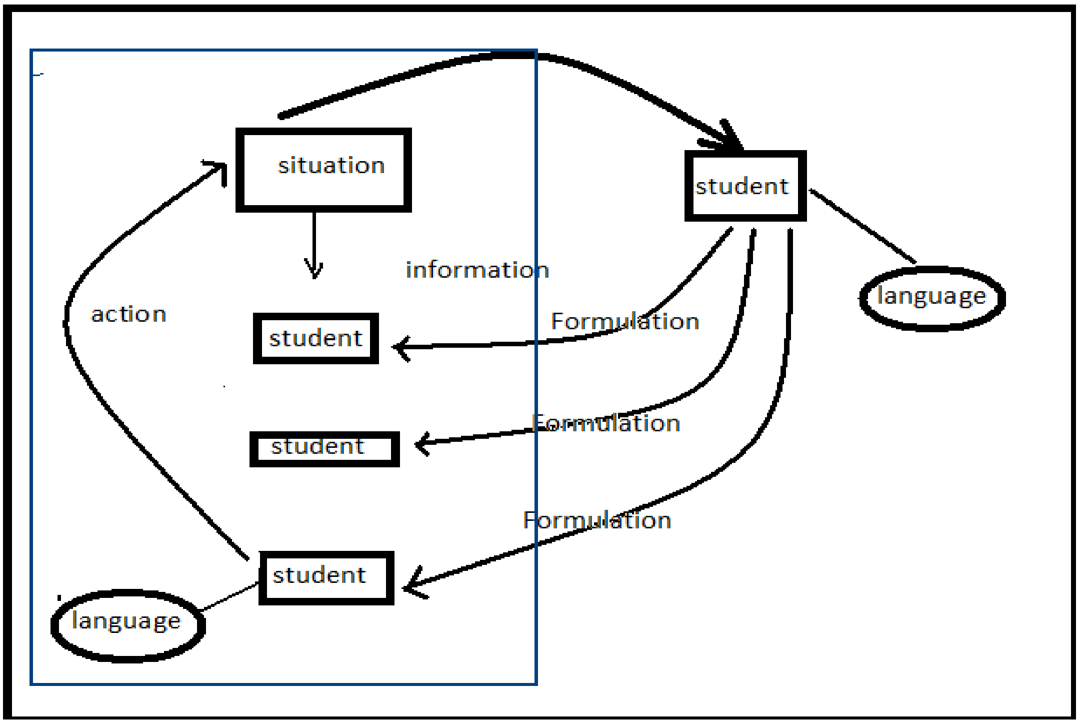

Figure 4: Scheme of the formulation situation described by TSD *(Brousseau, 1997). (Modified by the authors)*

The diagram in figure 4, depicts the student's ability to recognize, identify, break and reconstruct a linguistic system, in an environment in which communication is essential to be successful in formulating targeted knowledge (Brousseau, 1997).

3. Validation Situation: "At this moment, the organization of what has been studied and verified by the students occurs, if the resolutions obtained satisfy the expected, that is, if there was the construction of new knowledge" (Sousa, Alves & Fontenele, 2020, p. 13). At that moment, an empirical or cultural correction process is configured, to guarantee the adaptation, adequacy and relevance to the knowledge mobilized by the apprentices during the action and formulation process (Brousseau, 1997).

4. Institutionalization situation: "In it, the teacher and the student validate knowledge. It is at this stage that the teacher reveals his intentions in the face of didactic situations" (Sousa, Alves & Fontenele, 2020, p. 13). At this stage, an attempt is made to change individual knowledge to a more cultural and

historical dimension of scientific knowledge. Therefore, definitions and properties are presented in a more formal mathematical language. Figure 5 shows a relationship between the intention to teach and the knowledge transmission process.

The representation of the diagram in Figure 5, presents the work used in the resolution of a problem situation by the subjects involved in the activity, as well as the process of construction and elaboration of the didactic situations by the teachers, because according to Alves (2019) the mentioned elements demonstrate a clear influence of the teacher's action on the exposition of a mathematical concept.

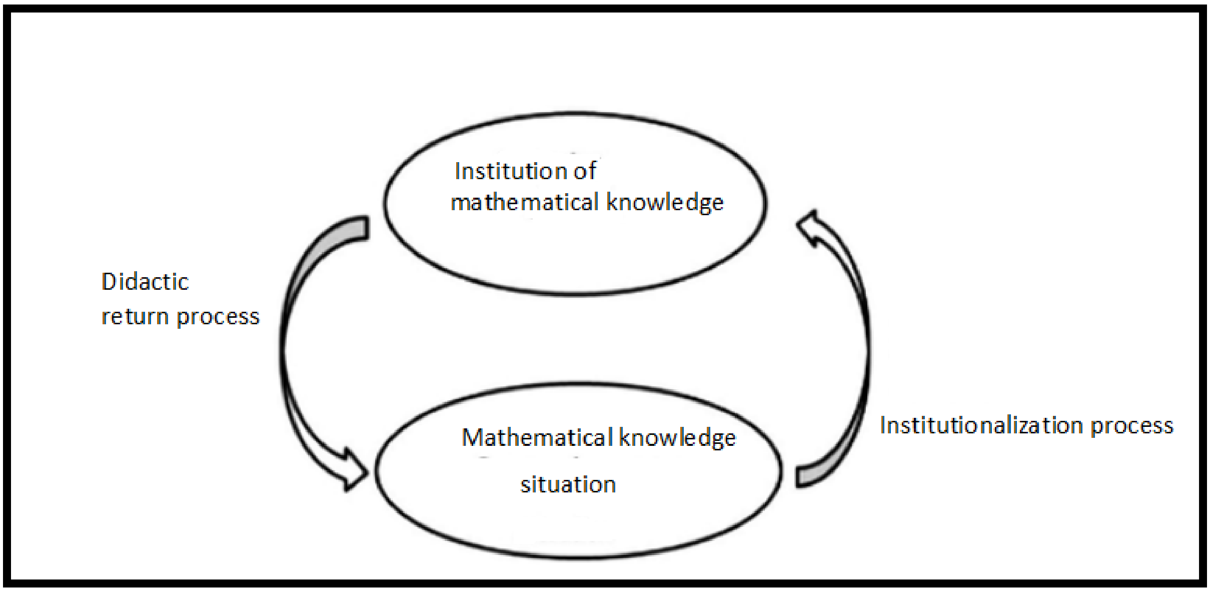

Figure 5. *Margolinas (2015) describes the dialectic between savoir / conaissance and its important role for Didactics of Mathematics. (Modified by the authors)*

With that, we can analyze the importance that the TSD attributes to the procedures that are related within the steps described here, promoting the participation of the student (academic) in order to contribute in the cognitive elaboration of the development of new mathematical knowledge. Furthermore, there is a possibility of approaching a certain mathematical concept, using both the research methodology and the teaching methodology in the initial and even continuing teacher training courses.

4. Methodological procedures

This investigation was scientifically based on the research methodology of Didactic Training Engineering (EDF), with the objective of designing a didactic resource for teaching the concept of the volume of a straight circular cylinder. A theory that has historically been associated with Didactic Engineering, the Didactic Situations Theory (TSD), was adopted as the teaching methodology, in which its dialectical faces (action, formulation, validation and institutionalization) are used to structure an example of application of a didactic situation about volumes within an ENEM exam problem.

According to Tempier (2012), a TSD allows the construction of learning situations when implemented in the classroom, in addition to allowing the teacher to appropriate the questions, explaining the connection between the game, the situation variables and the knowledge to be trained. The GeoGebra software software was used as a teaching resource to model the mathematical situation, improving the visualization and interpretation of the numerical and geometric properties of the selected problem.

The mathematical foundation, supported by previous knowledge and structured in a didactic sequence, according to the assumptions of Brousseau's dialectics, aims to know the epistemological obstacles of the learners during the development of the didactic situation, in addition to enabling such difficulties to be gradually overcome.

With regard to teacher training, it is expected to present elements for understanding the challenges of teaching the volume of a straight cylinder, developing situations to be implemented in the classroom that allow the construction of this knowledge by students, in addition to generating strategies that can be adapted and used in other teaching situations. In this study, a description of the possible solutions and behaviors of the students is presented based on a didactic situation. For the elaboration of the situation, the same process used by Santos and Alves was adopted, as follows:

> […] In the elaboration of these situations, we performed a descriptive and predictive action, in order to establish the possible behaviors and resolutions of the students, according to each phase of the TSD, in addition to the survey of some didactic hypotheses related to the proposed situations, characterizing a preliminary analysis the students' cognitive dimension (Santos & Alves, 2017, p. 453).

The following section, we present a problem with the ENEM Mathematics test in 2013, structured in the stages of the TSD, to exemplify a didactic situation model.

4. 1. Conception and Construction of a Didactic Situation visualized with the GeoGebra software

In this section, a proposal for a mathematical model for the teaching of volumes, structured in the Theory of Didactic Situations (TSD), of a problem taken from the ENEM tests with the contribution of the GeoGebra software for the construction and solution of the problem situation is presented. , emphasizing the visualization and description of a significant scenario for the learning of Mathematics.

Question 1 (ENEM 2013): In a water park there is a children's pool in the shape of a straight circular cylinder, 1 meter deep and a volume of 12 cubic meters, with base of radius R and center O. It is desired to build an island of leisure it dries inside this pool, also in the form of a straight circular cylinder, whose base will be at the bottom of this pool and with the center of the base coinciding with the bottom of the pool. The radius of the leisure island will be r. It is desired that, at the end of the construction, a volume radius of 4 cubic meters.

Considering 3 the approximate value of Pi, we ask: To satisfy the given conditions, the maximum radius r will be:

(A) 1,6
(B) 1,7
(C) 2,0
(D) 3,0
(E) 3,8

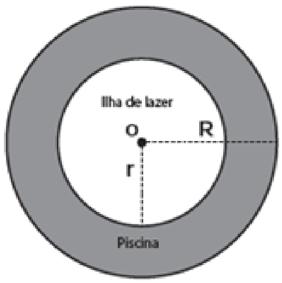

Figure 6: *ENEM Spatial Geometry issue of the year 2013 in Brazil. (Modified by the authors)*

Action Situation: This is the preliminary phase, in which the teacher should encourage students to look at the problem data and associate numerical and geometric aspects that are part of the statement of the question in Figure 6 that are configured with the new data presented in the construction of Figure 7, because according to Brousseau (2002), it is in the action situation that the student organizes the strategies and builds a representation of the problem situation that serves as a model to teach himself how to take the decisions to solve the issue . At that moment, the teacher should encourage the student to make use of all possibilities in the search for solving the problem, so that he also looks for dynamic solutions, using the GeoGebra software to abstract the mathematical idea of question 1, which describes a relationship functional between the volume of a swimming pool that has the shape of a straight circular cylinder and the volume of a leisure island, with the same shape. Initially, it is expected that the academic realizes that the volume of the pool after the construction of the island is related to the value of the radius measurement of both objects, since the center of the base of the two buildings occupy the same position. Thus, the apprentice may realize that to solve the question, it is necessary to determine the radius measurements and that this solution is interconnected with the initial volume of the pool and the final volume, since the statement of the problem makes a comparison between the initial capacity and the final capacity of the water volume. Figures 7 and 8 describe this relationship between the original volume of the pool and the volume of the pool after the construction of the island, based on the manipulation and changes in the construction of the GeoGebra software, which allows differentiating the numerical and geometric properties that will lead students to investigate the concept of cylinder volume.

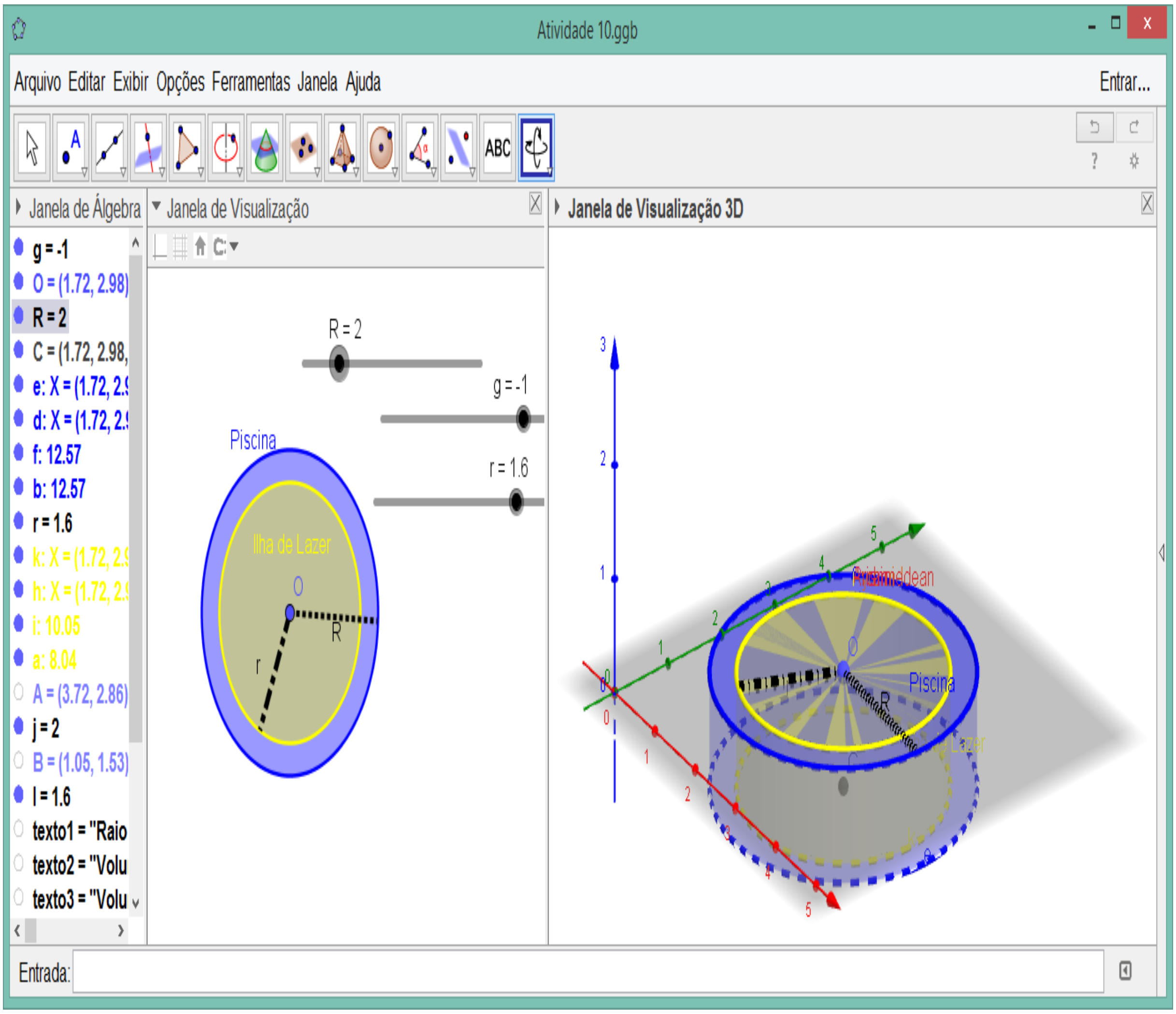

Figure 7. *2D / 3D visualization built on the GeoGebra software corresponding to question 1 of the ENEM. (Prepared by the Authors)*

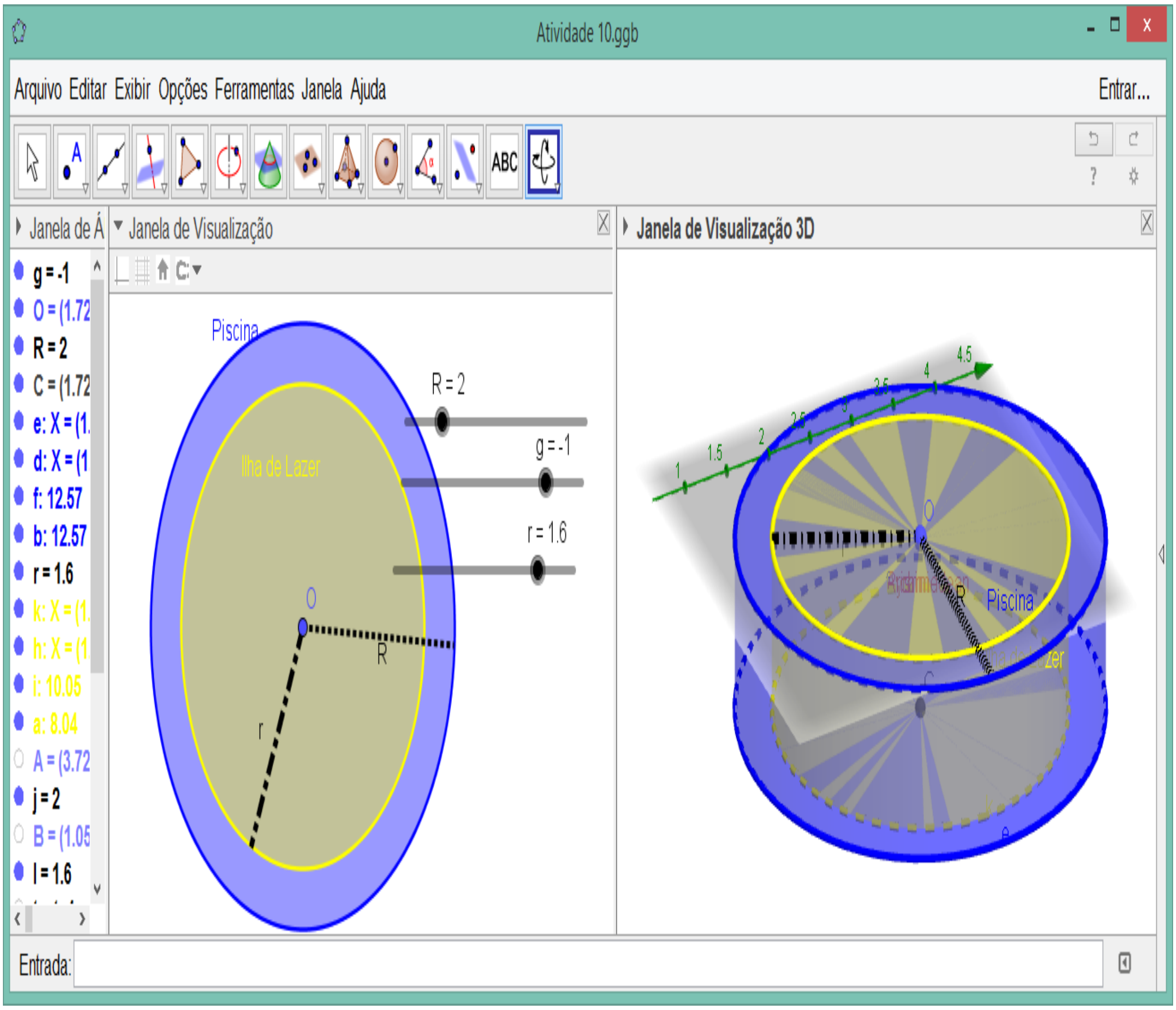

Figure 8. *2D / 3D visualization built on the GeoGebra software corresponding to question 1 of the ENEM. (Prepared by the Authors)*

Formulation Situation: It is the phase of information exchange between students and / or teacher, it is also characterized by descriptions that can be verbal or written. At this point, students should be encouraged to produce and develop assumptions from formal deduction and manipulation in the software. According to Brousseau (2002), the formulation consists of establishing a gradual language in which everyone can understand, considering the objects and the essential relations of a situation performed with reasoning and useful actions in an appropriate way. At this point, communication is paramount and the exchange of information will enable the student to conclude that when building the island the original volume of the pool decreases from 12m3 for all volumes 4m3, since the statement emphasizes that the space for water has at least 4m3, therefore, the the student can, through the relationship of these measures, realize that the volume of the island should be the result of the difference between the initial and final volume of the pool, thus 12m3 - 4m3 = 8m3. Thus, to arrive at the solution, he must present a model, by means of which it will be possible to calculate the radius of a circumference, and then calculate the volume of the cylinder. Through this knowledge, the student can conclude that by multiplying the base area by height, the capacity of any straight circular cylinder is obtained, which will be validated by the formula V = π. r2. H. In addition, the student must understand that to solve the question he can dynamically manipulate the radius measurement in the GeoGebra software, as it is variable and relates to the volume, presenting the structure that results in the solution of the problem. (See figure 9).

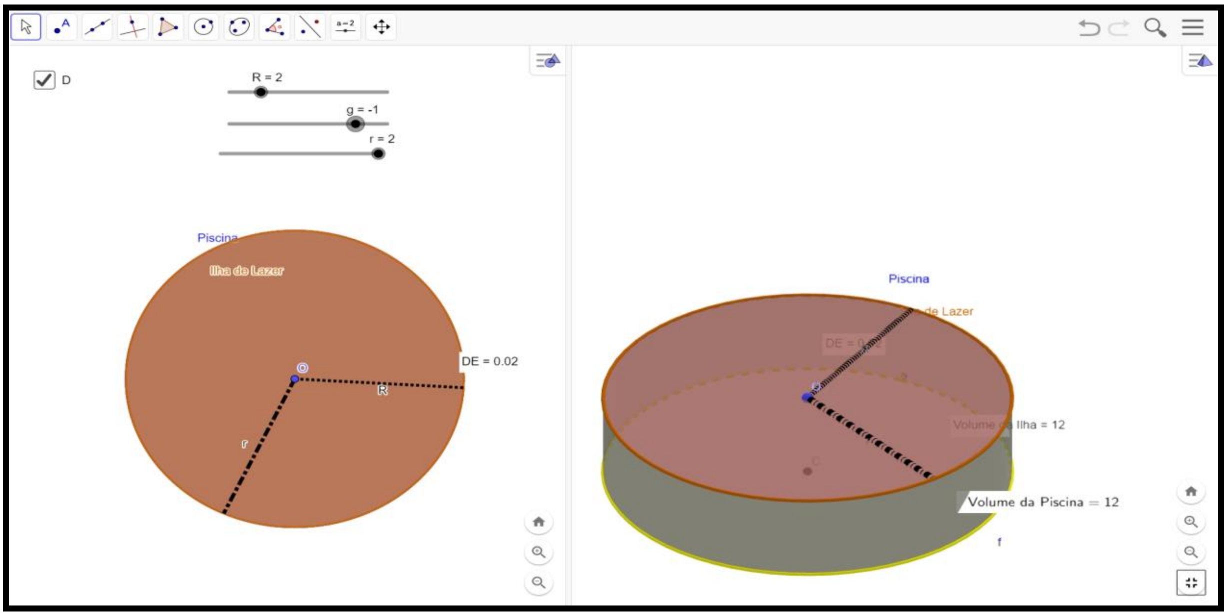

Figure 9. *2D / 3D visualization built on the GeoGebra software corresponding to question 1 of the ENEM. (Prepared by the Authors)*

Validation Situation: It is in this stage that the presentation of the strategies that the students used to arrive at the solution of the problem occurs, that is, it is the moment to show a structured mathematical model. In the validation it is expected that the student will be able to formulate sequences extracted from the arguments collected in the previous phases to obtain the structural form, as shown in Figure 9, thus justifying the actions carried out in the previous steps to solve the problem. Brousseau (2002, p. 15), calls attention to the fact that "validation motivates students to discuss a situation and favors the formulation of its implicit validations"

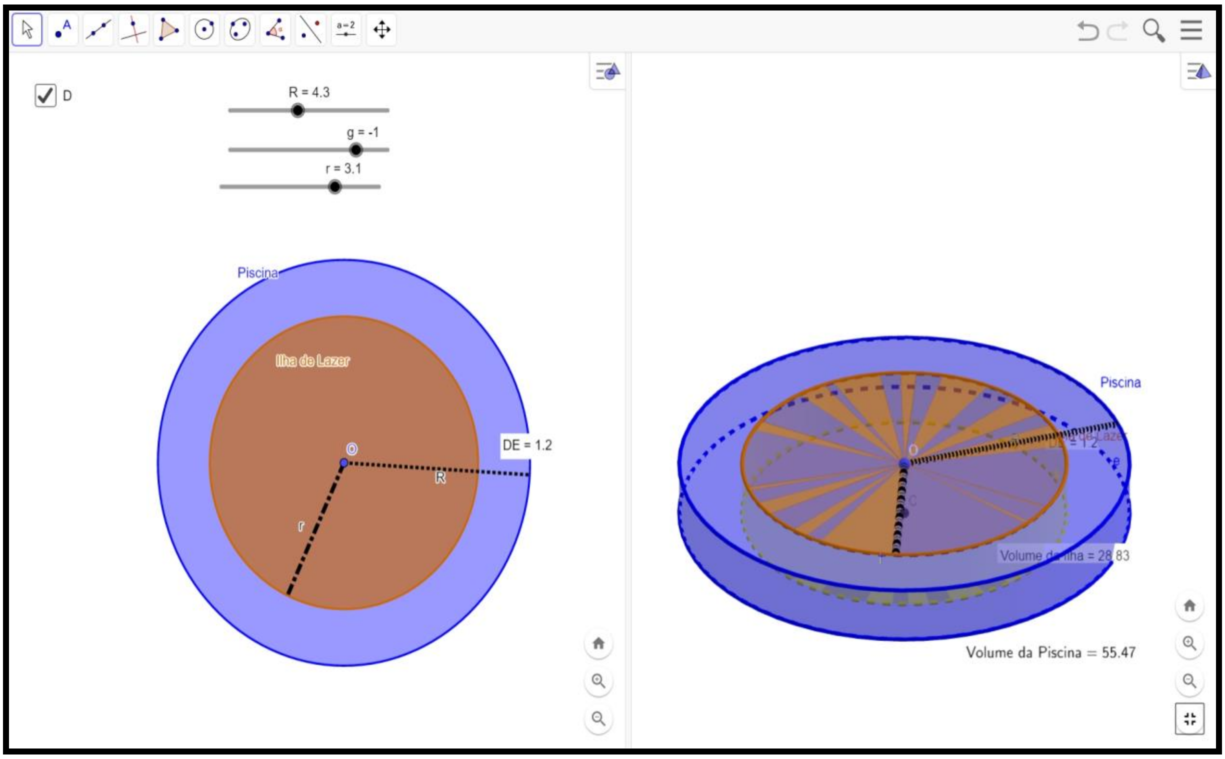

Figure 10. *2D / 3D visualization built on the GeoGebra software corresponding to question 1 of the ENEM. (Prepared by the Authors)*

Next, the apprentice is expected to apply the formula for calculating the volume of the straight circular cylinder to find the measurement of the radius of the pool and the island, and therefore solve the problem. To encourage the student, the teacher can suggest comparing the previously selected data and comparing it with the visualization possibilities offered by manipulating the construction of the GeoGebra software. In the window on the left side, the student can manipulate the radius measurements, clearly perceiving in the 3D view, that when reaching the radius value of the initial pool, corresponding to the volume presented in the statement of the question, the answer to the problem is found, which will be the result of the formulation V island = π. r2. h ≤ 8m3, thus presenting the final resolution of the proposed problem. (See figure 10).

Situation of Institutionalization: Institutionalization is shown in Figure 10, through the GeoGebra visualization windows, allowing a comparison with the mathematical model deduced from the statement of the question in Figure 6, and the model built on the computer. For according to Brousseau (2002, p. 215), “situations of institutionalization are those by which the cognitive status of knowledge is fixed conventionally and explicitly”. Therefore, it is the formalization phase of the resolution, mediated by the teacher with the objective of attaching the students' ideas and synthesizing the answers, thus, after instituted and validated, the new knowledge will pass to the status of the class's mathematical heritage, even if not constituting yet a social knowledge. In Figure 11, it can be seen that the exposed problem, despite its geometric representation, is related to the values of the radius measurements, not presented numerically in the statement of the question and the difference between the initial volume and the final volume of the pool.

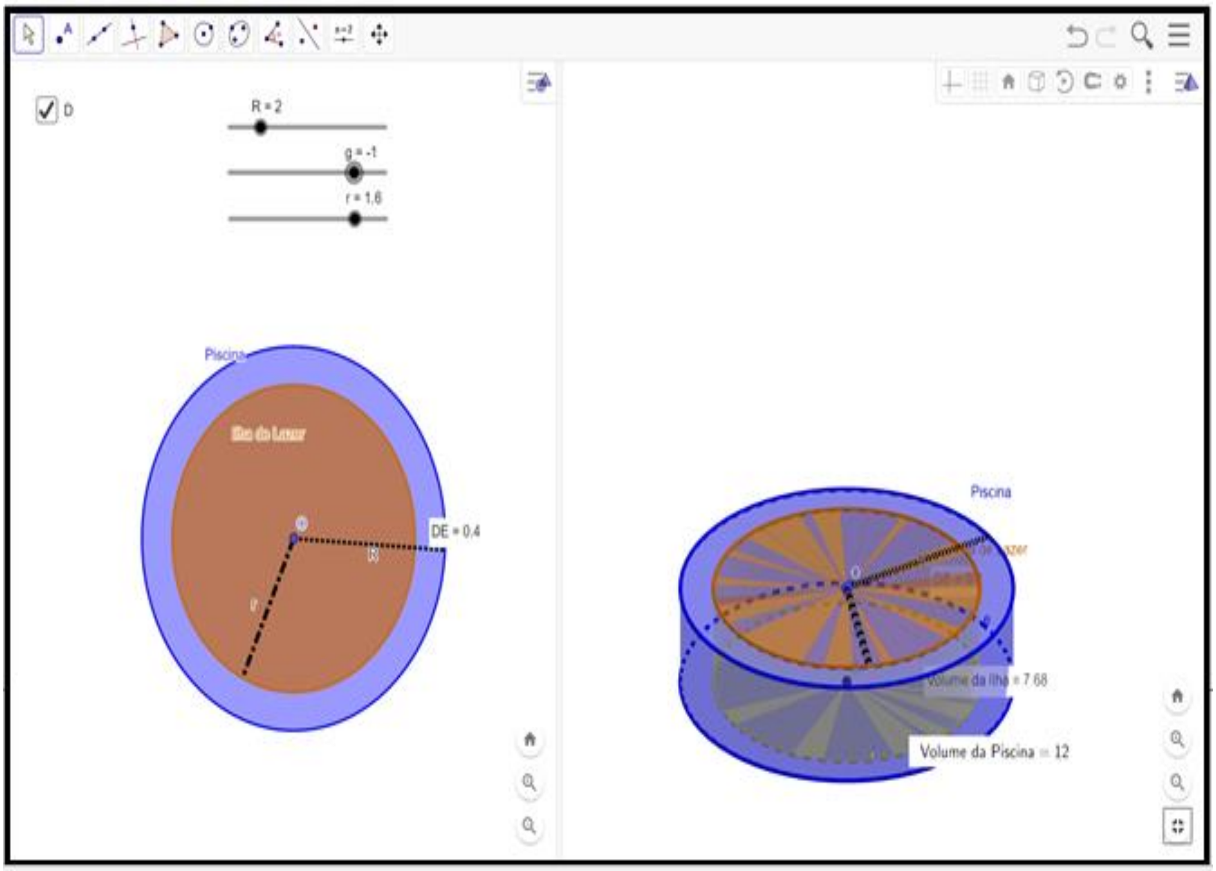

Figure 11. *Visualização 2D/3D construída no software GeoGebra correspondente a questão 1 do ENEM. (Prepared by the Authors)*

Finally, a final extension of the solution to the problem in Figure 6 is formalized. In this situation, the teacher can expose the steps of the resolution, reviewing and comparing it with the knowledge presented by the students in the initial phases. The theorem that validates the question is the cylinder volume, whose definition given by Lima (2011), is shown below: “Theorem 2: The volume of a cylinder is equal to the product of the base area by height”. (2011, p. 74).

Therefore, applying theorem 2, the following formula can be presented:

$V_{\text{Initial pool volume}} = \pi \,.\, R^2 \,.\, h$	$\text{Final Island Volume} = 12 - 4 = 8m^3$
$12 = 3 \,.\, R^2 \,.\, 1$	$V_{ilha} \leq 8m^3$
$3R^2 = 12$	$\pi \,.\, r^2 \,.\, h \leq 8$
$R^2 = \frac{12}{3}$	$3 \,.\, r^2 \,.\, 1 \leq 8$
$R^2 = 4$	$3r^2 \leq 8$
$R = \sqrt{4}$	$r^2 \leq \frac{8}{3}$
$R = 2$	$r \leq \sqrt{\frac{8}{3}}$
	$r \leq \frac{2\sqrt{2}}{3}$
	$r \leq \frac{2 \,.1{,}4}{1{,}7}$
	$r \leq \frac{2{,}8}{1{,}7}$
	$r \leq \mathbf{1,64}$

Figure 12: Proposed formal resolution scheme of the problem situation 1. *(Modified by the authors)*

It is concluded, therefore, that the maximum radius of the island will be close to 1.6 meters.

3. Conclusion

This article presents a proposal for a didactic situation for mathematics teachers in the light of the National High School Examination (ENEM), which offers suggestions that can positively impact teaching practices in the teaching of cylinder volume, as well as in the conduct and exposure of students. concepts included in this exam.

The research was based on the Didactic Training Engineering (EDF) methodology, given that it establishes a connection between research and teaching, enabling the researcher to carry out an analysis of structured didactic situations and to provide greater solidity to professional teaching training, as well as such as the design of didactic resources aimed at teaching mathematical concepts implemented in the classroom.

The Didactic Situations Theory (TSD) was adopted to structure the problem situation of ENEM, so that the teacher is able to develop and model situations that can be implemented in the classroom that promotes the student's role in the construction of their own mathematical knowledge, in addition to theoretical and methodological subsidies for understanding the elements necessary for teaching the volume of a cylinder and strategies that can be adapted and used in other didactic situations.

Furthermore, a class based on the four phases of the TSD and supported by the technology of the GeoGebra software for the construction of problem solving, provides a reflective, dynamic and more participatory environment for learners, generating debates that promote the gradual appropriation of the contextualization of mathematical knowledge, aiming to develop a situation that allows the teacher the appropriate management of the mathematical and didactic knowledge necessary to teach.

Regarding the potential of the GeoGebra software, it is believed that the manipulation of the object in the software can provide the student with formulation of resolution strategies, visualization of the geometric properties highlighted in the problem situation, in addition to promoting dialogue with other students in an attempt to validate the answer to the question addressed.

It is concluded, therefore, that this study presented significant perspectives in relation to the teaching and research methodology of EDF, based on the TSD, considering that this combination, allows the conception of a didactic resource for training and teaching that articulates theoretical knowledge to the practical, generating progress in the acquisition of mathematical knowledge by the students, in addition to presenting to the mathematics teacher a model that can promote the development of geometric reasoning present in the problem-situation selected from the ENEM tests. Thus, it is hoped that the results of this dissertation can contribute to the initial and continuous training of mathematics teachers, improving and perfecting the teaching of mathematical concepts in Brazilian schools.

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