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PRE-SERVICE PRIMARY SCHOOL TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE ON QUADRILATERALS

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Abstract: The aim of this study was to investigate pre-service primary school teachers' (PPSTs) pedagogical content knowledge (PCK) on quadrilaterals. In this study, the PCK components of knowledge of understanding students (KUS) and knowledge of instructional strategies (KIS) were used. The participants of the study consisted of 83 PPSTs studying at the primary education department of a university in Turkey. The illustrative case study method was used, while six scenarios were used as the data collection tool developed by researchers. The data obtained from open-ended scenarios were analyzed by using the summative content analysis technique. As a result of the study, it was observed that the KUS of the PPSTs about quadrilaterals was not on the desired level. Moreover, the KIS of the PPSTs was also not on the desired level. As a result of the study, it was observed that, in the process of eliminating the mistakes of students, the PPSTs preferred mainly the "*Direct Instructional*" method, which is based on traditional approaches and centers the teacher.

Key words: pedagogical content knowledge, quadrilaterals, teacher education, pre-service teacher, primary school.

1. Introduction

One of the fundamental aims of mathematics education is to understand the relationships between mathematics and the world. With geometry education, it is aimed that students understand geometric concepts in the best way and relate them to daily life (Ministry of National Education [MoNE], 2017). Geometry, which is composed of the words Geo and Metry, means the measure of the land. Humanity started to use geometry by the moment it started to make sense of the physical environment around it (Baki, 2014). It is believed that the first example if this started by the need of Egyptians to measure their fields based on retraction of flood water near the River Nile (Ball, 1960).

Geometry plays a role in understanding different topics of mathematics and serves as a bridge between various disciplines, such as art and architecture, and mathematics. According to the National Council of Teachers of Mathematics [NCTM] (2000), geometry contributes to development of the reasoning and problem-solving skills of students. Geometry also helps students understand abstract concepts (Duatepe, 2000). Therefore, from the very first years of education, teaching of geometry has remained a priority (Baykul, 2002).

Although geometry is highly important in mathematics education, students on all levels have many learning difficulties and misconceptions in this field of learning (Özerem, 2012). Quadrilaterals are particularly among the geometry concepts where students experience difficulty in learning (Okazaki & Fujita, 2007). Students experience problems in defining quadrilaterals, calculating their perimeter and areas (Özerem, 2012), determining models suitable for definitions of quadrilaterals (Mack, 2007), as well as establishing relationships among different quadrilaterals (Gal & Lew, 2008) and among the properties of a shape (Özerem, 2012).

Some of the reasons why students experience learning difficulties related to quadrilaterals are the complex relationships of quadrilaterals with each other (Fujita & Jones, 2006; Okazaki & Fujita, 2007). In the geometry literature, both inclusive and exclusive definitions of quadrilaterals are included (Usiskin & Griffin, 2008). For example, the trapezoid has an inclusive definition as *"a*

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quadrilateral with at least a pair of sides parallel," or an exclusive definition as *"a quadrilateral with only a pair of sides parallel"* (Öztoprakçı & Çakıroğlu, 2013). According to the exclusive definition of the trapezoid, only the shape numbered 4 below is a trapezoid, while all the shapes (square, rectangle, parallelogram) have the characteristics of a trapezoid if the inclusive definition is taken into consideration.

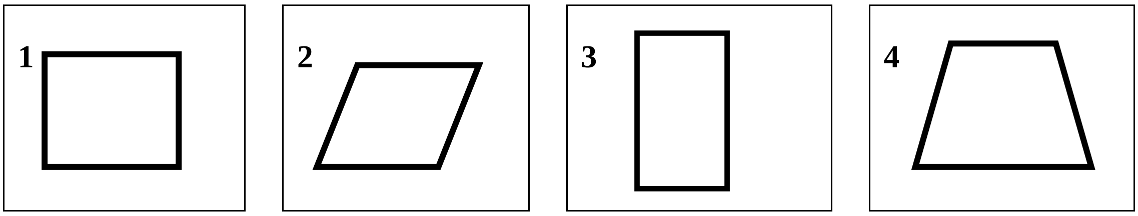


Figure 1. *Trapezoid models*

Therefore, teachers decide whether to use an inclusive or exclusive definition while teaching quadrilaterals in the classroom. Teachers should use inclusive definitions if students have reached cognitive maturity to understand the relationship between geometric concepts (Öztoprakçı & Çakıroğlu, 2013). Moreover, according to van Hiele's geometric thinking model, students need to reach level 2 (informal deduction) to establish a relationship between geometric shapes (Burger & Shaughnessy, 1986). On Level 0 (Visualization), individuals focus on the visual properties of geometric shapes, while on Level 1 (Analysis), they can analyze the properties and components of geometric shapes (Crowley, 1987). In this context, mathematics teachers have to teach quadrilaterals by considering the levels of cognitive development and geometric thinking of students. Otherwise, students will have difficulties in learning the concepts of quadrilaterals, and as a result, they will make mistakes. Therefore, teachers are also among the sources of the mistakes of students regarding quadrilaterals. Vocational inadequacy of the teacher is considered to be among the most significant reasons for the mistakes made by students in the geometry learning process (Confrey, 1990; Lim, 2011; Luneta, 2015). Additionally, several studies in the relevant literature (Ball, 1991; Baumert et al., 2010; Choy, Wong, Lim, & Chong, 2013; Even, Tirosh, & Robinson, 1993; Kahan, Cooper, & Bethea, 2003; Stewart, 2013) have revealed a positive relationship between the professional adequacy of the teacher and student success. Baumert et al. (2010) found a positive relationship between teachers' PCK and student achievement as a result of their findings on the structural equation model. In other words, teachers have the most significant responsibility in teaching students the definitions, properties and algorithms of quadrilaterals correctly.

Many studies have been conducted to explain the professional capacities of teachers, and the PCK model proposed by Shulman (1987) pioneered such studies (Depaepe, Verschaffel, & Kelchtermans, 2013). Shulman (1986, 1987) defined PCK as a type of knowledge that emerges as a result of the interaction between content knowledge and pedagogical knowledge. PCK involves determination and elimination of the mistakes and misconceptions of students, methods and techniques towards teaching concepts in the most effective form and different representations of concepts (Marks, 1990). In the relevant literature, there have been studies towards determining the PCK levels of pre-service teachers on several concepts (Ball, 1988; Şahin, Gökkurt, & Soylu, 2016; O'Hanlon, 2010; Tirosh, 2000). However, it is seen that studies on determining the PCK levels of form teachers or pre-service teachers on geometry, especially in relation to quadrilaterals, are fewer in comparison to those on other concepts (Depaepe et al., 2013; Stahnke, Schueler, & Roesken-Winter, 2016). In this context, this study aims to examine the PCK of PPSTs about quadrilaterals in terms of student mistakes. The following research questions were posed to guide the study:

1) On what level can PPSTs identify (recognize) students' mistakes about quadrilaterals? (KUS)
2) On what level can PPSTs propose solutions to eliminate students' mistakes about quadrilaterals? (KIS)
3) What are the instructional strategies that PPSTs often prefer in the process of resolving students' mistakes about quadrilaterals? (KIS)

2. Theoretical framework

2.1. Students' mistakes and misconceptions about quadrilaterals

Although geometry is highly important in mathematics education, students from early childhood to the university level have been reported to have many learning difficulties and misconceptions in this learning field (Clements & Sarama, 2000; Fujita & Jones, 2006; Hasegawa, 1997; Jung & Conderman, 2017; Mack, 2007; Monaghan, 2000). Quadrilaterals are particularly among the concepts where students experience difficulties in the process of learning geometry (Fujita & Jones, 2006; Okazaki & Fujita,2007).

Students experience problems in defining quadrilaterals, calculating their perimeter and areas (Özerem, 2012), determining models suitable for the definitions of quadrilaterals (Erez & Yerushalmy; 2006; Mack, 2007), as well as establishing relationships among quadrilaterals (Gal & Lew, 2008) and among the properties of a shape (Özerem, 2012). Mack (2007) stated that third grade students could not name the rotated form of squares and rectangles. Erez and Yerushalmy (2006) reported that fifth grade students thought, when rectangles are rotated, their properties change.

Gal and Lew (2008) stated that, although high school students with low level of success knew about the prototype form of a parallelogram, they were not aware that a square, a rectangle and an equilateral quadrangle are also parallelograms. Okazaki and Fujita (2007) expressed that even high school students are not aware that a square is also a rectangle and an equilateral quadrangle. Monaghan (2000) asked students aged from 11 to 16 what types of differences exist among various quadrilaterals. Some students failed to explain the differences among quadrilaterals. For example, these students stated that a square and a rectangle has common properties, but they could not explain their different properties.

Özerem (2012) concluded that some seventh-grade students multiplied the base by height and divided the result by two while calculating the area of a parallelogram. In the study by Huang and Witz (2013), some fourth-grade students confused the perimeter and the area of a rectangle. Kospentaris, Spyrou and Lappas (2011) stated that many high school and university students confuse congruence with area equivalence. Additionally, students believe that, when the visual properties of a quadrilateral change, its area also changes (Pitta-Pantazi & Christou, 2009). In other words, according to the students, when we transform a parallelogram into a rectangle, its area changes.

The reasons for the misconceptions of students regarding quadrilaterals may be listed to include the limited experiences of students related to concepts (Erez & Yerushalmy, 2006; Hasegawa,1997; Mooney, Briggs, Hansen, McCullouch, & Fletcher, 2018), limited representation of concepts in textbooks and curricular materials (Monaghan, 2000), incomplete subject content knowledge of teachers regarding concepts (Hasegawa,1997; Monaghan, 2000) and the complicated structure of quadrilaterals (Fujita & Jones, 2006; Okazaki & Fujita,2007). Monaghan (2000) stated that mistakes made about quadrilaterals originate from that expressions in curricular materials are usually prototypes. Hasegawa (1997) emphasized that general usage of prototype models and non-usage of definitions, examples and materials suitable for the student's level by the teacher were effective on students' mistakes related to quadrilaterals. Erez & Yerushalmy (2006) highlighted that the existing knowledge of students related to quadrilaterals is effective. The most significant factor in students' focus on prototype models of geometric shapes may be that teachers and textbooks usually prioritize prototype shape examples, and comprehensive concept definitions are not sufficiently prioritized (Fujita & Jones, 2006; Okazaki & Fujita, 2007).

2.2. Pedagogical content knowledge: Knowledge for teaching

In the early 1980s, chaos in the education system in America formed the basis for many studies (Carlsen, 1999). Many studies have shown that teachers play a critical role in making the education system more qualified (Ball, Thames, & Phelps, 2008; Cochran, De Ruiter, & Kin, 1993; Grossmann, 1990; Marks, 1990; Tamir, 1988). In this context, many teacher knowledge base models have been developed. One of the most important of models was developed by Shulman (1987). Shulman (1987) named problems regarding teacher education as "*missing paradigm*" and proposed the concept of PCK as a recommendation of a solution to these problems (Depaepe et al., 2013). Shulman (1987) described a unique combination of knowledge bases that a teacher should possess, with his PCK model that emphasizes the importance of teacher education. Shulman (1987) noted that both content knowledge and pedagogical knowledge are essential for PCK, and being able to use these at the same time is a critical aspect of a teacher. Shulman (1987) considered PCK as knowledge that distinguishes a teacher and a domain expert. One of the most prominent criticisms of the PCK model by Shulman (1987) was that it is not discipline-specific, and it is not based on experimental foundations (Ball et al., 2008). As a result of this, several models have been developed to reveal the professional capacities that teachers of language, mathematics and science should possess (Ball et al., 2008; Grossman, 1990; Fennema & Franke, 1992; Magnusson, Krajcik, & Borko, 1999; Schoenfeld, 1998). These models that were developed were constructed over the theoretical framework of the Shulman (1987) model (Carrillo-Yañez et al., 2018).

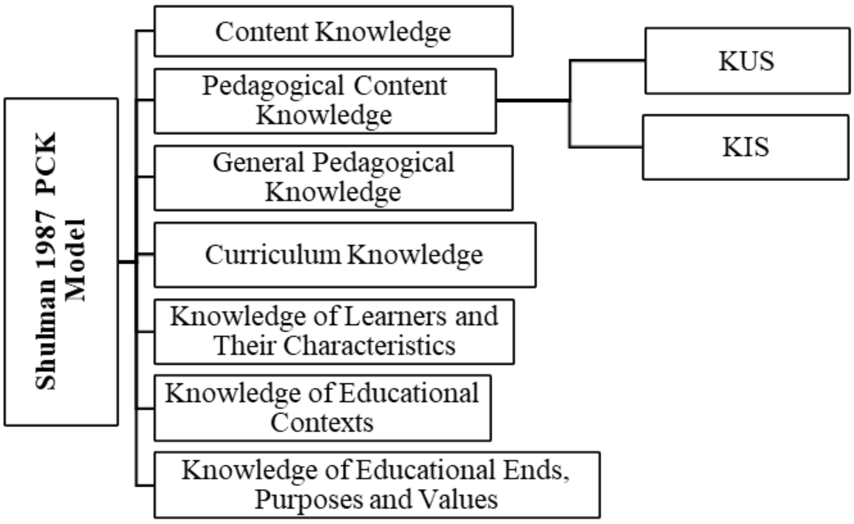


Figure 2. *Shulman (1987) PCK model*

Shulman (1987) included seven categories (Figure 2) in professional knowledge in teaching including PCK. Shulman (1987) noted that PCK is made up of the subcomponents of KUS and knowledge of instructional strategies. KUS is defined as the knowledge of teachers or pre-service teachers of students' preliminary knowledge about the learning concept, learning difficulties, mistakes, misconceptions and the reasons for these (Shulman, 1987). Additionally, the PCK component expressed as KUS by Shulman (1987) was named differently in other PCK models. This component of PCK was named in the literature as knowledge of students' understanding (Grossman, 1990), knowledge of content and students (Ball et al., 2008), knowledge of students' understanding of science (Magnusson et al., 1999), knowledge of students' understanding in science (Park & Oliver, 2008), students' understanding of the subject matter (Marks, 1990), knowledge of students' thinking (An, Kulm, & Wu, 2004) and knowledge of students (Cochran et al., 1993). KIS, which is the second component of Shulman's (1986) PCK was defined as the teacher's teaching method and technical knowledge, which are used in transferring content knowledge to students, eliminating the misconceptions of students and increasing the success of students (Ball et al., 2008; Cochran, et al., 1993; Magnusson et al., 1999). In other words, KIS is the knowledge that ensures teaching a concept to students in the most effective way.

3. Method

In this study, the qualitative research method of illustrative case study was used to examine the PCK of PPSTs on quadrilaterals in terms of student mistakes.

> *Illustrative case studies are descriptive. They utilize one or two instances to show what a situation is like. This helps interpret other data, especially when there is reason to believe that readers know too little about a program. These case studies serve to make the unfamiliar familiar and give readers a common language about the topic. The chosen site should be typical of important variations and contain a small number of cases to sustain readers' interest* (Davey, 1991, p.2).

The PCK of PPSTs in terms of quadrilaterals was descriptively discussed. The cases of this study included determining-defining student mistakes, producing solution recommendations for eliminating these mistakes and instructional methods.

3. 1. Participants

The participants of this study consisted of 83 PPSTs who were selected by the convenience sampling method (McMillian & Schumacher, 2010). The PPSTs were in the final year of their undergraduate education at the faculty of education at a university in Turkey. In the convenience sampling method, a group of subjects may be selected on the basis of accessibility and expedience (McMillian & Schumacher, 2010). In this context, the researchers carried out the study with pre-service teachers that were receiving education at the faculty of education where they work.

In Turkey, PPSTs complete their undergraduate education in eight terms. PPSTs, who participated in this study, took the courses *"Basic Mathematics 1"*, *"Basic Mathematics 2"*, *"Mathematics Teaching 1"* and *"Mathematics Teaching 2"* for mathematics teaching. Furthermore, during the last year of their undergraduate education, PPSTs are trained in primary schools within the scope of *"School Experience"* and *"Teaching Practice"* courses. During the internship training, PPSTs are given the opportunity to monitor all the educational activities carried out in schools and give lessons to students. In this study, the data were collected from the PPSTs at the end of the eighth semester of their undergraduate education. In other words, the data were collected after the PPSTs completed all the courses they needed to teach mathematics. Moreover, in the process of data collection, the real names of the PPSTs were not used in the text due to the ethical issues. In this context, the PPSTs who participated in this study were given codes from PT1 to PT83.

3. 2. Measure

In this study, the standardized open-ended interview method was used as a data collection tool. In this method, during the interview, pre-determined questions are asked to avoid participants' bias and subjectivity. This also increases transferability, by which studies that use standardized open-ended interviews may be replicated by other researchers. This method is usually preferred in cases when it is necessary to interview multiple people (Patton, 1987). In this context, an interview form consisting of six open-ended scenarios were applied to the PPSTs. The scenarios were prepared as an artificial storyline related to the investigated situation and intended to attract the attention of the participants. Therefore, through this scenario, the researcher could investigate a lot of matters at the same time with regard to the participants' PCK (Bütün, 2005).

At the first stage of the development process of the scenarios used in the study, the researchers created ten scenarios using their own experiences and the related literature (Ball, 1988; McCoy, 2016; Stecher et al., 2003). The scenarios that were developed were then examined by two mathematics education experts and a language expert to ensure their content validity and linguistic validity. Based on the expert opinions, one of the questions that measured similar skills was selected. After these adjustments, the final version of the interview form included six open-ended scenarios (App.1). Table 1 summarizes the geometry concepts that were used in the scenarios and how these scenarios were obtained. Moreover, the study also included which van Hiele geometry thinking level the geometric skills included in each scenario corresponded to. It is an issue which is open to criticism that, although the theoretical framework of this study emphasizes the field of learning geometry, it included scenarios related to perimeter and area measurements for quadrilaterals. NCTM (2000) includes geometry and measurement as two different content standards. In contrast, in the mathematics curriculum in Turkey where the study was carried out (MoNE, 2017), geometry and measurement are given under one content standard.

Two standard questions were used in each scenario to measure the PCK of the PPSTs. These questions were as follows: *"Consider the situation in this scenario. In this case given to you, are the answers of the students correct? If they are correct, explain why they are correct. (KUS)"* and *"If you think that the answers of the students are wrong, explain why they are wrong (KUS). How would you interfere with the student if you were in this situation? What would you do to eliminate this mistake?" (KIS).* If the PPSTs believed that the students had not made mistakes in the given scenario, they were expected to answer the first question. The PPSTs who answered this question also needed to explain why the answer of the student was correct. The PPSTs who answered the second question should identify and describe the student's mistake in the given scenario and explain how to eliminate this mistake.

Table 1. *Information about scenarios*

Scenario	Concept	Source	Explanation
Scenario 1 (Level 1)	Definition of square	Developed by researchers.	*The definition made by the teacher in this scenario is not only valid for a square. In addition to a square, this definition also expresses many geometric shapes such as an rhombus, regular pentagon and regular hexagon.*
Scenario 2 (Level 2)	Parallelogram examples	Developed by researchers	*The students selected only the prototype model of a parallelogram. The students did not know that a square and a rectangle are also parallelograms.*
Scenario 3 (Level 2)	Area of quadrilaterals	Developed by researchers	*The students generalized the formula that they used for calculating the areas of a square and a rectangle to other quadrilaterals (deltoid, equilateral quadrangle).*
Scenario 4 (Level 2)	Area concept (unit square)	The relevant literature (Stecher et al., 2003) was used	*The students made mistakes in calculating areas with 2x2 squares. The students firstly multiplied the side lengths, and they multiplied the result with 2 instead of 4.*
Scenario 5 (Level 2)	Relation between perimeter and area	The relevant literature (Ball, 1988) was used.	*The students thought that quadrilaterals with a larger perimeter would have larger areas.*
Scenario 6 (Level 2)	Perimeter and area of non-prototype quadrilaterals	The relevant literature (McCoy, 2016) was used.	*The students had difficulty in calculating the area of a non-prototype quadrilateral.*

3. 3. Data collection procedure

The data of this study were collected in the last semester of the four-year undergraduate education of the preservice teachers. At the first stage of the data collection process, the authors informed the PPSTs regarding the objective of the study and ethical principles. In this information, it was explained that the collected data would only be used for the purpose of this study and would not be shared with anyone. The data were collected in the classroom environment, and the scenarios were presented to the participants as printed copies. In each scenario, a sufficient space was allocated for the preservice teachers to answer the questions in the scenario. Without any time limitation, the participants were allowed to comfortably express their views. The participants took around 60-120 min to complete their responses to the scenarios. The also responded to the scenarios individually and without interacting with each other.

3. 4. Data analysis

The data obtained from the open-ended scenarios were analyzed through the summative content analysis technique. Summative content analysis enables us to categorize the answers of PPSTs according to the themes or categories already defined by the researchers (Hsieh & Shannon, 2005). In this study, themes were created for identification of mistake, instructional methods and elimination of mistake for each scenario. After then the codes for each theme were created, the themes and codes generated from the findings of the study were presented in frequency-percentage tables. To make the data analysis process more comprehensible, the data analysis frameworks formed for identification of mistake, elimination of mistake and instructional methods for each category are given in the tables below. These tables where the data analysis frameworks are summarized include themes for each category, explanations of these themes and examples of the participants' responses in these themes.

Table 2. *Themes for identification of mistakes (first scenario)*

Themes	**Description**	**Exemplary answer**
Correct identification of the mistake	A completely correct identification and description of the mistake.	PT16: *If the teacher had stated in the definition that a square has 4 sides, and these 4 sides are equal to each other, the students would not select the shape number 2. Additionally, if it had been given in the definition that the sides are perpendicular to each other, they would not select the shape number 3.*
Partial identification of the mistake	Use of incomplete statements while describing the mistake.	PT11: *The students made mistakes as the teacher used the expression "all sides are equal". While making a definition, the teacher did not state how many sides a square has.*
Failure to identify the mistake	A completely incorrect description of the mistake	PT1: *The shapes numbered 1 and 4 are squares, while those numbered 2 and 3 are not.*
No answer	Unanswered	--------------------------------------
No mistake	Stating that there is no mistake in the scenario	PT29: *The students answered correctly.*

PT16 correctly explained the mistake by stating that it originated from the teacher, and emphasis was not made in the definition on 4 sides and angle characteristics. While PT11 emphasized the number of sides of a square, as they did not mention the angle characteristics, it was assumed that they described the mistake as partially correct. As PT1 used a mathematically incorrect statement by stating that the shape number 4 which was a rectangle was a square, their answer was put under the code "*failure to identify the mistake*". PT29 stated that, in the given scenario, the students answered correctly, and they did not make any mistake.

Table 3. *Themes for elimination of mistakes (fourth scenario)*

Themes	**Description**	**Exemplary answer**
Correct elimination	A completely correct instructional and mathematical proposal.	PT62' s answer to fourth scenario (Table 7)
Partially correct elimination	A proposal that contains incomplete statements in instructional terms or mathematically	PT45: *I would first have them find the areas of the 2x2 unit squares. Afterwards, I would ask them to multiply the answer they found with how many 2x2 unit squares there are.*
Failure to eliminate	A completely incorrect or irrelevant proposal.	PT11: *The student found the perimeter of the shape by multiplying with 2. They should be taught that calculation of perimeter and calculation of area are different.*
No answer	Unanswered	-------------------------

PT62 produced a correct solution recommendation by making both instructionally and mathematically necessary explanations. PT45 followed an instructionally and mathematically correct approach towards elimination of the mistake. However, as PT45 did not state how these calculations should be mathematically made and what the correct result was, their response was considered to be in the category of "*partially correct elimination*". PT11 stated that the result that the student found was the perimeter of the shape, and thus, the relationship between area and perimeter should be taught. However, the perimeter of the shape was not 24 units but 28 units. Therefore, PT11 brought an incorrect solution recommendation as they could not describe the mistake correctly.

Table 4. *Themes for instructional methods*

Themes	**Description**	**Exemplary answer**
Direct instruction	These were the cases where preservice teachers directly expressed the definitions or properties of concepts.	PT8: …*To eliminate this mistake, while defining a square, the student should be taught that "it has 4 sides, side lengths are equal to each other, and the sides are perpendicular to each other. (Scenario 1)*
Expository teaching	These were teacher-centered approaches, where the preservice teachers tried to offer meaningful learning by using organizers.	PT18: *I would firstly give the definition of a square to the students. I would then show square-shaped objects around them. I would also show them non-square shapes and explain that they are named as triangle, rectangle, parallelogram. (Scenario 1)*
Discovery learning	These were student-centered approaches that the preservice teachers used to have children discover the properties of concepts with the help of guiding questions.	PT60: *I would firstly ask the students to calculate the length of each side. Afterwards, I would have them calculate the area of the rectangle. This way, the students would be aware of the mistake they have made. (Scenario 4)*
Manipulatives	These were the situations where the preservice teachers used concrete materials.	PT63: *I would bring materials in the shape of parallelograms to the classroom. With the help of these models, I would have them understand that only the shape numbered 3 is not a parallelogram. (Scenario 2)*
Question-Answer method	These were cases where the preservice teachers only directed questions to the students.	PT24: *I would ask the students about the properties of quadrilaterals. I would ask them what the shaped numbered 2 and 5 are. (Scenario 3)*
Other methods	Tutor, demonstration, daily life example, activities.	PT41: *The teacher should provide the student with tutor support. (Scenario 3)*
No answer	These were the situations where the preservice teacher could not propose a solution.	

PT8 used the method of "*direct instruction*" as they stated that definition should be given directly, PT18 used "*expository teaching*" as they used explanatory and comparative organizers, and PT60 used "*discovery learning*" as they allowed students to discover and fix their mistakes. The mistakes were aimed to be fixed by PT63 by using "*tangible materials*", by PT24 by asking questions to students and by PT41 by assigning tutors.

In this study, the PPSTs' written answers to all scenarios were analyzed by two researchers independently from each other for the reliability of the study using the formula ($\frac{\text{Agreement}}{\text{Agreement + Disagreement}}$) by Miles and Huberman (1994). The consistency between the coders was calculated as 0.91. So, it may be stated that the reliability of the study was highly acceptable. Additionally, the codes where the researchers had a disagreement were reviewed by the two researchers, and these disagreements were overcome.

4. Findings

This section includes findings derived from the PPSTs' answers to each scenario.

4.1. Identification of mistakes

Table 5. *Results regarding identification of mistakes*

	Correct identification of the mistake		*Partial identification of the mistake*		*Failure to identify the mistake*		*No answer*		*No mistake*	
	f	**%**	**f**	**%**	**f**	**%**	**f**	**%**	**f**	**%**
S1	29	34.93	48	57.83	2	2.4	2	2.4	2	2.4
S2	22	26.5	19	22.89	3	3.61	-	-	39	46.98
S3	19	22.89	28	33.73	10	12.04	7	8.43	19	22.89
S4	24	28.91	41	49.39	8	9.63	5	6.02	5	6.02
S5	-	-	6	7.22	4	4.81	14	16.86	59	71.08
S6	13	15.66	39	46.98	1	1.2	11	13.25	19	22.89

As seen in Table 5, a large part of the participants stated that there was no mistake in the second (46.98%) or fifth (71.08%) scenario. Additionally, 22.89% of the participants stated there was no mistake in the third and sixth scenarios. As in the case of many preservice teachers who stated that there was no mistake in the second scenario, PT55 and PT66 focused on the prototype form of a parallelogram. However, both a square and a rectangle satisfy the properties of a parallelogram. PT11 stated that there was no mistake in the third scenario by thinking that the area formula of the rectangle is valid for all quadrilaterals. PT52 and PT73 stated that there is a linear relationship between perimeter and area. For example, although the perimeters of a rectangle with side lengths of 2 cm and 6 cm and a square with side lengths of 4 are the same, their areas are different. Therefore, it may be stated that the inadequacies in the quadrilateral-related knowledge of the preservice teachers prevented them from noticing student mistakes.

PT55: *The students' answer is correct. A parallelogram is a shape of whose sides are parallel to each other. Shape 1 is a square, and shape 4 is a rectangle.* ***(S2)***

PT66: *The student's answer is correct, because the sides of a parallelogram do not cross each other perpendicularly as in shapes 1 and 4.* ***(S2)***

PT11: *The student gave the correct answer. The area is calculated by multiplying the lengths of two sides.* ***(S3)***

PT52: *The answer of the student is correct because in a geometric shape the perimeter and the area are directly proportional.* ***(S5)***

PT73: *It is correct. As the perimeter increases, the area also increases.* ***(S5)***

While the large majority of the participants were aware that there were mistakes in the given scenarios, a very small part of them were able to explain the mistakes in a mathematically accurate way. While no participant could correctly identify the mistake in the fifth scenario, very few were able to do so in the other scenarios. In the first scenario, PT4 correctly described the mistake by stating that the students selected the shapes that were suitable for the incorrect definition made by the teacher. In the second scenario, PT18 stated that the students focused on the prototype form of a parallelogram, and this is why they did not select the square or the rectangle. In the third scenario, PT9 identified the mistake correctly by explaining that the student generalized the area formula for a rectangle or a square to all quadrilaterals. In the fourth scenario, PT67 made a correct explanation by stating that multiplication should involve the area of a unit square rather than the length of one side. In the sixth scenario, PT45 correctly described the points the student could not comprehend while calculating the area of the given shape.

> ***PT4:*** *As Teacher Reyhan made the definition that a shape with equal sides is called a square, the students made mistakes. Therefore, the students selected the shapes numbered 1, 2 and 3 whose all sides were equal.* ***(S1)***

> ***PT18:*** *Since the students usually encounter shape 3, they did not regard the other shapes as parallelograms, although shapes 1 and 4 are also parallelograms.* ***(S2****)*

> ***PT9:*** *Tarik though that the area formula of a rectangle is valid for all quadrilaterals. This is because this formula results in correct answers regarding squares and rectangles.* ***(S3)***

> ***PT67:*** *The student found the length of the long side as 4 and the short side as 3 units. There are 12 unit squares in the shape. The student multiplied 12 and the unit square's side length, but he had to multiply 12 and the unit square's area.* ***(S4)***

> ***PT45:*** *It can be calculated. The student should have drawn a 4x4 square in a way to include the given shape within. He might not have thought that, afterwards, he needed to subtract the areas of the triangular pieces outside the shape from the area of the square.* ***(S6****)*

A large proportion of the participants who noticed that there were mistakes in the first (57.83%), fourth (49.39%) and sixth (46.98%) scenarios were able to determine the mistakes only partially correctly. A very small proportion of the participants completely failed to identify the mistake. In the first scenario, PT2 stated that, if the teacher used the expression *quadrilateral* while making a definition, the student would not make a mistake. On the other hand, an equilateral quadrangle’s all sides are also equal, but it is not necessarily a square. For this reason, in the first scenario, PT2 partially correctly identified the student’s mistake. PT50 made a partially correct discovery by not noticing that the students multiplied 12 with one side length instead of the area of the unit squares. In the sixth scenario, PT39 stated that the student could reach the solution by using their abstract thinking skill without any tools. However, as the participant did not explain what they meant by abstract operations, their response was accepted as incomplete. In the third scenario, PT78 explained the mistake incorrectly by stating that the student made a mistake because they considered the area formulae of a square and a rectangle to be same. PT79 identified the mistake incorrectly by giving a completely irrelevant answer (Figure 3). This was because PT79 gave as an example of a two-dimensional closed shape, a rectangular prism, which is, in fact, a three-dimensional object.

> **PT2:** *The students made mistakes as the teacher said “quadrilateral with equal sides” instead of “shape with equal sides”. The mistake was caused by the teacher’s expression.* ***(S1)***

> **PT50:** *To calculate the area, the side length of each unit square should be written in cm. The short side is 6 cm, and the long side is 8 cm. The area will be then 6×8=48.* ***(S4)***

> ***PT39:*** *The student thought that the area of the shapes could be measured with the help of a ruler and a goniometer. However, they could not think that it could be calculated by using a set of abstract mathematical operations without any tools.* ***(S6)***

> **PT78:** *A square and a rectangle are quadrilaterals, but their area formulae are different.* ***(S3)***

> **PT79:** *The answer is incorrect, because length does not change when a closed shape is opened. This is why the formula is wrong. For example, if we open this shape, the area of the shape turns out to be the same.* ***(S5)***

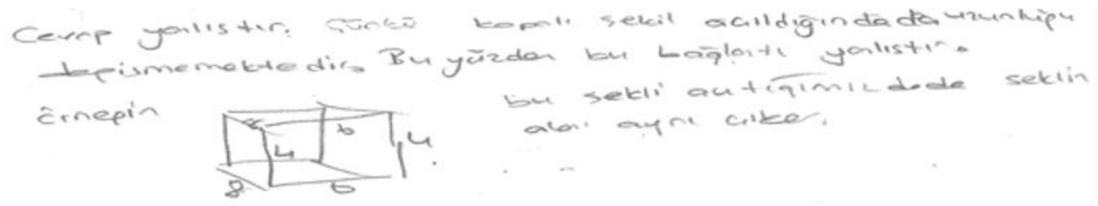


Figure 3. *Answer of PT79 to the fifth scenario*

4.2. Elimination of mistakes

Table 6. *Results regarding elimination of mistakes*

	Correct elimination		***Partially correct elimination***		***Failure to eliminate***		***No answer***	
	f	***%***	***f***	***%***	***f***	***%***	***f***	***%***
S1	9	10.84	41	49.39	22	26.5	11	13.25
S2	-	-	23	27.71	10	12.04	50	60.24
S3	-	-	22	26.5	16	19.27	45	54.21
S4	7	8.43	27	32.53	18	21.68	31	37.34
S5	-	-	1	1.2	3	3.61	79	95.18
S6	1	1.2	25	30.12	20	24.09	37	44.57

As seen in Table 6, very few participants were able to provide the correct solution recommendations to eliminate the mistakes in the given scenarios. Additionally, while no participant could form a correct solution recommendation in the second, third and fifth scenarios, only one (1.2%) could produce a correct one for the sixth scenario. PT8 brought a correct solution recommendation for the first scenario by stating that the mistake was caused by the definition made by the teacher, and the students would select only the first shape as a square if the definition had been made as a square has 4 sides, and the sides must be perpendicular to each other. In the sixth scenario, PT57 stated that the students could reach a correct solution by forming a square covering the given geometrical shape and finding its area by subtracting the areas of the triangle pieces from the area of this new shape (Figure 4). In the fourth scenario, PT62 provided a solution recommendation to eliminate the mistake by having the students understand that the unit squares did not consist of one unit, but they consisted of 2 units, and explaining how they could reach the correct solution.

> ***PT8:*** *Teacher Reyhan caused the students to fall into the misconception by defining the square as a shape with all sides equal. Therefore, the students chose all geometric shapes with equal side lengths. Shapes 1, 2 and 3 have equal side lengths. However, shape 2 is a triangle, and shape 3 is a parallelogram. When the square is defined as the shape of which four sides are equal and perpendicular to each other, such a situation will not occur. If the teacher had made the definition of the square more comprehensive, the students would have chosen only shape 1, and it would be the correct answer. (S1)*

> ***PT57:*** *This may be found by taken the spacing between two nails as one unit and surrounding the given shape by a rubber band. The students can find the area of the shape inside by subtracting the areas of the triangles from the area of the shape outside. (S6)*

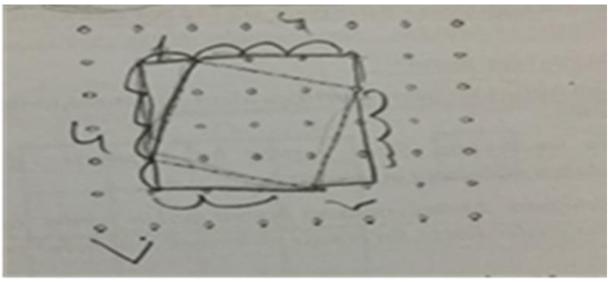
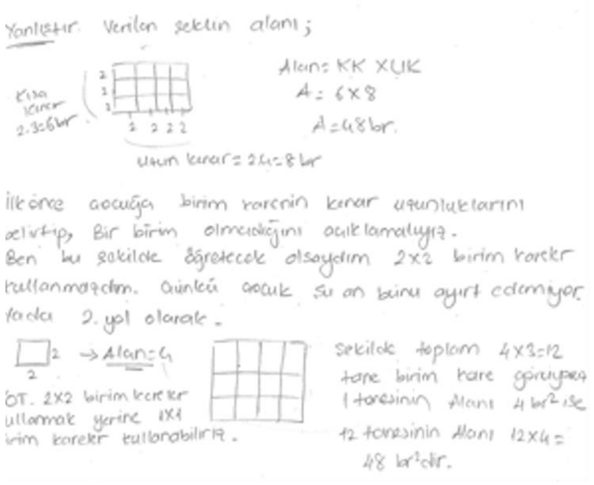


Figure 4. *PT57's drawing for scenario 6*

Table 7. *Answers of PT62 to the fourth scenario*

Participant	Answer to **scenario** 4	Translation of Quotation
PT62		*It is wrong. The area of the shape is Area = Short Side x Long Side* *First, we should tell the child that a side length of the unit square is not a unit. I would not use 2x2 unit squares if I were to teach this shape's area since the child cannot distinguish it right now.* *We can use 1x1 unit squares instead of 2x2 unit squares.* *There are 4x3 = 12 units in total in the shape. If the area of 1 unit square is 4, I will explain that the area of 12 unit squares is 48.*

The vast majority of the participants did not produce a solution recommendation towards eliminating the mistake in the second, third, fourth, fifth and sixth scenarios. Most of the participants who were able to bring solution recommendations for eliminating the mistakes either were able to bring partially correct solution recommendations or completely failed to eliminate the mistake. In the first scenario, PT10 brought a failed solution recommendation by not explaining how a square should be defined to eliminate the mistake. In the second scenario, PT83 failed to produce a solution recommendation for eliminating the mistake, because they stated that the diagonals of a rectangle cross each other with a perpendicular angle. However, rectangles whose diagonals are perpendicular to each other are squares, and this rule is not valid for all rectangles. In the fourth scenario, PT6 stated the necessity of teaching students the topic of area calculation, but they did not explain how this teaching process should be towards eliminating the mistake in the context of this scenario. In the third scenario, PT9 showed a correct approach towards eliminating the mistake. However, their response was considered partially correct as they did not make instructional explanations regarding how to calculate areas in shapes other than squares and rectangles.

PT10: *I would completely and correctly teach the definition of square for the students.* ***(S1)***

PT83: *The teacher should specify that the diagonals are perpendicular in a rectangle, but not in a parallelogram.* ***(S2)***

PT6: *Information should be provided to the student in terms of area calculation.* ***(S4)***

PT9:** The student generalized the formula they used in a square or a rectangle to other shapes. To overcome this, we need to show that the height of not every shape is a side of that shape, but some heights pass through the shape. **(S3)

4.3. Instructional methods

Table 8. *Results regarding the instructional methods*

Codes		S1	S2	S3	S4	S5	S6
Discovery learning	f	11	4	5	17	-	12
	%	13.25	4.81	6.02	20.48	-	14.45
Expository teaching	f	19	12	6	15	-	6
	%	22.89	14.45	7.22	18.07	-	7.22
Direct instructional method	f	41	11	19	17	4	18
	%	49.39	13.25	22.89	20.48	4.81	21.68
Manipulatives	f	1	2	2	3	-	10
	%	1.2	2.4	2.4	3.61	-	12.04
Question-answer method	f	-	2	3	-	-	-
	%	-	2.4	3.61	-	-	-
Other methods	f	-	2	3	-	-	-
	%	-	2.4	3.61	-	-	-
No answer	f	11	50	45	31	79	37
	%	13.25	60.24	54.21	37.34	95.18	44.57

As seen in Table 8, the participants who produced solution recommendations towards eliminating the mistakes in the scenarios mostly preferred the method of "*direct instructional*". They also used the "*expository teaching*" and "*discovery learning*" methods frequently in eliminating the mistakes. The participants also utilized the "*manipulatives*" and "*question-answer*" methods in eliminating the mistakes. On the other hand, the participants did not prefer methods that are more suitable for the level of primary school students such as *drama, analogy, educational games, digital storytelling* and *instruction with music*. PT8 in the third scenario and PT20 in the fifth scenario preferred the "*direct instructional*" method by directly reminding the students of rules, formulae and relations in eliminating the mistakes. In the fourth scenario, PT14 led the students to reach the result step by step by guiding them instead of directly stating the correct answer (*discovery learning*). In the second scenario, PT43 tried to eliminate the mistake by asking the students a question that would allow them to notice their mistake (*question-answer*). With the direct quotation of PT62 (Table 7), they argued that it must be explained to students that the unit squares are 2x2, and so, the area of any unit square is 4. Moreover, PT62 used the "*expository teaching*" strategy because they preferred to make educational explanations themselves to provide meaningful learning in eliminating this mistake. In the first scenario, PT61 stated that the mistake made by the students regarding the concept of a square could be eliminated with the help of three-dimensional tangible materials.

P8: *The students should be re-taught the area calculation formulae for geometric shapes. **(S3).***

P20: *I would tell the student that the formula they have found is not valid for all quadrilaterals. **(S5)***

PT14: *I would first have the students count the unit squares found in the long side. Afterwards, by emphasizing that the side length of each unit square is 2, I would have them calculate the* *length of the long side. Similarly, I would guide them to find the length of the short side. Finally, the students could reach the correct result by using the area formula.* ***(S4)***

P43: *I would ask the students why they did not select the shapes numbered 1 and 4.* ***(S2)***

PT61: *...I would try to materialize it with three-dimensional models.* ***(S1)***

5. Conclusion and discussion

In this study, preservice primary school teachers' PCK about quadrilaterals was examined in the context of intervening in children's mistakes. In the study, six open-ended scenarios were presented to the preservice teachers. The answers of the preservice teachers to the open-ended scenarios were analyzed in the context of identifying mistakes, eliminating mistakes and instructional methods.

5.1. Identification of mistakes

As a result of the study, it was observed that the PPSTs had difficulties in identifying student mistakes about quadrilaterals. The PPSTs were partially successful in scenario 1, scenario 2, scenario 3, scenario 4 and scenario 6. Nonetheless, they were unsuccessful in identifying the student's mistakes in scenario 5, in which the perimeter-area relation in a rectangle was discussed. In other words, it was observed that the KUS of the PPSTs about quadrilaterals was not on the desired level. Similarly, studies in the literature (Even & Tirosh, 1995; Fernández, Llinares, & Valls, 2013; O'Hanlon, 2010; Rieche, Leuders, & Renkl, 2019; Son & Sinclair, 2010; Schleppenbach, Flevares, Sims, & Perry, 2007; Tirosh, 2000) have shown that pre-service teachers and in-service teachers have difficulties in identifying student mistakes related to many mathematical concepts. Son and Sinclair (2010) stated that pre-service teachers experienced difficulties in explaining the mistakes of students related to geometry. Schleppenbach et al. (2007) reported that students in the classrooms of American and Chinese mathematics teachers made mistakes at similar frequencies. While the American teachers usually ignored the mistakes of the students, the Chinese teachers guided the students to think about their mistakes. Tirosh (2000) claimed that prospective elementary school teachers were unaware of the major sources of students' misconceptions regarding division in fractions. O'Hanlon (2010) found that pre-service secondary school mathematics teachers' KUS was weak. Contrary to the results of these studies, some studies in the literature (Chick, 2010; Gal, 2011; Gökkurt, Şahin, Erdem, Başıbüyük, & Soylu, 2015b) revealed that pre-service teachers' levels of KUS are sufficient. In Chick's (2010) study, the vast majority of mathematics teachers were able to recognize misconceptions about ratios. Gal (2011) carried out Problematic Learning Situations activities in geometry with a pre-service teacher. At the end of the activity, the pre-service teacher was successful in understanding and explaining the thoughts and misconceptions of students in geometry. Gökkurt et al. (2015b) determined that pre-service mathematics teachers were successful in identifying and explaining students' mistakes about the concept of a cone. If we evaluate the results of this study and the studies in the related literature together, we mat state that, for some concepts, prospective teachers are successful in identifying and explaining learning difficulties, but they fail in other concepts. However, in this study, it was observed that the PPSTs were unable to meet expectations in identifying the learning difficulties that students experienced with regard to quadrilaterals.

As a result of the study, it was observed that the content knowledge of the PPSTs was inadequate with regard to the concepts in the scenarios in which they had difficulties in identifying the student's mistakes. In other words, it may be stated that the two subcomponents of PCK, content knowledge and knowledge of understanding of students, are positively correlated with each other. For example, in Scenario 2, in which students chose only the prototype model for the parallelogram, PT55 stated that the answer of the students was correct. PT55 explained their statement by saying: *"Shape number 1 is a square, and shape number 4 is a rectangle"*. In other words, the pre-service teacher stated that the square and rectangle are not examples of parallelograms. Fujita (2012) made the inclusive definitions of the parallelogram as "*A quadrilateral which has two pairs of parallel lines*." However, when the definition of the parallelogram is examined, it is observed that the rhombus, rectangle and square meet the requirements of this definition. In other words, the rhombus, rectangle and square are also parallelograms. Therefore, it may be stated that the content knowledge of the pre-service teacher about the geometric shapes representing a parallelogram was insufficient. For this reason, the pre-service teacher could not identify the mistake of the student since the pre-service teacher themselves also had the same misconception as the student. In fact, many studies in the related literature have revealed that the content knowledge of teachers and pre-service teachers plays a vital role in understanding students' mistakes (Ball, 1991; Blömeke, Hsieh, Kaiser, & Schmidt, 2014; Krauss & Brunner, 2011; Pankow et al., 2016; Son, 2013). It was experimentally proven that content knowledge is necessary for the mathematics learning process of students to be fast and correct (Blömeke et al. 2014; Krauss & Brunner, 2011). Pankow et al. (2016) similarly determined that early-career mathematics teachers with strong content knowledge could detect student mistakes faster and more accurately. Son (2013) stated that subject content knowledge is necessary in understanding student mistakes related to quadrilaterals. However, they emphasized that a good level of subject content knowledge does not guarantee a good level of knowledge on the understanding of students. The relevant literature and the results of this study revealed that the content knowledge of teachers and pre-service teachers is an essential factor in identification, description and understanding the source of misconceptions or learning difficulties that students have about mathematical concepts.

5.2. Elimination of mistakes

As a result of the study, we saw that the PPSTs had difficulties in making precise educational explanations for elimination of the learning difficulties of students with regard to quadrilaterals. In general, the PPSTs either made inadequate instructional explanations or did not offer any solution recommendation. Moreover, very few PPSTs were able to make correct or partially correct instructional explanations for eliminating the student’s learning difficulties. In other words, the KIS of the PPSTs related to quadrilaterals was not on the desired level. Besides, in many studies (Ball, 1988; Cooper, 2009; Galant, 2013; Jakobsen, Ribeiro, & Mellone, 2014; Kleickmann et al., 2015) mentioned in the related literature, it has been concluded that pre-service teachers' KIS related to many mathematical concepts is inadequate. In this study, it was observed that the PPSTs who had difficulty in identifying the mistakes of students could also not produce a solution recommendation to correct these mistakes. Additionally, although some PPSTs could describe the mistake correctly, they could not produce the correct solution proposal for correcting the mistake. In other words, in order to have the KIS of PPSTs on the desired level, they should have enough KUS (Cooper, 2009). However, pre-service teachers’ possession of a high level of KUS does not guarantee that their KIS will be on a desired level. Masduki, Suwarsono and Budiarto (2017) reported that, although high school mathematics teachers were successful in explaining the mistakes of students regarding equations, they found it difficult to produce solutions to eliminate these mistakes. Likewise, Son and Sinclair (2010) stated that pre-service teachers had inadequacy in producing solution recommendations to eliminate mistakes, although they correctly recognized the mistakes students made in geometry. Gökkurt, Şahin, Soylu and Doğan (2015a) stated that, although pre-service mathematics teachers have moderate KUS in relation to geometric objects, they have insufficient knowledge of instructional strategies. Considering the related literature and the results of this study, it may be stated that, for producing solution recommendations to correct the mistakes of students, it is important for pre-service teachers to understand and identify the mistakes of students, but it is not enough by itself.

5.3. Instructional methods

As a result of the study, it was observed that, in the process of eliminating the mistakes of students, the PPSTs preferred mainly the "*Direct Instructional*" method, which is based on traditional approaches and centers the teacher. As it is known, PPSTs are responsible for the education of children in the 7-11 age range when they become teachers [MoNE, 2017]. When the Turkish mathematics curriculum is examined, we see that a contemporary philosophical approach has been adopted in which students are responsible for self-learning, discovering knowledge, questioning, problem-solving and reasoning [MoNE, 2017]. According to Piaget's (1936) theory of cognitive development, it is also expected from children in the concrete operational period to be able to discover information through concrete materials and various activities in the mathematics learning process (Şahin, 2012). Therefore, it may be stated that the PPSTs were unsuccessful in selecting and using methods-techniques that are suitable for both the philosophy of the Curriculum and the cognitive development levels of students. Similarly, Şahin et al. (2016) and Güler and Çelik (2019) stated that pre-service mathematics teachers used methods of making students memorize the rules to correct the students' mistakes related to numbers. However, Yavuz Mumcu (2017) found that pre-service teachers offered valid methods to correct the conceptual mistakes of students with regard to fractions, but they were generally inadequate in using these methods as appropriately for the situation.

6. Limitations and recommendations

This study investigated the PCK of PPSTs regarding quadrilaterals in the context of intervention with student mistakes. This study was limited to PPSTs enrolled at the faculty of education at one university. Additionally, the data of the study were obtained with the help of six open-ended scenarios. As a result of the study, it was seen that the PPSTs who were about to complete their undergraduate education were not on a desired level in terms of identifying possible student mistakes and producing solution recommendations towards eliminating these mistakes. In this sense, learning environments that will allow pre-service teachers to encounter student mistakes throughout their undergraduate education and provide them with opportunities to intervene with these mistakes should be designed. For this, in the "*Mathematics Teaching I-II"* courses, which are the only courses taken by PPSTs regarding teaching of mathematics, different instruments and methods such as case study, digital storytelling and vignette could be utilized. This study determined the PCK levels of PPSTs only regarding quadrilaterals. Further studies may investigate PCK levels or developments towards different geometry concepts such as angles, triangles, prisms, pyramids or symmetry.

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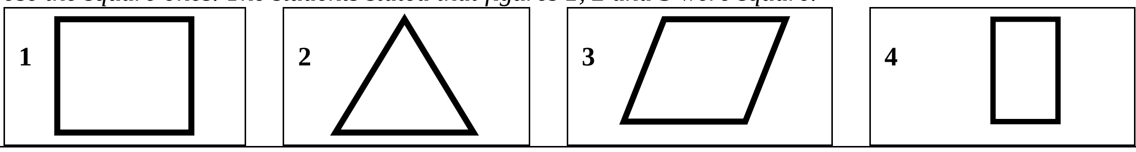
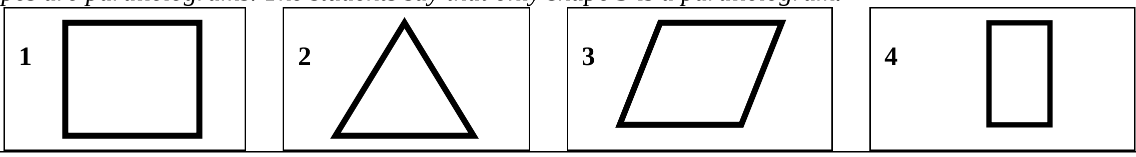
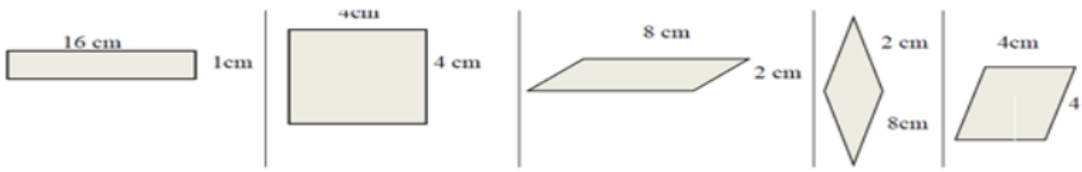
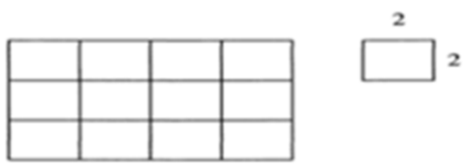
Appendix 1: Scenarios Used in This Study

N	Scenarios Used
1	*Teacher Reyhan was teaching the students the square shape. At first, after checking the students' preliminary knowledge about the square shape, she showed the students several square models. Later, Reyhan defined the square as "the shape with all sides equal." She then gave her students the following shapes and asked them to choose the square ones. The students stated that figures 1, 2 and 3 were square.* 
2	*Teacher Hasan gives to the students, who are taught the parallelogram for the first time, the following parallelogram models to check their preliminary knowledge. He wants the students to determine which of the shapes are parallelograms. The students say that only shape 3 is a parallelogram***.** 
3	*After teaching the subject of the area of quadrilaterals, teacher Tolga asked his students to draw different quadrilaterals with the same area. Tarık among the students drew the following shapes.* 
4	*You are teaching your students the area relation in a rectangle. You have noticed that many students can calculate the areas of rectangular shapes. Then, to be sure that your students can calculate the areas of rectangular shapes, you gave them the following shape which consists of 2x2 unit squares. Caner among the students said: "My teacher, the area of this shape is 24 because the long side is 4 and the short side is three units, and I get 12 out of here. Multiplying 12 and 2, we obtain 24. This will ultimately give us the area of the shape. "* 
5	One day, a student came very excitedly to you, and between you, the following dialogue took place. **Student:** My teacher, I think I found a new relation. I want to share this relation with you. **You:** Really? **Student:** Yes, my teacher. You have never mentioned this relation before. I found it myself. **You:** Ok, will you share the relation with me? **Student:** My teacher, if the perimeter of a closed shape increases, its area also increases. So, a shape with a more extended perimeter has a larger area.

6	Teacher Can is carrying out an activity related to the area and perimeter in polygons, on a geometry board with his students. Can asked the students to create various polygons with the help of the rubber bands given to them and calculate the areas and perimeters of these polygons. Haydar among the students said: *"My teacher, I created this shape, but the area and the perimeter of this shape cannot be calculated without using a ruler and a goniometer."*