Journal of Urban Mathematics Education
December 2020, Vol. 13, No. 2, pp. 60–86

Working Toward Linguistically and Culturally Responsive Math Teaching through a Year-Long Urban Teacher Training Program for English Learners

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This qualitative study examined how participating in-service teachers demonstrated linguistically and culturally responsive mathematical teaching (LCRMT) competences after they completed a year-long National Professional Development Program grant-funded project. A two-dimensional LCRMT framework was developed to measure participating teachers' mathematical and mathematics-related competences. The qualitative data source was from three in-service teachers' observations and interviews. The interview and observation data were analyzed using open and axial coding and activity systems. Three themes emerged: 1) mathematics-related content teaching practices, 2) tools to support mathematics learning, and 3) teachers' mindsets and attitudes towards English Learner (EL) teaching. The researchers then compared verbatim examples using activity systems to examine the following research question: How did participating urban in-service teachers apply linguistically and culturally responsive mathematics teaching competences for ELs learned at a university EL teacher training program to their actual mathematics teaching in the classroom? The results, in general, indicated that the urban in-service teachers demonstrated improvement of LCRMT strategies that they used in their actual mathematics teaching after they completed the university training. However, challenges in the areas of mathematical discourse competences and teachers' sociocultural beliefs toward ELs revealed the need for ongoing professional support.

KEYWORDS: activity theory, in-service teachers, LCRMT, teaching competences, urban education

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There is a need to improve teacher preparation programs for English Learners (ELs) in the United States, as a majority of U.S. teachers and teachers-to-be have little to no training to support ELs (Bunch, 2010). This lack of training is manifested in the secondary school dropout rate of 22.4 % for Latino/Latina youth, whose population doubles the national EL average (Janzen, 2008). In mathematics courses specifically, pre-service and in-service teachers struggle to adequately prepare ELs for academic success. In a study reporting the results of a K–8 mathematics methods course, Aguirre et al. (2012) posit the need to develop "robust forms of pedagogical content knowledge (PCK) for preservice teachers through a culturally responsive mathematics teaching approach" (p. 113). Such a change in educator training has the potential to greatly increase academic success in a large student population.

This study uses data from the National Professional Development Grant Program project titled Excellent Teachers for English Learners (ETEL), which collected data from pre- and in-service teachers in 2015. The ETEL project aimed at preparing teachers to improve ELs' academic achievement with a focus on mathematics. Mathematics is one of the major areas ELs are at risk of failing in school (Janzen, 2008). One of the reasons why ELs have an achievement gap may be a common misleading myth regarding ELs' mathematics learning: "The transition from social language to academic language is easier for ELs in math than in other subjects" (Kersaint et al., 2009, p. 60), like reading and social studies.

Another misleading assumption is that mathematics is a "culture-free" subject (Kersaint et al., 2009, p. 60) that may not have to deal with language. However, mathematics education deals both with everyday language and academic or technical mathematical language (Schleppegrell, 2007). Halliday (1978) points out that counting and calculating for mathematics draw on everyday mathematical language, such as "counting up" and "counting on," because counting is an everyday language mastered by ELs and may not need to be taught. On the other hand, ELs still need to acquire the academic language of mathematics when they enter school. One mathematics content pedagogical approach that seeks to address this is the introduction of ''new styles of meaning and modes of argument and of combining existing elements into new combinations'' (Halliday, 1978, p. 195). Halliday emphasizes the linguistic challenges of mathematics education when discussing the "mathematical register," which he defines as "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings" (p. 195). The language of mathematics not only requires a list of vocabulary, grammatical patterns, and equations with numbers and/or words with precise meanings, but also requires communicative competence or mathematics discourse sufficient for active participation in meta[cognitive]-mathematical thinking and reasoning procedures (Moschkovich, 2012). Teachers also need to grasp how mathematics content is structured with a particular set of mathematical language/discourse and mathematics pedagogy to make their teaching more equitable and effective for ELs (Tran, 2014).

Understanding the need of teachers' mathematics discourse and pedagogical competency development in the field of Teaching English to Speakers of Other Languages (TESOL), this study examines how multidimensional mathematics teaching competences learned at a university EL teacher training program have or have not been applied to teachers' K–6 mathematics teaching for ELs. A research question guided this study: How did participating urban in-service teachers apply linguistically and culturally responsive mathematics teaching for ELs learned at a university EL teacher training program to their actual mathematics teaching in the classroom?

Theoretical Framework

In this section, we review a theoretical framework starting with linguistically and culturally responsive teaching (LCRT) and expanding it to linguistically and culturally responsive mathematics teaching (LCRMT).

Development of LCRT Framework

A basic goal of EL teacher education programs is to prepare linguistically and culturally responsive (LCR) teachers. Highlighted in this framework is *linguistically responsive* teaching (Lucas et al., 2008), recognizing that most research in teacher education has predominantly emphasized *culturally* responsive/relevant teaching/pedagogy (CRT; Gay, 2010; Ladson-Billing, 2014). This predominant CRT emphasis has resulted in shadowing or downplaying the role of language in ELs' academic achievement (Nieto, 2002). We also note that teachers need to develop linguistically responsive pedagogy because "language cannot be separated from what is taught and learned in school" in any content classroom (Lucas et al., 2008, p. 362). Bonner and Adams (2012) emphasize culturally responsive mathematics teaching when teacher education programs "prepare prospective teachers for being able to reach children where they are and make children's mathematics learning experiences strong" (p. 35). Hymes (1972) mentions "linguistic competence as just one kind of cultural competence" (as cited in Byram, 1997, p. 8). Such a focus on the cultural aspect does not give the due understanding of major roles academic and social languages play in ELs' learning (Nieto, 2002). In this study, we emphasize linguistically responsive teaching up front, which is tied with culture and power.

Background of LCRT Framework Development

In 2013, a research-based LCRT framework was developed (Song & Simons, 2014). The LCRT framework was built on works from Van Dyne and his associates'

(2012) four factor model of cultural intelligence (CQ), which was influenced by van Ek's (1986) sociocultural and communicative competence work. The four factors of their CQ model were "metacognitive, cognitive, motivational, and behavioral" cultural intelligences (Van Dyne et al., 2012, p. 295). Metacognitive CQ is a strategic factor that involves awareness of oneself and others, planning on how to approach a cultural situation, and checking the process of monitoring what one does and how it actually plays out in relationship to what one expects to happen. Cognitive CQ is a knowledge factor (i.e., one's actual knowledge about particular cultural issues and differences, including cultural norms). Motivational CQ is a drive factor, involving being interested, confident, and driven to adapt to another culture. Behavioral CQ is an action factor or an actor—an individual's ability to modify their verbal and nonverbal behaviors in order to interact with others from different cultural backgrounds. Thus far, however, CQ has not been assessed in relationship to TESOL, only in staff development and leadership training (Van Dyne et. al., 2012).

In this study, our aim is not to explore linguistic competence as part of cultural competence but rather the opposite. Our focus is on helping teachers develop linguistically (and culturally) responsive teaching competences. In general, teachers lack knowledge on language systems or linguistics, and they do not understand the benefits of valuing the linguistic diversity ELs bring into the classroom (Fillmore & Snow, 2002; González & Darling-Hammond, 1997). Teachers with ELs need to receive more systematic and intensive preparation on what it means to possess and demonstrate linguistically responsive teaching competences, which embrace pedagogical and cultural competences (Kim et al., 2018).

van Ek (1986) introduced a model of communicative ability with six competences (linguistic, sociolinguistic, discourse, strategic, socio-cultural, and social) that emphasizes the role of the native speaker as a model for a language learner. Outside of linguistic competence, the other five competences consider the influence of sociocultural and pedagogical aspects in developing communicative competence (Song & Simons, 2014). Lucas and her associates (2008) introduced six linguistically responsive teaching principles for authentic EL teaching: a) every day and academic/technical language, b) teachable but challenging content concepts, c) roles of social interaction between ELs and other peers in the classroom, d) roles of ELs' native languages, e) inclusive classroom climate for minimal anxiety, and f) linguistic form and functions. Lucas and Villegas (2010) designed a linguistically responsive teaching framework that complements van Ek's communicative principles and Lucas et al.'s LCRT principles that emphasizes knowledge of languages, second language acquisition principles, and ELs' linguistic and cultural diversity. Each of these LCRT elements represents a commitment made by teachers to become more aware and considerate of what ELs can bring to the classroom and challenges that they routinely face (Song & Simons, 2014). These frameworks represent dimensions of linguistic, pedagogical, and cultural competences, but they are limited by their one dimensionality.

In light of this, Nguyen and Commins (2014) introduced a two-dimensional LCRT framework. Nguyen and Commins proposed linguistic, pedagogical, and cultural competences as the first dimensional constructs and knowledge in depth; higher-order thinking skills such as application, analysis, and synthesis; and implementation as the second dimensional constructs, which are yet to be elaborated. In 2016, an LCR**M**T framework, based on Nguyen and Commins' (2014) two-dimensional LCRT model, evolved after the first author of the current study explored extensive research on linguistically and culturally responsive teaching as well as mathematics teaching for ELs (Aguirre et al., 2012; Fillmore & Snow, 2002; González & Darling-Hammond, 1997; Halliday, 1978; Janzen, 2008; Lucas et al., 2008; Schleppegrell, 2007; Song & Coppersmith, 2017).

Development and Expansion of LCRMT Framework

The newly developed LCRMT framework has two dimensions: a mathematics-related dimension and a meta[cognitive]-mathematics dimension. The first mathematics-related dimension for EL teachers includes the following competences: a) mathematics content competence, b) mathematics discourse competence, and c) mathematics pedagogical competence. The second meta[cognitive]-mathematics dimension's features address larger contexts of mathematics teaching practices, mainly through EL teachers' "how-to-actors": a) acquiring and demonstrating knowledge in depth, b) developing and applying procedural demands and reasoning skills, and c) examining and developing socio-political teacher beliefs (Table 1). The three how-to-actors start with the action verb (e.g., acquire, demonstrate, develop, and examine), unlike Nguyen and Commins' (2014) noun phrases in the second-dimensional construct. The LCRMT constructs can be used to measure the three competences of the first dimension. The following characterizes details of the LCRMT framework.

LCRMT's first-dimensional mathematics-related competences are interdependent with the meta-mathematics how-to-actors when preparing, delivering, and reflecting on mathematics instruction for ELs (Uribe-Flórez et al., 2014). Teachers need to maintain a focus on mathematics reasoning (i.e., develop and apply mathematics procedural demands and reasoning under mathematics content competence) as well as language development by recognizing "how ELs express their mathematical ideas as they are learning English, and teachers can maintain a focus on mathematical reasoning as well as on language development" (Moschkovich, 2012, p. 18). Teachers need to allow ELs to use their native languages to understand individual mathematics-related words (acquire and demonstrate knowledge in depth) so they can expand their mathematics reasoning. Nguyen and Commins (2014) emphasized the importance of teachers' sociopolitical endeavors because teaching cannot be neutral when they prepare content resources and pedagogies (i.e., examine and develop sociopolitical teaching beliefs under mathematics pedagogical competence).

The first author of the current study renamed and rearranged the first- and second-dimensional features of LCRT when developing LCRMT. For the first dimension, the LCRT (D. Nguyen & N. Commins, personal communication, December 9, 2014) had a) linguistic, b) pedagogical, and c) cultural competences. For LCRMT, the first author added mathematics content competence and renamed linguistic competence to mathematics discourse competence and pedagogical competence to mathematics pedagogical competence to be more inclusive and mathematics-related. LCRT's cultural competence was incorporated into LCRMT's third how-to-actor: examine sociopolitical beliefs.

The second-dimensional features are "actors" that help teachers acquire, prepare, deliver, assess, and reflect on their mathematics teaching more inclusively to better serve ELs. These three how-to-actors start with action verbs, such as acquire, develop, apply, demonstrate, and examine, in order to support and measure the first dimension of mathematics-related competences (Table 1). For example, when preparing a mathematics lesson on a unit of weight (acquire knowledge in depth under mathematics content competence), an LCR mathematics teacher should explore their ELs' academic and cross-cultural funds of knowledge when exploring mathematics pedagogies (examine sociopolitical beliefs under mathematics pedagogical competence; Moll, 2015). Teachers may need to check if the ELs are from countries in which they use different metrics (e.g., the metric measurement system rather than the American standard measurement system). Teachers also need to recognize the learning needs of ELs and cultivate strategies "implemented with respect to the [math] languages students bring to the class" (acquire knowledge in depth under mathematics discourse and pedagogical competences; Uribe-Flórez et al., 2014, p. 236).

The new assessments for Common Core State Standards require "students to show their work and explain it" (Crouch, 2015, p. A7). In order for ELs to develop the skillset necessary to do so, teachers need to support them when they solve equations and word problems in class (develop procedural demands under mathematics content competence). In addition, the National Council of Teachers of Mathematics (2000) has set priorities for constructivist teaching, under which students work to problem solve through reasoning and communication (develop procedural demands and reasoning under mathematics pedagogical competence; Swars et al., 2015). The term "mathematical register" relates to LCRMT's second-dimensional how-to-actors by choosing particular language forms to aid understanding about how mathematics content competence is formed and includes multiple "semiotic systems and grammatical patterns" unique to mathematics discourse competence (Schleppegrell, 2007, p. 141).

EL teachers should be aware that the "symbols, oral language, written language, and visual representations such as graphs and diagrams" (Schleppegrell, 2007, p. 141) used in the mathematical register may present linguistic challenges (acquire and demonstrate mathematics knowledge in depth under mathematics content and discourse competences; Halliday, 1978). EL teachers need to conquer these linguistic challenges and portray mathematics concepts by helping ELs develop procedural demands and reasoning skills. Varying ways of illustrating mathematics concepts and ideas through language forms and other avenues (i.e., develop procedural demands under mathematics discourse competence) makes mathematics learning more accessible for ELs (Halliday, 1978; Schleppegrell, 2007). As many mathematics ideas and concepts do not translate easily, EL teachers need appropriate mathematics pedagogical skills for instructing students in technical mathematics discourses (Schleppegrell, 2007). Teachers also need to help ELs understand that there are diverse registers (Pimm, 1987) and that they have to learn when to use particular mathematical languages (e.g., everyday versus technical mathematical language) with proper reasoning processes (Aguirre et al., 2012). Table 1 illustrates the nine constructs of the LCRMT framework with examples and references.

Table 1
Nine Constructs of Two-Dimensional LCRMT Framework for Mathematics Teachers with ELs

Math-Related vs. Meta-Math Actors	Math Content Competence	Math Discourse Competence	Math Pedagogical Competence
1. Acquire & Demonstrate Knowledge in Depth	1a. Teachers acquire knowledge of math content, math symbols, notations (+, -, ×, ÷, =, ⁄), math operations, & visual representations (Schleppegrell, 2007) 1b. Teachers demonstrate knowledge of math content curriculum and standards (Grossman et al., 2005).	1a. Teachers demonstrate knowledge of math language and math register in teaching ELs (Halliday, 1978) 1b. Teachers acquire knowledge of technical content discourse (Aguirre, et al., 2012) 1c. Teachers acquire natural/ nontechnical math discourse (Schleppegrell, 2007).	1a. Teachers acquire culturally relevant math content pedagogy knowledge (Turner & Drake, 2016) 1b. Teachers demonstrate teaching strategies to support ELs' math language acquisition (Richards, 2013).
2. Develop & Apply Metacognitive Procedural Demands	2a. Teachers develop teachers' understanding of math concepts and procedural demands (Turner & Drake, 2016) 2b. Teachers apply knowledge of math concepts to word problems with reasoning process (Janzen, 2008; Moschkovich, 2012).	2a. Teachers apply meaning of math concepts to procedures and reasoning strategies (what if & therefore; Aguirre, et al., 2012) 2b. Teachers attribute process of part-to-whole and nonreversible math operation rules (Janzen, 2008; Turner & Drake, 2016).	2a. Teachers apply procedural and reasoning skills using cooperative and inquiry-based learning strategies (Janzen, 2008; Moschkovich, 2012) 2b. Teachers create situated learning experiences based on ELs' linguistic and cultural resources (Gee, 2016).
3. Examine & Develop Cross-Cultural & Sociopolitical Beliefs	3a. Teachers examine and develop cross-culturally "just" beliefs in teaching math for ELs (Turner & Drake, 2016) 3b Teachers explore ELs' funds of knowledge for academic learning (Moll, 2015).	3a. Teachers reject an English-only orientation and apply equitable attitude toward different language use (Austin, 2009; Liggett, 2014) 3b. Teachers develop social discourse for ELs' conversational and academic math language advancement (Moschkovich, 2012).	3a. Teachers integrate equity-based pedagogy into their teaching (Flores & Rosa, 2015) 3b. Teachers create safe, welcoming climate with minimal anxiety (Krashen, 2003; Pappanihiel, 2002) 3c. Teachers examine cross-cultural curriculum knowledge within situated context (ounce & pound vs. gram & kilogram).

Research Methods

We chose a qualitative case study design and used the data collected from observations and interviews in which we examined three urban in-service teachers' applications of EL-specific mathematics teaching strategies that they learned at a university EL teacher training program to their actual mathematics teaching in an elementary school setting. The research question that guided this study was the following: How did participating urban in-service teachers apply linguistically and culturally responsive mathematics teaching competencies for ELs learned at a university EL teacher training program to their actual mathematics teaching in the classroom?

Study Context

The two main activities of the ETEL project were a) six tuition-free TESOL courses for the ETEL cohort of pre- and in-service teachers to obtain state TESOL certification and b) five full-day summer institute professional development (PD) sessions followed by monthly PDs in the area of mathematics. The LCRMT constructs were integrated into each of the six TESOL courses: a) linguistics for enhancing teachers' mathematics content discourse and acquiring mathematics knowledge in depth, b) principles of second-language acquisition to support ELs' content and discourse competencies and in-depth knowledge acquisition, c) sociolinguistics and cross-cultural communication to enhance teachers' discourse competence and examination of sociopolitical teaching beliefs, d) assessment and instructional material development that emphasizes pedagogical competence and development of procedural demands, e) EL teaching methods that focus on three mathematics-related competences and three meta[cognitive]-mathematics how-to-actors, and f) practicum for EL teachers that is focused on three mathematics-related competences with all of the three meta[cognitive]-mathematics how-to-actors. The participating urban teachers were taking two TESOL courses per semester in 2015: two courses in the spring (linguistics and second-language acquisition courses), two in the summer (sociolinguistics and assessment courses), and two in the fall (methods and practicum courses). The participants also attended a five-day PD summer institute that included activities to improve their mathematics content knowledge, content discourse, and content pedagogical competences for teaching ELs. When taking TESOL methods and practicum courses, participants were required to prepare, implement, and reflect on mathematics teaching cases using the LCRMT framework as their guide.

A mathematics consultant and a state EL specialist from the ETEL team examined the elementary school mathematics curriculum utilized by the in-service teachers to verify that it followed World-Class Instructional Design and Assessment (WIDA) English Language Development standards (WIDA). These standards provided the language supports used during instruction, which were based on the six

English proficiency levels present in the four language modalities (reading, writing, speaking, and listening; WIDA, n.d.). These two mathematics and English-language teaching specialists additionally chose key mathematics concepts, such as place value, number sense, and fractions, for the mathematics PDs along with EL-specific teaching practices for mathematics content, discourse, and pedagogical competences. Some mathematics teaching methods included inquisitive, hands-on, and collaborative activities and backward assessments. Tools such as manipulatives, sentence stems, multi-language mathematics vocabulary word banks, word-walls, games, number tables, and graphic organizers were also introduced to and utilized by these participating urban teachers.

Participants and Sampling Methods

Seven urban in-service teachers participated in the 2015 ETEL grant project. The researchers contacted the seven in-service teachers through emails to arrange follow-up visits in 2016 after their ETEL graduation in December 2015. Three (42.9%) of the in-service teachers agreed to participate in follow-up observations and interviews. The three in-service teachers, who completed the university ETEL grant project and agreed to participate in this qualitative study, were from two schools. Ms. Happ is a female sixth-grade teacher who taught at School 1. Mr. Mack is a male third-grade teacher and Ms. Bishop is a female second-grade teacher who taught at School 2. Ms. Happ taught mathematics for the sixth graders in School 1 for five years. Mr. Mack taught third and fourth graders in School 2 for four years. Ms. Bishop taught the second graders for two years in School 2. Pseudonyms were used for the teachers' names.

The EL population in School 1 and School 2 were diverse (more than 10 nationalities and more than nine languages). The ELs were from South/Latin American countries and U.S. territories (Colombia, Mexico, and Puerto Rico) as well as Bosnia, Ghana, Iran, Nepal, Somalia, Syria, and Vietnam. Their English proficiency levels were varied (level 1 to level 4). All three urban in-service teachers were monolinguals, though the two teachers in School 2 tried to use Spanish translations for new words that they got from Google Translate, which was a method introduced in the TESOL methods and practicum courses. However, the teachers did not even try to translate other languages, such as Bosnian and Vietnamese, even though more than 30 percent of the ELs were from Bosnia and Vietnam. Purposive sampling was chosen for this study in order to gain information "of central importance to the purpose of the research" by "focusing in depth on a small number of participants" (Patton, 1990, p. 169).

Instruments and Data Collection

An observation protocol was designed with three sections: 1) heading (grade, date/time, mathematics content, and school), 2) low inference, which included time and field notes describing what actually happened, and 3) high inference, which contained the observer's interpretation, comments, and reflection (see Appendix A). The observation protocol was developed so the observers (two researchers) could collect low inference data (factual data; i.e., what actually happened) as well as high inference data (opinions and interpretation data; e.g., I think/assume/argue). Three observations, each lasting between 30 minutes to one hour, were conducted for each teacher at times convenient for the teachers. These observation periods allowed the researchers to explore if and how the teachers were utilizing what they had learned in the TESOL courses and the mathematics PDs in the ETEL learning community. Each observation occurred one to two weeks apart, and the researchers took field notes using the observation protocol.

An interview protocol was developed to explore the teachers' mathematics teaching practices with two main questions: 1) What have been some success indicators of your mathematics teaching in the classroom with ELs? and 2) What have been some challenges when you teach mathematics to ELs? (see Appendix B). Two 30-minute interviews were conducted with each of the three in-service teachers, taking into consideration the demands on their schedules. The researchers took the notes and recorded the interviews using an iPad. The recorded interviews were transcribed.

Data Analysis

This basic qualitative data analysis involved two phases. The first phase was informed by a grounded data analysis approach (Charmaz, 2010). The second phase was informed by activity theory (Yamagata-Lynch, 2010). The grounded data analysis was processed by the two researchers. First, they open coded the observation and interview data individually and compared the coded data regularly to examine the commonality and criticality. Then the researchers collaboratively produced a coding book with key codes, descriptions, and categories (Saldaña, 2016) after they had open coded individually. Using open and axial coding (Strauss & Corbin, 1998), the researchers explored and obtained verbatim examples to identify emerged themes (Saldaña, 2016). Each researcher identified individual codes and themes that emerged from the first phase through the grounded theory approach. Then, we compared the findings from the collaborative coding process with the elements of the activity systems (Yamagata-Lynch, 2010).

Engeström (1987) clarified that two complex human systems (e.g., the university teacher education program versus the public-school classes) can be compared within activity system triangles (Figure 1). The activity system originated from

Leont'ev's (1978) cultural historical activity theory, which was later refined by Kaptelinin and Nardi (2009). Engeström (2001) clarified the activity system triangles in which human activities take place, such as school environments and job contexts. The elements of activity system triangles are interdependent, and they are subjects (people doing the acting), objects (goals people produce by their acting), and labors (actual labors using tools to reach the goals; Hardman, 2015). Adopting this activity system triangle framework, the researchers analyzed how the teachers (subjects) performed (labored) in the classroom with ELs and investigated if they met the learning objectives (i.e., ELs' achievement in the content areas). In addition, the researchers analyzed what teachers used as tools when they actually labored to support ELs.

The second phase of data analysis intentionally focused on the data sets for the comparison of the two activity system triangles (Figure 1) using analyzed data from the classroom observations for actual teaching context and follow-up interviews for school context. Figure 1 was created by the researchers based on Engeström's (1987) model of activity systems, which demonstrated the juxtaposition of the two activity systems (i.e., university system and actual classroom system) using the triangles. The researchers examined if the three teachers utilized their learning from the university PD training context (Activity System A in Figure 1) when they actually taught mathematics in their classroom context to ELs (Activity System B in Figure 1) by adopting Engeström's (1987) "model of interacting activity systems" (as cited in Forbes et al., 2009).

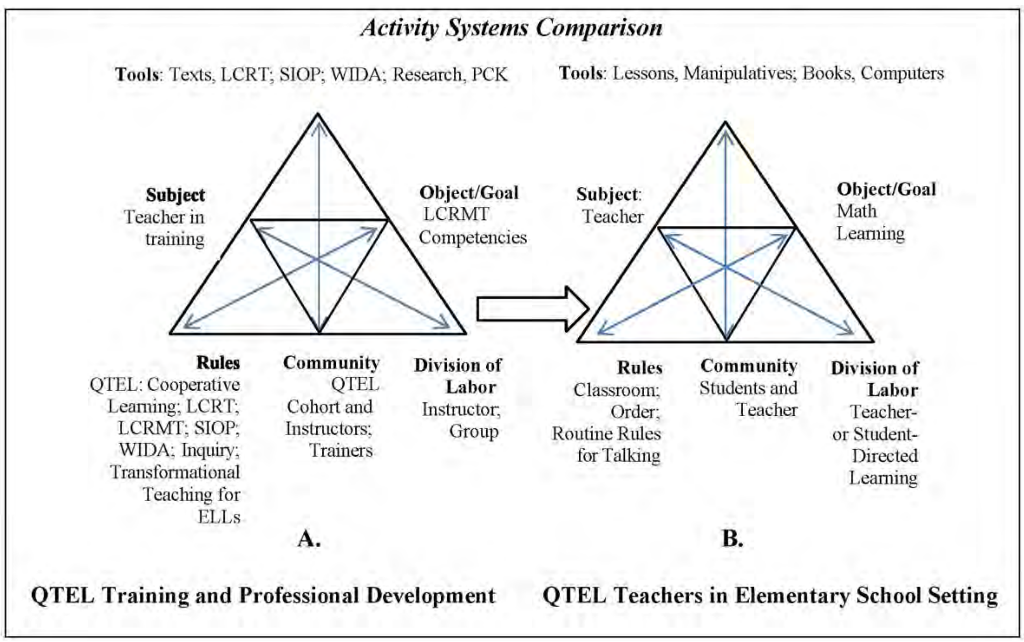

Figure 1. Activity Systems: From University ETEL Community to Actual Classroom Community

Through comparisons and contrasts of the key codes from the data using the two activity systems, two patterns converged: (a) Transfer of ETEL LCRMT Training to Elementary Mathematics Classrooms and (b) [Non] Transfer in Progress/Incomplete. These first and second patterns were related to whether the LCRMT competences that the teachers learned at the university ETEL community (Activity System A) were transferred or not to the elementary classroom community (Activity System B), either as a "rule" toward reaching the goal in the elementary school system or the object in the university system.

Trustworthiness

Trustworthiness in research represents the rigor of the research procedure and how convincing the finding is (Gorard, 2013; Shank, 2006). In this study, trustworthiness was achieved by triangulating the qualitative data of observations and interviews through cross-checking between the researchers using open and axial coding. Trustworthiness was also checked by triangulating interview and observation data. A descriptive audit trail of data collection and analytical process also ensured that the study kept to the ethical process of data collection, which included appropriate IRB procedures, informing participants of their rights, and obtaining consent forms. Finally, the thick description of study context and participants' own words in the result section further established the trustworthiness of this study.

Results

One major phenomenon across the data sets was that the three in-service teachers demonstrated if their LCRMT competences learned from university ETEL trainings were transferred to their mathematics teaching, including a counter-story that was in progress or incomplete. Three themes emerged from the collaborative coding process: 1) Mathematics-related content teaching practices, 2) Tools to support mathematics learning, and 3) Teachers' mindsets and attitudes towards EL teaching.

Participating teachers' narratives were analyzed along with the interview and observation data under each of the three emergent themes. Each of the three emergent themes were connected to the nine LCRMT constructs and the four main units of the activity system (i.e., objects, rules, tools, and subjects) to measure the goal of the university triangle system as well as of the school triangle system illustrated in Figure 1. For example, mathematics-related content teaching practices also address LCRMT's first how-to-actor (acquire and demonstrate knowledge in depth) under mathematics content, mathematics discourse, and mathematics pedagogical competences (i.e., first dimension of LCRMT [Table 1]). Tools to support mathematics learning referred to the second how-to-actor under the three competences (develop and apply procedural demands). For example, for a lesson on the part-to-whole fraction method, a teacher leads the ELs to justify the mathematical concept by using their native languages in the part-to-whole vocabulary (half, one-third, three quarters) and for the multi-language sentence starters (demonstrate knowledge in depth and procedural demands under mathematics content, discourse, and pedagogical competences). Teachers' mindsets and attitudes referred to the third how-to-actor (examine and develop cross-cultural and sociopolitical beliefs) under the three competences of the LCRMT (e.g., ELs and their families' funds of knowledge as well as teacher examination of their own attitudes towards ELs). The emergent themes and related activity system elements were incorporated or integrated when analyzing the three teaching cases of Ms. Happ, Mr. Mack, and Ms. Bishop with the sample narratives and researchers' overviews and interpretations. The researchers emphasized the areas that contained key discourses and LCRMT competences using italics and parentheses. When reporting the findings, the researchers integrated each of the three teaching cases into the interdependent activity system model with the main units of subjects, objects/goals, rules, and labors/tools (Figure 1) and the LCRMT model with nine constructs in its two dimensions (Table 1).

Mathematics-Related Content Teaching Practices

The following described the first theme: Mathematics-related content teaching practices with the main units of the activity system model and LCRMT constructs. *Mathematics-related content teaching practices* can be connected to the activity system *object/goal* regarding what teachers (subjects) are working towards, as well as LCRMT's first how-to-actor (acquire and demonstrate knowledge in depth). Ms. Happ, a sixth-grade mathematics teacher with 14 students, of whom six were ELs (43%), utilized mathematics content teaching practices to deepen her students' mathematics content competence. Ms. Happ's content and language objectives were presented in a conversation that took place after she frontloaded and taught the foundational *mathematical key vocabulary words* of a lesson, such as "expression," "equation," and "variables," before introducing the content and language objectives of the lesson so that students could better understand the *goal/concept of the lesson.*

Happ: Share some of the ideas we discussed yesterday. Yesterday we learned "expression" and "equation." What's the difference between these two?

Dan: An expression does not have an equal sign. An equation does have an equal sign.

Happ: Is that right Team Green? Yes. Does anybody have a trick?

Mary: "Equation" has "equ" in "equal."

Happ: Today, I am adding a word, "variables." Why do we use variables in math?

Mabel: Variables, Umm, an unknown number.

Happ: You mean that variables are letters that represent unknown numbers?

Happp: Today, we will use variables in a different way. Anyone have a question on expression, equation, and variables?

Happ: Look at the word wall. Take a look at objectives here and on the board.

Content Objective: I can write mathematical expressions and equations.

Language Objective: After reading a story problem or a verbal expression, I can represent it with a mathematical expression or equation.

Mr. Mack, a third-grade teacher with 21 students, of whom 11 were ELs (52%), started reviewing addition (mathematics content) and pronouns. He was trying to teach pronouns before introducing addition by asking questions. This grammar lesson helped students understand the mathematical word problem, which was the *goal and objective*:

Mack: Jane and I had 4 pencils each last Wednesday, and [Jane] brought another 4 pencils this morning. How many pencils did we [Jane and I] have all together this morning?"

Mack: Who can tell me which pronoun is used for Jane?

Jerry: "She."

Mack: Can you answer with a complete sentence replacing a pronoun for "Jane and I" in the first sentence?

Jerry: Uum. Jane and I had 4 pencils each last Wednesday, and *she* brought another 4 pencils this morning.

Mack: Great! Who can have a pronoun for "Jane and I" in the last sentence? [He was pointing at the last sentence.]

Mary: We. (pause) How many pencils did *we* have all together this morning?

Mack: Great.

Because he is an elementary teacher, Mr. Mack integrated the grammar lesson of pronouns before introducing addition, which demonstrated an interdisciplinary approach. An interesting phenomenon was that Mr. Mack did not call on any ELs to answer his questions. Even if Mr. Mack called on ELs, they might have failed to have a correct answer if they did not understand what "each" means. Because Mr. Mack did not frontload the lesson with a description of what "each" means, ELs in his class could have thought that "Jane and I" in the question had four pencils altogether rather than four individually.

Ms. Bishop, a second-grade teacher with 19 students, of whom eight were ELs (42%), taught fraction and parts-to-whole relation as a review. The data did not have any evidence of her covering mathematics content knowledge and mathematics-related discourse (objects/goals), but she only had 30 minutes for the mathematics class. Even with only 30 minutes, Ms. Bishop might have frontloaded the key vocabulary words, but she instead rushed into the content teaching (i.e., fractions with tools [e.g., apple cutouts, orange slices]) without checking if each EL understood the content discourses (mathematical academic language and direction/nontechnical language). Both Mr. Mack and Ms. Bishop demonstrated the typical phenomenon observed in many elementary mathematics classes: teachers prepared some hands-on activities that might work for every student (i.e., one-size-fits-all activities).

Tools to Support Mathematics Learning.

Ms. Happ talked through each mathematical problem set and *allowed students to work in groups* (content pedagogical competence) and to *express their ideas with reasoning* (develop procedural demands) to the whole class. One salient feature in Ms. Happ's class was her adoption of *teaching tools* from the ETEL training, which encouraged teachers to utilize *tools to support mathematics learning* by sharing content and language objectives (objects/goals) and explicitly using "I" statements, inquiry-based teaching, and *cooperative learning strategies.* For example, Ms. Happ engaged students by using tools such as *collaborative learning strategy*, which involves setting up group learning (i.e., "Green Team, Blue Team, and Yellow Team"). This was a pattern consistently observed in her lessons and one that matched with an LCRMT construct (mathematics pedagogical competence). Furthermore, Ms. Happ instructed the students by asking them to "give a reason why" when she posed a mathematical word problem (develop procedural demands). To this end, Ms. Happ tried to emphasize *reasoning* rather than simply the solving of the mathematical equation.

> Happ: I loved how you said, "four hundredths" and not "point zero four." How do we know it's four hundredths divided by *n* instead of *n* divided by 0.04?
>
> James: You're dividing four hundredths by *n*.
>
> Happ: Is your 0.04 the dividend or the divisor? Say it on a count of 3 — who can explain?
>
> John: I think it is dividend because zero and four hundredths go inside the box.

Ms. Happ regularly used *think-aloud, thinking prompts, comprehensible input, scaffolding,* and "*making thinking visible*" to engage students in *reasoning* through modeling, repeated questioning, and creating instructional structures through collaborative learning opportunities. She kept engaging students by encouraging them to think and express their *reasoning* through "how, why, and/or because" thinking prompts (develop procedural demands under mathematics discourse competence).

Mr. Mack also demonstrated his LCRMT competences in the emergent theme "tools to support mathematics learning" learned from the ETEL PD. Mr. Mack structured mathematics classes with a variety of *cooperative learning activities,* from whole-class instruction "on the carpet" to pair work, group work, and individual work at computers. Mr. Mack showed a "phone-a-friend" collaborative strategy as a tool to support mathematics learning. Mr. Mack's students also participated in a "I have, who has?" call-out activity during guided practices at the carpet area. Students additionally rolled foam dice in pairs and wrote a mathematics addition/multiplication problem along with the answer.

Despite effectively utilizing LCRMT competences in many of his lessons, Mr. Mack did not show differentiated teaching strategies for ELs to improve their mathematics learning. Most of the students who were asked for answers were non-ELs. Additionally, when checking their mathematics worksheets, most of Mr. Mack's ELs with low English language proficiency did not have the correct answers to his weight unit, demonstrating that they did not receive adequate support during the unit. Mr. Mack did attempt to address this weakness in his instruction however.

With researchers' feedback of how ELs were struggling to acquire the language of mathematics during the follow-up interview, Mr. Mack attempted *several strategies to make mathematics discourse more accessible* to ELs. For example, *he infused Spanish into a mathematics teaching activity* for students following a suggestion from the observer, who had been his previous TESOL instructor, about the benefits of using *ELs' native languages* when introducing new vocabulary. These examples illustrated how Mr. Mack utilized *tools to support mathematics concepts and reasoning* and tried to adopt a multitude of teaching strategies (phone-a-friend, cooperative learning strategies, and hands-on activities) to support ELs, as learned at the ETEL PD. Even if this was not enough to demonstrate the teacher's socially just beliefs, Mr. Mack utilized subjects' (engaged ELs) tools (labored with tools to support mathematics learning) on the second day of the observation.

Ms. Bishop also appropriated a *variety of tools to support mathematics learning*. On the first day of the observation, she used a children's book to teach fractions, introducing *parts-to-whole*. During the lesson, she would read then stop and ask specific questions from the book, in which there was a story related to fractions. Ms. Bishop also drew a pizza and let six students come up to the board and divide it into the six pieces. The students wrote their names in each of the six slices/pieces of the pizza (tools for understanding mathematics concepts, parts-to-whole). She then used an apple cutout and explained fractions and parts-to-whole. After using the apple cutout, she used actual oranges for the students to peel and portion into segments. In these activities, she tried implementing *meaningful mathematics activities with tools using a children's book and props/realia* that were taught during the university mathematics PD.

The ELs in Ms. Bishop's class, as well as in Mr. Mack's class, struggled to use mathematics vocabulary in sentences to complete their assignments. Both teachers tried to introduce *mathematics content concepts* using *tools to help students* *understand mathematics concepts*, but ELs were not able to compose and/or solve any word problems. It seemed that the teachers might have thought that their one-size-fits-all teaching strategies could work for ELs. However, when researchers walked around the class while ELs were completing worksheets, they observed that ELs were not able to complete assignments with the correct answers. The teachers should have explored the needs of each EL and developed their mathematics pedagogy intentionally.

In one of Ms. Bishop's lessons, some ELs whose English proficiencies were low did not comprehend the basic content knowledge of fractions with the directions that were given. Ms. Bishop talked and modeled most of the time, and the students were supposed to follow her directions. This excerpt from Ms. Bishop's class showed her *attitude* and *effort*; she wanted to prepare for the students, including ELs, by showing many shapes:

You will work on deciding whether pieces are *equal* or *unequal*. First, *circle* "equal" or "unequal." Look at this one. This is a *rectangle* that *is divided into* four parts. Does *each* of the four parts, *one fourth* equal or unequal? [She modeled.] Use the circle and other shapes.

Unfortunately, even in the directions above, there were a number of the mathematical vocabulary words, such as "equal," "unequal," "rectangle," "is divided into," "each," and "one fourth," that should have been taught before telling the ELs what to do and how to do it. She could have used multi-language (i.e., translated) directions as well as multi-language word banks with the pictures. She could have also shown the final teacher-made sample projects in multiple languages for the ELs and asked ELs to demonstrate their understanding of her directions.

In summary, from the analyzed observation and follow-up interview data, the three urban in-service teachers demonstrated that they were able to prepare and deliver mathematics lessons that contained mathematics content, discourse, and pedagogical competences by utilizing mathematics-related content practices and tools to support mathematical concepts and reasoning. The three teachers also demonstrated through their efforts and attitudes that they would like to develop professionally to support ELs. However, the teachers' efforts and attitudes did not result in differentiating their mathematics teaching explicitly and intentionally enough to produce outcomes that would demonstrate their ELs' mastery of mathematics concepts and reasoning skills. The three teachers adopted mathematics teaching tools, such as inquiry-based, student-centered cooperative learning strategies, children's literature, realia, and ELs' native languages to show their effort, but their cross-cultural and/or sociopolitical beliefs about ELs were not evident in their teaching practices.

LCRMT Competences: Incomplete and in Progress

Despite demonstrating multiple levels of transfer of ETEL training in the elementary mathematics classrooms of three participating teachers, we noticed that there were areas that these teachers needed further support in in order to comprehensively and appropriately apply their learning from the University PD (Activity System A) into their individual educational settings (Activity System B). First, the teachers' EL-specific LCRMT support was not always consistent. Ms. Happ utilized content and language objectives, cooperative learning and inquiry-based strategies, pre-vocabulary activities, and cognates as indicators of successful transfer of the ETEL training. However, it was observed that her ELs (43% of the class) inconsistently responded to her questions at times. As the excerpts from Ms. Happ's classes show, most of the students who responded were non-ELs. She could have specifically asked the ELs questions in order to provide more opportunities for them to show their level of mathematics understanding (i.e., tools to support their mathematics content and discourse learning). Although Ms. Happ included mathematics content, discourse, and pedagogical competences by demonstrating depth of mathematics knowledge and procedural demand levels in her lessons, there was not much evidence of her making intentional efforts to demonstrate an equity-based attitude and effort for ELs. Second, the intentional use of language supports, for example, by using everyday mathematical language or ELs' first languages (i.e., mathematics discourse competence and teacher's cross-cultural and multilingual beliefs), was not always observed, which would have helped ELs with low English proficiency to better comprehend the mathematical language used in class. Furthermore, the three teachers indicated more linguistic challenges (Halliday, 1978) in the area of mathematics discourse than in the mathematics content pedagogical competence. Finally, Ms. Happ, as well as the other participating teachers, mostly used English-only verbal questioning (i.e., teacher's attitude toward ELs' language and cultural backgrounds) in their teaching strategies. They needed to further examine and develop their own sociopolitical and multilingual beliefs when teaching mathematics to ELs and prepare multi-language texts, vocabulary banks, and direction using translation applications (e.g., Google Translate).

In Mr. Mack's mathematical problem, "Jane and I had 4 pencils each last night, and Jane had four pencils this morning. How many pencils do they have in total?," ELs did not understand what "each" meant (mathematics discourse competence). After a phone-a-friend activity, Mr. Mack asked the students to answer an addition/multiplication question and had the following interaction:

Mack: Angela [who is an EL student], can you answer this question?

Angela: It is, I think, 8. 4 plus 4. Addition. [Angela displayed her white board showing 4 + 4 = 8.]

Shawn: No, it is 12. Jane had 4 and I had 4 last Wednesday. Jane had 4 pencils this morning. 4 + 4 + 4 equals to 12. I can use addition and multiplication.

Mack: Why did you think you can use multiplication?

Shawn: Multiplication because of three same numbers, 4, 4, 4.

Angela, an EL student, did not notice "each" but instead saw two instances of "four" and came up with "eight" for an answer until Shawn (a native English-speaking student) listed three fours. It was obvious that Angela had understood the concept of addition but did not understand the grammatical meaning of "each" in a mathematical word problem sentence (tools to support mathematics learning under mathematics discourse to develop reasoning and procedural demands). Mr. Mack used Shawn's reasoning, "because [there were] three same numbers, 4, 4, ,4" to explain multiplication (teacher's belief) to students. Mr. Mack did not stop and explain to Angela why "eight" could not be a correct answer.

As shown in the excerpt above, Mr. Mack could have guided the students learning addition by asking them to count the number using the number-line up to four then explaining what "each" meant. Then students could have counted up to another four, and Mr. Mack could have asked, "How many pencils did Jane bring this morning?" After this, he could have let the ELs count up to another four. Mr. Mack could have also explained how this problem of addition or "counting up to" could be used for multiplication by illustrating the three fours and explaining what "each" meant in that specific context. Like Angela, some ELs came up with "eight" in this word problem instead of "twelve" because they did not understand the word "each." Without giving time for ELs to have more practice on the multiplication concept, Mr. Mack introduced another activity on *place value* and wrote the two-digit number multiplied by a one-digit number: 14 x 4. Mr. Mack could have given more opportunities to allow the ELs to have more practice on word problems that included mathematical discourses that they needed to understand.

In another lesson on weight units, Mr. Mack focused on the unit vocabulary words of "ounce," "pound," and "ton" along with their symbols (oz, lb, t, respectively). He displayed a picture of a pencil when explaining 1 oz, a picture of a can of green beans when explaining 1lb, and picture of a small car when explaining 1 t, assuming the ELs would understand the concepts of these units. Mr. Mack could have used more resources, such as a comparison chart with the metric measurement system and the U.S. customary system (e.g., gram and kilogram versus ounces and tons). In addition, Mr. Mack could have brought a scale (realia) to the class so the ELs could actually measure a pencil, a piece of paper, an empty can, and so forth. That way, even if ELs were confused with the U.S. customary system, they could guess or estimate the difference between the two measurement systems.

In Ms. Bishop's classes, transferring the concepts from the read-aloud to the hands-on activity with oranges to writing an equation was a challenge for most of the ELs. She could have whispered to the ELs to check for their understanding and, if needed, directly intervened to better assist them in understanding the mathematics concepts under study. Most of Ms. Bishop's ELs were newcomers whose English proficiency levels were low. Ms. Bishop might have provided mathematics-related pre-activities to help beginning level ELs to understand the main vocabulary words, such as "equal," "unequal," "divided by," and "one fourth," and used fractions with pictures and number sentences as well as students' native languages and multi-language word banks. Ms. Bishop could have also allowed ELs to write word problems in their native languages and translate them with partners. She might have also allowed the ELs who spoke the same native languages to use Google Translate to discuss what these words meant. While walking around the classroom, the researcher found that Ms. Bishop's ELs were not writing mathematics word problems because they did not understand the mathematics concepts and key vocabulary words used in the lesson well enough to do so.

The three urban in-service teachers' mathematics teaching practices indicated that they tried to adopt *mathematics-related teaching practices, tools to support mathematics concepts and reasoning,* and *their attitudes and beliefs* to be linguistically and culturally responsive. However, they did not differentiate or alter their mathematics teaching practices based on the needs of their ELs. Additional language teaching supports were needed for the ELs to meet content and language objectives. As seen in these cases, the teachers did not sufficiently understand the role of developing mathematics discourse and pedagogical competences for serving ELs, indicating a gap between the ETEL training activity triangle system and the elementary school activity triangle system (Figure 1). Explicit language supports using ELs' native languages or intentional efforts to call on them during lessons would have provided deliberate opportunities for ELs to think and respond to numerous questions. To build on ELs' funds of knowledge (Moll, 2015), teachers should provide intentional opportunities and proper tools to monitor ELs' understanding of mathematics concepts and reasoning given proper attitudes and beliefs.

Finally, the three participating teachers needed to follow the institutionally established protocols in their instruction in terms of classroom management and the curriculum process. When asked during an interview what she would like to change in her teaching, Ms. Bishop shared a challenge concerning divided instructional time due to recess occurring in the middle of the mathematics class. She said, "Sometimes I struggle getting them back on track, especially my ELs. I have to revisit the vocabulary we've just taught. Some ELs asked what an apple is." This particular interview excerpt exemplified teachers' linguistic challenges (Halliday, 1978) directly related to a tight schedule. More time is needed for ELs to develop mathematics discourse and vocabulary work. Ms. Bishop's teaching case showed that she tried some LCRMT strategies, such as implementing language and content objectives, adopting children's literature, using realia, and utilizing cooperative learning strategies, but she was limited by institutional routines (time limits from the school building and classroom structures), and a *lack of intentional supports with differentiated mathematics teaching practices* to serve each of the ELs whose needs were different.

Similarly, Mr. Mack focused primarily on classroom management by using routines and pacing to keep order. While working at tables of four on worksheets to follow up on the day's mathematics lesson, some ELs had difficulty understanding the weight unit and accompanying mathematics word problems (mathematics discourses and goals). However, instead of going over challenging areas of the daily mathematics lesson, Mr. Mack moved the class on to multiplication games, another routine activity, before the class was over despite the fact that some ELs did not have the correct answers to the weight unit worksheet. Mr. Mack missed the opportunity to reteach the mathematics concepts discussed earlier in class due to a focus on institutional routines. This routine was also found in Ms. Bishop's second-grade class, as "teacher talk" and teacher-driven activities dominated the instruction most of the time. Ms. Bishop had a tight classroom structure, with routines and school rules for teamwork and student movement. Students, especially ELs in her classroom, responded to questions only by nodding, with/without thinking, reasoning, and/or questioning, and most of the ELs did not have correct answers to the problems and were unable to write any word problems. Specific differentiated language supports based on ELs' English language proficiency levels (e.g., bilingual word bank and having an EL with low English language proficiency partnered with an EL with high English language proficiency who spoke the same native language and could explain the concept in their shared native language) were not observed.

In the examples above, we compared and contrasted the Activity Theory System for the university ETEL triangle and the actual school triangle. Although goals and tools learned at the university PD were maintained and some good practices were transferred, there were noticeable weaknesses that indicated the need for additional teacher resources, awareness, and intentional support in the teachers' activity system in order to serve ELs more equitably and effectively.

Discussion

The purpose of this study was to examine whether or not participating teachers applied the linguistically and culturally responsive mathematics teaching competences when they taught mathematics in their classrooms, especially for ELs. Transferring strategies and pedagogies teachers learn during their university studies to the actual teaching context requires intentional effort and commitment. Ms. Happ said, "Yeah, I was trying to use anchors and sentence stems for each mathematics lesson. I think university helped me become more aware of it." She continued, "For each lesson, I list the language and content objectives. I prepare content and language objectives because of the university courses. I'm applying many teaching activities I did not know before." During classroom observations and interviews, we took a closer look at how the three teachers incorporated linguistically and culturally responsive mathematics teaching competences learned at the university EL teacher training program into their mathematics classes for ELs.

Using mixed methods to analyze the research question, "How did participating urban in-service teachers apply linguistically and culturally responsive mathematics teaching competencies for ELs learned at a university EL teacher training program to their actual mathematics teaching in the classroom?," we tried to examine, triangulate, blend, and/or integrate the qualitative data to support the research question. We also analyzed these integrated data to examine the juxtaposition between what the teachers had learned at the ETEL trainings and how they applied the university training to their mathematics teaching in their K–6 classes. When applying the two activity system triangles (Engeström, 1987; Yamagata-Lynch, 2010), the data from observations and interviews revealed that teachers employed LCRMT practices learned in the university training (Activity System A) but that they did not fully transfer the LCRMT competences learned at the university into their actual mathematics teaching practices (Activity System B). In the collaborative coding process, three themes emerged*:* a) mathematics-related teaching practices, b) tools to support mathematics learning, and c) teachers' attitudes and beliefs.

In summary, we found that teachers used tools to support mathematics-related and meta[cognitive]-mathematics teaching practices, but they still needed to develop linguistically and culturally responsive mathematics teaching activities and assessments in order to truly help ELs enhance their mathematics content knowledge and its application as well as their critical thinking skills. We also found that they were constrained by personal, classroom, and school management structures. In addition, although the participating teachers demonstrated use of LCRMT strategies, they did not specifically tailor them to the needs of ELs, assuming instead that good strategies might work for every learner. In fact, Ms. Happ mentioned, "I strongly believe that ELs are learning from my math classes because I am applying all of the ETEL strategies, I mean LCRMT strategies." What she was saying was that she was assuming that learning would happen to ELs without checking if they were actually engaged and learned in her mathematics classes. In reality, deep learning did not consistently happen.

The qualitative data exhibited teacher support for students' engagement through the use of cooperative communication strategies, such as quack-quack, shoulder partners, and phone-a-friend, but teachers should have exhibited their competency and commitment for targeting ELs with equitable attitudes towards ELs. In general, all of the three urban in-service teachers applied LCRMT strategies, showing mathematical content, discourse, and pedagogical competences along with procedural and reasoning skills when teaching mathematics in their classrooms. The teachers, however, could have prepared linguistically and culturally responsive mathematics resources and provided more opportunities for ELs in order to support them intentionally with differentiated language anchors and assessment strategies. The participating teachers showed a shift to becoming more competent in mathematics content and pedagogy in terms of "acquiring knowledge in depth" and "developing procedural demands" but also showed the need for more PD in order to fully examine and develop mathematical discourse in sociocultural contexts where ELs could feel more comfortable or familiar. In addition, the school system (Activity System B) did not fully support the LCRMT competences learned at the university ETEL trainings (Activity System A) because the system demanded its own routinized activities and assessments that did not always match with intentional LCR mathematics teaching strategies for ELs.

Conclusion with Implications

Despite the limitations of a small sample size and short duration of research, this study provides meaningful insights for linguistically and culturally responsive mathematics teaching. First, the research process shared a two-dimensional LCRMT framework (Table 1) that could be adopted by practicing teachers in their own contexts. Second, through data collection and analysis, we compiled narrative examples under the three themes that emerged during the data analysis. Finally, this study made a meaningful connection with the activity systems model, illustrating the juxtaposition between the university teacher training and its transfer or non-transfer to the actual classroom context for ELs.

We would like to conclude this paper with implications that might emphasize mathematics competences that move away from mastering a list of mathematical vocabulary words with precise meanings to using collaborative and communicative participation (Moschkovich, 2012). For such a change to happen, teachers may need to "recognize and support ELs to engage with the complexity of language in math classrooms" (Moschkovich, 2012, p. 22). These recommendations conflate with the findings of this study; the participating teachers implemented the tools to engage the ELs, which might help ELs understand the complexity of mathematical language. The findings also revealed the non-transferred competences at the observation sites. For example, our observational data showed that more non-ELs participated in lessons and replied to the teachers' queries than ELs. Unfortunately, the teachers on occasion failed to adequately engage ELs with mathematical language in a way meaningful to them. Materials and professional development should be prepared more explicitly to support teachers with knowledge of when to move from traditional to technical mathematical ways of communication and when and how to develop and implement precise mathematical discourse (Schleppegrell, 2007).

Replication studies are needed to examine how ELs learn to read and understand different mathematical texts in word problems. When working with ELs, it is important to distinguish between children who are competent readers in their first languages and those who are not. Those who are competent in their first languages are able to read and comprehend not only mathematical texts, but also mathematical word problems, whereas those who are not competent in their first languages may be able to read mathematical texts but may take a longer time to read and understand mathematical word problems (Moschkovich, 2012). In line with Horn et al.'s (2008) research, the findings of this study highlight the importance of understanding that there are gaps between what teachers are learning when in university and what teachers actually do in the field. We have identified transferred learning and non-transferred gaps in the field of LCRMT for ELs with suggestions for future professional training and research on this topic.

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