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A Conceptual Model for the Interaction of Mathematical and Financial Literacies*

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Abstract

The concept of financial literacy, which has become popular as a 21st-century skill, is also a new field for mathematics education. Financial literacy, which has taken place in PISA since 2012, has not only been presented as a separate course and has also increased in density in related courses such as mathematics through tasks, curriculums and tests. The aim of this study was to present an interaction model in which mathematical and financial literacy competencies can be presented in a common framework. in the organization of this model, literacy literature, consisting of both mathematical literacy and financial literacy models, as well as the PISA literacy framework, were used. The focal point of the model was not individual skills but is the dimensions of this interaction that should be considered in the organization or analysis of an educational or an instructional document (tasks, applications, curriculums, etc.). While this model is built on the basis of the PISA literacy framework, it has some differences with respect to the components of the dimensions. It is expected that the model contributes to the financial literacy education integration initiatives, studies on the financial literacy education and the analysis of measurement tools of financial literacy, such as PISA.

Keywords: Mathematics Education, Mathematical Literacy, Financial Literacy, PISA, Financial Literacy Education, Conceptual Model

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INTRODUCTION

The rise in the importance of practical skills in mathematics education has not only provided opportunities to set contextual relationships but also encouraged interdisciplinary studies (Blum, 2002). In this manner, the process skills in learning and teaching of mathematics has become a meaningful part of mathematics, which supports the idea that mathematics is a tool that can be used in real life. Moreover, this helps students develop positive attitudes toward mathematics (McCrone & Dossey, 2007). One of the areas with which mathematics is intensely related is finance (Lusardi, 2012). In addition to creating macroeconomic indicators, mathematics plays an important role in the basic financial behaviors and decisions of individuals (Lusardi, 2015). Mathematical operations and concepts (equations, functions, models, representations, etc.) are used as tools in financial studies (Lusardi, 2012). Looking at this situation from the field of education, it can be observed that mathematics education is a productive field for the development of individuals' financial skills. Moreover, financial contexts extend the practical area of mathematics education (Ozkale, 2018).

It is thought that this interaction between mathematical and financial fields can be examined in the field of education from the perspective of literacy. Briefly, the concept of literacy states that individuals exhibit their knowledge and skills in real-life situations. It is a popular research area with many sub-fields, which include mathematical and financial literacies (OECD, 2016a). The concept of mathematical literacy (ML), which emphasizes the in-depth analysis of mathematical knowledge and skills in particular situations, is an important area in mathematics teaching and has a close relationship with other literacy types with its practical structure (OECD, 2016a). In this context, addressing financial literacy (FL) in mathematics teaching opens up a wide area for mathematics literacy, and it is also valuable for the integration of FL as a 21st-century skill into the education system (Lusardi, 2015). FL is a new discipline for mathematics education. In this respect, the creation of a model that demonstrates the relationships between FL and ML can serve as a guide for both document creation and document analysis in both the areas. The aim of this study was to design a conceptual framework that deals with both the concepts of mathematical and financial literacies. For this purpose, related studies including conceptual approaches and the Program for International Student Assessment (PISA) documents considering the fields were examined. Inspired by all the documents, the Interaction Model of Mathematical and Financial Literacies (IMMFL) has been developed. The focal point of the model is not individual skills but the dimensions of this interaction that should be considered in the organization or analysis of an educational or an instructional document (tasks, applications, curriculums, etc.). In this respect, this study was expected to guide further studies which consider this interaction in both the areas.

In this paper, in order to better indicate the organization of the model and the need in the field, first, we provide information about the concepts of mathematical and financial literacies and their theoretical frameworks. Then, we demonstrate their reflections in the area of education and how they are handled in PISA. Furthermore, we detail the framework of the model, as well as its visual meaning, dimensions, components, subcomponents and the relationships between them.

AN OVERVIEW ON MATHEMATICAL AND FINANCIAL LITERACIES

Mathematical literacy

The main factor that has triggered mathematical development from the past to the present is the search for solutions to the problems people face in life. The reason why mathematics education is in schools is that mathematical knowledge and skills can be used in real life. ML is about being able to use mathematics in real-life situations effectively and meaningfully (Jablonka, 2003). ML emphasizes the skills of mathematical understanding, reasoning (Kilpatrick, 2001) and thinking with mathematical notation (OECD, 2016a) in real-life situations. On the other hand, it is valuable for ML that the contribution of mathematics to the development of the civilization (McCrone & Dossey, 2007) as well as an important discipline are understood (OECD, 2016a). ML, which touches different aspects of mathematics education, is a valuable research topic in this field (Julie, 2006). Van de Walle, Karp and

William (2007) summarize ML as understanding mathematics, doing mathematics and being interested in mathematics. In this context, to mathematize a problem, represent it mathematically, produce different solutions and choose the most appropriate of them, as well as use communication skills in doing mathematics can be evaluated in ML.

Although the theoretical explanations regarding a concept give an idea of this concept, a model that sets out the framework of this concept facilitates its understanding with visuals. One of these frameworks developed for the ML is Pugalee's ML model, which uses two concentric circles to depict the components of ML (Pugalee, 1999). The model is shown in Figure 1.

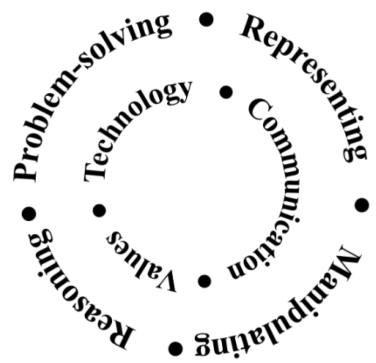

Figure 1. Model of mathematical literacy of Pugalee (Pugalee, 1999)

According to the model, the larger circle represents the processes of ML, and the inner circle is the factors that affect these processes. According to Pugalee (1999), these two concentric circles depict the interrelatedness of the enablers and processes in the evolution of ML. Additionally, the interaction of these two circles enables the development of ML.

Representing refers to the transformation of problem cases into mathematical forms (models, visuals, notations, etc.). *Manipulating* refers to the calculations and algorithms necessary for identifying the solutions to the problems. *Reasoning* describes how mathematical ideas and operations are supported by arguments and require the questioning of mathematical models and solutions and relating them with mathematical concepts. *Problem-solving* is the main reason for dealing with mathematics. Knowledge and skills in problem-solving are recruited from a mathematical perspective. Problem-solving is not a result but a mathematical process. *Technology* can be used for explanations, representations, demonstrations, calculations in the processes of ML. In the problem-solving process, effective *communication* skills are required for explanations and thoughts of the individual. For this process to be carried out correctly, it is necessary to be interested in and have positive attitudes towards mathematics and consider it a necessary tool for the solution of real-life problems.

Kilpatrick (2001), who has conducted many studies in this field, grounds ML on five basic principles: (1) *Conceptual understanding* refers to the meaning of knowledge and relates knowledge to mathematical concepts and having adaptable skills to new situations (Skemp, 1978). (2) *Procedural fluency* describes realizing the mathematical operations necessary for the solution of the problem. (3) *Strategic competence* refers to selecting the most appropriate solution among many. (4) *Adaptive reasoning* refers to the questioning of the logical thinking on which solutions are based in order to support and explain it with different mathematical arguments. (5) *Productive reasoning* refers to the emergence of conceptual relationships in mathematical thinking and questioning in order to create more than solutions and transfer the processes to other situations (Julie, 2006; Kilpatrick, 2001; Pugalee, 1999).

Mathematics is a science that is related to many areas of life. Accordingly, ML prepares the grounds for individuals who handle deep mathematical thinking in the problem-solving processes related to many real-life situations (Papanastasiou & Ferdig, 2006; Pugalee, 1999). Nowadays, many problems faced by individuals in daily life are also closely related to the financial area. In this context, FL, which is a basic concept in this study and in which mathematics has intense interaction, will be explained below in brief.

Financial literacy

Everyone has to balance their needs and desires according to their income and assets. From this perspective, in the financial circle, there are jobs to earn money, investments and savings that constitute their assets, and outgoings related to their desires and needs. In this balancing, economic circumstances such as borrowing, risking and gaining may occur. Accordingly, the process of the financial decision-making of an individual is an important part of real life. Moreover, gaining proper financial behaviors is seen as a literacy skill. FL is about being able to use knowledge and skills to make proper financial decisions with confidence (Lusardi & Mitchell, 2011). Within the framework of these explanations, FL is recognized as a 21st Century skill (Lusardi, 2015).

It can be stated that the concept of FL has four basic dimensions. These are *financial knowledge, skills, behaviors and affecting factors* (Lusardi, 2012; Ozkale, 2018). These four dimensions have complex relationships and affect each other. In describing financial knowledge, the realities of the financial world, basic financial concepts, risk and reward situations, as well as mathematical knowledge, can be mentioned (Lusardi, 2012; Shim et al, 2010). Similar to ML, to describe financial skills, numeracy, financial reasoning, problem-solving, communicating, and using technology can be expressed (OECD, 2014). Furthermore, the dimension of financial behavior includes optimum opportunity, financial management and planning, as well as getting proper advice and providing financial assurance and security (Financial Literacy and Education Commission [FLEC], 2016). While the individuals indicate their literacy skills in the financial field, there are several factors that direct them. These include the individual's perceptions and emotional factors as well as their social environments such as family, peers and the media (Jorgensen & Savla, 2010; Shim et al. 2010).

FL is essential competency for all the individuals and can be used to make important decisions in all the steps in life. FL skills can guide proper financial behaviors in many situations such as when a child saves their pocket money for a bicycle, a teenager plans their spending on education or marriage, and an adult increases their savings to prepare for retirement.

Reflections of financial literacy in education

People's level of FL affects the society as well. For this reason, FL is involved in educational policies. One of the clearest examples of this is that it is one of the topics discussed at the summits of the Group of Twenty (G20) (OECD, 2017). It has been seen that everyone needs FL regardless of their country, age, gender and occupation. However, the FL ratio is not high around the world (Orton, 2007; Klapper, Lusardi and Van Oudheusden, 2014; Lusardi & Mitchell, 2011; OECD, 2014). A number of countries in the Organization for Economic Co-operation and Development (OECD) have recognized this need and taken initiatives to ensure that their citizens, especially children and adolescents, receive education regarding FL (Ontario, 2010). In addition, some organizations, especially OECD, conduct researches to measure and improve the FL status of individuals and communities (Atkinson & Messy, 2012; Lusardi, 2012; Lusardi & Mitchell, 2011). Financial literacy education (FLE) can be seen as an auto-control mechanism for a sustainable and healthy financial system in the society. The fact that school-age students tend to develop and learn and that their behaviors become more evident at these ages have triggered the integration of FLE into their curriculum and other teaching environments (Frisancho, 2019). The initiatives of the integration of FL into the education systems have been taken in many countries such as Australia, Brazil, Canada, New Zealand, Singapore, South Africa, the Netherlands, the United States and the United Kingdom. (Aprea et al., 2016; Ontario, 2010; Worhington, 2004).

There are two approaches for FLE initiatives. The first is designing a separate FL course (California Department of Education, 2015; Frühauf & Retzmann, 2016). The second is the integration of FL into related courses such as mathematics. This integration requires the analysis of both the fields in detail in terms of theoretical and practical. In the process of designing these initiatives in particular, it can be observed that an intensive content preparation phase has been used (Ontario, 2010). It can also be observed that these initiatives extend to time, literature reviews, and analysis of the current status, pilot implementations, and many studies in related disciplines. Moreover, many resources and material support has been provided to stakeholders. Additionally, the developments of students have been monitored (OECD, 2014; Ontario, 2010).

Mathematical and financial literacies in PISA

PISA, an international test that measures the basic literacy skills of students around the age of 15, measures how ready the students are for life (OECD, 2016a). PISA exams, as well as ML, are presented for the students' reading and science literacies. Notably, for the PISA exam held in 2012, FL skills were included in the program (OECD, 2014). What is essential to PISA is not knowledge but literacy. The fact that different qualifications are involved in it shows that PISA is not an ordinary qualification exam. PISA aims to determine if individuals have the compact skills that are necessary in today's world. Furthermore, the PISA exams consider the abilities to understand and interpret knowledge in real-life problems, not curriculums of countries. Accordingly, PISA is a reference for countries' educational policies and curricula.

In PISA, problem situations for literacy skills are handled in three dimensions: *context* (which is the grounds for the problem), *content (*which selects the concepts) and *processes* (which indicates the students' problem-solving skills). Under this common structure, there are different components for each literacy type. The structure of both ML and FL in PISA are described below.

The framework of mathematical literacy in PISA

The PISA questions are presented in a real-life problems. For this purpose, questions are organized based on a context. In the context, specific contents are enabled that the student can use. Then, the students are asked to complete certain processes. For example, a question about price options can be organized through the story of being a member of a gym (OECD, 2016b). In this example, the pricing can have an algebraic structure. Accordingly, in the question, a mathematical principle of change and relationship develops. In order to find out which is the most advantageous membership option, the student may be asked to determine the algebraic or graphical modelling, perform mathematical reasoning or estimate and calculate the results. The structures of ML conducted in PISA, including the dimensions of content, context and process, are shown in Figure 2.

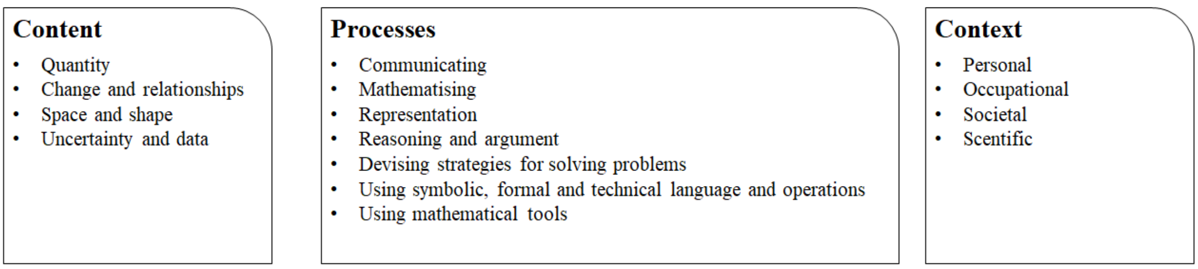

Figure 2. The framework of mathematical literacy in PISA (OECD, 2016a)

The content dimension specified in ML provides a framework for mathematical concepts: quantities (numbers and operations), change and relationships (algebraic structures), space and shapes (geometric concepts), uncertainty and data (mathematical concepts and topics about probability and statistics). In the dimension of context in the ML domain of PISA, environments in which a person performs their tasks in real life are considered. With this distinction, individual, occupational and scientific activities and problems in social life of a person are taken as a basis. Furthermore, in the dimension of process, explanations about mathematical skills are preferred. These abilities can be summarized as understanding the meaning of the problem, developing strategies for solutions, using mathematical language (including mathematical notations) and using necessary tools.

The framework of financial literacy in PISA

The OECD (2016b) states that social permeability has increased and that families and individuals who are forced to establish new order due to immigration or other situations should consider their financial circumstances, which also affects the social order (Nam, Lee, Huang & Kim, 2015; OECD, 2016b). For this reason, OECD has pioneered FLE initiatives since the early 2000s and added FL as a separate domain to PISA examinations. The main purpose of the FL domain of PISA is to measure the ability of 15-year-old students, which will increase their financial responsibility as well as the understanding of financial concepts and financial situations in different areas such as home, work, school and social life (OECD, 2016b). The framework of FL domain in PISA is shown in detail in Figure 3.

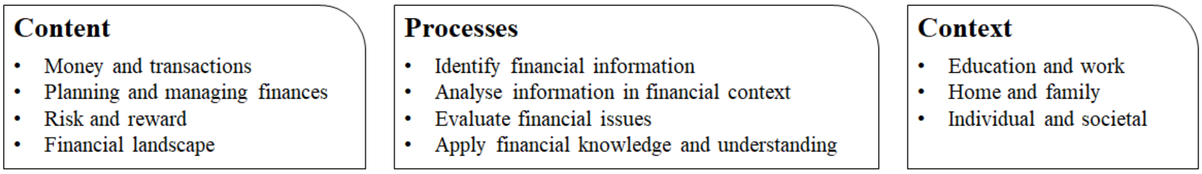

Figure 3. The framework of financial literacy in PISA (OECD, 2016b)

In the content of the FL domain in PISA, the concepts of monetary transactions, financial management and planning, risk and reward as well as financial landscape have been remarked upon. The components in the dimension of content are formed nested financial situations. For example, in the context of travelling to a different country, different tasks are asked in the components of content, such as exchange money (money and transactions), spreading of expenditures (financial management and planning) and understanding the value scales of money (financial landscape) (OECD, 2014). In the dimension of context, similar to other literacy types, different environments in which the person is located are indicated. In the dimension of process, the meaning of financial concepts, data analysis, and evaluation of financial situations and use of obtained information are expressed.

TOWARD AN INTERACTION MODEL

OECD (2014) stated that ML is a prerequisite for FL. However, in the 2015 report, the relationship between ML and FL was discussed in terms of content, and it was stated that the two literacy intersected only in the arithmetic field (OECD, 2016b). However, considering the conceptual grounds and criteria of competency, it is seen that there is a wide set of interactions between ML and FL (Cole, Paulson, Shastry, 2014; Jayaraman, Jambunathan, & Counselman, 2018). From this point of view, we are of the view that this set of interaction can be handled in two aspects. First, the abilities of both ML and FL are similar and in support of each other. Processes such as reasoning, problem-solving, effective communication, and using technology are the abilities discussed in both the literacy areas (Lusardi, 2015; OECD, 2016a; Pugalee, 1999). Second, the relationships in the conceptual grounds of FL and ML are intense and complex (Dituri, Davidson, Marley-Payne, 2019). The two have many common concepts, processes and skills. Mathematical knowledge and skills are needed in many financial situations such as interest transactions, budgeting, exchange money, rental options, evaluating investments, estimating prices, pension processes and conscious shopping. For example, in order for a person to regulate and pay their taxes, they have to first make and balance their budget accordingly. For this process, the concept of percentage may be used intensively. Furthermore, many processes such as presenting data, estimating of budget balance and reasoning for payment options may be benefited by this.

Besides, it has been stated that FLE and mathematical literacy support each other and contribute to each other's conceptual development (Ontario, 2011; Sole, 2017). FL skills have been integrated into the mathematics curricula of many countries, including Canada (Ontario), Singapore, and the United States (California). This integration requires both expertise in these areas and pedagogical perspectives. Therefore, reflecting this interaction into the curriculum, conceptual relationships and common skills should be done properly (Ontario, 2010).

It can be stated that in PISA, the content and context components of the dimensions of both the domains do not have much similarity, and both the domains focus on their fields. However, in real-life problems, many financial contexts can be designed, which are the basis of mathematical content and processes. On the other hand, PISA demonstrates similar skills in the dimension of the process of both the domains. First, it can be said that common processes such as meaning problems and thinking deeply, producing reasonable solutions, evaluating results and expressing thoughts get remarked upon in both the domains. In this study, after considering the literature of both literacies and their structures in PISA, we wished to design a common model for the interaction between them. The three dimensions of content, process and context are similar in the domains of PISA. Therefore, the components of the dimensions of the new model were organized with the perspective of FL, using the FL framework and mathematical contents and processes.

Financial context

The main idea in the generation of IMMFL is to deal with the mathematical knowledge and skills which can be evaluated within the framework of FL. Therefore, we focused only on financial contexts. In PISA, the dimensions of the context of both the domains address different roles of individuals in real life around their areas. In ML, the personal, occupational and societal contexts are stated (OECD, 2016b). Similarly, FL uses the components of education and work, home and family as well as individual and societal context (OECD, 2016b). Considering that these components are noted similarly, it can be said that the contexts are formed based on the different roles of individuals in real-life situations. In designing the dimension of context of IMMFL, it is thought to limit the dimension with financial situations. The components of the dimension of financial context are based on the FL literature.

It is thought that the method, which is based on roles of people in PISA, may cause confusion because a concept may be used in more than one role. Therefore, in the designing of financial contexts, the intensity of the contexts in the FL literature and the framework of the top contexts which provides integrant role are more effective. In addition, the reports of some countries or states, which integrate FL into mathematics education, are also considered for the financial contexts. The notions determined from the resources are shown in Figure 4.

	Artur (2011)	Aprea et al. (2016)	Blue, Grootenboer & Brimble (2014)	California DoE (2015)	Hung et al. (2009)	Huston (2010)	Klapper, Lusardi & V. Oudheusden (2014)	Kozup & Hogarth (2008)	Lusardi (2012)	Lusardi & Mitchell (2011)	Lusardi & Mitchell (2014)	Lusardi et al. (2010)	Marcolin & Abraham (2006)	Ontario (2010)	Orton (2007)	Schuchardt et al. (2009)	Shim et al. (2010)	Sole (2014)	Williams (2007)	Worthington (2004)
Earning	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Salary	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Overtime	✓	✓			✓						✓		✓	✓					✓	✓
Part time	✓	✓												✓	✓					
Inheritance		✓									✓									
Investing and saving	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Interest	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Currency		✓											✓	✓						
Bond/Share	✓	✓			✓	✓	✓	✓		✓	✓		✓	✓	✓			✓		✓

Spending	✓	✓	✓	✓			✓		✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓
Credits	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Credit card	✓	✓		✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓		✓
Renting	✓	✓		✓							✓		✓	✓			✓			✓
Shopping	✓	✓	✓	✓			✓		✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓
Borrowing	✓	✓				✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓
F. management and planning	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Budget	✓	✓		✓	✓						✓		✓	✓	✓		✓	✓	✓	✓
Buying/renting	✓	✓		✓	✓			✓	✓	✓	✓	✓	✓	✓	✓		✓		✓	✓
Tax	✓	✓	✓	✓	✓			✓	✓		✓	✓	✓	✓	✓			✓	✓	✓
Insurance	✓	✓	✓	✓	✓	✓		✓			✓	✓	✓	✓	✓	✓		✓	✓	✓
Retirement	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Future	✓	✓	✓	✓	✓	✓	✓	✓			✓		✓	✓	✓	✓	✓	✓	✓	
Consulting		✓	✓		✓			✓		✓	✓	✓	✓	✓	✓		✓		✓	✓

Figure 4. The resources for the components of the dimension of financial context

The components discussed in Figure 4 are financial notions and cases in the general literature. The resources are studies which discuss the definition and framework of FL as a concept and present ideas and data regarding FLE. In this context, the components of investing and saving and financial management and planning exist in the all sources. Besides, income (salary), interest, credit and retirement are key concepts in all the sources. The components of the financial contexts can be used together to explain the same financial situations. For example, the purchase of a computer is a spending situation, but it may also necessitate looking at financial management and plans to check the budget.

The frameworks regarding the contexts in the literature reveal four main components of the dimension of financial context of IMMFL. These are *(1) earning, (2) investing and saving, (3) spending and (4) financial management and planning.* These components have a wider representation of the contextual notions in the literature. They also represent the financial experiences that individuals face in their financial processes.

Furthermore, the notions can be evaluated under more than one component of the dimension. For example, the concept of interest may be considered in the components of investing and saving, spending as well as financial management and planning. In the case of buying a home, if the interest rate of the loan is mentioned, it can be mentioned in the component of financial management and planning, after considering the payment process of the loan. Similarly, the concept of interest, which is used to evaluate existing savings in the bank, can be mentioned in the component of investing and saving. Accordingly, it can be said that the notions mentioned in the components are not stable.

Content

The dimension of mathematical content mentioned in PISA consists of four components, which are also considered in ML studies (Van de Walle, Karp, & Williams, 2007; OECD, 2016a). These components are included in IMMFL as well. However, the component of *financial content* is added in the dimension. Many financial concepts may be placed in mathematical contents. For example, the concept of interest may be placed in the component of change and relationships or the concept of the percentage may be placed in the component of quantity. However, in order to be aware of financial matters such as banking transactions, financial institutions and credit ratings, some specific financial concepts cannot be related to mathematical concepts. Therefore, the component of financial content is added to the dimension of content.

Processes

In the processes of the domains of FL presented in PISA, Bloom's taxonomy is taken into consideration and the steps of understanding, analyzing, evaluating and applying are placed in a non-hierarchical order (OECD, 2016b). ML processes in PISA follow the skills which focus on using mathematical concepts in different contexts. Therefore, while the financial circle is considered more in terms of the contexts in the interaction between both the literacy types, mathematical processes are considered more in terms of the dimension of processes. This indicates that the components of the dimension of processes of IMMFL are similar to ML processes (Pugalee, 1999). In the process skills of IMMFL, how to approach a problem situation is considered. Accordingly, in a financial situation, the skills of meaning (thinking), decision-making and acting (applying) can be considered (OECD, 2016b). IMMFL has process skills which are appropriate for this sequence. In the process, mathematical knowledge and skills, as well as financial environment and situations, are taken into account. It would be wrong to consider only the financial or the mathematical process. These processes are composed using financial and mathematical literacies studies, the framework of PISA and the mathematics education literature.

THE INTERACTION MODEL OF MATHEMATICAL AND FINANCIAL LITERACIES (IMMFL)

In this study, the basic knowledge, skills and dynamics of both the fields were considered. IMMFL includes the relationships of the circle and explains its editing. IMMFL looks like the shape of an enzyme structure. The model of IMMFL is shown in Figure 6. Accordingly, the contents and financial contexts are integrated with the processes with the intention to reveal the proper behaviors.

Financial context

In the dimension of the financial context, different behaviors of individuals are considered which, unlike PISA, do not edit the context as locations where people are in. In this section, the explanations of the dimensions of IMMFL are provided and their positions in the model are determined.

Earning In the literature, considering the context of earning, working, income (salary), financial support, scholarship, part-time working, overtime and legacy are mentioned. A person must work to earn permanent income. According to the agreement of the work, overtime pay, payment by work hours or a bonus system may be used. Except them, a person may be supported financially by their family or someone else. In addition, legacies from the family are also considered as an income (Aprea et al., 2016; Lusardi, 2015). The profit individuals make from investment items are also considered as earnings.

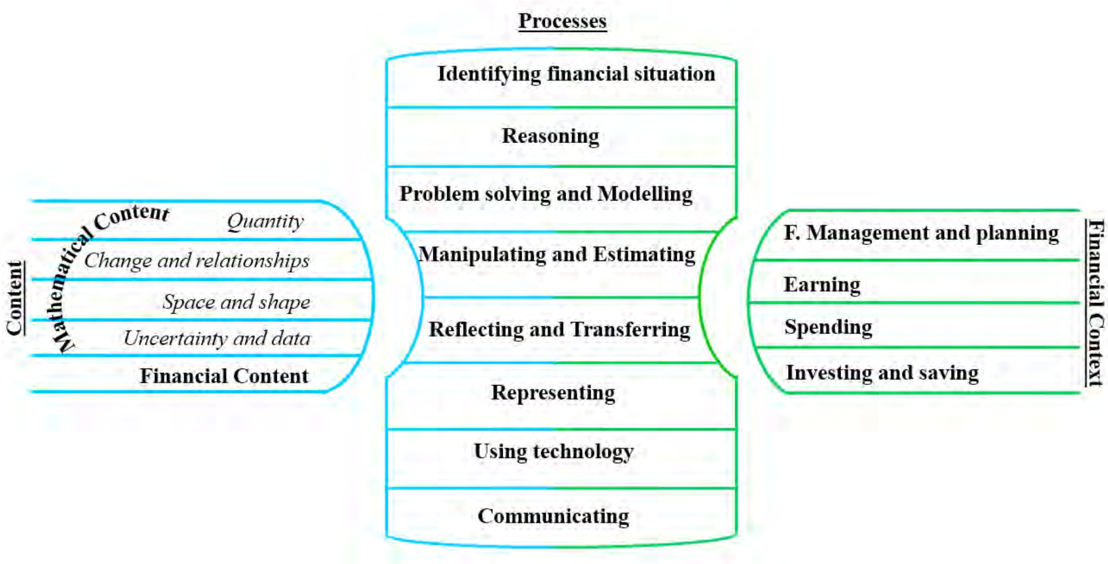

Figure 5. The interaction model of mathematical and financial literacies (IMMFL)

Investing and saving Individuals may save a certain amount of their earnings for other expenditures or investments. The use of a money box from an early age is one of the important marks in FL, and piggy bank is one of the symbols in this field. The habit of moneybox can contribute to the balancing of individuals' expenditures and their saving. Regarding investment items, individuals may wish to evaluate their savings as deposits, foreign currency, precious metals, real estate, stock exchange, bonds, trade or any other field and provide additional earnings.

Spending Individuals spend money to satisfy their needs. In the circle of spending, daily needs such as food and clothing, periodical needs such as holidays and long-term needs such as house or vehicle can be expressed. On the other hand, it is also important to determine whether the wishes of the individuals are needs. In today's world, expenditures go beyond people's earnings. For this reason, the usage of credit and credit card is common. In this manner, individuals can divide and delay their payments. However, this situation leads them to get used to using money easily and fasten the borrowings.

Financial management and planning Considering financial situations, financial management and planning is a requirement for the balance of earning-spending-savings and investing. Accordingly, the outgoings must be determined according to the income. Some aspects of the component of financial management and planning are selecting the right financial options, saving money for proper investments and preparing for the expected and unexpected situations such as education, unemployment, accident, retirement and the future.

Content

Mathematical contents

-Quantity Numbers, operations, and clusters are considered in the component of quantity.

-Change and relationships Proportion relations, algebra and patterns, functions, equations, series and sequences are considered in the component of change and relationships.

-Space and shape Geometry and all the measurements are considered in the component of space and shape.

-Uncertainty and data Possibility, statistics, analyses and management of data are considered in the component of uncertainty and data

Financial content The component consists of some financial situations that are not interested in mathematics, such as the usage of credit and credit card, credit rating and banking transactions. First, the specific financial situations in the financial context need to be defined. Therefore, students should also be provided with financial contents to make sense of the financial contexts. Looking at IMMFL from the perspective of mathematics education, it can be said that it, although possibly smaller than the mathematical contents, is needed. Particularly, the contents included in the FL domain of PISA regarding financial management and financial landscape can be considered in this component.

Processes

In the organization of the components of the processes of IMMFL, the FL perspective provides a new meaning to the definitions of the components. However, some components seem to be close to each other, and their differences and main points are noticeable in their definitions.

Identifying financial situation As regards the organization of the dimensions of IMMFL, the financial situation first needs to be understood in order to run mathematical skills in the financial contexts. The spectrum of financial concepts is broad and complex. In order to make sense of a specific financial problem, it is necessary to figure out the situation before interpreting it mathematically. For this, financial concepts are discussed rather than mathematical concepts and skills. For example, in a problem created in the context of house loan, the sub-concepts and information such as credit, interest, compound interest, payment schedule, and delay should be organized. Moreover, before the process of calculating the compound interest mathematically, it is necessary to make sense of the concepts such as principal (capital), interest and monthly payment.

Reasoning Understanding the differences of the processes is important to figure out the organization of IMMFL. However, the scope of the skills in the process of reasoning makes this difficult. Reasoning skills are seen in all the processes of thinking, decision-making and acting. Reasoning is the mathematical meaning of a situation and the handling of objects and thoughts with mathematical notations (Schoenfeld, 1992). Reasoning includes mathematical processes such as relating, comparing, evaluating, proving and conceptual as well as procedural understandings. In the setting of reasoning skills, the use of these processes in the financial area are considered. Moreover, the kind of reasoning in the literature (Russel, 1999; NCTM, 2000; English, 1997; Van de Walle, Karp, & Williams, 2007; Martin & Kasmer, 2009) and processes of PISA are handled in IMMFL as sub-components (OECD, 2016a).

-Relating In general it refers to connecting new knowledge with existing schemes (Piaget & Ebrary, 2002). In IMMFL, relating refers to connecting the conceptual and procedural schemes created for the existing mathematical concepts to financial concepts and situations, and they are put together for the solution. The relationship between compound interest and the concept of exponential function can be given as an example. The compound interest calculations are within an exponential function structure. In the calculation of the compound interest, the increase of the total amount is not linear due to the exponential function structure. The scheme of students about the exponential functions can be useful for the compound interest calculations. This scheme can be a stimulus for revealing the features of compound interest. The structure of function $S = P\ x\ [1 + r]^n$ is similar the function $y = a^x$. The elements of the compound interest calculation, n: time, r: the rate, P: principal and S: amount, can be related to the logic of the basic exponential.

-Comparing It is the evaluation of several situations, such as two similar or more options with their positive and negative aspects. Many examples can be given to explain this, such as the comparison of two cars with different criteria and the economic comparison of monthly and annual subscriptions. Comparing is a type of reasoning used commonly in financial situations. In order to choose the optimal one, the existing options must be compared. The criteria or aspects which are used in the comparison should be determined. For example, when deciding which investment item to choose, different criteria such as profit margin, reliability, risk-reward situations and continuity can be determined.

-Evaluating This refers to deciding on a situation. As an example, in order to decide the limit of a credit card, many details such as the amount of monthly spending, installment options, frequency of cash usage, travel, traffic insurance and school expenses should be evaluated.

-Conceptual understanding and procedural understanding *Conceptual understanding* refers to the establishment of meaningful relationships between concepts, while *procedural understanding* refers to running processing steps, algorithms and mathematical language (Hiebert & Leferve, 1986; Star, 2005). In the process of problem-solving, the relationships between concepts should be made meaningful and the process skills should be run. In the processes, both the skills support each other and are necessary for permanent learning. As an example, in the case of an investment of deposits in a bank, the correct calculations and correct algorithms regarding the profit, time and rate are procedural understandings, while the understanding of relationships between profit and the rate and the effect of time on the amount and the interpretation of it all is conceptual understanding.

The process of reasoning is an umbrella term for a wide range of mathematical thinking. Therefore, the types of reasoning can be expanded as follows: algebraic and geometric reasoning, proportional reasoning, spatial sense, quantitative reasoning and interpreting skills (NCTM, 2000).

Problem-solving and modelling Problem-solving is an important component of mathematical processes. Problem posing, designing and solving can be expressed as successive parts of this process. The need for intensive and systematic reasoning for financial situations and unclear solutions are regarded as a problem (Torp & Sage, 2002; Lesh & Zawojewski, 2007). Moreover, while balancing and visualizing the monthly budget can be considered a problem, the calculation of kitchen expenditures in the budget is not one. Similarly, selecting a loan according to different criteria and determining payment terms are problems, while the calculation of compound interest maybe not be one. Even though modeling is included in the problem-solving process in different sources, it is stated that algebraic and graphical models are used extensively in financial contexts, and these models have positive effects in understanding and explaining the financial situations (Worthington, 2004; Ontario, 2010). Blum (1993) states that mathematical modeling should be considered as a separate process skill that contributes to the problem-solving process. Algebraic and graphical modeling can be used to mathematize and interpret the financial situation in a problem (Lesh & Doerr, 2003). The main purpose of modeling is to help reveal the mathematical meaning in a context. Accordingly, the modeling process can be built within an equation, a graph or a drawing. In addition to being a cognitive process, modeling can be used as a visual which facilitate solutions. For example, the question of how many different choices can be possible in a probability can be illustrated with visual modeling. This situation is not regarded as mathematical modeling. However, visual modeling is useful, especially concerning the meaning and solution of financial situations (Ontario, 2010).

Manipulating and estimating Manipulating refers to computational skills, algorithm use, measurement conversion, data manipulation, and the meaning of these processes. Data manipulation can be described as controlled monitoring of the change of data and its properties (Pugalee, 1999). Computational strategies and the process of trial and error learning can also be considered within the manipulation. Within the framework of FL and ML, manipulating is expressed as trying a large number of data, monitoring the changes and their effects on the data, calculating and analyzing algorithms (OECD, 2016b; Ontario, 2010; Pugalee, 1999). Moreover, manipulating is inherent in mathematics, and financial arguments are very convenient for this manipulation. The meaning of changing a large number of data using financial instruments can be revealed by manipulating (Aprea et al., 2016). Furthermore, manipulating can be useful in the processes where people should select the best of the many available options. For example, manipulating can be a convenient process for clarifying the effects of the multivariate situation in the calculation of a loan. Numerous manipulations can be performed through technology in order to examine the changing principal amount, total amount or interest amount and monthly payments because of the changing interest rate and time. Additionally, manipulations help in the conceptual understanding of changes, and, therefore, manipulating is a necessary process for finding out the optimum choice as well. As can be seen, the components of the processes of IMMFL progress together and feed each other.

Estimating is also an important process in financial solutions (Ackermann & Eberle, 2016). Posing and testing hypotheses for the solutions and making estimations by calculating allows for the development of mathematical abilities. Both estimation and computational strategies are commonly used skills in mathematical processes (Jablonka, 2003; Ontario, 2010). For example, in the case of a restaurant business, many situations such as the total capacity, amount of daily food, frequency of using of required equipment and policy on pricing can be estimated.

It is important to note that estimating can be used with other processes such as manipulating, evaluating or using technology. Manipulating and estimating are skills that feed each other in financial situations. Being familiar with financial situations, making many calculations regarding them, encountering many financial problems and dealing with familiar algebraic expressions strengthens estimating skills. Besides, using estimates and manipulations together with modelling enriches the understanding of the structure of financial problems. For this reason, IMMFL separates the components of manipulating and estimating from the components of reasoning and problem-solving.

Reflecting and transferring Mathematics is the main tool for finance. Reflecting (which is related to different disciplines) is shown in mathematical approaches as the calling of knowledge (Wheatley, 1992). In order to activate knowledge, it should be transferred to a new context and used there. This cognitively renews the concept enriches its meaning. In this way, the usability of knowledge is increased and the effect of literacy skills is expanded. Reflecting and transferring refers to the transfer of mathematical knowledge and skills to the financial environment as well as the enrichment of mathematical knowledge and skills by use in the financial field. Transferring knowledge to the new field is a common skill of both the literacy areas and is one of the processes of IMMFL. For calculating the membership fee, manipulation is performed according to many variables such as the time of payment, the number of installments or the type of services evaluated within the membership. Furthermore, algebraic skills are used in the context of pricing. This effort in the situation also leads to the development of algebraic skills. To provide another example, a person who wants to calculate the salary they will receive in the new year can calculate it by using the concept of percentage with the declared increase and inflation rates. Thus, the person reflects their mathematical skills in order to figure out a financial situation. Besides, a person in the financial area who works on stock market graphs can improve their reading and interpretation skills on data. Accordingly, financial and mathematical skills can be mutually reflected in the other field.

Representing It is important to use both mathematical and financial language in order to understand and solve a financial problem. In this process, symbols, graphs, tables, equations, algebraic expressions, geometric shapes and financial representations can be observed. Representing refers to a system that can replace the mathematical object and mathematical thinking. In IMMFL, representation classification developed by Nakahara (2008) benefited from the model proposed by Lesh and Doer (2003). Accordingly, in IMMFL, mathematical notations, numbers and symbols are given as as symbolic representation, conceptual explanations are given as linguistic representation, shapes, tables and graphs are given as visual representation, instructional tools are given as manipulative representation and concrete materials and situations are given as realistic representation.

Using technology When organizing IMMFL, it is thought that the component of technology can be especially of use in problem-solving, communication and computation skills as a supporter and a tool. The component of using technology in IMMFL refers to the use of calculators, computers and related applications for the processes of calculating, manipulating and communicating.

Communicating Communicating refers to understanding the financial status, calculations and conditions and explaining them to oneself as well as others while working together. Accordingly, an individual should first perceive the financial situation verbally, and be able to explain their thoughts, evaluations and calculations on a situation clearly and comprehensibly by using the correct language around them. For example, if a family buys a television for their home, the members of the family should be able to explain their expectations and financial status to each other. However, for this to occur, each of them should know the concepts, conditions and calculations. They can evaluate this situation together as well.

CONCLUSION

IMMFL was created as a model to examine the relationship between both kinds of literacies from the perspective of mathematics education. The main purpose of the organization of this model was to present a framework for the researchers who study the two areas together. In particular, IMMFL intends to help the integration of FL into mathematics education. In IMMFL, there are components that should be considered in the typology of a task and not in the literacy characteristics that the students should have. The components and sub-components within these dimensions can be increased and developed. IMMFL has a structure that provides the grounds for relational learning. Therefore, besides having FLE as a separate course, its integration into related courses such as mathematics may indicate the usability of knowledge in real-life contexts. In this respect, it is thought that IMMFL can be used in the design and analysis of tasks, book reviews and curriculum studies, and especially in the studies yet to be carried out regarding the integration of FL into mathematics education.

In the window of FLE, IMMFL is seen as an important tool to determine the relationship between the content, financial context and processes because this structure is not only valid for mathematics but also each literacy field. Therefore, the structure of IMMFL may be used for separate FL courses and their documents as well. From this perspective, the relationships between each component and each dimension are valuable subjects of study. This is also useful because it shows the wide spectrum of mathematics to the students.

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