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OBSERVING THE EFFECTS OF HANDS-ON ACTIVITIES ON MATHEMATICS TEACHER CONFIDENCE IN GEOMETRY AND STATISTICS: A CASE STUDY

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Abstract

Middle school mathematics teachers often do not emphasise statistics and geometry in their lesson plans for a variety of reasons. However, these are integral topics in mathematics, and using direct lecture or hands-on activities may have different effects on teacher confidence when approaching these subjects. Using qualitative and quantitative methods, anxiety was found to improve with lecture-based workshops, while confidence and effectance was found to improve with manipulatives-based workshops. The results showed some surprising and unsurprising reasons for these conclusions.

Keywords

Teacher education; mathematics anxiety; manipulatives; geometry; statistics

Review of the Literature

Geometry and statistics are polarising mathematical subjects: students and teachers will swear by these as the only mathematics topics that make sense, while others emphatically denounce these topics as the beginning of the end for their mathematical learning. For example, pre-service secondary mathematics teachers "may believe statistics is difficult and … this may impact their perceptions about their ability to teach statistics content" (Lovett & Lee, 2017, p. 300). Additionally, Hedges and Harkness (2017) found that student opinions fell into three categories: "(1) statistics is mathematics, (2) statistics is like geometry, and (3) statistics is almost not mathematics" (p. 349). Also, prior methods taught to teachers could manifest negatively towards mathematics, particularly geometry (Fonseca & Cunha, 2011). Clearly, then, these are important milestones in high school mathematics, and the purpose of this study will explore the background and strength of the teachers in charge of relaying this information to students on their way to those courses—middle school mathematics teachers (defined here as 4–8 grade levels). To be more exact, the research question will be as: How does the style of learning geometry and statistics in a professional development setting affect the confidence of middle school teachers?

Manipulatives for professional development

Research has focused on determining pedagogical content knowledge in teachers with regard to more technologically based concepts (Niess, 2005). Specifically, manipulatives (considered a type of technology) have been shown to inspire stronger problem-solving skills, knowledge retention, and justification when compared to learning mathematics strictly through symbolic representation (Carbonneau et al., 2013). Similarly, research has also shown that one of the more effective ways to measuring teacher content knowledge and confidence is through a professional development atmosphere (Baxter et al., 2014; Sarama et al., 2016). However, simply including manipulatives, a mathematics education term for hands-on materials, into a classroom setting or lesson does not necessarily indicate correct implementation or strength in teacher understanding (Moyer, 2001). In fact, understanding is directly related to the experience people have with mathematics (Sarama et al., 2016; Schoenfeld, 1994), which makes it imperative for these lessons to be maximised in both efficiency and engagement. Allowing students (and teachers) to spend more time brainstorming and testing ideas

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Pages 31–42

independently has been shown to lead to a more overall productive and effective class period and learning experience (Schifter, 1996), and teachers have similar needs (Stipek et al., 2001).

Teacher confidence and math knowledge

Beyond content knowledge, confidence is the main aspect of teachers to be assessed in this study. The three focal points here will be confidence in learning mathematics, mathematics anxiety, and effectance motivation in mathematics. Mathematics anxiety can be defined as “a feeling of tension, apprehension, or fear that interferes with math performance” (Ashcraft, 2002, p. 181). Effectance motivation hinges on the philosophical idea that a person does not rely on external factors for learning “involved in learning because [they] wanted to be” (Zakaria & Nordin, 2008, p. 27), which research has found correlates positively with self-motivation, self-efficacy, behaviour, achievement, and persistence (Zimmerman, 1998). Confidence, then, relates in that it is the “single most important predictor of math accuracy” (Morony et al., 2013, p. 1). Research shows that when teachers understand “how their anxiety interfered with their mental function during mathematics activities, [it can help] them learn how to stay focused and relaxed while solving problems” (Cohen & Leung, 2004, p. 1084). Research shows that teachers with traditional views about how to learn mathematics have lower self-confidence, and even find less enjoyment in mathematics, than those with inquiry-based beliefs (Stipek et al., 2001). Similarly, there is a general lack of confidence for middle school teachers towards the math content they teach (Beswick et al., 2006). These ideas are vital, because any “lack of confidence in teachers of primary mathematics may arise from weak understanding of the subject, limiting the teacher’s ability to extend and enhance children’s progress” (McCullouch, 2016, p. 9).

Emphasis on geometry and statistics

Certain subjects within the K-12 mathematics spectrum are more difficult for some teachers than others. For example, “since statistics has been introduced into mathematics curricula, often without adequate attention paid to teachers’ professional development, they often depict insufficient understanding and a fragile knowledge for teaching statistics” (Henriques & Ponte, 2014, p. 1). Research shows that teaching practice can affect the confidence and training of teachers in statistics, both veteran and prospective (Fitzmaurice et al., 2014). Therefore, teacher education is essential to improve this knowledge base to meet instructional goals in the school system (Batanero et al., 2011). Similarly, research shows that positive learning environments and teacher views towards geometry lead to better achievement of classroom goals within that subject, as well (Ly & Malone, 2010). A very real problem has been noted in the past, as “many younger teachers these days have had a poor training in geometry both at school and at the teacher training colleges” (Lundsgaard, 1998, p. 236), and while geometry education through visualisation has improved over time “for early geometry … it is often neglected when students get older” (Jones et al., 2017, p. 562). Research has shown that despite advances in methodology and technology aimed primarily for geometry classrooms, many teaching styles have remained the same and have resulted in a continuous cycle of underdeveloped geometric knowledge, and often pre-service teachers have a preconceived notion of geometry simply based on both expectation and memories (Barrantes & Blanco, 2006).

Methodology

The first week (manipulatives-based) consisted of eight participants from five surrounding schools: four from 5th grade, one from 6th grade, two from 8th grade, and one that covered 6th, 7th, and 8th grade. The second week (worksheet-based) consisted of seven participants from five surrounding schools: one from 4th grade, one from 5th grade, two from 6th grade, one from 7th grade, one from 8th grade, and one that covered 7th and 8th grade. Average grade level was 6.125 for the first group, and 6.214 for the second group. Teacher demographics were as follows: nine Caucasian, four African-American, one Hispanic, and one mixed. Partners were paired up in both cases, with no one working individually, but no two people were paired together more than once. Sessions lasted two hours each for five days, for a total of 10 professional development hours. Both weeks covered the same material, but in different way, shown in Table 1.

Table 1: ***Workshop: Topics Per Week***

Workshop Day	Workshop Topic
Monday (Day 1)	Box-and-Whisker, Measures of Central Tendency
Tuesday (Day 2)	Fallacies Comparing Volume and Surface Area
Wednesday (Day 3)	Probability Basics and Expected Outcome
Thursday (Day 4)	Direct and Inverse Relationships, Ratios, and Linear Graphs
Friday (Day 5)	Triangle Proportions and Statistical Sampling

For the entire study, the sample is only n=15, and in reality, each sample has n=8 and n=7, respectively. This is hardly enough data to run vigorous statistical analysis based on positively and negatively coded survey responses, as normal distribution is not accounted for—some addition information is necessary. However, the interpretations for this quasi-experimental study are based on mixed methods, using the quantifiable data from such surveys to complement, rather than supplement, the qualitative responses given in the free response sections of pre- and post-surveys. The qualitative data is the primary source of information. The sample was not completely random in selection—contact was made to teachers and administrators in the region just before the summer break to seek out teachers who were interested in the study and did not have other professional development commitments. The sample consists of teachers taken from the southern portion of a state in the southern United States. These teachers did not know the purpose of the study until the full workshop was over, and were then informed how participants were experiencing the workshop in the other week they were not involved with, so as not to influence post-survey responses in prideful favour of their own workshop experience or teaching philosophy.

The survey is a slightly edited version of the Fennema-Sherman Mathematics Attitude Scale (Fennema & Sherman, 1976), particularly modified from a more recent version used in a lower-primary teachers study as the descriptions were extremely relevant (Ren et al., 2016), including a 1 to 10 scale with some reverse coding for authentic responses (see Appendix A). One question (about being assessed in mathematics) was removed for irrelevance to the context of the study. Further, this survey has been slightly edited from the already-modified version of Ren et al. (2016) to fit middle school teachers exploring the specific mathematics topics of geometry and statistics (foreign as a singular-subject class in the middle grades, rather than high school). This format parallels the modified intention from 2016 that asked general lower education major teacher opinions on mathematics. Also included is a mathematics subject-specific ranking system for confidence, limiting only the topics relevant for this study; included in Appendix B. The purpose of the survey, both before and after the week of workshop activities, was to view any changes in confidence towards these topics. Three disciplines were covered in the survey: confidence in learning mathematics (10 statements), mathematics anxiety (12 statements), and effectance motivation in mathematics (11 statements).

The participants were asked which workshop days they felt were most impactful and least impactful, including explanations. An additional portion of the post-survey allowed for extended qualitative comments about positives and negatives of applying that particular week's workshop style to their classroom in the future. Those responses are used, combined with the data from the surveys, to create a narrative of teacher opinions on their confidence in teaching the two less-familiar subjects of statistics and geometry. Some additional concept problems were included as well, and this qualitative portion of the survey is included in Appendix C. Week 1 involved the hands-on workshop, while Week 2 was based around direct lecture and slides. Both weeks involved a group dynamic of pairing and group discussions. Both weeks mirrored the same material covered day by day as the basis of the lesson (although the discussions could take organically different directions). These topics were: box-and-whisker plots relating measures of central tendency, fallacies included in comparing volume and surface area, observing natural probability occurrences related to expected value, experiments comparing direct and inverse variation, and statistical sampling using ratios and triangles. Figure 1 shows some collaboration with teachers in that aspect. Video and audio were taken consistently through both styles of workshops, capturing initial feedback and overall group discussion over the course of each week.

Initial findings

The first week

The atmosphere in the room felt more positive in the first week (superficially: more nodding, smiling, and discussing) compared to the second week. Many comments showed teachers with self-awareness in improved confidence and content knowledge. In the first week (manipulatives-based), two particular workshop days were widely considered to contain the most effective lessons: the volume/surface area project received approval for "surprising results" and "relevance with current classroom lessons", and the direct/inverse relationships project was cited as providing a more specific foundation of definitions regarding variation and proportions. The projected probability board game lesson was voted least helpful, mainly due to confusing terminology and general unfamiliarity with probability.

When asked to describe positives or negative aspects about hands-on activities in general versus direct instruction strategies, many immediately considered their future students and turned the thoughts inward, as learners themselves. One teacher remarked, "All hands-on projects were awesome. I learned several new terms in the process. Those hands-on keep my interest." Another commented, "There's opportunity to perk interest in the math with the learning taking place and an interaction with the activity. [For direct instruction], guide to what was lacking in [getting started] on the project-based learning." Still another teacher admitted that "a con [of hands-on] would be the time and cost involved in putting it together. The con of [direct] is that even though you may learn the material you may not have an honest, real understanding of it". Often, the teachers described direct instruction as "boring" or "predictable", even though the workshop for the first week had no direct instruction involved. During the hands-on activities, many of the teachers appreciated the value of using this strategy, and many agreeing verbally that it was "fun", to teach these concepts. One teacher said: "You do a lot of standards in one practice…We waste a lot of time trying to get the foundation together, whereas here you … covered a lot of standards in one activity." Another teacher cited classroom management as a benefit to hands-on activities: "It gives kids the opportunity to get up. Because, you know, there's a problem with kids' behaviour, because I'd let them get up sometimes, but these activities we can let them get up even more, where they're not just sitting idle to get in trouble."

The second week

A common remark during the second week was that the material covered was much deeper than the teachers' previous exposure to such topics. The volume/surface area lesson did not have nearly the same standout impact on the teachers, and the group instead considered the direct/inverse relationship lesson to be the weakest. Some supporting comments called the direct/inverse lesson "confusing", "brand new", and containing "different definitions than taught before". The general consensus was that proportional variation was a concept they were not as clear on as they previously believed. One teacher admitted that the format of the workshops was nearly irrelevant, stating, "I generally tend to have trouble starting. My mind is trying to grasp the information at times because it is so new that I may miss out on the introduction of added information." Yet many teachers were more positive about direct lecture formats, calling it "a fair way to teach large classes" where "all the necessary information is likely to be given to the students in a comprehensible way", but notably did not reference student understanding in the comment. Another wrote that manipulatives lead to "problem-based learning [which] might not gather all the needed information". At one point on the fourth day (direct and inverse relationships topic) the whiteboard was full of variables and proportions. One teacher stopped the group to say: "I want to say something about being an elementary school teacher, in that this kind of stuff, just working with numbers and seeing that is not going to make any difference for them. But applying it … when you start doing a study or an experiment or something like that, that's when they're going to get it."

Often, the group conversations led to deep questions that went beyond the original content of the lessons. For example, an intense discussion arose about the idea of a number on a number line actually having width, or taking up any space on the line itself. In general, the group agreed that by the end of the workshop many of them actually felt shakier about their knowledge because of how much they realised they did not know as strongly as they previously believed, or at all. They also acknowledged that exploring with manipulatives may be helpful, but more importantly this could also leave open plenty of room for knowledge gaps. For example, during the geometry lesson, one teacher pointed out that without using manipulatives, "often, not all the time, but often these prisms are not sitting on its base, so here you've given us one sitting on the base. If it's laying down on its side, they mistake the height as still going vertical". At the same time, the teachers more quickly identified how to find three-dimensional length and width dimensions when they had diagrams rather than physical objects in front of them.

A limited quantitative comparison

In addition to the findings from above with teacher comments and coded recordings of the workshops, provided next is the quantitative data from the survey (again, see Appendix A). The extensive list, shown in Appendix D, categorises the questions in order given on the survey, defined strictly by the three domains tested in the Fennema-Sherman Scale, and shows the average change in each week for that particular question per appropriate week. In Table 2, these domains alone are averaged to see effect (limited, of course, by sample size) on the parties involved in the two different workshop formats. Based on the coding of the survey prompts, positive values indicate an improvement, while negative values indicate a regression.

Table 2: ***Workshop Topics Per Week***

Week 1	Domain	Week 2
0.075	Confidence	-0.45714281
-0.260416667	Anxiety	0.714285808
0.568181818	Effectance	0.233766227

No confidence intervals can be included because of the limited sample size. However, as further support of the comments indicated in the earlier Week 1/Week 2 sections, anxiety improvement provided the largest difference between workshop styles, by one whole point. In the manipulatives-based week (Week 1), anxiety slightly worsened, but anxiety improved during the lecture-based week (Week 2). This overall difference was the most striking quantitative comparison. Effectance improved during both weeks, but less so in the lecture-based week. Confidence hardly changed during the manipulatives-based week, and actually decreased during the lecture-based week—an accurate reflection of teacher comments. Other differences between Week 1 and Week 2 can also be noticed. Effectance was .33 less in Week 2 than in Week 1. Confidence was .53 less in Week 2 than in Week 1. Anxiety was nearly .98 more in Week 2 than in Week 1.

When breaking down the survey on an individual question basis in Table 3, the actual number of domain statements that saw an average increase, decrease, or no change from beginning to end of the workshop are tallied. Once again, the sample size is small, but when coupled with many teacher statements during the workshops, some trends are noticeable. Note that these values are not individual scores per each teacher, but per each question type. To help with interpretation: in row one, for example, during the manipulatives-based week (Week 1), all 10 confidence prompts were averaged collectively in the pre-survey, and similarly in the post-survey, to get two single values representing the group's confidence before and after the workshop collectively. Six of the confidence prompts yielded a higher value in the post-survey compared to the pre-survey, and four confidence prompts yielded a lower value in the post-survey compared to the pre-survey.

Table 3: Rating Change Per Question

Domain	# Increase	# Decrease	# Unchanged
Week 1			
Confidence	6	4	0
Anxiety	5	7	0
Effectance	8	3	0
Week 2			
Confidence	2	8	0
Anxiety	7	4	1
Effectance	7	3	1

Conclusions and implications

By the nature of this project, active workshops can certainly contain a small group of individuals in order to foster an environment that promotes dialogue and close group collaboration. This workshop was no different, and the results (while difficult to construe to all populations because of the severely limited sample size) can be used to shape similar workshops in other districts and/or universities. The following are concise findings after coding the qualitative responses and pairing them with the quantified survey responses:

1. Anxiety decreased during the week of manipulatives, while confidence decreased during the week of lecture. Anxiety was the most affected attributed when comparing overall between the week of manipulatives and the week of lecture.
2. Confidence and effectance both increased during the week of manipulatives, while effectance and anxiety both increased during the week of lecture. Dialogue reinforced the fact that confidence dropped over the entirety of the week of lecture.
3. The topics of direct/inverse relationships and volume/surface area received more positive reception when using manipulatives rather than lecture strategies. Proportional reasoning in general (which included direct/inverse relationships) was perceived as a more difficult topic regardless of instruction format.

While more content can be covered in a deeper sense when lecture is the primary mode of teaching to improve anxiety towards the topics, teachers in the lecture group voiced their lowered confidence because of their prior superficial knowledge base on the topics, as more difficult questions were posed that led into unfamiliar territory. Many teachers knew the advantage of hands-on teaching in their classrooms, but they either had budgetary concerns (another topic altogether) or were not sure the most effective and creative ways to carry out such lessons. However, the implications of the above results imply and support that a mixed methods professional development workshop may be best for districts to use for their teachers. Anxiety is improved with lecture-based instruction, but hands-on training is necessary to improve confidence (and to a lesser extent, effectance). Hopefully, the experience of this project can be used as a springboard for more intensive, larger studies to evaluate how to best provide professional development opportunities to future middle school teachers that cannot forgo teaching statistics and geometry concepts as a fundamental base for their students. Further, this case study can serve as the beginning for studies in the future to determine how best to improve mathematical professional development, and larger samples can allow for more accurate trends to extend out into surrounding classrooms.

References

Ashcraft, M. H. (2002). Math anxiety: Personal, educational, and cognitive consequences. *Current Directions in Psychological Science*, *11*(5), 181–185. https://www.mccc.edu/~jenningh/Courses/documents/math_anxiety.pdf

Barrantes, M., & Blanco, L. J. (2006). A study of prospective primary teachers' conceptions of teaching and learning school geometry. *Journal of Mathematics Teacher Education*, *9*(5), 411–436. https://doi.org/10.1007/PL00021938

Batanero, C., Burrill, G., & Reading, C. (2011). Teaching statistics in school mathematics—Challenges for teaching and teacher education: A Joint ICMI/IASE Study (pp. 407–418). Springer.

Baxter, J., Ruzicka, A., Beghetto, R., & Livelybrooks, D. (2014). Professional development strategically connecting mathematics and science: The impact on teachers' confidence and practice. *School Science and* Mathematics, *114*(3): 102–113. https://doi.org/10.1111/ssm.12060

Beswick, K., Watson, J., & Brown, N. (2006). Teachers' confidence and beliefs and their students' attitudes in mathematics. In P. Grootenboer, R. Zecenbergen & M. Chinnappan (Eds.), *Identities, cultures and learning spaces: Proceedings of the 29th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 68–75). MERCA.

Carbonneau, K., Marley, S., & Selig, J. (2013). A meta-analysis on the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology*, *105*(2), 380–400. https://doi/org/10.1037/a0031084

Cohen, R., & Leung, P. (2004). Math-anxious, elementary teachers' change processes in a graduate course aimed at building math confidence and reducing anxiety. In D. E. McDouball & J. A. Ross (Eds.), *Proceedings of the Twenty-sixth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Toronto: OISE/UT (1079–1086). PMENA.

Fennema, E., & Sherman, J. (1976). Fennema-Sherman mathematics attitudes scales: Instruments designed to measure attitudes toward the learning of mathematics by females and males. *Journal for Research in Mathematics Education*, *7*(5), 324–326. https://doi.org/10.2307/748467

Fitzmaurice, O., Leavy, A., & Hannigan, A. (2014). Why is statistics perceived as difficult and can practice during training change perceptions? Insights from a prospective mathematics teacher. *Teaching Mathematics and its Applications, 33*(4), 1–19. https://doi.org/10.1093/teamat/hru010

Fonseca, L., & Cunha, E. (2011). Preservice teachers and the learning of geometry. In A. Pytlak, T. Rowland & E. Swoboda (Ed.), *Proceedings for the 7th Congress of the European Society for Research in Mathematics Education*. ESRME.

Hedges, S., & Harkness, S. (2017). Is gaise evident? College students' perceptions of statistics classes as 'almost not math'. *Statistics Education Research Journal*, *16*(1), 337–356.

Henriques, A., & Ponte, P. (2014). Preparing teachers to teach statistics: Developing professional knowledge and practice. In K. Makar, B. de Sousa & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the Ninth International Conference on Teaching Statistics* (ICOTS9, July, 2014). International Statistical Institute.

Jones, K. Maschietto, M., & Doze, J. (2017). Introduction to the papers of TWG04: Geometry education. Paper presented at the *10th Congress of the European Society for Research in Mathematics Education*.

Lovett, J., & Lee, H. (2017). New standards require teaching more statistics: Are preservice secondary mathematics teachers ready? *Journal of Teacher Education*, *68*(3), 299–311. https://doi.org/10.1177/0022487117697918

Lundsgaard, V. (1998). General considerations on curricula designs in geometry. In C. Mammana & V. Carmelo (Eds.), *Perspectives on the teaching of geometry for the 21st century. An ICMI study* (pp. 235–242). Kluwer Academic Publishers.

Ly, R., & Malone, J. (2010). Teachers' perceptions of geometry instruction and the learning environment in Years 9–10 ESL Classrooms. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education: Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia*. MERGA.

McCullouch, J. (2016). Contribution of teacher confidence to 'excellent' mathematics teaching. *Philosophy of Mathematics Education Journal*, *30,* 1–12.

Morony, S., Kleitman, S., Lee, Y. P., & Stankov, L. (2013). Predicting achievement: Confidence vs self-efficacy, anxiety, and self-concept in Confucian and European countries. *International Journal of Educational Research*, *58,* 79–96. https://doi.org/10.1016/j.ijer.2012.11.002

Moyer, P. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47*(2), 175–197. https://doi.org/10.1023/A:1014596316942

Niess, M. L. (2005). Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge. *Teaching and Teacher Education*, *21*(5), 509–523.

Ren, L., Green, J., & Smith, W. (2016). Using the Fennema-Sherman mathematics attitude scales with lower-primary teachers. *Mathematics Education Research Journal*, *28*(2), 303–326. https://doi.org/10.1007/s13394-016-0168-0

Sarama, J., Clements, D., Wolfe, C., & Spitler, M. E. (2016). Professional development in early mathematics: Effects of an intervention based on learning trajectories on teachers' practices. *Nordic Studies in Mathematics Education*, *21*(4), 29–55.

Schifter, D. (1996). A constructivist perspective on teaching and learning mathematics. *Phil Delta Kappan*, *77(*7), 492–499.

Schoenfeld, A. (1994) Reflections on doing and teaching mathematics. In A. Schoenfeld (Ed.), *Mathematical thinking and problem solving* (pp. 53–70). Routledge.

Stipek, D., Givvin, K., Salmon, J., & MacGyvers, V. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*, 213–226. https://doi.org/10.1016/S0742-051X(00)00052-4

Zakaria, E., & Nordin, N. (2008). The effects of mathematics anxiety on matriculation students as related to motivation and achievement. *Eurasia Journal of Mathematics, Science & Technology Education*, *4*(1), 27–33.

Zimmerman, B. J. (1998). Academic studying and the development of personal skills: A self-regulatory perspective. *Educational Psychologist*, *33*, 73–86. https://doi.org/10.1080/00461520.1998.9653292

Appendix A: Confidence Survey[1, 2]

Rate the following statements, from Strongly Disagree (0) to Strongly Agree (10). Neutral is (5).

1. Most math topics I can handle OK., but I feel a tendency to mess up geometry and/or statistics.
2. I would never get nervous during a geometry- or statistics-focused lesson in the classroom.
3. When a question is left unanswered in a geometry- and/or statistics-based class, I would continue to think about it afterward.
4. Geometry and/or statistics are enjoyable and stimulating to me.
5. My mind goes blank and I am unable to think as clearly when doing geometry and/or statistics.
6. I am sure that I can learn geometry and/or statistics.
7. I don't understand how some people spend so much time on geometry and/or statistics and seem to enjoy it.
8. I'm not the type to do well with geometry and/or statistics.
9. Generally I have felt secure about attempting geometry and/or statistics.
10. I am usually at ease during geometry and/or statistics lessons in the classroom.
11. I am challenged by geometry and/or statistics problems that I can't understand immediately.
12. I consider geometry and/or statistics my weakest subject.
13. I have a lot of self-confidence when it comes to geometry and/or statistics.
14. The challenge of geometry and/or statistics problems does not appeal to me.
15. I think I could handle more difficult geometry and/or statistics topics.
16. Geometry and/or statistics make me feel uncomfortable, restless, irritable, or impatient.
17. I don't usually worry about being able to solve geometry and/or statistics problems.
18. I am at ease teaching geometry/statistics topics.
19. I get a sinking feeling when I think of trying hard geometry and/or statistics math problems.
20. Giving a geometry- and/or statistics-based lesson to my class would scare me.
21. I'm not good at geometry and/or statistics.
22. Geometry and/or statistics usually make me feel uncomfortable or nervous.
23. Geometry and/or statistics puzzles are boring.
24. I can perform well with geometry and/or statistics topics.
25. It wouldn't bother me at all to teach more geometry and/or statistics topics.
26. Once I start trying to work a geometry and/or statistics puzzle, I find it hard to stop.
27. Figuring out geometry and/or statistics problems does not appeal to me.
28. When a geometry and/or statistics problem arises that I can't immediately solve, I stick with it until I have the solution.
29. Geometry and/or statistics make me feel uneasy or confused.
30. I would rather have someone give me the solution to a difficult geometry and/or statistics math problem than to have to work it out for myself.
31. Geometry and/or statistics don't scare me at all.
32. For some reason, even though I lesson prep, geometry and/or statistics seems unusually hard for me to teach.
33. I want to cover the minimum material in geometry and/or statistics units of my classes.

[1]Reverse coded: Q1, Q2, Q7, Q8, Q10, Q12, Q14, Q17, Q18, Q21, Q23, Q25, Q27, Q30, Q31, Q32, Q33.
Not reverse coded: Q3, Q4, Q5, Q6, Q9, Q11, Q13Q15, Q16, Q19, Q20, Q22, Q24, Q26, Q28, Q29.

[2]Confidence in Learning Math: Q1, Q6, Q8, Q9, Q12, Q13, Q15, Q21, Q24, Q32.
Math Anxiety: Q2, Q5, Q10, Q16, Q17, Q18, Q19, Q20, Q22, Q25, Q29, Q31.
Effectance Motivation in Math: Q3, Q4, Q7, Q11, Q14, Q23, Q126, Q27, Q28, Q30, Q33.

Appendix B: Topic-specific Ranking

Directions: Please rate the following mathematics topics according to how confident you would be teaching your students each topic.

Please use the following scale:

1	2	3	4	5	6
Not confident at all					*Completely confident*

1. Averages, Mean, Median, Mode, Quartiles	1	2	3	4	5	6
2. Multiplication	1	2	3	4	5	6
3. Number Patterns	1	2	3	4	5	6
4. Shape Properties	1	2	3	4	5	6
5. Fractions	1	2	3	4	5	6
6. US Measurement System	1	2	3	4	5	6
7. Probability	1	2	3	4	5	6
8. Decimals	1	2	3	4	5	6
9. Order of Operations	1	2	3	4	5	6
10. Statistical Sampling	1	2	3	4	5	6
11. Division	1	2	3	4	5	6
12. Perimeter & Area	1	2	3	4	5	6
13. Volume & Surface Area	1	2	3	4	5	6
14. Tables & Graphs	1	2	3	4	5	6
15. One- and Two-Step Equations	1	2	3	4	5	6

Appendix C: Qualitative Post-Survey

1. Which topic do you think improved your knowledge base the most? What about the least? Please explain on both accounts.
2. Please describe *at least* one pro and one con for both hands-on, project-based learning, and direct-instruction, lecture-based learning.
3. What are some of the struggles that your students may have with both geometric and statistical concepts in class? Try and be specific, whether it is content-based or skills-based issues with students.
4. A student is talking with a friend about rolling up a piece of paper into a cylinder to hold sand from the playground. His friend wants to roll the paper differently, thinking he can get it to hold more sand. Discuss your confidence in helping these students with this problem, and briefly explain ideas to consider when solving this kind of math problem.
5. A fair coin is tossed ten times, and the probability needs to be calculated for at least one head appearing, then at least two heads appearing. Without solving, write down thoughts and ideas to consider when solving this kind of math problem. Include your confidence in exploring this type of question.
6. A group of students is using their own heights and shadows to estimate the height of a nearby building (using its shadow). However, the group is arguing about how to set up the proportion. One says the person's shadow corresponds to the building's shadow, and equivalently the person's height corresponds to the building's height. Another says this is wrong, and instead the person's shadow corresponds to the person's height, and equivalently the building's shadow corresponds to the building's height. Another student decides it does not matter which item is compared to which, as "it will all work out proportionally". Briefly explain the idea of proportionally reasoning to these students based upon their preconceptions.
7. The data set of $N = 30$ ordered observations as shown below is examined for outliers:

   35, 98, 102, 112, 112, 205, 275, 290, 387, 400, 465, 465, 466, 517, 551, 628, 680, 699, 811, 834, 846, 902, 912, 939, 947, 981, 1016, 1050, 1135, 1474

   To the researcher, 35 and 1474 look fishy, and they wonder if those values are actually outliers. Describe your process in analysing this problem; doesn't have to be specific solving.
8. Draw an example of a graph that represents a relationship that is:
   1. Direct but not proportional.
   2. Direct and proportional.
   3. Inverse but not proportional.
   4. Inverse and proportional.

Appendix D: Extensive Survey Data[3]

Q#	Week 1	Domain Tested	Week 2
1	0.25	Confidence in Learning Math	-1.285714
2	-0.625	Math Anxiety	-0.285714
3	1.5	Effectance Motivation in Math	0
4	1.625	Effectance Motivation in Math	1.2857143
5	0.375	Math Anxiety	1.7142857
6	-0.25	Confidence in Learning Math	1.2857143
7	0.25	Effectance Motivation in Math	0.2857143
8	0.875	Confidence in Learning Math	-0.428571
9	0.5	Confidence in Learning Math	-0.857143
10	-1.875	Math Anxiety	2.2857143
11	1	Effectance Motivation in Math	0.8571429
12	-1.5	Confidence in Learning Math	-2.142857
13	0.5	Confidence in Learning Math	-0.571429
14	-0.125	Effectance Motivation in Math	-0.714286
15	0.5	Confidence in Learning Math	-0.142857
16	0.875	Math Anxiety	0
17	-0.375	Math Anxiety	1.7142857
18	-0.25	Math Anxiety	0.1428571
19	-1.625	Math Anxiety	-0.285714
20	0.125	Math Anxiety	2.2857143
21	-0.5	Confidence in Learning Math	-0.428571
22	0.5	Math Anxiety	-0.285714
23	0.5	Effectance Motivation in Math	1
24	1.375	Confidence in Learning Math	0.5714286
25	-0.75	Math Anxiety	-0.285714
26	1.125	Effectance Motivation in Math	0.5714286
27	0.75	Effectance Motivation in Math	-0.285714
28	1	Effectance Motivation in Math	0.1428571
29	-0.125	Math Anxiety	1.2857143
30	-0.625	Effectance Motivation in Math	-0.857143
31	0.625	Math Anxiety	0.2857143
32	-1	Confidence in Learning Math	-0.571429
33	-0.75	Effectance Motivation in Math	0.2857143

[3]Questions #1-33 are listed strictly by which domain is tested, with the average change between pre- and post-surveys shown for Week 1 (left) and Week 2 (right).