How to Design Teaching Materials to Improve the Ability of Mathematical Reflective Thinking of Senior High School Students in Indonesia?

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ABSTRACT
This research is a research and development which produces teaching materials. The research method consists of stages, namely: an introduction (literature study, observation, and interviews), product development of teaching materials, test by a team of experts, and restricted trial to the eleventh-grade students of senior high school in Cimahi, West Java, Indonesia. It was concluded that the teaching materials through scientific approach with what if not strategy load some activities including observing, trying, reasoning, asking, and concluding. The activities of asking using what if not strategy involve changing the data, adding the data, changing the data with the same question, and changing the question to the same data. In the teaching materials, it is presented some contextual problems in accordance with the experience of students. The design of teaching materials consists of three parts, namely: the beginning involves cover, preface, and table of contents; the content section: containing the title of the chapter dealing with the enumeration rules as a representation of the contents of the chapter; and closing: as a reflection and evaluation of the material presented on teaching materials for the next revision. Teaching materials can enhance the ability of reflective thinking of the students with medium criteria.

Keywords: teaching material, mathematical reflective thinking, scientific approach, what if not strategy

INTRODUCTION
The students’ mathematical thinking ability becomes the focus in learning mathematics in school. To make students having good mathematical thinking skills, teachers must be able to facilitate students in developing their potential. According to Sumarmo (2000) to develop the ability of mathematical thinking in learning, teachers should encourage students to engage actively in discussions, ask questions, answer questions, and explain and argue for each answer given. Sabandar (2008) suggests that learning mathematics is closely related to the activity, learning, and thinking, because the characteristic of mathematics is patterns of thinking and organizing the logical verification using the terms which were defined carefully, clearly, and accurately.

The ability of mathematical reflective thinking is one of the high-level thinking which is important to develop for the students in the era of globalization because the level of complexity of the problems in all aspects of modern life is getting higher. According to Garrison, Anderson, and Archer (2001) if the students’ ability of reflective thinking is developed, it will encourage them to seek the truth, be open-minded and be tolerant of new ideas, be able to analyze a problem well, to think systematically, be inquisitive, be mature in thinking, and be able to think critically and independently.

The importance of reflective thinking is also proposed by Rudd (2007) who states that reflective thinking encourages students to think of during the problem-solving process because the reflective thinking provides an
opportunity to re-examine and find the best way to achieve the goal. To develop the students’ mathematical reflective thinking, Herman (2007) argues that the mathematical tasks in the learning process should be able to make the students observe and explore the phenomena of mathematics so that it requires students to think optimally within its capabilities.

However, as evidenced by research Vaske (2001); Black (2005); Choy and Cheah (2009) students will have no ability of critical thinking since their teachers face difficulty in integrating critical thinking adequately into the students’ daily practice because it needs such a reflection. Critical thinking and reflective thinking are often used synonymously. Critical thinking involves a wide range of thinking skills leading toward desirable outcomes and reflective thinking focuses on the process of making judgments about what has happened.

The ability of students’ mathematical reflective thinking was still rarely trained by teachers in learning (Choy & Oo, 2012; Moss, 2010). Based on data obtained by Nindiasari (2011) on one of the high schools, it showed that more than 60% of students had not given satisfactory results in working on the problems that load the indicators of the mathematical reflective thinking process. It shows that the reflective thinking process had not been familiarized on the students, so their ability of reflective thinking becomes low. This opinion is supported by the findings Choy and Oo (2012); Kurniawati, Kusumah, Sumarmo, and Sabandar (2014). Teachers must be creative in providing challenging tasks for students to improve their reflective thinking and scientific attitudes (Suryawati & Osman, 2018). Expert teachers attempted to understand students thinking (Zhu, Yu, & Cai, 2018).

The principle of learning in the curriculum of 2013 emphasizes on a paradigm shift, namely (a) the students are told becomes the students find out; (b) the teacher as the only source of learning becomes learning based on a variety of learning resources; and (c) a contextual approach becomes the process approach as strengthening of the use of scientific approach. Based on Kemdiokbud (2013), scientific approach facilitates students to acquire knowledge and skills based on a scientific method: observing, trying, reasoning, asking, and concluding. Students are directed to the process knowledge, discover and develop their own concepts with regard to the subject matter so that it provides an opportunity for students to cultivate high-level thinking skills.

Reference Brown and Walter (2005) stated that generally, mathematical thinking begins with given statements so that we just trained to resolve the issue of the statement. However, it should be better if we give another statement rather than just accept the statement. Thus, the student’s high order thinking skills will thrive. One of the learning strategies that can develop the high-level ability of mathematical reflective thinking is what if not strategy. This strategy can develop reflective thinking activities of students in analyzing problems, contrasting conditions on the problem, and checking the correctness of completion.

Based on the explanation above, it seems that the ability of reflective thinking for students is so important that teachers need to develop teaching materials. The problem statements of this research are formulated as follows (a) How to develop teaching materials through a scientific approach with what if not strategy to improve the students’ ability of reflective thinking? (b) What is the impact of the use of teaching materials through a scientific approach with what if not strategy on the students’ ability of reflective thinking?

LITERATURE REVIEW

Teaching Materials Design

Teachers should design innovative mathematical learning by giving students the opportunity to explore their abilities (Hendriana, 2017). Teachers can design teaching materials that can develop students’ thinking skills. Teaching materials is a set of learning tools that consists of subject matter, methods, and attractive and systematically arranged evaluation in order to achieve the expected competencies. Teaching materials prepared properly can determine the success of the students in understanding the material studied (Widodo & Jasmadi, 2008).

Good teaching materials shall include at least the user study, the competence to be achieved, subject content, supporting information, exercises, work instructions, evaluation and response to evaluation (Prastowo, 2012). Some things to consider in making instructional materials are as follows (a) provide interesting examples and illustrations in supporting the presentation of learning materials; (b) provides the possibility for students to respond and
measure their mastery of the material through questions, tasks, and so on; (c) the material presented relates to the students’ environment (contextual); (d) the language used is simple and does not confuse students.

Teaching materials consist of various types, such as handouts, books, modules, and the student worksheet. In this study, teaching materials developed is in the form of worksheets with enumeration rules materials. This material is chosen because it is assumed to be difficult by students to understand. They are confused to determine the concept of permutations and combinations which are part of the enumeration rules. To overcome the students’ difficulties, it is necessary to design instructional materials that can help students develop reflective thinking ability.

The first part of teaching materials is cover. The cover consists of title explaining that this is a teaching materials used in learning, the curriculum logo applied in learning is Curriculum 2013, the approach used in designing teaching materials is scientific approach and what if not strategy, the teaching materials is used for secondary school students at Class XI of Semester 2. The logo of institution providing funding for designing teaching materials is The Ministry of Research, Technology, and Higher Education (Kemenristek Dikti) and the identity of the teaching materials writer.

The second part is preface. It contains Gratitude for compilation of teaching materials design and supports from other parties. First Paragraph contains Gratitude and Thanks to God for compilation of the teaching material. The second paragraph is purpose of compilation of the teaching materials, which is, to develop students’ reflective thinking ability through scientific approach and what if not strategy on enumeration rules that students find them difficult. Third paragraph contains Gratitude to the superintendent and staff at Kemenristek Dikti and other parties for providing financial supports in designing and compilation of the teaching materials.

The third part of teaching materials is table of content containing details of enumeration rules in mathematic course. Table of content contains page numbers to facilitate readers in finding every chapter of the teaching materials to learn. Table of contents contains information of page number of preface, table of contents, teaching materials of enumeration rules and closing. Details of material from the teaching materials are enumeration rules, factorial, permutation, combination, Newton binomial, and probability theory. Contents of permutation consist of permutation with other different and same elements and cyclical permutation. Probability theory contains total of event appearances and multi-events appearance probability.

Fourth part of the teaching materials is Contents. They contain description of enumeration theory. First part of Contents contains information of school and students’ names. There are also direction and sequence of material to be learnt by students. There are information about school name, class and semester, subjects, date and time allotted. There also direction for students about how to use the teaching material/learning techniques, which are; group discussion to understand every material, the discussion is led by a group leader, posing problems to understand the material more deeply, and if students have difficulties, they can ask the teacher for help assisted by the group leader. Sequence of material about enumeration theory is also written in part of Content, which is the theory of multiplication, factorial & permutation, combination and Newton binomial, and probability theory.

Scientific Approach and What If Not Strategy

Reference Kemdikbud (2013) states that the scientific approach is a learning process in which students are encouraged to think logically, coherently, and systematically because the real learning is this scientific process (science) itself. The learning using scientific approach comprises five components as follows: observing, trying, reasoning, asking, and concluding. Brown & Walter (2005) designed the strategy is called what if not. This strategy emphasizes the manipulation of some elements of the problem and its effect through five stages.

First is choosing a starting point. This stage requires students to start choosing what issues will be raised. The problems posed could be a mathematical theorem or application. Secondly, listing attributes. In this second step, students enroll the properties or attributes of the problem set by the teacher. Third is what if not. Activity at this stage is students perform manipulation of the facts. Fourth, asking a question. At this stage, the students propose new problems of a given problem. Fifth is analyzing the problem. Students at this level analyze or resolve the question of new problems. This activity can foster reflective thinking students by examining the back of the truth of the answers.

Some of the techniques that can be used to create questions with that what if not strategy are: changing information or data to the original question; adding information or data to the original question; changing the value of the data provided, but still retains its original condition or problem situations; changing the situation or the condition of the original question, but keeping the data or information contained in the original question.

The display of teaching materials with scientific approach and what if not strategy is presented through Figure 1.
Figure 1 is a display of teaching materials on observing activities on a scientific approach to permutation materials with different elements. Students observe the problem of queue numbers. Here is one of the problems given in the teaching materials.

A bank officer prints a customer queue number consisting of three numbers from numbers 1, 2, 3, and 4. How many queue number options can be made from:

a. The first three numbers?

b. Four numbers available?

From these problems students may have experienced queues at the bank in life. They think to understand the problem, and then they try to solve the problem. In this experimental activity the students’ reflective thinking ability begins to grow when it raises the idea of what needs to be done so that problems can be solved. Here is presented one of the tried activities in teaching materials.

Figure 2 is a resource display of the activity trying to solve the problem (a) regarding the queue number. To solve the problem, students are asked to look for many queuing numbers consisting of three numbers that can be formed if the bank officer uses the first three numbers, namely 1, 2, and 3. Students are given a column to try to resolve the issue and be instructed in order to get an exact picture of the settlement. The instructions given are six order of queue numbers. Students are asked to try to find various forms of order of queue number to get 6 correct arrangement.

Figure 3 is a display of the activity of trying to solve the problem (b) regarding the queue number. Students are given instructions in trying to resolve the problem. If the bank officer uses the four numbers available, namely 1, 2, 3, and 4, the number of queue arrangements consisting of four numbers can be formed is 24. Students try to find
various forms of queue number arrangement of the four numbers. After the students have successfully obtained the arrangement, they are then directed to the activities of reasoning. The following is presented in the scientific activities of reasoning in the teaching materials.

**Figure 4** is the display of instructional materials on the reasoning activity in solving the problem (a) regarding the queue number. After the students gain by experimenting with the many sequences of queue numbers from the first three digits 1, 2, and 3 are 6 ways, then they are directed to using the factorial concept. Many arrangements that can be formed from 3 numbers are 6 ways that can be written into $3! = \frac{3 \times 2 \times 1}{1!}$. On the concept of factorial $3! = 3!$, furthermore $1! = 0! = (3 - 3)!$. The final shape of the pattern becomes $\frac{3!}{(3-3)!}$. The complete equation can be written into $6 = \frac{3 \times 2 \times 1}{1!} = \frac{3!}{1!(3-3)!}$.

Students are asked to check the truth of the equation whether it is correct or there is a mistake to grow their reflective mathematical thinking ability. After students understand the solution of problem of part (a) to the concept of permutation, they can apply the same concept to solve problem of part (b).

**Figure 5** is the display of instructional materials on the reasoning activity in solving the problem (b) regarding the queue number. After the students gain by experimenting with the many sequences of queue numbers from the first four digits 1, 2, 3, and 4, there are 24 ways, then they are directed to using factorial concepts. Many arrangements that can be formed from 4 numbers are 24 ways that can be written into $4! = \frac{4 \times 3 \times 2 \times 1}{1!}$. On the concept of factorial $4! = 4!$ and $1! = 1!$ so can be formed into $1! = (4 - 3)!$, furthermore $4! = \frac{4!}{(4-3)!}$. The final shape of the pattern becomes $\frac{4!}{(4-3)!}$. The complete equation can be written into $24 = \frac{4 \times 3 \times 2 \times 1}{1!} = \frac{4!}{1!(4-3)!}$.

In order for students’ reflective mathematical thinking ability to flourish, they are asked to examine the truth of the pattern and explain according to their own understanding. Furthermore, students are informed that the pattern and is the concept of permutation. When students are familiar with the pattern, they are asked to ask two different questions about the problem. To make time effective, they were asked to answer one of the two questions they raised. It aims to train their reflective thinking in checking whether the questions they raised can be solved or need to be added to other information so that questions can be answered. The following shows the appearance of the activity on the scientific approach in teaching materials.
Figure 6 is the display of the query activity on teaching materials on queue number problems. Students are asked to compose two new questions from the problem of queue numbers. Then select one new question and complete it with the description of the rules used. In the teaching materials are provided columns for students to write two questions and columns to answer either of the two questions. Students can use the what if not strategy to compose new questions about queue number problems. The following presents various possible new questions about queue number problems that students can address through the what if not strategy.

1. Changing information or data to the original question.
   If the bank officer wants to print the client’s queue number consisting of 4 numbers from the numbers 1, 2, 3, and 4. How many queue number options can be made from the first three numbers?

2. Adding information or data to the original question
   If the bank officer wants to print a customer queue number consisting of three numbers of 1, 2, 3, and 4 with the condition that each number can be repeated. How many queue number options can be made from the first three numbers?

3. Changing the value of the data provided, but still retains its original condition or problem situations.
   If the bank officer wants to print a customer queue number consisting of three numbers from 1, 2, 3, 4, and 5, then how many queue number options can be made from the 5 numbers available?

4. Changing the situation or the condition of the original question, but keeping the data or information contained in the original question.
   If the bank officer wants to print a customer queue number consisting of four numbers from 1, 2, 3, 4, and 5. How many queue number options can be made from the 5 numbers available?

After students finish compiling and answering questions, they are then directed to conclude material on the concept of permutation. The following shows the activity view concluded on the teaching materials.
Figure 7 is an inferred activity display of the problem of queue numbers. Students are instructed that the number of different 3-digit arrangements that can be formed from the 3 numbers available is \( \frac{3!}{(3-3)!} = 3! = 6 \) which is called 3-element permutations, written into \( P(3,3) = \binom{3}{3} = \binom{3}{3} \). The number of different 3-digit arrangements can be formed from the 4 numbers available is \( \frac{4!}{(4-3)!} = 4! = 24 \) and written into \( P(4,3) = \binom{4}{3} = \binom{4}{3} \). Based on the pattern the students are asked to draw conclusions from the many permutations of k elements and the number of permutations of elements of the available elements. The student concludes that the number of permutations k elements is k! While the number of permutations k elements of the available n elements is \( P(n,k) = \frac{n!}{(n-k)!} \).

Through concluding activities students are expected to understand that the concept of permutation is concerned with determining many arrangements based on the order or location of an object so that they can use the concept of permutation in solving the problem. The picture display of the above permutations is representative of the material of the enumeration rules in teaching materials using a scientific approach with the strategy of what if not. For the view of the material the combination will not be much different from that displayed with the permutation material. It is expected that with learning activities on this teaching materials students better understand the use of permutation concepts and combinations that they find difficult.

The Ability of Mathematical Reflective Thinking

According to Dewey (1993) reflective thinking is the consideration of a conviction, the reason that supports the knowledge, and the conclusion which is the estuary of knowledge. Students who have the ability to think reflectively can control the learning process actively; assess what they know, what they need to know, and how they bridge the gap during the learning process.

Reference Hmelo and Ferrari (1997) revealed that reflective thinking can help developing students’ higher thinking skills, namely: connecting new knowledge to previous understanding; abstract and concrete thinking, having a specific strategy in solving problems, understanding their own thought processes, and learning to build strategy.

Teachers can use certain ways in developing students’ reflective thinking to solve problems; they are: ask the students to present the information contained in the problem; compare the different values of an event, idea or the same symptoms; provide an alternative solution and write the different results; play the role, role reversal, try to see what was omitted and inconsistent; insert ideas that do not appear in a text; remove or omit information; ask the question “What if ...?”; and (h) try to identify the assumptions.
RESEARCH DESIGN

The design of this study is a research and development focused on the development of teaching materials through a scientific approach with the what if not strategy of the students’ ability of and reflective thinking. Teaching materials were validated by two experts. Data collections tools used were validation sheets and interview guidelines.

All data collected from the validation process would be analyzed descriptively and qualitatively. The results of the analysis of the validation sheet and interview guides were used as reference materials to make revisions in order to obtain teaching materials through a scientific approach with what if not strategy which meets valid criteria.

Teaching materials were valid and then they tested in class XI students at one of high school in Cimahi. The procedure of this study consisted of three phases (Sukmadinata & Nana, 2006), namely (a) a preliminary study; (b) product development of teaching materials; and (c) testing expert and limited. Research and development of teaching materials were carried out in three high schools namely SMAN 2, SMAN 3, and SMAN 4 in Cimahi, West Java, Indonesia which had implemented Curriculum 2013 in mathematics. The subjects were students of class XI in the school.

The procedure of designing teaching materials can be displayed in Figure 8. In the field survey section conducted visits to schools to obtain information from teachers about the difficulties faced by students of mathematical material. Results from the field survey obtained the students’ information difficulties in understanding the enumeration rules that contain the concept of permutations and combinations. In addition, students’ reflective thinking ability are rarely trained in solving problems.

Based on field survey conducted literature review to obtain theories of learning in overcoming students’ mathematical learning difficulties and train their reflective thinking ability. One effort is to design teaching materials using scientific approach with what if not strategy. The literature review result is a draft teaching materials with a scientific approach with what if not strategy to improve students’ reflective thinking ability. The draft also contains reflective thinking instruments. According to Putra (2016) in preparing the instrument of reflective thinking must be adjusted with the indicators in order to obtain a valid test.

Draft teaching materials are given to the expert team for analysis and validation. If the draft has not been valid, it will be refined according to the expert team’s suggestion. Drafts that have been valid by expert team conducted limited trials of ten high school students who have studied the material. The trial results are limited as a guide to improve the teaching materials to obtain valid teaching materials.

The teaching materials with a scientific approach with what if not strategy can be used in learning to train students’ reflective thinking ability. The improvement of students’ reflective thinking ability was obtained from the test scores before and after using the teaching materials.

RESULTS AND DISCUSSION

Activities performed on preliminary study consisted of a literature study and observation of the schools. The results obtained from literature review were helpful to add knowledge and study of learning theory (scientific approach with what if not strategy), materials taught (the rules of enumeration) in the preparation of teaching materials, the indicators of test of the ability of reflective thinking, and manufacture of sheet assessment for a team of experts. Furthermore, the observation was conducted in three high schools mentioned in the research design section. The following are the preliminary studies which were conducted.

Literature Review

In literature, the activities undertaken were to analyze journal articles, books, and theories about the abilities of reflective thinking, scientific approach with what if not strategy. From the results of the literature, it can be obtained some indicators of the ability of reflective thinking to the material of enumeration rules.

Based on the findings of Mursidi and Muhsetyo (2012) and Haryadi, Mardiyan, and Saputro (2014) students still had difficulty in solving problem related to the concept of the rule of enumeration (permutations and combinations). They were confused on using permutation or combination formula to solve the problems of a given problem. To overcome the difficulties of the students, teaching materials need to be created to enable students to understand more the concept of enumeration rule with the appropriate approach, the scientific approach with what if not strategy.

According to Putra, Herman, and Sumarmo (2017) a scientific approach with the strategy of what if not can improve students’ high thinking level ability that is mathematical problem posing ability. In doing problem posing students need reflective thinking ability in checking the truth of the problem presented in the material of enumeration rules.
Figure 8. Procedure of designing teaching materials
Experts have an opinion on indicators of mathematical reflective thinking ability. Nindiasari (2011) has made a summary. Question dealing with the ability of mathematical reflective thinking of the enumeration rules materials refers to the indicators: distinguish between relevant and irrelevant data about material of multiplication rule; analyze and clarify answers about material of product rule; generalize and analyze generalizations about permutations and combinations of materials; interpret a case based on the concept of chance; check the validity of an argument on the material chance; and draw an analogy between the two cases which have similar chance.

**Observation to Field**

Based on interviews with math teachers in three of the schools, it was obtained the information: material in the second semester of eleventh-grade students of science classes which are suitable to develop into teaching materials are enumeration rule, consisting of the product rule, permutations, combinations, and chance.

The difficulties experienced by students were due to the presentation of the material from the book source that is used only to make students follow the example of completion of the given problem so that when the questions were changed, they become confused to define the concepts used whether it is permutation or combination.

Teacher efforts to minimize the difficulties of the students to understand the material of permutations and combinations by providing the motivation to always study material had been taught, to ask questions of the material which are difficult to interpret, to discuss the exercise and to use the worksheets provided by the publisher. This effort had not run maximally. Thus, it is necessary to develop teaching materials to develop students’ reflective thinking ability.

Based on observations and interviews conducted with teachers at those three schools, it can be concluded that the teaching materials need to be developed through a scientific approach with what if not strategy to improve the ability of reflective thinking of high school students.

**Preparation of Teaching Material**

Teaching materials which were arranged in the form of worksheets for students were about the materials of enumeration rules. The initial design of teaching materials includes: the front part consists of a cover, preface, and a table of contents: display the page number of the teaching materials are made.

The body section consists of a crosshead of the enumeration rules that represent the content of the chapter, namely rule multiplication, permutations, combination, Binomial Newton, and Chance. Next is the contextual issue. At the beginning of the presentation of the material given the problems that exist in the everyday life of the students were resolved by the given instructions to find the concept. Teaching materials were adjusted to the scientific approach that includes activities to observing, trying, reasoning, asking, and concluding.

In the observing activity, pictures or problems were presented to observe students as an introduction to the concepts to be learned. In the trying activity, students were asked to solve problems in teaching materials. In the reasoning activity, students were asked to check the correctness of the solution and provide an explanation of each step performed.

In the asking activity, students were asked any questions or problems regarding the given image. Students can use what if not strategy in asking questions. Through this activity of asking this question, students’ reflective thinking ability can develop when examining the truth of the question. If they find an error, they can realize it and then fix it.

In the concluding activity, students were asked to make a conclusion of the concepts learned with their own understanding. In conclusion section, the students give reflection and evaluation of the presentation material on teaching materials that they used to work on in order to make improvements.

**Validation of Teaching Materials from Experts Team**

Assessment of teaching materials according to a team of experts from the aspect of the contents had fulfilled the five elements in the scientific approach: observing, trying, reasoning, asking, and concluding in teaching the concept of the problem. From the display of teaching materials, it looks simple. Questions on mathematical reflective thinking need to be added so that the student’s ability could thrive. Based on the results of the assessment team of experts, these materials were already eligible tested on students.

To determine the feasibility of teaching materials used formula percentage, as follows:

\[
N = \frac{k}{Nk} \times 100%
\]
The criteria for eligibility of the teaching materials are set, as follows:

- High eligible: 83.5% - 100%
- Eligible: 64% - 83%
- Eligible enough: 44.5% - 63%
- Not eligible: 25% - 44%

The assessment of both the experts is as shown in Table 1.

Based on Table 1, the percentage of the content of the teaching materials provided by expert 1 is 83.33 and the percentage of contents given by expert 2 is 77.08. According to the eligibility criteria of the teaching materials, this percentage is at intervals of 64% - 83% which means the content of the teaching materials is worth using.

The assessment of feasibility of presentation of teaching materials given by expert 1 is 85.94. This percentage is located at intervals of 83.5% - 100 which means the presentation of teaching materials according to expert 1 is very feasible to use. While the percentage of feasibility of presentation of teaching materials provided by expert 2 is 79.69. This percentage is at intervals of 64% - 83% which means the presentation of teaching materials is worth using.
Based on Table 3, the percentage of linguistic feasibility of the materials provided by expert 1 is 81.67 and the percentage of the content given by expert 2 is 80.00. According to the eligibility criteria of the teaching materials, this percentage is at intervals of 64% -83% which means the language of the teaching materials is worth using. Overall, according to the two experts stated that the teaching materials viewed in terms of content, presentation, and linguistics are feasible to be used to improve students’ reflective mathematical thinking. In accordance with Zulyadaini (2017) students worksheets validated by expert teams are on eligible criteria.

### Restricted Trial

Teaching materials that had been validated by two supervisors were further shown to teachers at three schools where trials conducted to provide advice. Given the teaching materials is still new for the teachers; their assessment of the teaching materials are in accordance with the approach used and measuring abilities will be developed.

Teaching materials can be tested, which is in eleventh-grade students in Mathematics and Science classes of SMAN 2, SMAN 3, and SMAN 4 in Cimahi, West Java, Indonesia. Teaching materials through a scientific approach with what if not strategies regarded to the enumeration rules that had been validated, and then given to students in learning during nine meetings. Students were given an explanation regarding the presentation of teaching materials so that they can solve the problems properly.

In teaching materials, there are five scientific activities that must be done by students who observing the subject, trying to solve the problem, reasoning for the troubleshooting steps, asking about the problems with the strategy of what if not, and concluding from the material they had learned. After finishing studying all the material of enumeration rules in teaching materials, they were asked to fill a scale of opinion regarding the presentation of the teaching materials they had learned.

Students’ work on teaching materials is shown in Figure 9.
1. **MENGAMATI**

Amati permasalahan mengenai **nomor antrian** berikut ini.

Seorang petugas bank ingin mencetak nomor antrian nasabah yang terdiri atas tiga angka dari angka 1, 2, 3, dan 4. Berapa banyak pilihan nomor antrian yang dapat dibuat dari:

a. Tiga angka pertama?

b. Empat angka yang tersedia?

**Figure 9.** Students observe queue number problem

2. **MENCoba**

Untuk menyelesaikan permasalahan tersebut, coba lakukan kegiatan berikut ini!

a. Apabila petugas bank menggunakan tiga angka pertama, yaitu 1, 2, dan 3. Banyaknya susunan nomor antrian yang terdiri dari tiga angka dapat dibentuk, antara lain:

<table>
<thead>
<tr>
<th>Penyelesaian:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terdapat 6 susunan nomor antrian, yaitu: 123, 132, 231, 213, 321, dan 312.</td>
</tr>
</tbody>
</table>

**Figure 10.** Students try to answer questions from problems that have been observed in part 1. a

b. Apabila petugas bank menggunakan empat angka yang tersedia, yaitu 1, 2, 3, dan 4. Banyaknya susunan nomor antrian yang terdiri dari tiga angka dapat dibentuk, antara lain:

<table>
<thead>
<tr>
<th>Penyelesaian:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terdapat 24 susunan nomor antrian, yaitu: 1234, 1243, 1324, 1342, 1423, 1432, 2134, 2143, 2314, 2341, 2413, 2431, 3124, 3142, 3214, 3241, 3412, 3421, 4123, 4132, 4213, 4231, 4312, dan 4321.</td>
</tr>
</tbody>
</table>

**Figure 11.** Students try to answer questions from problems that have been observed in part 1. b

**Figure 9** shows the activity of observing problems about queue numbers. Students are expected to understand well the information of the problem.

**Figure 10** shows that the student can determine many order of queue numbers consisting of 3 digits on the number 1, 2, and 3 as many as 6 namely 123, 132, 231, 213, 321, and 312.

**Figure 11** shows that the student can determine many order of queue numbers consisting of 3 digits on the number 1, 2, 3, and 4 as many as 24 namely 1234, 1243, 1324, 1342, 1423, 1432, 2134, 2143, 2314, 2341, 2413, 2431, 3124, 3142, 3214, 3241, 3412, 3421, 4123, 4132, 4213, 4231, 4312, and 4321. Students use structured methods to get answers.
3. MENALAR

Penyelesaian permasalahan tersebut juga dapat menggunakan konsep faktorial. a. Apabila petugas bank menggunakan tiga angka pertama, yaitu 1, 2, dan 3.

Banyaknya susunan nomor antrian yang terdiri dari tiga angka dapat dibentuk adalah 6.

Konsep Faktorial:

\[
6 = 3 \times 2 \times 1 = \frac{3 \times 2 \times 1}{1} = \frac{3!}{1!} = \frac{3!}{(3-3)!}
\]

Periksalah kebenaran dari konsep faktorial tersebut! Sertakan alasan Anda.

\[
\text{Jadi sesuai, maka (3-3)! = 0! = 1! berasal 1!}
\]

**Figure 12.** Students reason about the factorial concept in question 1. a

b. Apabila petugas bank menggunakan empat angka yang tersedia, yaitu 1, 2, 3, dan 4.

Banyaknya susunan nomor antrian yang terdiri dari tiga angka dapat dibentuk adalah 24.

Konsep Faktorial:

\[
24 = 4 \times 3 \times 2 \times 1 = \frac{4 \times 3 \times 2 \times 1}{1} = \frac{4!}{1!} = \frac{4!}{(4-3)!}
\]

Periksalah kebenaran dari konsep faktorial tersebut! Sertakan alasan Anda.

\[
\text{Jadi sesuai, Konsep Faktorial, Konsep Semua Konsep Seharusnya}
\]

**Figure 13.** Students reason about the factorial concept in question 1. b

**Figure 12** shows that students check the correctness of the factorial concepts that can be used in finding solutions to problems. Students argue that there is a mistake in the concept that \( \frac{3!}{1!} \) should be \( \frac{3!}{(3-3)!} \) though \( 1! = 0! = 1 \) but has a different meaning. In the activities of reasoning, students’ reflective thinking ability can develop when examining the truth of answers, according to the opinion of Sumarmo (2000); Garrison, Anderson, & Archer (2001); and Rudd (2007).

**Figure 13** shows that the students declared the factorial concept used correctly. Students understand to obtain results 24 can use the factorial concept that is \( \frac{4!}{(4-3)!} \). The next activity the students do is make a question. Sumarmo (2000) stated that one of the activities in developing students’ reflective thinking ability is by making a question. Hu, Xing, & Tu (2018) advise teachers to guide students in asking questions about concepts they are learning.
Figure 14 shows another question from the student regarding the previous problem:

1. If the bank officer uses the first three numbers of 1, 2, and 3. Use another way to determine the number of repeating queue numbers that can be arranged?

2. Use another way to specify multiple queue numbers consisting of 4 digits that can be formed from numbers 1, 2, 3, and 4 without repetition?

Garrison, Anderson, and Archer (2001) suggest that students’ reflective thinking ability can develop when analyzing problems and examining the correctness of answers. The questions raised by students vary according to their thinking skills. According to Herman (2007) in the learning process students are directed to explore problems according to their abilities. Then, students are asked to answer questions they have written.

Figure 15 shows students’ answers based on the questions they have written. Another way students use in solving queue number problems is the rules of multiplication.

1. Answer: $3 \times 2 \times 1 = 3! = 6$
2. Answer: $4 \times 3 \times 2 \times 1 = 4! = 24$

Through this questioning activity, students’ reflective thinking ability develop when answering their own questions. Sumarmo (2000) stated in developing reflective thinking ability can also be done by asking students to answer questions. Rudd (2007) discloses the reflective thinking process occurring during problem solving when reviewing answers and finding the right way of completion.

The teacher guides the students so as not to be mistaken in solving the problem. Zhu, Yu, and Cai, (2018) said teachers should understand the thinking process of students in overcoming the problems they face. Furthermore, the student’s activity is to conclude the concept of permutation that they have understood.
Figure 16 shows that students can conclude the concept of permutation with teacher guidance. The student concludes that the number of permutations $k$ elements is $P(k, k) = k! P_k = k! (k-k)! = k!$. While the number of permutations $k$ elements of the available $n$ elements is $P(n, k) = \frac{n!}{(n-k)!} = $.

Table 4. Students Opinion Criteria

<table>
<thead>
<tr>
<th>Percentage (P)</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>81.00% - 100%</td>
<td>Very Good</td>
</tr>
<tr>
<td>61.00% - 80.00%</td>
<td>Good</td>
</tr>
<tr>
<td>41.00% - 60.00%</td>
<td>Not Good</td>
</tr>
<tr>
<td>20.00% - 40.00%</td>
<td>Very Bad</td>
</tr>
</tbody>
</table>

Figure 16 shows that students can conclude the concept of permutation with teacher guidance. The student concludes that the number of permutations $k$ elements is $P(k, k) = k! P_k = k! (k-k)! = k!$. While the number of permutations $k$ elements of the available $n$ elements is $P(n, k) = \frac{n!}{(n-k)!} = n!$. These teaching materials can help develop students’ reflective thinking ability when examining the correctness of the concepts in solving the problem.

Then, the students give an assessment of the teaching materials through the scale of opinion. Students provide an assessment by choosing numbers between 1 to 5 as desired. The results of students’ assessments are analyzed using the following percentage ($P$) formula:

$$P = \frac{\text{averagevalue}}{\text{maximumvalue}} \times 100\%$$

Student’s opinion criteria according to Ratumanan & Laurens (2003) shown in Table 4. Percentage score of opinion scale of students from three schools are 69.09%, 67.50% and 67.06%. Based on Table 4, the score percentage is between 61.00% - 80.00%. It shows that the students’ opinions on teaching materials were good. Students agreed that teaching materials can encourage active learning, identify the mathematical concepts, work resilient, train creative thinking, critical, and reflective mathematically. Students also agreed that the teaching materials is easy to understand, encourage learning, challenges students to work in different ways, includes relationships between mathematical concepts, and train students deducing from the material being studied.

**Improvement of Mathematical Reflective Thinking**

To know the improvement of students’ reflective thinking ability using teaching materials, students are given reflective thinking ability test consisting of six questions. The test is based on indicators of reflective mathematical thinking. Here’s a matter of reflective mathematical thinking on the material enumeration rules are trained on students based on indicators that have been set.

1. Indicator: distinguish between relevant and irrelevant data about material of multiplication rule.
   Problem: There are seven ceramics each red, white, green, yellow, blue, purple, and black. These seven ceramics will be arranged on the floor. Consider the following questions and answer the questions given: 1) Is the chance for the formation of a green ceramic stack in the center larger than the chances of a ceramic arrangement in the middle instead of red? Give your explanation along with the formula and calculations used! 2) What is the chance of the formation of red and yellow ceramic piles

2. Indicator: analyze and clarify answers about material of product rule; (c) to generalize and analyze generalizations about permutations and combinations of materials.
   Problem: From city P to city C is traversed by 4 types of cars. Harry wanted to go to town B through city C. Then, he returned again to city P through city C without using the same car. Determine the many ways Harry chose car? Determine the many ways Harry chose car! Is the information relevant to determine the
many ways Harry chose a car? What information should be added? Provide an explanation that underlies your answer!

3. Indicator: generalize and analyze generalizations about permutations and combinations of materials.
   Problem: Consider the following permutation and combination relationships.
   \[ \frac{9}{2!P}=72 \quad \text{and} \quad 2!C=72 \]
   \[ \frac{9}{3!P}=336 \quad \text{and} \quad 3!C=336 \]
   \[ \frac{8}{3!P}=840 \quad \text{and} \quad 4!C=840 \]
   \[ \frac{6}{5!P}=720 \quad \text{and} \quad 5!C=720 \]
   It can be generalized that \( nP = rC \). Analyze the generalization and then prove it along with the concept used!

4. Indicator: interpret a case based on the concept of chance.
   Problem: There are seven pieces of ceramics each red, white, green, yellow, blue, purple, and black. These seven pieces of ceramic will be stacked on the floor. Consider the following statements and answer the questions:
   a. Is the chance that the green ceramic arrangement in the center is larger than the chances of the ceramic arrangement in the middle instead of the red? Give your explanation along with the formula and calculations used!
   b. What are the chances of forming a ceramic pile with two red and yellow ceramic pieces together? Give an explanation accompanied by the formula and calculation used!

5. Indicator: check the validity of an argument on the material chance.
   a. In a closed bag, there are 5 yellow marbles and 10 red marbles. Then take 1 marbles randomly. Is it true the odds of red marbles taking 2 times the chances of yellow marbles being captured? Provide an explanation accomplished by the formula used.
   b. Suppose in the bag is taken marbles at once. Is it true that the odds of 2 red marbles and 1 yellow marbles are greater than the chances of getting 2 yellow marbles and 1 red marble? Provide an explanation accompanied by the formula used.

6. Indicator: draw an analogy between the two cases which have similar chance.
   Problem: There are 5 red shirts, 4 white shirts, 3 blue shirts, and 2 green shirts. Choose the right one from the following statement with an explanation.
   The number of ways these clothes are distributed to 12 children if each child gets a different colored shirt, similar to the many ways of composing:
   a. Words formed by letters in MATH.
   b. Books in the shelf consisting of 2 books of mathematics and 1 physics book if there are 5 books of mathematics and 3 books physics.

Before students learn the material of enumeration rules using teaching materials, they are asked to answer the first six questions containing reflective thinking indicators. This activity is called pretest. The average pretest result obtained by 31 students in SMAN 2 is 4.83. The average pretest result obtained as many as 35 students in SMAN 3 is 4.73. Meanwhile, the average pretest result obtained as many as 37 students in SMAN 4 is 4.76. This data indicates that students’ mathematical ability in the three schools is not significantly different.

After the pretest, students learn the material of enumeration rules using teaching materials with a scientific approach with what if not strategy to develop their reflective mathematical thinking skills. The lessons were conducted during 10 classroom meetings. After students understand the material about the enumeration rules, then they are asked to do the test of reflective mathematical thinking skills. This activity is called posttest. The average posttest result obtained by 31 students in SMAN 2 is 11.15. The average posttest result obtained by 35 students in SMAN 4 is 10.51. The average posttest result obtained by 37 students in SMAN 3 is 10.68. The average posttest result obtained by 37 students in SMAN 4 is 10, 51.

To determine the N-gain \( g \) used formula according to Hake (1999) as follows:

\[
< g > = \frac{\text{posttest} - \text{pretest}}{\text{maximumscore} - \text{pretest}}
\]
Maximum score is 14. The N-gain criteria used are shown in Table 5.

Table 6 presented the results of the pretest, posttest, and N-gain students at three schools. Based on Table 6, it is shown that an improvement in the ability of mathematical reflective thinking of the students from the third grade is medium. Students had not been accustomed to solve problems concerning the data to distinguish relevant and irrelevant, analyze the answers, generalize and analyze generalization, interpret a problem, check the truth of a matter, and drew an analogy of two cases of the matter. Through the use of these materials, it can be said that the ability of the students' mathematical reflective thinking had increased quite well.

It is necessary to test statistics using SPSS v.19 software to ensure that there is an improvement in the ability of mathematical reflective thinking before (pretest) and after (posttest) using teaching materials at all three schools. Statistical test results are shown in Table 7. The statistical testing criteria using SPSS v.19 is when Sig. (2-tailed) greater than 0.05 indicates no increase in the ability of mathematical reflective thinking of students before and after using teaching materials, whereas if Sig. (2-tailed) smaller than 0.05 indicates an increase in the ability of mathematical reflective thinking of students before and after using teaching materials. In Table 7 we see that the value of Sig. (2-tailed) at each school is 0.000 smaller than 0.05. This shows that there is a significant improvement in students’ ability of mathematical reflective thinking before and after using teaching materials.

Students in SMAN 2 have improved better than students in SMAN 2 and SMAN 4. This is because SMAN 2 is a school with high criteria. Students in SMAN 3 have better improvement than students in SMAN 4. This is because SMAN 3 is included in schools with medium criteria, while SMAN 4 is included in schools with low criteria. But the difference in the score they have is not very significant, because the three schools are both in a good school group. This instructional material is appropriate for students at high, medium, and low criteria schools in an effort to improve reflective mathematical thinking.

**CONCLUSION**

In developing teaching materials through a scientific approach with what if not strategy, it loaded the activities of observing, trying, reasoning, asking, and concluding. In the activity of asking using what if not strategy was through changing the data, adding the data, changing data with the same question, or changing the question with the same data. In the teaching materials, it is presented some contextual problems in accordance with the experience of students. The design of teaching materials consist of three parts, namely (1) the beginning involves a cover, preface and table of contents; (2) the content section: containing the title of the chapter dealing with the rules of the enumeration as a representation of the contents of the chapter; (3) contextual problems that exist in the students’ life; and (4) Closing: as a reflection and evaluation of the material presented on teaching materials for the next revision.
The teaching materials that had been eligible based on the team of experts and had tested in class XI students in three schools with a high, medium, and low criteria can increase the ability of reflective thinking of the students with medium criteria. Students in schools with high criteria obtained a higher score increase than students in schools with moderate and low criteria. Students in schools with criteria are having a higher score than students in schools with low criteria.

In teaching materials, students were guided to understand the concept of the problem and resolve to ask questions, detailed questions, and prepare questions before, during, and after solving problems. Students were also required to check the correctness of the answer given so their reflective thinking ability would be trained. Through the presentation of these materials, the ability of reflective thinking of the students became well trained and developed.

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