Marzano's New Taxonomy as a framework for investigating student affect

Jeff Irvine
Brock University

ABSTRACT

In 1998 Marzano proposed a taxonomy of learning that integrated three domains or systems: the self system, which involves student motivation; the metacognitive system, involving goal setting and planning; and the cognitive system, required to complete the task at hand. Although extant for 20 years, a paucity of studies have utilized this taxonomy, even though employing Marzano's taxonomy as a framework is particularly appropriate for studies involving student affect. This study provides an exemplar of the use of Marzano's taxonomy as a framework to investigate the impact of a classroom intervention using active and social strategies to enhance student participation. Further, this paper provides suggestions for employing Marzano's taxonomy in other areas for practising teachers, teacher educators, and educational researchers.

Keywords: Marzano's New Taxonomy, engagement, attitude, theory-to-practice.

INTRODUCTION

In 1998, Marzano proposed a taxonomy of learning domains that integrated three *levels* of processing: self (including motivation), metacognitive, and cognitive (Marzano, 1998; Marzano & Kendall, 2007). Marzano's New Taxonomy (MNT) differs from previous taxonomies in that it comprises three interrelated domains whereas the well-known Bloom's (Bloom et al., 1956) taxonomy addressed only the cognitive domain. Revisions to original Bloom (Anderson & Krathwohl, 2001) added metacognition, but only as a passive knowledge domain to be acted upon by the active cognitive domain.[1]

Unlike Bloom, MNT is not a strict hierarchy but instead is two-dimensional, encompassing: "(a) flow of processing and information and (b) level of consciousness required to control execution based on flow of information, and level of consciousness" (Irvine, 2017, p. 2). In top-down fashion, initially the self system engages, making decisions about whether to engage in a new task. This is followed by the metacognitive system that sets goals and strategies. Finally, the cognitive system engages at whatever levels are appropriate to resolve the task. Although Marzano specifies a hierarchy among the three systems, there is no strict hierarchy within the cognitive system.

The three active systems of MNT—self (including motivation), metacognitive, and cognitive—act on three passive knowledge domains: information, mental procedures, and psychomotor procedures, as shown in Figure 1 (Appendix A). In Marzano's model, the self system engages first, making a decision about whether to engage in a new task or continue with the present task. The metacognitive system then engages to identify goals and select strategies. Once these goals and strategies are determined, the cognitive system carries out the cognitive activities required to address the task. While no feedback mechanisms are explicitly included in MNT, the self system continues to monitor the desirability of continuing with the current task compared to other alternatives, and the metacognitive system monitors processes to determine efficacy.

The systems of MNT can be further subdivided by strategy, as shown in Figure 2 (Appendix A): Self-system strategies examine importance, self-efficacy, emotional response, and overall motivation; metacognitive system strategies comprise goal specification, process monitoring, and monitoring for clarity and accuracy; and cognitive system strategies encompass storage and retrieval, analysis, and knowledge utilization processes.

The flow of processing is illustrated in Figure 3 (Appendix A). Marzano also argues that his taxonomy is hierarchical based on levels of consciousness, which increase as one proceeds up the taxonomy. For example, retrieval processes may be automatic, requiring a very low level of consciousness; however, knowledge utilization requires significantly more conscious thought, as does goal setting by the metacognitive system, while self system involvement and decision-making requires even more.

Marzano and Kendall (2008) published *Designing and Assessing Educational Objectives* to help educators apply the taxonomy, although the work's instructional strategies are somewhat basic and need enhancement and augmentation before using them in classroom situations.

Because MNT explicitly addresses self system constructs (such as motivation and emotions), it is appropriate to investigate whether instructional strategies based on this taxonomy can positively influence student attitude and engagement, as well as student achievement in mathematics. Although Marzano and Kendall (2008) outlined ways that MNT could be applied to learning, specifically in designing and assessing educational objectives, scant empirical

[1] For a detailed comparison of MNT and revised Bloom, see Irvine (2017).

research was found. Indeed, no applications of MNT were found for secondary school education or secondary school mathematics education. This is surprising because MNT has the potential to address attitudes and engagement—dimensions of learning that have been identified as critical for student success and well-being (Clarkson, 2013).

REVIEW OF THE LITERATURE

Since Marzano identifies the self system as the first system to engage, followed by the metacognitive system and then the cognitive system, the discussion below reflects Marzano's sequencing in Figure 3 (Appendix A).

Self System: Decision to Engage

Marzano's self system (see Figure 2, Appendix A) includes four subsystems that involve examining: importance, efficacy, emotional response, and overall motivation. Marzano considers motivation to be a superordinate category that combines emotional response, efficacy, and importance across three dimensions of task engagement: (a) students believe the task is sufficiently important, (b) students believe they can successfully complete the task, and (c) students have a positive emotional response in relation to the task (Irvine, 2017).

Marzano's conception of motivation is based on expectancy-value theory (Wigfield & Eccles, 2000), self-efficacy (Bandura, 1997; Pajares, 1997), and, in the case of mathematics, MWB (Clarkson et al., 2010). The following section examines each subsystem of the self system in greater detail.

Examining Importance: Expectancy-Value Theory

Expectancy-value theory suggests that students' task selection, persistence, and achievement are predicated on two things: a belief that they will succeed and the value they assign to the task (Eccles, 1994, 2005, 2009; Eccles & Wigfield, 1995, 2002; Wigfield & Eccles, 2000). In other words, task selection is based on students' perception of: (a) difficulty with the task and (b) the ultimate cost of the task (Eccles & Wigfield, 2002; Eccles et al., 1993; Eccles et al., 1998). The relationship between expectancy-value theory and self-efficacy therefore is that students' perceived ability to complete a task influences their decision to undertake the task. While Ball et al. (2016) note that self-efficacy and expectancy essentially represent disparate theoretical constructs, it can be difficult to distinguish them and their associated factors for research purposes (Irvine, 2018).

The importance component of Marzano's self system is a central concept of expectancy-value theory. Marzano asks students to respond to questions such as: How important is this to you? Why do you think it might be important? Can you provide some reasons why it is important? How logical is your thinking with respect to the importance of this?

Examining Efficacy: Self-Efficacy Theory

The self system's second subsystem is examining efficacy. Self-efficacy (Bandura, 1997; Pajares, 1997) involves individuals' perceptions about their capability to accomplish a task. Regarding mathematics, Middleton and Spanias (1999) identified a relationship between perceived mathematical abilities and intrinsic motivation. S. Ross (2008) found that the impact of self-efficacy was greater than other motivational variables such as goal orientation, intrinsic motivation, or an instrumental versus relational view of instruction. Self-efficacy is domain and task specific (Bandura, 1997). Unfortunately, self-efficacy is very difficult to change, especially in the short term (J. Ross, 2009). Because of its domain- and task-specificity, students' self-efficacy will differ for different subjects (e.g., mathematics vs. English) and for different tasks within each subject. Such factors make self-efficacy a difficult variable to manipulate in the short or intermediate term.

In relation to self-efficacy, Marzano poses questions such as: How good are you at this? How well do you think you can do on this? Can you improve at this? How well can you learn this? How logical is your thinking about your ability to do this?

Examining Emotional Response

The third subsystem of the self system is examining emotional response. This subsystem identifies affective considerations as being important in the overall decision to engage. Regarding emotional response, Marzano asks questions such as: What are your feelings about this? What is the logic underlying these feelings? How reasonable is your thinking? These questions tend to involve affective dimensions, as well as cognitive questions concerning reasonableness. A major component of emotional response is interest, which can be construed as an emotion, as affect, or as a schema (Reeve et al., 2015).

If considered an emotion, "interest exists as a coordinated feeling-purposive-expressive-bodily reaction to an important life event" (Reeve et al., 2015, p. 80). Interest is activated by the opportunity for new information or greater understanding. With regards to feeling, interest involves an alert, positive feeling; in terms of purpose, it creates a motivational urge to explore and to investigate; as an expression, interest widens the eyelids, parts the lips slightly, and notably stills the head; and in terms of bodily changes, it decreases heart rate. Collectively, this coordinated pattern of reactivity facilitates attention, information processing, stimulus comprehension, and learning (Reeve et al., 2015, p. 80).

A second way of viewing interest is as affect or mood. The two dimensions of affect are pleasure/displeasure and activation/deactivation. The goal of instruction is to place the student's affect/mood in the pleasure-activated quadrant, increasing motivation and stimulating engagement. The third way of viewing interest is as an emotion schema, which is "an acquired, process-oriented, highly individualized, and developmentally rich construct in which an emotion is highly intertwined with appraisals, attributions, knowledge, interpretations, and higher-order cognitions such as the self-concept" (Reeve et al., 2015, p. 82). This conceptualization of interest is closely related to identification of value that enables a shift from situational interest to individual interest (see discussion below). Interest is a predictor of engagement and has been shown to replenish motivational and cognitive resources when an interested student is engaged in an activity.

Interest is positively and reciprocally correlated with self-efficacy (Bong et al., 2015), self-concept (Durik et al., 2015), and self-regulation (Sansone et al., 2015), and is also related to valuing of content (Kim et al., 2015). The value that students place on particular content is related to their level of interest for that content. Kim et al. (2015) also demonstrated that interest and value have an impact on engagement and achievement, with self-efficacy acting as a moderator variable. For specific content, it has also been shown that value impacts interest. The greater the value that students place on particular content, the higher the likelihood they will demonstrate interest in that content (Ainley & Ainley, 2015).

The four-phase model of interest development (Hidi & Renninger, 2006) presents a taxonomy of interest development. This model postulates that initial interest is triggered by a situation or topic (triggered situational interest), which may be fleeting, and may be positive or negative. If interest in the situation becomes more sustained (maintained situational interest), this phase is characterized by positive student focus and persistence with the material. If students develop emerging individual interest, they are likely to independently re-engage with the material or classes and ask curiosity questions, building stored knowledge and stored value about the material. Finally, at the well-developed individual interest stage, students willingly re-engage with the content, self-regulating to reframe questions and seek answers. This level is characterized by students' positive feelings towards the material, perseverance through frustration and challenges, and actively seeking feedback on their learning. The four-phase model has abundant research evidence supporting it. The present research study focused on the first two levels of the four-phase model—triggered situational interest and maintained situational interest—with the hope that some students will become sufficiently engaged in the material to proceed to the higher two stages of the model.

Examining Overall Motivation

The last subsystem examines overall motivation. Marzano's concept of overall motivation is a synthesis of importance (expectancy-value), self-efficacy, and emotional response. In this, Marzano is consistent with Hannula's (2006) model of attitude as well as Di Martino and Zan's (2009) three dimensions of attitude. Marzano's treatment recognizes that students may be motivated across all three of these dimensions, or some subset of them. Therefore, the strength of a student's motivation will vary depending on the number of dimensions (importance, self-efficacy, emotional response) that are engaged at a specific point in time. Thus, the level of motivation can and will fluctuate across tasks as well as within tasks. Students may approach a task with high motivation but become disinterested as the task progresses. Alternatively, students may approach a task with low initial motivation but become more motivated while engaging in the task due to increased self-efficacy and confidence that they can successfully accomplish that task.

Questions posed by Marzano in relation to overall motivation include: How interested are you in this? How motivated are you to learn this? How would you explain your level of interest in this? How reasonable is your thinking about your motivation for this?

Instructional strategies that support the self system and motivation include: choice, open questions, connections to real life, RAFT (role, audience, format, topic), journals, placemat, PMI (plus, minus, interesting), and explicit questioning about aspects of motivation.

Motivation and Achievement in Mathematics

There is substantial evidence, although not complete agreement, that motivation in mathematics is positively correlated with mathematics achievement (Hannula, 2006; Koller et al., 2001; Malmivuori, 2006). This correlation is also bidirectional (Koller et al., 2001; Middleton & Spanias, 1999), in that such increases in motivation resulted in increases in achievement, which stimulated further increases in motivation. Further, in a study on streaming students in secondary schools into applied (non-university track) courses, Maharaj (2014) found that "student achievement often has more to do with motivation than innate intelligence" (para. 1). Therefore, when students are unsuccessful in mathematics achievement, the result is decreased motivation, which leads to further low achievement and continued decreases in motivation.

Teachers' beliefs and practices significantly influence students' motivation, particularly in mathematics. For example, Middleton (1995) found that teachers who emphasize content acquisition instead of considering student motivation tend to decrease student motivation in mathematics; when the subject of mathematics is "intrinsically motivating" to some but not all students, "individual differences among students, and the ways in which mathematics education complements these differences, determine … the degree to which mathematics is perceived as motivating" (p. 255). Since motivation impacts mathematics achievement, teachers' attitudes towards mathematics and their choice of instructional strategies are important dimensions of influencing student achievement (Middleton & Spanias, 1999). Student motivation typically decreases over a student's academic career (Middleton & Spanias, 1999). Cotic and Zuljan (2009) found that both student cognition and student affect in mathematics were influenced by instructional strategies that involved problem solving and problem posing.

Because motivation is a superordinate category and therefore very broad, the current study specifically addressed two subcategories of motivation: student attitudes and engagement. The study's duration was approximately 4 weeks. A seminal study by McLeod (1992) found that engagement can be positively influenced in relatively short time periods, while attitude requires longer periods of time to be affected. Therefore, the two subdimensions of motivation were specifically selected as the target of the classroom intervention.

Metacognitive System: Planning and Goal Setting

The second system in MNT is metacognition, defined by Marzano as a separate system, based on four subsystems: goal specification, process monitoring, monitoring clarity, and monitoring accuracy. The positioning of metacognition in MNT as the second system to engage is consistent with earlier work by McCombs and Marzano (1990).

Metacognition has been defined as "the knowledge about and regulation of one's cognitive activities in learning processes" (Veenman et al., 2006, p. 3). In a comparison of MNT and revised Bloom's taxonomy (RBT), Irvine (2017) contrasts the treatment of metacognition in the two taxonomies stemming from Flavell's (1979) division of metacognition into (a) "declarative knowledge about cognition" and (b) self-regulation, involving "control monitoring and regulation of cognitive processes" (Irvine, 2017, p. 5). This dualistic treatment is found in RBT's approach to metacognition (Anderson & Krathwohl, 2001) in comparison to MNT, as RBT places metacognition in the domain of knowledge. While Anderson and Krathwohl (2001) noted some disagreement surrounding metacognition's categorization under declarative knowledge, they maintain that metacognition underpins every cognitive process. Still, such positioning remains inconsistent, as Anderson and Krathwohl label certain aspects of metacognition as "processes" while RBT assign metacognition to the knowledge domain (Irvine, 2017). The stance in RBT is consistent with researchers who treat metacognition as declarative knowledge (Veenman et al., 2006). However, Veenman et al. (2006) point out that metacognition subsumes a number of distinctly different constructs, of which declarative knowledge is only one.

In MNT metacognition is considered separate active system, based on Flavell's (1979) second substrate of self-regulation. Jans and Leclercq (1977) defined metacognition as active judgments that happen throughout learning. Similarly, metacognitive dimensions such as defining learning goals and monitoring progress towards those goals are dimensions of student self-regulation (Nunes et al., 2003). The current study used metacognitive strategies to promote student self-regulation and as autonomy supports for students.

A literature review by Veenman et al. (2006) found studies that support the positioning of metacognition both as domain specific as well as general, and argue such inconsistent positions may reflect the studies' respective grain size. For instance, studies assigning metacognition a "fine grain size" (e.g., for reading strategies) place it in RBT; those involving a "coarser" grain size (e.g., for problem-solving) adopt Marzano's position (Irvine, 2017, p. 5).

Such differing interpretations of metacognition thus have different implications. Because RBT classifies metacognition in the domain of knowledge, metacognition becomes a passive agent that is acted upon; Marzano, in turn, categorizes metacognition on a higher scale in MNT (second only to the self system) as a significant, active domain. Overall, metacognition is a key element in the sequence of processes, bounded by motivation to undertake a task (self system) and the incitement of cognitive processes needed for the task. RBT offers few examples that illustrate the appropriateness of metacognition as declarative knowledge (Anderson & Krathwohl, 2001); MNT, however, recognizes the more active aspects of metacognition, such as setting goals (Irvine, 2017).

Other research evidence supports the positioning of metacognition as an active rather than passive system. Hattie (2009), in his synthesis of more than 800 meta-analyses of factors affecting student achievement, found an effect size of 0.56 for teaching goal-setting strategies, and an effect size of 0.69 from teaching metacognitive strategies. Meijer et al. (2006), when developing their metacognitive taxonomy, also considered metacognition to be an active strategy.

Veenman et al. (2006) point to the importance of teaching metacognitive strategies to enhance student learning, and they identify three research-affirmed principles for successful metacognition instruction: embedding metacognitive instruction in the content matter to ensure connectivity, informing learners about the usefulness of metacognitive activities to make them exert the initial extra effort, and prolonged training to guarantee the smooth and maintained application of metacognitive activity. Veenman et al. refer to these principles as the WWW&H rule: what to do, when, why, and how (p. 9).

Marzano and Kendall (2008) apply a rather simplistic version of these principles in their text concerning design and assessment of educational objectives, in which they limit metacognition to goal setting, process monitoring, and monitoring clarity and accuracy. Their text ignores other metacognitive strategies such as anticipation guides, think aloud, timed retell, plus/minus/interesting (PMI), and ticket to leave. A number of instructional strategies can be tailored to address any of the three systems specified in MNT.

Marzano's dimensions of metacognition (goal specification, process monitoring, monitoring clarity, and monitoring accuracy) omit some important aspects; namely, planning and evaluating. Meijer et al. (2006) identify these aspects as components of the highest level of metacognition. Because metacognition plays an important role in MNT as well as in Marzano's theory of behaviour, this study implemented metacognitive instructional strategies throughout the intervention. Once the metacognitive system has set goals and formulated a plan of action, the cognitive system engages to analyze and perform the required task.

Cognitive System: Performing the Task

The third system of MNT is the cognitive system, with four sublevels: retrieval, comprehension, analysis, and knowledge utilization. Cognition is "the mental action or process of acquiring knowledge and understanding through thought, experience, and the senses"

("Cognition," 2020, para. 1). Cognition has been identified as an important component of all student learning. Therefore the cognitive system was present in all control and treatment lessons of the MNT intervention. The MNT intervention involved modifying or adding to base lessons to explicitly focus on metacognitive and self-system attributes, in addition to the cognitive activities already included in the lessons.

Prior knowledge has been identified as the key cognitive factor in learning mathematics (Milic et al., 2016). Cognitive competence has been shown to be significantly related to mathematics achievement as well as students' self-rating of mathematical ability (Milic et al., 2016). Of particular note is the notion that "cognition is always for action" (Nathan et al., 2016, p. 1692) since the instructional intervention in this study took an active stance with respect to student learning, which may be different than the more passive mathematics lessons that students had experienced up to this point in their academic careers.

MNT identifies four levels within the cognitive system (lowest to highest): retrieval, comprehension, analysis, and knowledge utilization. Marzano states that they are ordered based on the level of processing required. This position is supported by Nokes and Belenky (2011) who claim that knowledge utilization that supports far transfer requires a significantly higher level of processing than other cognitive tasks. The two lower levels (retrieval, comprehension) share similarities with the corresponding levels of RBT. Below is a discussion of the four levels of the cognitive system, beginning with the lowest level, retrieval.

Cognitive System: Retrieval

Retrieval, the lowest level, involves the activation and transfer of knowledge from permanent memory to working memory, usually done without conscious thought. This retrieval may take the form of recognition or recall. Recognition is a simple matching of a prompt or stimulus with information in permanent memory. Recall involves recognition and production of related information. Marzano and Kendall (2007) give the example of selecting a synonym for a word (recognition) contrasted with producing the definition of a word (recall).

Cognitive System: Comprehension

The next level of MNT is comprehension, which consists of two subsystems: integrating and symbolizing. Integrating involves taking knowledge in a microsystem form and producing a macrosystem form for that knowledge. This may involve deleting extraneous information, replacing specific propositions with more generalized ones, or constructing a single proposition to replace a set of less general propositions. Symbolizing involves creating symbolic representations of knowledge, in both linguistic form and imagery. The linguistic form is semantic, while the imagery form involves mental pictures or physical sensations to support cognition. Thus, teachers may frequently employ graphic organizers, which combine both the semantic and imagery forms for a specific knowledge set.

Cognitive System: Analysis

The third level of the cognitive system in MNT is analysis, which has several sublevels: matching, classifying, analyzing errors, generalizing, and specifying (predicting). Matching involves identification of similarities and differences. Matching has been identified by Atkinson et al. (2000) as a critical component of learning from worked examples. Matching is also important in near transfer (Nokes & Belenky, 2011) and in learning through comparison (Rittle-Johnson & Star, 2011). Classifying requires organizing knowledge into meaningful categories. Thus, classifying involves identifying defining characteristics, identifying superordinate and subordinate categories, and justifying these categories. Classifying is used in concept comparison throughout formal education (Rittle-Johnson & Star, 2011). Analyzing errors involves the accuracy, reasonableness, and logic of knowledge. Generalizing is the process of constructing new generalizations or inferences from knowledge that is already known. Rittle-Johnson and Star (2011) point out that generalizing typically involves examination of a range of specific cases in order to identify commonalities and critical features. Finally, specifying (predicting) extends a known generalization to other similar situations, and draws conclusions about these new situations.

Cognitive System: Knowledge Utilization

The highest and most complex level of the cognitive system in MNT is knowledge utilization, which has four sublevels: Decision making, problem solving, experimenting, and investigating. The knowledge utilization level is unique to MNT, and no similar level exists in RBT, although Bloom's synthesis category has elements of some of the subcategories of knowledge utilization, without specifically addressing knowledge utilization. Decision making requires selecting among two or more alternatives. This involves thoughtful generation of alternatives and selecting among them based on sound criteria. Problem solving is a cognitive process directed at achieving a goal when no solution method is obvious to the problem solver. Problem solving has also been described as a situation having an initial undesired situation, a desired end situation, and an obstacle preventing the movement from the initial situation to the end situation (Irvine, 2015).

Thus, problem solving requires identification of obstacles, generating alternative ways to accomplish the goal, evaluating the alternatives, and selecting and executing the optimal alternative. Experimenting requires the generation and testing of hypotheses to understand or explain a phenomenon, typically from primary data collection. Alternatively, investigating relates to generating and testing hypotheses based on secondary or historical data.

Instructional strategies that specifically address the cognitive system include concept attainment, problem posing, timed retell, jigsaw, open questions, explicit questioning, what/so what double entry, decision trees, and flowcharts. The sublevels of knowledge utilization may also serve as significant motivational factors since they have a more active stance for students and involve activities such as investigation and problem solving. All learning involves cognition; however, cognitive strategies may be used as vehicles to stimulate student engagement and interest.

MNT AS A FRAMEWORK FOR INVESTIGATING STUDENT AFFECT: AN EXAMPLE

A mixed methods study (Teddlie & Tashakkori, 2009) examined a set of classroom activities ("the MNT intervention") using MNT as the theoretical framework (Figure 4, Appendix A). This study consisted of student surveys, which were analyzed quantitatively; student post-intervention interviews, analyzed qualitatively; and teacher pre- and post interviews, as well as 20 classroom observations by the researcher. The study involved three classes of

Grade 10 Academic Mathematics at one high school in Ontario, Canada. One class functioned as a control and did not receive the MNT intervention lessons. The two treatment classes received lessons that focused on motivation and metacognition while covering the same content as the control class.

This study was consistent with Veenman et al.'s (2006) three principles in that the metacognitive instruction is embedded in the mathematics unit involved in the study; students are made aware of the metacognitive strategies being used; and metacognitive strategies are embedded throughout the instructional intervention to help foster maintained application of the strategies.

The MNT intervention utilized activities explicitly linked to an MNT sublevel of the self system and the metacognitive system (see Appendix B for details of the linkages). Prior to implementation teachers were given professional learning time to understand the MNT intervention and make suggestions with regard to its implementation. The intervention was based on reform mathematics principles (Moyer et al., 2018). Technology was readily available and utilized where appropriate since the school was a "bring your own device" (BYOD) school.

Method

Teachers delivered all lessons to their own classes. With respect to instruction, treatment classes received lessons with instructional strategies based on the self and metacognitive domains, comprising two classes, and the control class received lessons without a focus on metacognitive and self systems.

Throughout the intervention, the researcher was available as a resource but did not engage in any classroom teaching. The researcher observed approximately 25% of classes over the duration of the study, to support implementation fidelity. Observed classes were assessed for fidelity of implementation against seven criteria identifying the degree to which the lessons reflected the expectations of the MNT intervention: matching given sequencing of topics; inclusion of all elements of the MNT intervention; instructional strategies; responses to student questions; use of manipulatives; use of technology; and responsiveness to student needs. This method of assessing fidelity of implementation was chosen over self-report surveys (O'Donnell, 2008) and was reinforced through data obtained from teacher post-intervention interviews.

The unit on quadratic functions and quadratic equations was identified by the researcher as the most appropriate for the study, based on an analysis of the units in the course as well as comparisons with other secondary mathematics courses. Grade 10 was selected based on the relative homogeneity of prior knowledge, since all students had completed the Grade 9 Academic Mathematics course. In addition, confounding factors such as the transition from Grade 8 to Grade 9, and attending a new (and usually larger) school were minimized since the students had attended the same school in the prior academic year. This unit is one of four units in the course, with the others being linear systems, analytic geometry, and trigonometry. The quadratics unit was the second unit taught in the semester, after linear systems.

Before the treatment, all students (both in treatment classes and the control class) completed surveys on attitude and engagement on computer, smartphone, or tablet. Students completed weekly reflections, while teachers completed daily reflections, with all reflections being done online. Summative assessments occurred twice, with one midway through the unit and the other at the end of the unit, along with a rich assessment task. The summative assessments were created by the teachers involved in the survey. Both summative assessments consisted of written paper-and-pencil tests, scored with marking schemes. The researcher reviewed both assessments prior to their administration. The rich assessment task was designed by the researcher and assessed with a rubric constructed by the teachers involved in the study, with researcher input. After the unit was completed, students again completed online surveys on engagement and attitude.

After completion of the treatment, five student volunteers were identified to participate in audiotaped interviews. Permission forms were given for parental consent. Five students volunteered, and all were interviewed after receiving completed permission forms. All students were assigned pseudonyms when information was reported in the results section. At the conclusion of the study, both teachers participating in the research were interviewed again, using a separate targeted interview guide.

In summary, this study sought to examine whether instruction based on MNT that explicitly targeted dimensions of student metacognition and motivation had positive impacts on student engagement, attitude, and achievement.

Selected Results: Engagement—Quantitative Findings

Both before and after the intervention, students in both the treatment classes and the control class completed online surveys from the *Dimensions of Student Engagement Survey©* (DSES; Reeve, 2013). The DSES is a 39-question Likert scale survey (1=strongly disagree to 5=strongly agree). All surveys were completed online, during class time. The DSES has four subscales: cognitive, behavioural, emotional, and agentic engagement. For this study, the DSES had a Cronbach's α of 0.95.

Pre–Post Comparisons

DSES scores for students in the treatment classes (T_{Total}) are shown in Table 1 (Appendix A). Irvine (2020) reports the intervention's pre–post results as follows:

> Pre- and post measures of engagement for T_{Total} resulted in a statistically significant positive effect size of 0.54 (M=0.527, SD=0.694, $t(45)=5.29$, $p<0.001$); such an effect size is identified as medium (Cohen, 1992) and indicates that the MNT intervention had a positive impact on student engagement. In addition, all four of the engagement subscales of the *DSES* had statistically significant increases. … [Eighty-four percent] of students in T_{Total} showed increases in overall engagement scores (M=0.44, SD=0.816, min=-1.46, max=3.48), self-reported. Overall engagement and all subscales showed statistically significant; the greatest increase occurred for the agentic engagement subscale (Cohen's d=0.73). Agentic engagement is student self-advocacy subdimension, involving students self-identifying interests and preferred learning environments. (p. 19)

Treatment–Control Comparisons

Irvine (2020) did not find a significant difference in student engagement scores in pre–post results for the control class, nor for any of the subscales (Table 1, Appendix A):

> Prior to the MNT intervention, the control class showed a significant differential advantage over T_{Total} (M= 0.34, SD= 0.158, t(66)=2.140, p=0.036). After the MNT intervention, no significant differences were found for the control class (M=-0.24,

> SD=1.024, $t(21)=-1.100$, $p=0.284$). Therefore, no change was found for the control class (not receiving the MNT intervention lessons), while both treatment classes showed statistically significant increases in engagement. (p. 21)

Selected Results: Attitude—Quantitative Findings

All students in both the treatment classes and the control class completed the *Attitude Towards Mathematics Inventory*© (ATMI) both before and after the MNT intervention. The ATMI (Tapia & Marsh, 2005) is a 40-question Likert scale survey (1=strongly disagree to 5=strongly agree) with four subscales: enjoyment, self-confidence, value, and motivation. For this study, the ATMI had a Cronbach's α of 0.978.

Pre–Post Comparisons

ATMI scores for students in the treatment classes (T_{Total}) are shown in Table 2(Appendix A). Irvine (2020) reports the intervention's pre–post results as follows:

> For T_{Total} (treatment students) a statistically significant medium effect size of 0.32 was found (M=0.270, SD=0.0870, t(45)=3.110, $p=0.003$) .Among the subscales, the only statistically significant increase was for the self-confidence subscale. [Seventy-six percent] of students in T_{Total} showed a positive increase in their attitudes towards mathematics (p.23).

Treatment-Control Comparisons

Irvine (2020) found only one significant change for the control class with respect to attitudes.

> For the control class only the self-confidence subscale (M=3.38, SD=0.660, t(21)=-2.608, $p=0.016$) was significant, and found a *negative* change in attitudes toward mathematics [Table 2, Appendix A]. Prior to the MNT intervention, no statistically significant differences in attitudes were found between the control class and T_{Total} (M=-0.007, SD=0.192, $t(66)=-0.037$, $p=0.970$). After the MNT intervention, T_{Total} had a statistically significant increase in attitude scores compared to the control class (M=0.381, SD=0.1372, $t(66)=2.781$, $p=0.007$). (p. 26)

Qualitative Results

A convenience sample of five volunteer students were interviewed after completion of the intervention. The interviews were recorded and the transcribed interviews were analyzed using content analysis (Krippendorff, 2013) as well as constructivist grounded theory (Charmaz, 2014). A sample of five is insufficient to formulate theories; however, the student comments supported the quantitative results. Students indicated that the classroom activities were enjoyable and interesting, and that the students were more engaged in their own learning compared to the regular classroom instructional strategies, which typically consisted of traditional, teacher-centred lessons.

Ethical Considerations

All participants in this study received information letters and returned signed consent forms prior to the commencement of the study. Students who volunteered to be interviewed received and returned an additional, separate consent form prior to the interviews taking place. The university Research Ethics Board approved this study (file #17-096).

DISCUSSION

This study used MNT as its framework, which integrates the affective (self) system, the metacognitive system, and the cognitive system into a coherent whole (Figure 1, Appendix A). This differs from other taxonomies that typically address only one system. For example, RBT (Anderson & Krathwohl, 2001) addresses only the cognitive system and relegates metacognition to a passive information role. Further, Marzano postulates a hierarchical integration of self, metacognitive, and cognitive systems (Figure 3, Appendix A) that emphasizes the sequential nature of system engagement, with primacy being given to the self system, which encompasses student motivation. This is followed by engagement of the metacognitive system, an active system involving goal setting, planning, and monitoring. Finally, the cognitive system engages to address and resolve the task. The study described in this paper demonstrates that MNT is a viable framework for studies involving motivation (self system) and metacognition. While gains in engagement and attitude were observed, the structure of the intervention did not specifically follow Marzano's sequencing of self, metacognitive, and then cognitive systems, since each lesson included both self and metacognitive dimensions. However, the efficacy of such instructional features mitigated the potential to modify student affective dimensions in a positive way.

The MNT framework has the potential to enrich practice in a number of areas. One of the major implications for practice is to raise awareness of the linkages among the three systems of the MNT framework: self (motivation), metacognition, and cognition.

Schools and Teachers

For current mathematics teachers, the framework provides a template to develop units or subunits of mathematics content that provide a specific focus on one or more systems, particularly student motivation and metacognition. Through teachers' awareness of the importance of these dimensions over and above the mathematics content, a more student-focused and student-engaged classroom climate will develop (see, for example, Irvine, in press-a, in press-b). In-service professional learning opportunities need to be provided for practicing teachers to become aware of the MNT framework and its implications.

An additional constraint is that bridging the theory-to-practice gap has frequently been problematic (e.g., Nuthall, 2004). This can be attributed to a number of factors, including time to learn and implement the innovation, ease of implementation, and clear and direct relationships between theory and practice (Farley-Ripple et al., 2018). Frequently, workplace socialization and school culture mitigate against successful implementation (Allen, 2009; Lattimer, 2015). Yet, "educational research will not have any practical value if it does not affect teaching and learning in classrooms, no matter how brilliant the design or how magnificent the result" (Wang et al., 2010, p. 105). By providing teachers with a complete unit instructional intervention, including classroom activities and lesson plans, and by giving teachers "on-demand" professional learning and support when requested, this study mitigates these traditional barriers to theory-practice implementation.

As Irvine (2020) points out, this study's MNT instructional intervention presents a practical model for educational programs seeking to effect changes in student attitudes or engagement, as well as a framework to develop comparable learning units and/or activities. Mathematics teachers may also adopt this study's instructional intervention to develop similar approaches for other units in Grade 10 Academic Mathematics (i.e., trigonometry, analytic geometry, linear systems). The MNT framework could be adopted to plan an entire mathematics course, on a trial basis, and the outcomes could be investigated further (Irvine, 2020).

Teacher Educators

Teacher educators would benefit from knowledge of the MNT framework and its relationships to higher order thinking skills (HOTS) and deep learning. With respect to MNT (Figure 2, Appendix A), HOTS include all the sublevels of the metacognitive system, all sublevels in the cognitive domain of knowledge utilization, and the sublevels "generalizing" and "specifying" of the cognitive domain of analysis. The sublevel "specifying" refers to predicting and may include formulating a hypothesis. Formulating hypotheses will also fall into the knowledge utilization categories of experimenting and investigating. Lower order thinking skills would consist of the lower two levels of MNT and the sublevels of analysis not noted above.

As well, MNT makes explicit the roles of student motivation and metacognition in learning. These concepts could then be included in the curricula for pre-service teachers of mathematics. Since there is now a significant body of research on student attitudes in mathematics (e.g., Pepin & Roesken-Winter, 2015), the MNT framework provides a structure for introducing these concepts into pre-service courses, as well as a viable framework for lesson planning with an emphasis on one or more MNT systems.

Educational Researchers

For educational researchers the MNT framework provides a structure for the construction of studies in one or more of the dimensions of the framework. The study described in this paper illustrates the utility of the MNT framework for investigating affective dimensions. As a first step, a complete Grade 10 Academic Mathematics course could be developed and implemented for a full semester. In doing so, a large body of exemplar materials would be available, and interactions among variables could be investigated.

The framework would also be useful in structuring studies on student cognition in mathematics or in other subject areas, as well as multi-system studies linking two or more MNT systems. The knowledge utilization level of the cognitive system in MNT (problem solving, investigating, decision-making, experimenting) has particularly rich potential for research studies. Having access to a rich and well-developed framework provides researchers with a structure that is understandable to the participants in a study and may be more easily communicable to any non-researchers involved.

CONCLUDING REMARKS

The MNT framework is external to the students' locus of control, providing a framework for teachers and educators to develop instructional strategies to positively influence student behaviours. In 2010, Clarkson et al. proposed the concept of mathematical well-being (MWB), that provides a five-stage taxonomy based on an internal conception of students' locus of control:

1. Being aware of and accepting mathematical activity;
2. Responding positively to mathematical activity;
3. Valuing mathematical activity;
4. Having an integrated and conscious value structure for mathematics; and
5. Being independently competent and competent in mathematical activity. (p. 117)

Each level of Clarkson et al.'s taxonomy describes student behaviours and motivation towards mathematical activity that delineate changes occurring in student beliefs (as indicated by student behaviours) towards the utility and value of mathematical activities. MWB provides an enlightening differentiation among the five levels of students' mathematical beliefs. However, MWB, in its current form, is not an effective framework for developing instructional strategies to support students' progression among the levels. Indeed, Clarkson et al. cite the need for developing and examining effective instructional techniques in their summary of future research required to further develop the MWB construct and move it from theory to practice.

The framework provided by MNT is demonstrably useful for structuring research initiatives. It is also valuable for enriching the knowledge base of in-service teachers, pre-service teachers, and teacher educators. While MNT has been extant for 20 years, the dearth of studies utilizing this framework is surprising. This is particularly true in the area of student affect, which is a burgeoning area of research, especially in the field of mathematics (Hannula, 2015; Pepin & Roesken-Winter, 2015; Schoenfeld, 2015). However, the MNT framework is also a valuable theoretical framework for studies beyond student motivation and affect, as well as studies examining the linkages among motivation, metacognition, and cognition.

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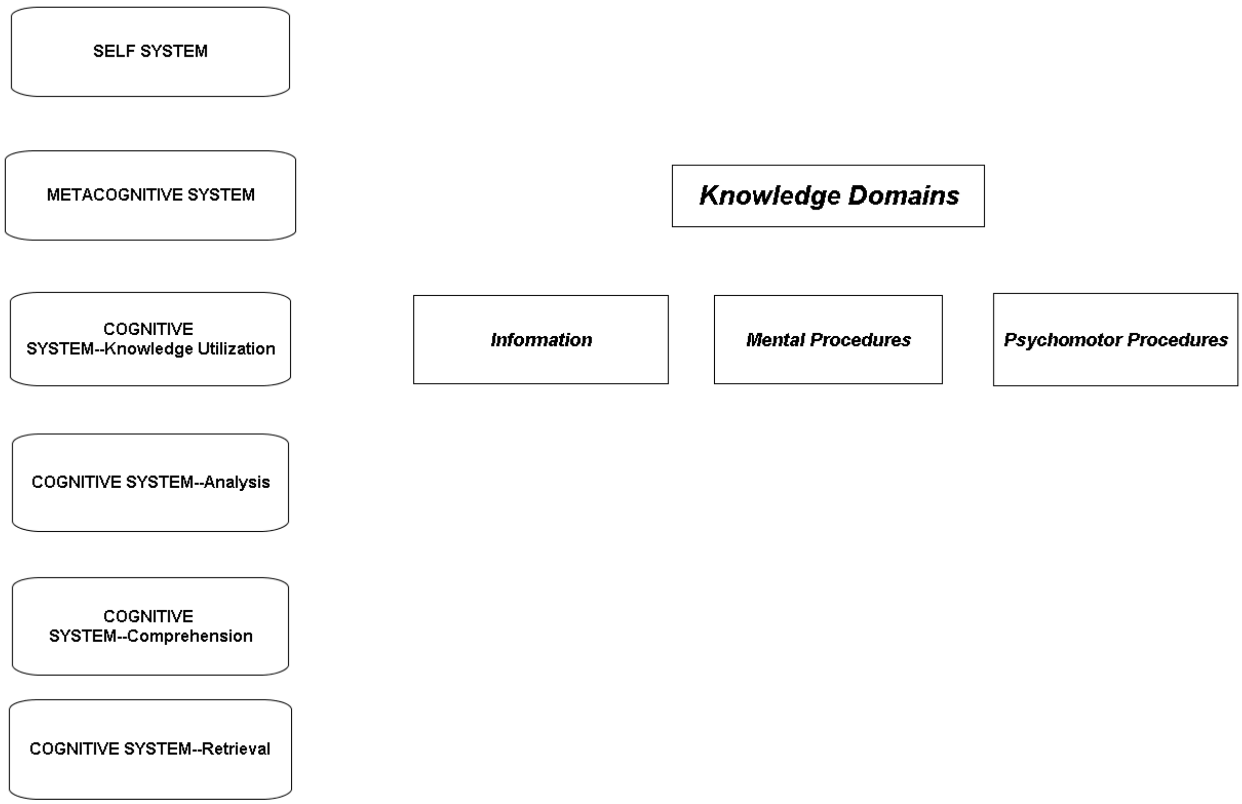
APPENDIX A: FIGURES/TABLES

Figure 1

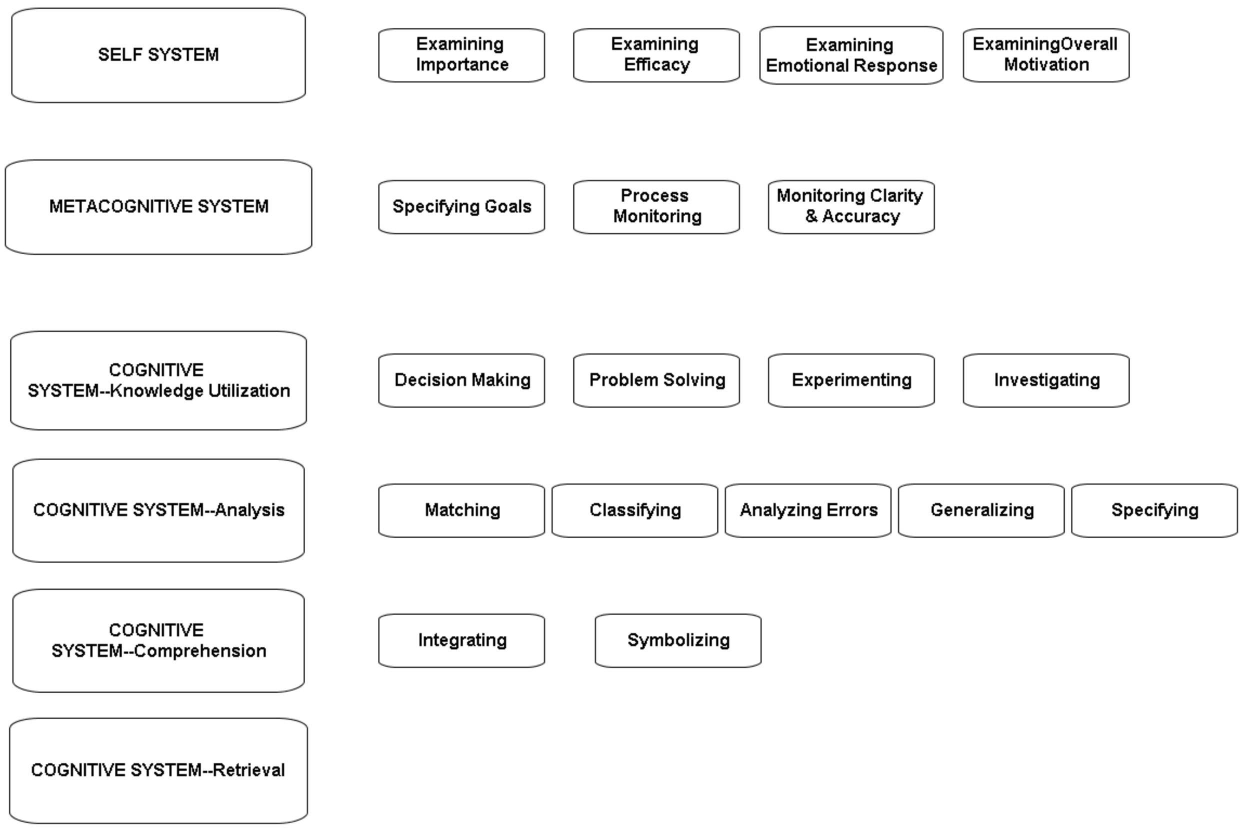
Marzano's New Taxonomy of Educational Objectives



Note. From Marzano, R., & Kendall, J. (2007). *The new taxonomy of educational objectives* (2nd ed.). Corwin Press. Reproduced with publisher's permission.

Figure 2

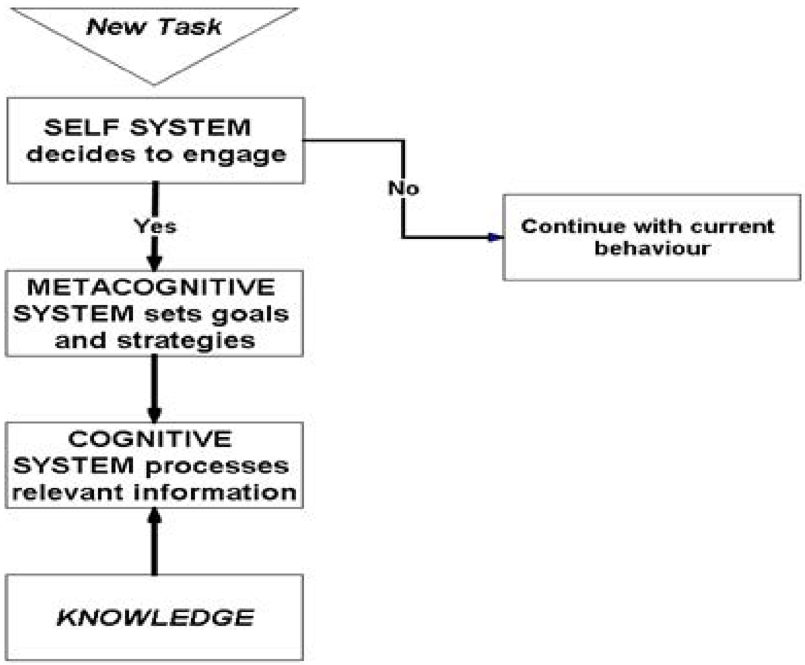
Marzano's New Taxonomy Showing Sublevels



Note. From Marzano, R., & Kendall, J. (2007). *The new taxonomy of educational objectives* (2nd ed.). Corwin Press. Reproduced with publisher's permission.

Figure 3

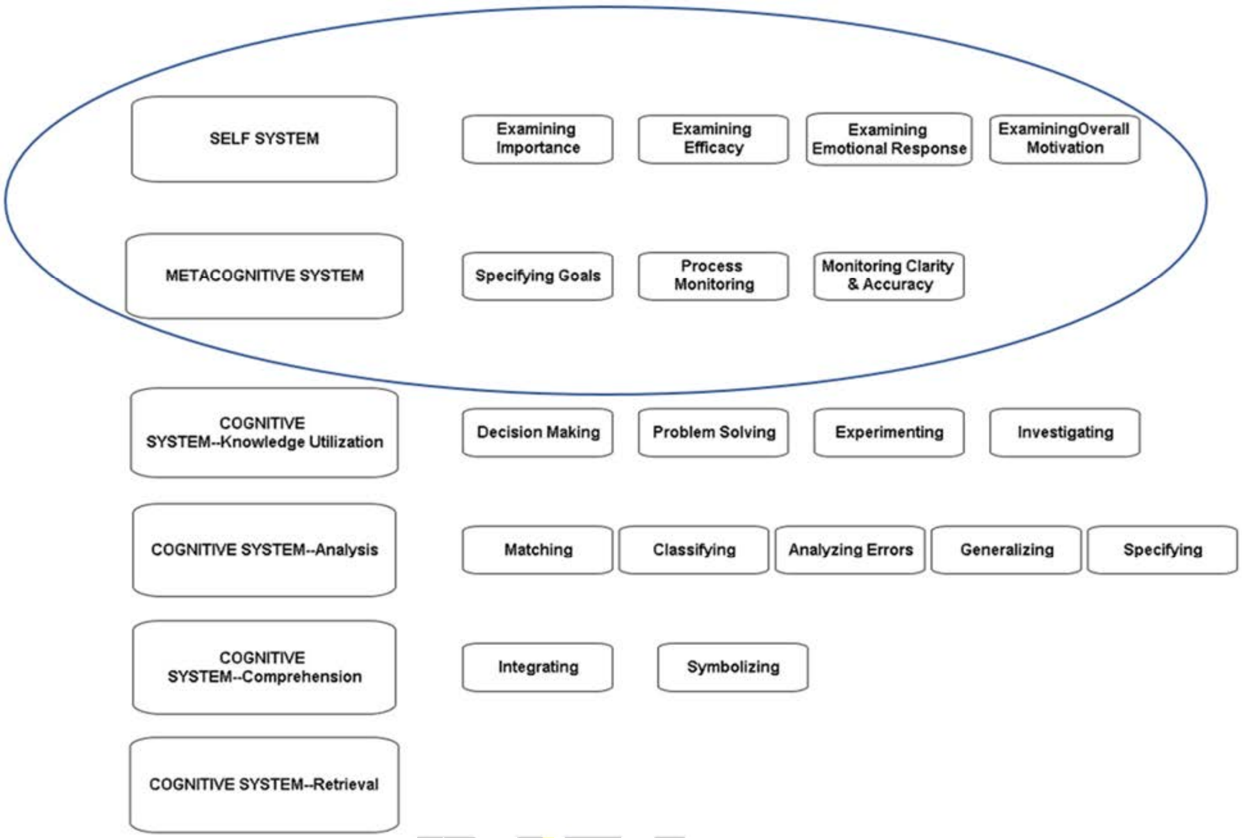
Flow of Processing in Marzano's New Taxonomy



Note. From Marzano, R., & Kendall, J. (2007). *The new taxonomy of educational objectives* (2nd ed.). Corwin Press. Reproduced with publisher's permission.

Figure 4

Affective Dimensions Addressed in Marzano's New Taxonomy Self and Metacognitive Systems



Note. From Irvine, J. (2020). Positively influencing student engagement and attitude in mathematics through an instructional intervention using reform mathematics principles. *Journal of Education and Learning*, *9*(2), 48–75. Reproduced with permission.

Table 1

Dimensions of Student Engagement Scores Post-Intervention

	Treatment			Control	
	n	Sig.	Cohen d	*n*	Sig.
Engagement (full scale)	46	<0.001***	0.54	22	0.284
Emotional	46	<0.001***	0.65	22	0.392
Behavioral	46	0.006**	0.38	22	0.708
Agentic	46	<0.001***	0.73	22	0.069
Cognitive	46	0.005**	0.31	22	0.329

Note. **significant at p=0.01; ***significant at p=0.001.

Table 2

Attitudes Towards Mathematics Inventory Scores Post-Intervention

	Treatment			Control	
Category	*n*	Sig.	Cohen d	*n*	Sig.
Attitude (full scale)	46	0.003**	0.32	22	0.472
Value	46	0.123	–	22	#
Enjoyment	46	0.220	–	22	0.563
Motivation	46	0.358	–	22	0.329
Self-Confidence	46	0.003**	0.33	22	0.016*

Note. **significant at p=0.01; *significant at p=0.05; # t and significance cannot be computed since mean difference is 0.

APPENDIX B: LINKING CLASSROOM INTERVENTION TO MARZANO'S NEW TAXONOMY

Additions to base problems unless indicated as replacement (R)

Expectations	Learning Goals			Metacognition Focus	Self Focus
– determine, through investigation with and without the use of technology, that a quadratic relation of the form $y = ax^2 + bx + c$ (a>0) can be graphically represented as a parabola, and that the table of values yields a constant second difference (***Sample problem:*** Graph the relation $y = x^2 - 4x$ by developing a table of values and plotting points. Observe the shape of the graph. Calculate first and second differences. Repeat for different quadratic relations. Describe your observations and make conclusions, using the appropriate terminology.); – identify the key features of a graph of a parabola (i.e., the equation of the axis of symmetry, the coordinates of the vertex, the y-intercept, the zeros, and the maximum or minimum value), and use the appropriate terminology to describe them;	*Students will learn the basic properties of parabolas and be able to describe these properties using appropriate mathematical language *Students will learn how to apply quadratic regressions to data sets *Students will learn how to use finite differences to determine equations of quadratic functions	Minds On	Carousel • crocodile river • handshake problem • pizza cuts • logpile	• Anticipation Guide	• Likert scale: interest • Groups • Placemat: Tell me everything you know about linear relations
		Action	Whole class • Use the method of finite differences to find equations for each pattern $y = ax + b$ (linear) $y = ax^2 + bx + c$ (quadratic) •	• Think Aloud • What do we want to know; what do we know; how can we connect these	• Likert scale: importance
		Consolidate/Debrief Homework: *Parabolas in Real Life*	• Extend the pattern to negative x's using your equations • Terminology (vertex, max/min, axis of symmetry, intercepts, domain, range)	• Journal entry • How well was your plan achieved? Did it require any modifications ?	• (R) *Connecting Cube Quadratics* • Homework Crossword puzzle terminology + *Parabolas in Real Life*
– collect data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology (e.g., concrete materials, scientific probes, graphing calculators), or from secondary sources (e.g., the Internet, Statistics Canada); graph the data and draw a curve	*Students will learn how to collect and model data that can be represented by a quadratic relation	Minds On	Groups Use technology to graph an example from *Curve Fitting* and discuss appropriate models	• Pairs • What/So What plan solution method	• Graphic organizer • Motivation
		Action	Groups Apply quadratic regressions to obtain equations for data given in *Curve Fitting*	• Groups • What/So What revisit	• Journal entry: How confident are you that you can solve problems involving quadratic relations • Choice

of best fit, if appropriate, with or without the use of technology (***Sample problem:*** Make a 1 m ramp that makes a 15° angle with the floor. Place a can 30 cm up the ramp. Record the time it takes for the can to roll to the bottom. Repeat by placing the can 40 cm, 50 cm, and 60 cm up the ramp, and so on. Graph the data and draw the curve of best fit.);		Consolidate/Debrief	Groups Debrief *Parabolas in Real Life*	• Journal entry: • How well was your plan achieved? Did it require any modifications?	• Emoji scales: • overall motivation • efficacy • interest • importance
– identify, through investigation using technology, the effect on the graph of $y = x^2$ of transformations (i.e., translations, reflections in the *x*-axis, vertical stretches or compressions) by considering separately each parameter *a*, *h*, and *k* [i.e., investigate the effect on the graph of $y = x^2$ of *a*, *h*, and *k* in $y = x^2 + k$, $y = (x - h)^2$, and $y = ax^2$];	*Students will learn the effect on the graph of a quadratic function of modifying a parameter in $y = a(x - h)^2 + \mathrm{k}$,	Minds On	Jigsaw Use technology to investigate the effect of various values of parameters • $y = ax^2$ • $y = -ax^2$ • $y = x^2 + \mathrm{q}$ • $y = (x - p)^2$	• What/So What • Why does each parameter change result in the transformation of the graph	• Choice • Choose group for jigsaw
		Action	Whole class Practice with various $y = a(x - p)^2 + \mathrm{q}$	• Use strategy of example, think-pair-share discussion, worked questions, then repeat	• On a scale of 1 to 10, identify how well you understand the impact of changing parameters
		Consolidate/Debrief	Whole class Summarize transformations Individual Journal entry: summarize the transformations of $y = a(x - p)^2 + \mathrm{q}$ and the impact of parameters	• Journal entry • Given a specific $y = a(x - h)^2 + \mathrm{k}$ describe the steps you would take to graph it	• (R) *Quadratic Aerobics*
-explain the roles of a,h, and k in $y = a(x - h)^2 + \mathrm{k}$, using appropriate terminology to describe the transformations, and identify the vertex and axis of symmetry -sketch, by hand, the graph of $y = a(x - h)^2 + \mathrm{k}$ by applying transformations to the graph of $y = x^2$ [Sample problem: Sketch the graph of $y = -(x - 3)^2 + 4$, and verify using technology	*Students will learn how to sketch and connect graphs and equations $y = a(x - h)^2 + \mathrm{k}$, using appropriate mathematical terminology	Minds On	Groups Matching graphs and equations	• Groups • Placemat • Sketch graphs from given equations and verify accuracy with technology	• Snowball PMI • Role of a, h, k in $y = a(x - h)^2 + \mathrm{k}$
		Action	Individual Sketch graphs for various values of parameters	Groups Matching graphs and equations • Verify using technology	• (R) inside/outside circle: generate equation and explain impact of parameters

-determine the equation in the form $y = a(x-h)^2 + \mathrm{k}$ of a given graph of a parabola			$y = a(x-p)^2 + \mathrm{q}$		
		Consolidate/Debrief	Pairs Think-Pair-Share to construct questions matching graphs, equations, and information (domain, range, intercepts, vertex, axis of symmetry) Inside/Outside Circle to share with others	• Groups • What/So What • Effect of various parameter changes, how to recognize them, how to verify them	• Likert scale: interest
– expand and simplify second-degree polynomial expressions [e.g., $(2x + 5)^2$, $(2x - y)(x + 3y)$], using a variety of tools (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil) and strategies (e.g., patterning);	*Students will learn how to expand and simplify second degree expressions, with and without manipulatives	Minds On	Groups Use algebra tiles for some basic expansions	• Pairs • Order algebra tile pieces to show expansion and vice versa	• Groups • Discussion • Why is this/might this be important to me?
		Action	Whole Class Algebraic expansions Student practice	• Groups • Graffiti • Step by step expansion using algebra tiles, then algebraic expansions	• Journal entry • How useful is this to me?
		Consolidate/Debrief	Individual Inside/outside circle: Student generated examples of expansions Journal entry: Create an example of each type of expansion	• Journal entry • Give examples of expansions in both directions with and without algebra tiles	• Journal entry: my favourite expansion and why
– factor polynomial expressions involving common factors, trinomials, and differences of squares [e.g., $2x^2 + 4x$, $2x - 2y + ax - ay$, $x^2 - x - 6$, $2a^2 + 11a + 5$, $4x^2 - 25$], using a variety of tools (e.g., concrete materials, computer algebra systems, paper and pencil) and strategies (e.g., patterning);	*Students will learn how to factor polynomial expressions *Students will learn how to recognize and factor special cases	Minds On	Groups 1)Use algebra tiles for simple factoring *Using Algebra Tiles* 2) Whole class: Construct a decision tree for factoring	• 1)Verify factorizations by expanding • 2) Matching steps for an example	• 1) Likert scale How fun is algebraic manipulation • 2) Graphic organizer • Emotions
		Action	Whole class 1)Algebraic treatment of trinomials, perfect squares, difference of squares 2)Jigsaw practice	• 1) Recognition • What type of factoring is it • 2) Pairs • Timed retell • Given a card with a factorable expression on it, explain how to factor	• 1) Groups • cartoon placemat • different groups get different types of factoring • 2) Four corners • Different types of factoring at each corner (multiple questions on same type)
		Consolidate/Debrief	1) Individual practice Journal Entry: Explain the relationship between expanding and factoring	• 1) Pairs • One partner factors, the other partner	• 1)Emoji scales: • overall motivation • efficacy • interest • importance

			2) Groups: Write a script to explain to a classmate how to factor (various expressions)	verifies by expanding • 2) Game of *Facto*	• 2) Journal entry • How confident are you that given an expression to factor, you can factor it and verify your answer
– express $y = ax^2 + bx + c$ in the form $y = a(x – h)^2 + k$ by completing the square in situations involving no fractions, using a variety of tools (e.g. concrete materials, diagrams, paper and pencil);	*Students will learn how to complete the square, with and without manipulatives	Minds On	Groups Use algebra tiles to complete *Make a Square*	• Think Aloud • What do we know, what do we want to know, how are they related	• Graphic organizer • Emotions
		Action	Whole class Algebraic complete the square examples	• Pairs • Matching steps for a numerical example	• Groups • Choice apply completing the square to various expressions
		Consolidate/Debrief	Individual Practice completing the square Groups Think Aloud: What information can we obtain by completing the square	• Ticket to leave • Given a numerical example, outline the steps in completing the square	• Ticket to leave • Choose one of three expressions and complete the square
– determine, through investigation, and describe the connection between the factors of a quadratic expression and the x-intercepts (i.e., the zeros) of the graph of the corresponding quadratic relation, expressed in the form $y = a(x – r)(x – s)$;	*Students will learn how to determine the zeros of a quadratic relation and connect them to x-intercepts and equations expressed in the form $y = a(x – r)(x – s)$;	Minds On	Groups Matching zeros from graphs with zeros from algebra	• Groups • Outline a plan to convert to $y = a(x – r)(x – s)$	• Likert scale • How confident are you that you can convert among forms
		Action	Whole class Algebraic intercepts by factoring Intercepts using technology	• Pairs • Matching graphs and equations	• Likert scale • How interesting do you find these conversions
		Consolidate/Debrief	Individual Practice finding intercepts algebraically and writing quadratic functions in the form $y = a(x – r)(x – s)$; Ticket to leave: Given values in $y = a(x-p)^2 + \mathrm{q}$ Rewrite in form $y = a(x – r)(x – s)$;	• Journal entry • How can you be confident that you converted correctly	• Likert scale • How important do you think these conversions are to you
– determine the zeros and the maximum or minimum value of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) or from its	*Students will learn how to determine features of a quadratic relation (x-intercepts, maximum/minimum) from its graph and from its equation	Minds On	Groups Michaela problem Watch Detroit Airport video 1 Brainstorm some questions that you might ask about the	• Groups • Graphic organizer • Complete the Polya organizer	• Graphic organizer • Right angles for interest, efficacy, importance

defining equation (i.e., by applying algebraic techniques);	*Students will learn how to connect algebraic and graphical techniques to real life situations and identify restrictions		fountains, and what information you would need to answer them Then watch video #2		
		Action	Groups Solve real world problems using a variety of techniques	• Groups • Solve, referring to Polya organizer	• Use computer software or graphing calculators to solve problems
		Consolidate/Debrief Homework	Whole class Polya plan Identify restrictions based on real life situation Watch Detroit Airport video #3	• Ticket to leave • Summarize plan, modification, restrictions, how to recognize	• Graphic organizer • motivation
– explore the algebraic development of the quadratic formula (e.g., given the algebraic development, connect the steps to a numerical example; follow a demonstration of the algebraic development [student reproduction of the development of the general case is not required]);	*Students will learn how to develop the quadratic formula and apply it ti find zeros of functions and x-intercepts of quadratic relations	Minds On	Whole class Sample algebraic solution by factoring	• Groups • Graph using technology, estimate zeros • How confident, accurate are zeros	• Groups • Graph using technology and estimate zeros • Discussion • How confident are you that the zeros are correct and accurate
		Action	Whole class Algebraic development of quadratic formula with values for a,b,c Algebraic development of quadratic formula with a,b,c Worked examples	• What/So what • Relate algebraic steps to numerical example	• Likert scale • importance
		Consolidate/Debrief	Individual practice	• Timed retell • Explain the steps for a numerical example	• Graphic organizer • Emotions
– solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over	*Students will learn how to model real life situations using quadratic functions *Students will learn how to solve quadratic models to answer real life questions	Minds On	Groups Dan Meyer basketball video	• Self-select jigsaw • Plan solution	• Open problems • Choice
		Action	Whole class Problems worked examples	• Execute plan • Gallery walk	• Likert scale • Confidence
		Consolidate/Debrief	Groups getthemath.org basketball problem	• Ticket to leave • Explain plan and execution, restrictions	• Likert scale • Importance

what time interval is the height of the ball greater than 3 m?).					
– interpret real and non-real roots of quadratic equations, through investigation using graphing technology, and relate the roots to the *x*-intercepts of the corresponding relations; – sketch or graph a quadratic relation whose equation is given in the form $y = ax^2 + bx + c$, using a variety of methods (e.g., sketching $y = x^2 – 2x – 8$ using intercepts and symmetry; sketching $y = 3x^2 – 12x + 1$ by completing the square and applying transformations; graphing $h = –4.9t^2 + 50t + 1.5$ using technology);	*Students will learn how to interpret real and non-real roots of quadratic equations *Students will learn how to graph quadratic relations using a variety of methods	Minds On	Jigsaw Graph various quadratics with real integer, real decimal, non-real roots using technology	• Anticipation guide v2	• Emoji scales: • overall motivation • efficacy • interest • importance
		Action	Groups Find the zeros algebraically or explain why this is not possible	• Groups • Relate roots to graphs and identify patterns	• Likert scale • interest
		Consolidate/Debrief	Groups Sketch graphs using a variety of techniques (complete the square; factor to find roots; use technology to graph to find roots; table of values Gallery Walk to share solutions	• Journal entry • How can you tell how many real roots a quadratic equation will have?	• Graphic organizer • motivation
– solve quadratic equations that have real roots, using a variety of methods (i.e., factoring, using the quadratic formula, graphing) (***Sample problem:*** Solve $x^2 + 10x + 16 = 0$ by factoring, and verify algebraically. Solve $x^2 + x – 4 = 0$ using the quadratic formula, and verify graphically using technology. Solve $–4.9t^2 + 50t + 1.5 = 0$ by graphing $h = –4.9t^2 + 50t + 1.5$ using technology.).	*Students will learn how to solve quadratic equations that have real roots, using a variety of methods	Minds On	Groups Build a box	• Four corners • Choose solution method	• Groups • Discussion: • Importance • Efficacy • Interest • motivation
		Action	Whole Class Worked examples	• Groups • Solve a problem by at least two different methods	• Likert scale • efficacy
		Consolidate/Debrief	Groups Given a problem solving flowchart, identify the various features and then apply to problems *Problem Solving Flowchart v2*	• Timed retell • Explain at least one method to partner	• Ticket to leave • Choose one problem and present solution
– compare, through investigation using technology, the features of the graph of $y = x^2$ and the graph of $y = 2^x$, and determine the meaning of a negative exponent and of zero as an exponent (e.g., by examining patterns in a table of values for $y = 2^x$; by applying the exponent rules for multiplication and division).	*Students will learn to interpret the meaning of exponents of 0 and exponents of a negative integer *Students will learn how to extend the exponent rules to exponents of 0 or a negative integer	Minds On	Groups placemat Compare the graphs of $y = x^2$ and $y = 2^x$ • domain • range • intercepts • max/min	• Anticipation guide v3	• (R) Groups • *Money Maker*
		Action	Whole class Use Table feature of graphing calculator to develop values for	• Matching • Information to $y = x^2$ *or* $y = 2^x$	• Groups • *Exponent Facto*

			exponents of 0 or negative integers Groups Practice evaluating, exponent laws		
		Consolidate/Debrief	Individual Journal entry: Give several examples of evaluating powers with integer exponents and the exponent laws	• Inside/outside circle • Information to $y = x^2$ *or* $y = 2^x$	• What/So What • List some examples and worked solutions
Review		Minds On	Groups Construct a summary page for quadratic functions		
		Action	Inside/Outside Circle Use Think-Pair-Share to each construct and confirm 3 questions involving quadratic functions Use Inside/Outside Circle to share with classmates	•	• These periods can be inserted as needed for consolidation, skill building, formative assessment. They do not have to be used as full classes, but a total of 75x2=150 minutes may be used in whole or in part.
		Consolidate/Debrief	Groups Solve max/min problems and quadratic equation problems	•	•
Consolidate periods (2)		Recommended: use pairs and groups: gallery walks, jigsaw, inside/outside circle, carousel, think-pair-share, create questions, open questions		• These periods can be inserted as needed for consolidation, skill building, formative assessment. They do not have to be used as full classes, but a total of 75x2=150 minutes may be used in whole or in part.	
RAT		Groups *The painted cube problem*		Groups *The painted cube problem v3*	
Test					
Total 26 classes				• Could include a mixed practice day prior to review/test	