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The Process of Structure Sense of Group Prerequisite Material: A Case in Indonesian Context

Junarti* Universitas Negeri Semarang, INDONESIA
Y.L. Sukestiyarno Universitas Negeri Semarang, INDONESIA
Mulyono Universitas Negeri Semarang, INDONESIA
Nur Karomah Dwidayati Universitas Negeri Semarang, INDONESIA

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Abstract: This study was to support the understanding of the set structure, binary operations, and their properties as a prerequisite of group theory material categorized as 9 structure senses. This study aimed at investigating the process of students' structure sense in recognizing the structure of mathematical properties or objects as a prerequisite of group theory material. A task-based case study by exploring 9 categories of structure senses through three integrated process frameworks in the questionnaire was employed in this study. It involved 26 students who had obtained a prerequisite of group theory material and would take abstract algebra course. The choice of subjects was determined based on the results of the questionnaire, in which it identifies the type of structure sense processes. There were 6 out of 26 subjects were chosen. The 6 subjects consisted of 2 subjects from the first path process, 2 subjects from the second path process, and 2 subjects from the third path process. Then, the 6 subjects were interviewed. The choice of 2 subjects for each path process was because it used a fixed comparison theory. Then, the data were validated by using triangulation methods by comparing the students' work on assignments and questionnaires as well as audio recordings of interviews. The results show the tendency of the process of structure sense was more dominated by students from the second type of path process, in which the subjects still depend on the well-known structure of the properties or mathematical objects in the form of sample questions. The subjects were unable to understand definitions in order to construct structures of properties or mathematical objects.

Keywords: *Structure sense, task-based case study, group theory material, set structure, binary operatio.*

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Introduction

A severe problem of mathematics education, according to Dubinsky et al. (1994), deals with abstract algebra specifically about group theory. The difficulty of students in abstract algebra or algebraic structure is partly due to the lack of understanding of structural concepts in arithmetic (Hoch & Dreyfus, 2004; Linchevski & Livneh, 1999; Van der Klis, 2017). The cause of this difficulty is due to cognitive barriers related to the students' inability to recognize and feel the structure of mathematical properties or objects (Hoch & Dreyfus, 2004; Jupri & Sispiyati, 2017; Linchevski & Livneh, 1999; Novotna et al. 2006, Octac, 2016). In addition, the cognitive obstacle in recognizing structure of conceptual knowledge which involves understanding the facts of basic arithmetic, mathematics ideas, and procedures of solving problems, also becomes important thing to improve the understanding of the structure of abstract algebra (Tekin Sitrava, 2018).

Structural understanding is closely related to structural awareness. Structural awareness supports a significant shift between arithmetic and algebraic thinking (Mason et al., 2009). This structural awareness becomes an essential part of abstract algebra teaching and learning. According to Papic (as cited in Mason et al., 2009), structural awareness that is raised can encourage students to become aware of repetition and growth as structures that can then be used to extend the sequence.

Structural awareness supports the growth of the ability of structure senses. The ability of structure senses is an intuitive ability to symbolic expressions, including skills to interpret, manipulate, manage, and perform symbols in different roles, and is considered the key to success in learning algebra (Jupri & Sispiyati, 2017). Whereas Sugilar et al.

* **Corresponding author:**
Junarti, Universitas Negeri Semarang, Doctoral Program, Mathematics Education Program. ✉ junarti@ikippgribojonegoro.ac.id

(2019) define structure sense as the sensitivity of mathematical structures. Sensitivity to structures (structure senses) affects students' algebraic thinking abilities (Apsari, 2015) and help understand well the operation or nature of algebra (Sugilar et al., 2019).

Students are said to be able to demonstrate the ability of algebraic structure senses in secondary schools if they can: (1) recognize well-known structures in the simplest form, (2) handle compound terms as a single entity and through appropriate substitution recognize the well-known structures in more complicated structures, and (3) able to manipulate the structures correctly (Hoch & Dreyfus, 2006). Furthermore, it is also said that structure sense is a continuation of number sense and symbol sense (Novotna & Hoch, 2008).

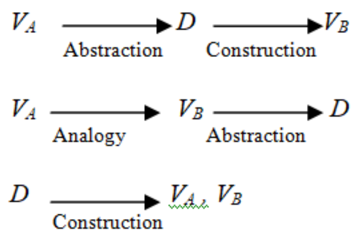
The inability to recognize the structure of the set elements is because the structure of the set elements is not in the form of numbers, so it is difficult to do numerically and the structure cannot be felt (Junarti et al., 2019). The set elements, binary operations, and their properties become a fundamental part of starting learning abstract algebra. When students have not been able to recognize the structures contained in a set, until they can recognize their elements to be manipulated in binary operations; even if they are standard binary operations, they will have difficulty determining the results of their operations. Therefore, students must be able to recognize the simplest form of set elements, as well as compound terms in the set as a single entity of the set element selected to be able to substitute precisely the form of a well-known binary operating structure in a complex form and be able to manipulate it as well as possible.

The introduction of the structure sense above will continue to the introduction of algebraic structures at the university. Algebra at university is like in abstract algebra courses, for example, in group theory material. The prerequisites of group theory material are the understanding of standard binary operations, non-standard binary operations, and binary operations defined in the Cayley's table that are applied to a set. Furthermore, students must be able to recognize the similarities and differences in binary operations presented in the form of formulas or tables, be able to understand the definition of identity elements, be able to link identity elements to determine inverse elements, be able to associate commutative properties in showing associative, identity, inverse characteristics, and to maintain the order both its quality and quantity, as well as being able to spontaneously mention the element of identity. This ability has been developed by Hoch and Dreyfus (2004), Novotna et al. (2006; 2008), and Simpson and Stehlikova (2006) in studying algebra at universities. Oktac (2016) also added that there are difficulties of students in making structural shifts that occur spontaneously, therefore there is a need of appropriate learning strategies to help students understanding Abstract Algebra.

The term 'structure' is widely used and most people think that they do not need to explain what it means (Novotna & Hoch, 2008). However, Junarti et al. (2019) explain the need to feel the existence of a structure that is used in any related structural changes such as the relationship between elements of the set or from the process of abstraction in understanding between concepts. The related structure, for example, between two elements of the set chosen, when going to carry out certain binary operations, then what will be considered in the process of abstracting an appropriate definition in order to be able to link the two elements of the set, students are still overshadowed by numerical forms or similar examples. The process illustrates an error in the process of abstraction or an error in extracting an example in operating two elements numerically. This event shows that there are still obstacles in abstracting definitions to extracting well-known examples into the structure of newly recognized mathematical objects.

Oktac (2016) and Novotna et al. (2006) asserted that at the university level, especially in the case of binary operations in abstract algebra, there are two stages for the development of structure sense based on the two steps identified by Simpson and Stehlikova (2006) which include structure sense in the elements of the set and the idea of binary operations (abbreviated as SSE) and structure sense in binary operating properties (abbreviated as SSP).

A study conducted by Stehlikova (as cited in Novotna et al., 2006), about the arrangement of mathematical knowledge in advanced mathematics, describes a student who knows a specific arithmetic structure as a development process from the dependence of new structures on ordinary arithmetic until independence arises gradually. The stages formed in recognizing the structure sense depend on the students' previously well-known ability. The stages of each individual in the process of understanding are different, as explained by Novotna et al. (2006). There are at least three paths to understand the structure as follows.



The first two paths represent the abstraction of the specific properties of one or more mathematical objects to form the basis of the definition of a new abstract mathematical object (Novotna et al., 2006). Then Oktac (2016) explains that, like the first line, it is extracted from well-known structures to make basic resolutions from which abstract concepts are built in a general context. The second path shows the extraction properties from well-known structures to a generalization and then to a definition. The third is following the concept of construction through logical deduction from its definition (Harel & Tall, as cited in Novotna et al., 2006).

In a well-known structure, illustrated as conventional examples, it usually refers to the opinions of mathematicians and the mathematical community which are part of the curriculum or are local content of the teacher's attention to facilitate students (Oktac, 2016). The examples of a well-well-known structure of mathematical properties or objects given are the student's capital to recognize the structure of new mathematical properties or objects in sets, binary operations, and their properties. The process of recognizing structure senses related to sets, binary operations, and their properties is the focus of this study.

From the elaboration above, the purpose of this study was to investigate the process of students' structure sense in recognizing the structure of mathematical properties or objects as a prerequisite of group theory material by adapting the three paths to understanding the structure of Novotna et al. (2006). This study was a pre-research to recognize the structure sense as a prerequisite of group theory material about the set, binary operations, and their properties.

Methodology

Research Goal

This study aimed at investigating the process of students' structure sense in recognizing the structure of mathematical properties or objects as a prerequisite of group theory material.

Sample and Data Collection

The design of this study is a task-based case study in an exploratory form. The three paths of the process of structure sense from 26 students were explored based on 9 categories of structure sense, which fulfilled the prerequisite of group theory material. The nine categories of structure sense consist of 4 structure senses in the set elements and the idea of binary operations (SSE) and 5 structure senses in binary operating properties (SSP) (Simpson and Stehlikova, 2006). The results of the 26 students' assignments have been classified into 4 SSE and 5 SSP. Meanwhile, to investigate the process of structure sense was done through questionnaire which explores the type of path (path) (Novotna et al., 2006) of students when doing assignments during three meetings.

The subjects in this study were students who would take abstract algebra course. The research subjects were chosen because they have passed the prerequisite courses of abstract algebra and are assumed to be able to understand the set concepts, binary operations, and their properties.

After all subjects were given assignments and questionnaires, the results of assignments and questionnaire were classified based on three frameworks of the three types of paths. There were 2 students from each path. The subjects representing each path were chosen by using a fixed comparison (Creswell, 2017), with the consideration that the nominal number of choices on the path was the most, or at least, the same or close to the same category. If there is no same category, then the minimum nominal number of paths is close to the same. Next, to investigate the process of structure sense, in-depth interviews were conducted with the 6 subjects. All selected subjects were interviewed and the results of the interview were displayed as interview passages to verify research data.

Classification of process structure senses with three frameworks based on the three types of paths is presented in Table 1 as follows.

Table 1. The Classification of Structure Sense Process into Three Paths

Type of Path	Path	The Description of Indicator (adapted from Novotna et. al. as cited in Octac, 2016, p. 311)
Path-1	VA ⟶ D ⟶ VB Abstraction Construction	Students can extract structures of well-known mathematical properties or objects to abstract definitions in constructing new/unfamiliar mathematical structures or objects
Path-2	VA ⟶ VB ⟶ D Analogy Abstraction	Students can extract the structure of well-known properties or mathematical objects through analogy to construct new/unfamiliar mathematical properties or objects, then they will be able to abstract definitions
Path-3	D ⟶ VA , VB Construction	Students can construct structures of well-known and unwell-known mathematical properties or objects through logical deduction from definitions well-known by students

The investigation of the process of structure sense to the set, binary operations, and their properties was done through the assignments given to the students. Each of the 9 categories of structure sense was described and the indicators were formulated. The formulation of indicators includes 3 types of task problem characteristics. The characteristic of the first problem is to use the set written in the form of membership conditions such as "$G = \{x \mid x = a\sqrt{b}$, a is integer and b is natural number} in additionoperation." The second characteristic of the problem is to use a set of numbers with non-standard binary operations such as "$x \otimes y = 5x - 6y$". The third characteristic of the problem is to use finite sets in which its elements consist of symbols such as "$\{N, X, Y, XY\}$" with binary operations presented in the Cayley table (Novotna et al., 2006). The form of the task is a structured essay. The questions of the first, second, and third assignment are presented in the following table.

Table 2. The Presentation of Tasks

<table>
<tr><th>The Characteristics of Task</th><th>Task 1</th><th>Task 2</th><th>Task 3</th></tr>
<tr><td>I</td><td>Let $B = \{x \mid x$ is natural numbers $> 15\}$ in addition operation. Prove whether it meets the nature of closed, associative, identity, and inverse!</td><td>$G = \{x \mid x = a\sqrt{b}$, a is integer and b is natural number} with addition operations. Prove whether it meets closed, associative, identity, and inverse!</td><td>Let $\mathbb{Q}$ be the set of rational numbers, and suppose that $G = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$. Prove whether it meets closed, associative, identity, and inverse!</td></tr>
<tr><td>II</td><td>Consider the set of integers $\mathbb{Z}$. Defined: $a \oplus b = a + b + 2$ for each $a, b \in \mathbb{Z}$. Show that $(\mathbb{Z}, \oplus)$ meets the nature of commutative, associative, identity, and inverse!</td><td>If $(\mathbb{Z}, \oplus) \ni x \oplus y = x + y - 4$, Show that $(\mathbb{Z}, \oplus)$ meets the nature of commutative, associative, identity, and inverse!</td><td>If $\mathbb{R}$ is a set of real numbers with $(\mathbb{R}, \otimes) \ni x \otimes y = 5x - 6y$, Show that $(\mathbb{Z}, \oplus)$ meets the nature of closed, associative, identity, and inverse!</td></tr>
<tr><td>III</td><td>The well-known binary operations "°" are defined as follows:
<table>
<tr><td>°</td><td>a</td><td>b</td><td>c</td></tr>
<tr><td>a</td><td>a</td><td>b</td><td>c</td></tr>
<tr><td>b</td><td>b</td><td>c</td><td>a</td></tr>
<tr><td>c</td><td>c</td><td>a</td><td>b</td></tr>
</table>
Show that $H = \{a, b, c\}$ of the "°" operation meets the nature of the closed, associative, identity, and inverse!</td><td>The well-known binary operations "Δ" are defined as follows:
<table>
<tr><td>Δ</td><td>e</td><td>a</td><td>b</td><td>c</td></tr>
<tr><td>e</td><td>e</td><td>a</td><td>b</td><td>c</td></tr>
<tr><td>a</td><td>a</td><td>b</td><td>c</td><td>e</td></tr>
<tr><td>b</td><td>b</td><td>c</td><td>e</td><td>a</td></tr>
<tr><td>c</td><td>c</td><td>e</td><td>a</td><td>b</td></tr>
</table>
Show that $P = \{e, a, b, c\}$ of the "Δ" operation meets the nature of the closed, associative, identity, and inverse!</td><td>The well-known binary operations "*" are defined in the following table:
<table>
<tr><td>*</td><td>N</td><td>X</td><td>Y</td><td>XY</td></tr>
<tr><td>N</td><td>N</td><td>X</td><td>Y</td><td>XY</td></tr>
<tr><td>X</td><td>X</td><td>N</td><td>XY</td><td>Y</td></tr>
<tr><td>Y</td><td>Y</td><td>XY</td><td>N</td><td>X</td></tr>
<tr><td>XY</td><td>XY</td><td>Y</td><td>X</td><td>N</td></tr>
</table>
Show that $K = \{N, X, Y, XY\}$ of the "*" operation meets the nature of the closed, associative, identity, and inverse!</td></tr>
</table>

Another instrument used in this study was questionnaire. It was consisted of 21 questions/statements of responses about the process experienced when students worked on an integrated task item which were designed by integrating the 9 categories of structure senses. There were three options of answers of the questionnaire which represented the three paths as in Table 1. The three types of paths are described by adapting the Novotna's description (as cited in Oktac, 2016) and the content was validated by 2 experts. The nine structure sense categories for prerequisite of group theory material were adapted from Simpson and Stehlikova (2006), which are presented in Table 3 below.

Table 3. The Descriptions of nine structure sense categories for prerequisite of group theory material

Category	Description
SSE-1	Recognise a binary operation in well-known structures
SSE-2	See elements of the set as objects to be manipulated / understand the closure property.
SSE-3	Recognize a binary operation in “non-familiar” structures
SSE-4	See similarities and differences of the forms of defining the operations (formula, table, other).
SSP-1	Understand ID in terms of its definition (abstractly).
SSP-2	See the relationship between ID and IN: ID → IN
SSP-3	Use one property for another: C → ID, C → IN, C → A.
SSP-4	Keep the quality and order of quantifiers
SSP-5	Apply the knowledge of ID and IN spontaneously

Note: Abbreviations ID, IN, C, A stand for identity, inverse, commutative property, associative property. (Simpson & Stehlikova, 2006)

Analyzing of Data

The technique of data analysis in this research is specifically presented in the following flowchart.

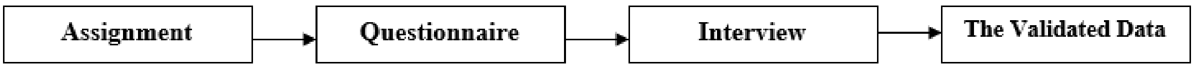


Figure 1. Data Analysis Flowchart

To validate the data, a triangulation technique was done by comparing the results of the assignment and the results of the questionnaire from the subjects. Then the results were used as a basis for the interview. The interview was recorded with audio recording instrument. The interviews were used to determine the extent to which the stages of work on the task that has been integrated into the questionnaire related to the path used when understanding the structure they are experiencing.

All of the data were reduced through selecting, focusing, and classifying similar data into 9 categories of structure sense and three process pathways used by the subjects. Furthermore, the data were simplified and the unnecessary parts were discarded. Then, the data were described and presented. At last, the research findings were verified and the conclusions were made.

Findings / Results

The results of this study are presented in two parts. The first part, Table 4, presents the results of the questionnaire from each student. The second part, Table 5, is about the exposure of 9 categories of structure sense and the type of problem. It presents the results of the process of searching structure sense done by the 6 subjects representing three paths.

1. *The Overall Research Data Presentation*

The following table presents the results of the questionnaire from each student. It deals with the type of paths chosen by the students in structure sense.

Table 4. The Distribution of the Paths Chosen by Students based on Questionnaire

Number of Students who chose Path -1	Number of Students who chose Path -2	Number of Students who chose Path -3	Number of students who wrote in the blank column	Number of students who did not answer	Number of Students who chose Path -1	Number of Students who chose Path -2	Number of Students who chose Path -3	Number of students who wrote in the blank column	Number of students who did not answer
-	-	-	14	7	-	-	-	21	-
-	3	7	9	2	4	-	-	15	2
4	-	-	15	2	-	5	5	7	4
-	13	7	1	-	-	6	4	7	4
-	10	-	4	7	1	8	11	1	-
-	7	-	10	4	2	2	10	-	7
-	3	-	4	14	2	-	-	19	-
1	11	-	9	-	2	14	-	5	-
-	3	-	4	14	-	6	-	2	13
-	1	-	-	20	-	7	-	3	11
4	4	2	4	7	-	2	-	11	8
-	5	4	12	-	-	6	-	5	10
-	4	-	5	12	-	5	-	4	12
9	64	20	91	89	11	61	30	100	71

From the Table 4 above, it was found that the dominant path chosen by students was path-2, which is about 23%. 4% of students chose Path-1, and 9% of students chose Path-3. While the other 64% consists of 35% writing the reason that the steps carried out always repeat the examples in the book repeatedly to be able to do the work of the assignment; and 29% did not write their choices. Path-2 shows the ability of subjects to extract the well-known structure of properties or mathematical objects through analogy to be able to construct structures that are not well-known yet, and then they will be able to abstract definitions.

The next result of this study is about the three paths which were chosen from each category of structure sense and corresponding task questions. It is presented in this Table 5 below.

Table 5. The Distribution of Path Choices in Terms of 9 Categories of Structure Sense and the Types of Task Questions

The Category of Structure Sense	Type of Task	Number of Students who chose Path -1	Number of Students who chose Path -2	Number of Students who chose Path -3	Others	The Category of Structure Sense	Type of Task	Number of Students who chose Path -1	Number of Students who chose Path -2	Number of Students who chose Path -3	Others
SSE-1	Type 1	3	5	4	14	SSP-2	Type 2	-	6	-	20
SSE-2	Type 1	5	5	1	15	SSP-2	Type 3	-	3	3	20
SSE-2	Type 2	4	5	4	13	SSP-3	Type 1	-	3	3	20
SSE-2	Type 3	4	2	4	16	SSP-3	Type 2	-	4	2	20
SSE-3	Type 2	-	5	2	19	SSP-3	Type 3	-	6	3	17
SSE-4	Type 1	1	4	1	20	SSP-4	Type 1	1	6	3	16
SSP-1	Type 1	-	6	2	18	SSP-4	Type 2	-	3	1	22
SSP-1	Type 2	2	3	1	20	SSP-5	Type 1	1	2	4	19
SSP-1	Type 3	1	1	3	21	SSP-5	Type 2	-	2	5	19
SSP-2	Type 1	-	4	1	21	SSP-5	Type 3	-	3	2	20
						Percentage		4%	15%	10%	71%

The Table 5 above shows the choice of the path from each of the 9 categories of structure sense and type of task questions. From the Table 5, it can be seen that there is a tendency for subjects to recognize SSE-1, SSE-2, SSE-4 using paths that are spread over 3 types of paths but are still dominated by path-2. Meanwhile, to recognize SSE-3, the students chose 2 types of paths, and path-2 is still widely chosen by students. Whereas, in recognizing SSP-1, SSP-4, and SSP-5, the students chose 3 types of paths, but still dominated by path-2. To recognize SSP-2 and SSP-3, the students used two paths that are spread into 2 types of paths i.e., path-2 and path-3, and path-2 is still dominating.

From the point of view of the type of task problem characteristics, the second type of problem, which is about recognizing non-standard binary operations, there is a tendency that the subjects were unfamiliar with them. Thus, the introduction of SSP-1, SSP-2, and SSP-3 is more likely to use path-2. Further, from the Table 5, it is proven that there is no one chose path-1 for SSP-1, SSP-2, and SSP-3. This is because the subjects do not recognize the structure of the nature of the identity element, the inverse element, or link the commutative nature to the proof of the element of identity/inverse.

The answers of questionnaire from 6 students which were identified from the type of path chosen in the category of structure sense, and their types of problems are presented in Table 6 below.

Table 6. Subject Initials and Structure Understanding Paths

No.	Subject Initials	The Path of Structure Understanding	9 Categories of Structure Sense
1.	RN	Path-1	SSE-1(1), SSE-2(1), SSE-2(2), SSE-2(3),
2.	DA	Path-1	SSE-1(1), SSE-2(1), SSE-2(2), SSE-2(3),
3.	IS	Path-2	SSE-2 (2), SSE-1(1), SSP-1(1), SSP-2(2), SSP-3(1), SSP-3(2),SSP-3(3), SSP-4 (1), SSP-4(2), SSP-4(3), SSP-5(1), SSP-5(2), SSP-5(3)
4.	DN	Path-2	SSE-2(1), SS-2(2), SSE-2(3),SSE-3(2), SSE-4 (3), SSP-1(1), SSP-1(2), SSP-2(1), SSP-2(2), SSP-2(2), SSP-3(1), SSP-3(2), SSP-4(1), SSP-5(1)
5.	SE	Path-3	SSE-1(1),SSE-2(1), SSE-2(2), SSE-2(3), SSE-3(3), SSP-2(3), SSP-3(1), SSP-3(3), SSP-4(1), SSP-5(1)
6.	SN	Path-3	SSE-2(3), SSE-4(3), SSP-1(1), SSP-2(1), SSP-3(1), SSP-3(3), SSP-4(1), SSP-4(2), SSP-5(1), SSP-5(2), SSP-5(3)

Note: SSE-1 (1) and so on explains the subjects' choice of path-1 for statements categorized as SSE-1 for the first type problem.

The RN and DA subjects had the same answer, choosing path-1 only on 4 questionnaire statements related to the categories of SSE-1(1), SSE-2(1), SSE-2(2), SSE-2(3), while for other statements they chose option (d).

The IS subject chose path-2 for 13 questionnaire statements related to the categories of SSE-1(1), SSE-2 (2), SSP-1(1), SSP-2(2), SSP-3(1), SSP-3(2), SSP-3(3), SSP-4(1), SSP-4(2), SSP-4(3), SSP-5(1), SSP-5(2), SSP-5(3), and for other statements, IS chose option (d).

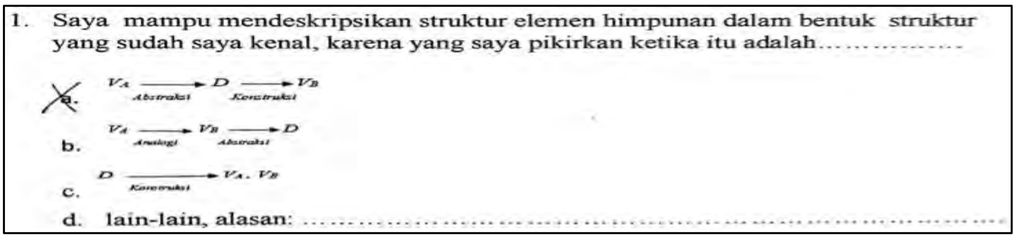
The DN subject chose path-2 for 14 questionnaire statements related to the categories of SSE-2(1), SS-2(2), SSE-2(3),SSE-3(2), SSE-4 (3), SSP-1(1), SSP-1(2), SSP-2(1), SSP-2(2), SSP-2(2), SSP-3(1), SSP-3(2), SSP-4(1), SSP-5(1), and for other statements, DN chose option (d).

The SE subject chose path-3 for 10 questionnaire statements categorized as: SSE-1(1),SSE-2(1), SSE-2(3), SSE-3(3), SSP-2(3), SSP-3(1), SSP-3(3), SSP-4(1), SSP-5(1), and for other statements, SE chose option (d). The SE subject also showed other choices in path-1 for 2 statements in the category of SSE-2(2) and SSP-1(3) and chose path-2 for 2 statements in the category of SSE-3(2) and SSE-4(3). The SN subject chose path-3 for 11 questionnaire statements related to the categories of SSE-2(3), SSE-4(3), SSP-1(1), SSP-2(1), SSP-3(1) , SSP-3(3), SSP-4(1), SSP-4(2), SSP-5(1), SSP-5(2), SSP-5(3), chose path-2 for 10 statements, path-1 for one statement, and for other statements SN chose option (d).

2. *The Presentation of the process of structure sense investigation of each path*

a. *The investigation of Structure Sense Process with Path-1*

The subjects who used path-1, their answers of the questionnaire and the work assignments were in line. The characteristic of the first question is described as follows. This is the screenshot of the RN and DA's answer of the questionnaire:


1. Saya mampu mendeskripsikan struktur elemen himpunan dalam bentuk struktur yang sudah saya kenal, karena yang saya pikirkan ketika itu adalah................

a. $V_A \xrightarrow{\text{Abstraksi}} D \xrightarrow{\text{Konstruksi}} V_B$

b. $V_A \xrightarrow{\text{Analogi}} V_B \xrightarrow{\text{Abstraksi}} D$

c. $D \xrightarrow{\text{Konstruksi}} V_A, V_B$

d. lain-lain, alasan: ....

Figure 2. The screenshot of RN and DA's answer of the questionnaire

Question: I am able to describe the structure of set element in the form of a well-known structure, because I think that ....

RN and DA chose Path-1 to describe the structure of the set elements in the form of a well-known structure because they can extract the well-known structure of the set of real numbers to abstract the definition of the set of real numbers in constructing the well-known structure of the set elements. Thus, the subjects were able to work on assignments that had the same characteristics as the questionnaire statement. The followings are the screenshots of the work assignments by RN subject compared to the DA's.

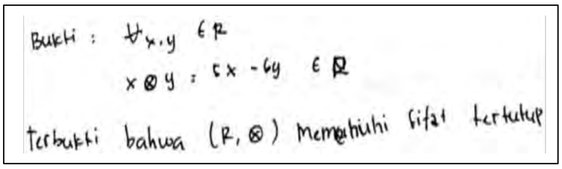


Figure 3. The Screenshot of RN subject's answer

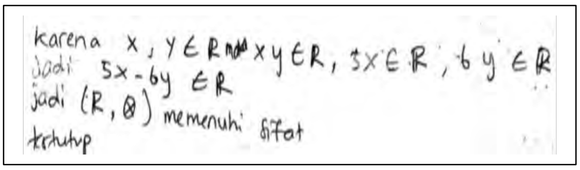


Figure 4. The Screenshot of DA subject's answer

From Figure 3 above, RN's answer shows that the selected element is suitable with the well-known set that is the set of Real numbers. If it is clarified with the answer of the questionnaire by choosing path-1, it shows that when determining the set of elements selected, RN still relies on well-known examples, so that RN can abstract the definition of the set of Real numbers. The following is excerpt of interview with RN:

R : Why did you choose Path-1 when you want to recognize the structure of a set of real number?

RN : Because I have already well-known the set.

R : What do you know from the form of the set?

RN : It is the common set of real numbers, ma'am. So, I immediately understood the definition of the set.

R : If you understand, why did you still use the well-known/familiar structure when working on this task?

RN : Because I am still doubtful about my work, ma'am.

R : Which one?

RN : $5x - 6y$ in R, ma'am.

R : Why do you doubt that the result of $5x - 6y$ are contained in R?

RN : *(Silence and looked confused)* I don't know, ma'am.

Note: R = Researcher

From the interview excerpts with RN, it shows that x and y are elements of a set of real numbers, but when x is multiplied by *5* and *y* is multiplied by *6*, then subtracted, RN is confused. This shows that RN is familiar with SSE-2 and not familiar with SSE-3.

From Figure 4 above, which is about DA's answer, it shows that when it is linked to the SSE-2 category, DA recognized the set element to do manipulation and it shows that the closed nature was in accordance with the well-known set, but the selected element did not represent the randomness set of elements. So, the conclusion is that DA had not fulfilled the correct answer, because the selected real set elements did not represent the randomness of the set of elements of real numbers. The following is the interview excerpt with DA.

R : Why did you choose Path-1 when you want to recognize the structure of a set of real number?

DA : Because at that time I was inspired by previous examples, ma'am.

R : So, is your work correct?

DA : I'm sure it's correct, ma'am.

R : What about the set of real number elements that you choose?

DA : I didn't choose the elements, but I immediately wrote that $5x - 6y$ contained in R, Ma'am.

R : Why are you sure that $5x - 6y$ is contained in R?

DA : Because if *x* is real and *y* is real, then $5x - 6y$ must be real *(DA shows a confident expression)*

R : If you are sure, why didn't you write the elements *x* and *y* selected to represent the randomness of the R element?

DA : Oh yes, ma'am, sorry ma'am, I'm not careful *(DA shows a regretful expression)*

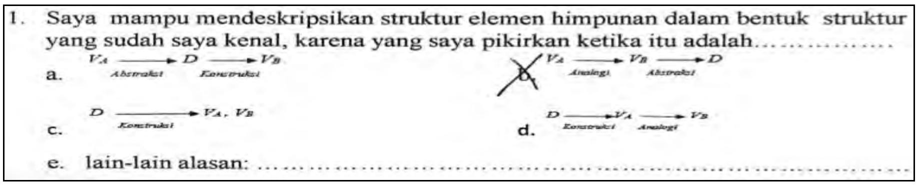
Note: R = Researcher

From the interview excerpt from DA subjects, it shows that DA still did not recognize the structure of the randomness of set element as a guarantee of the randomness result of binary operations. The path that the subject experienced when working on a task was in accordance with the meaning that the subject was unable to construct the structure of the set of elements chosen to support the results of binary operations. This shows that the construction process carried out by DA was still correct and was familiar with SSE-2 and SSE-3.

From the two subjects, RN and DA, they thought that the path chosen was to describe the same process, but when they interpreted their thoughts, their answers were different. If we examine the results of path-1 selection, it illustrates that the subjects of RN and DA still depend on well-known structures. RN and DA still show their dependence on well-known examples or close to the form of the problem they would be working on. RN was familiar with the structure of SSE-2 but was not familiar yet with SSE-3. While DA was already familiar with SSE-2 and SSE-3, however, the selected set of real numbers were not written down for reasons of inaccuracy.

b. The investigation of Structure Sense Process with Path-2

IS and DN chose path-2 when responding to the questionnaire statement related to recognizing the structure of well-known set elements as presented in the answer below.


1. Saya mampu mendeskripsikan struktur elemen himpunan dalam bentuk struktur yang sudah saya kenal, karena yang saya pikirkan ketika itu adalah................

a. $V_A \xrightarrow{\text{Abstraksi}} D \xrightarrow{\text{Konstruksi}} V_B$

b. $V_A \xrightarrow{\text{Analogi}} V_B \xrightarrow{\text{Abstraksi}} D$

c. $D \xrightarrow{\text{Konstruksi}} V_A, V_B$

d. $D \xrightarrow{\text{Konstruksi}} V_A \xrightarrow{\text{Analogi}} V_B$

e. lain-lain alasan: ....

Figure 5. The screenshot of IS and DN's answer of the questionnaire

Question: I am able to describe the structure of set element in the form of a well-known structure, because I think that ………

IS and DN chose path-2 to describe the structure of the set elements in the form of well-known structures. They could extract the structure of a set of real numbers that were already well-known to abstract the definition of a set of real numbers. The results of this questionnaire were clarified with the results of the task work as presented in the figure below.

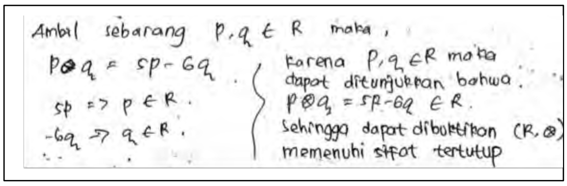

Ambil sebarang p, q ∈ R maka,
p⊗q = 5p − 6q
5p => p ∈ R.
−6q => q ∈ R.
karena p, q ∈ R maka dapat ditunjukkan bahwa p⊗q = 5p − 6q ∈ R.
Sehingga dapat dibuktikan (R, ⊗) memenuhi sifat tertutup

Figure 6. The Screenshot of IS subject's answer

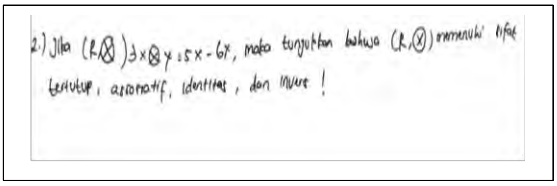

2.) Jika (R,⊗) ∃ x⊗y = 5x − 6y, maka tunjukkan bahwa (R,⊗) memenuhi sifat tertutup, asosiatif, identitas, dan invers!

Figure 7. The Screenshot of DN subject's answer

Figure 6 is about IS' answer. It shows that the elements taken are correct and in accordance with the well-known set. Meanwhile, the questionnaire answer on path-2 shows that IS can extract well-known structural properties or mathematical objects through analogy to construct unfamiliar structural properties or mathematical objects, then IS can abstract definitions. After comparing the two answers of the assignment and the questionnaire, it was revealed that to recognize the structure of the number set elements and to abstract the definition of a set of well-known real numbers, IS still depends on well-known examples or structures as analogy processes. The following is the interview excerpt with IS:

R : Why did you choose Path-2 when you want to recognize the structure of a set of real number?

IS : Because I followed the example that I knew, ma'am.

R : So, do you think that your answer is correct by following the example?

IS : Yes, ma'am. *(with nodding head)*

R : Are you sure?

IS : Yes, sure, ma'am.

R : Why didn't you just use the definition or rules?

IS : I am still confused if I immediately use the definition, ma'am.

R : Why?

IS : I'm not confident, ma'am. *(smiling)*

Note: R = Researcher

From the interview excerpt, it is clarified that the answer of the questionnaire and the assignment was in line. From the IS' work, it can be seen that IS wrote the answer correctly and accurately, because IS wrote the results of the operation with the information of the elements of each variable contained in the set of Real numbers appropriately. The choice of path-2 was in accordance with the answers of the interview, that IS followed the well-known examples to answer the question without thinking about definition. Therefore, it is concluded that IS was able to recognize SSE-2 and SSE-3 using path-2.

Figure 7 is about DN's answer. DN did not provide any answer. This shows that DN was unable to answer the question. After it was compared to the result of the questionnaire, DN still depends on well-known structures or similar examples of assignments. Thus, this subject showed the inability to abstract definitions without adequate examples or in other words the subject did not recognize the structural elements in the set of real numbers. The following is the interview excerpt with DN:

R : Why did you choose Path-2 when you want to recognize the structure of a set of real number?

DN : I always follow the previous examples, ma'am.

R : Why didn't you do your assignment?

DN : Even though I have followed the previous examples, I'm still confused, ma'am.

R : Why? Which part makes you confused?

DN : I'm confused with the results of the operation. It is subtracted but from two different variables.

R : Do you know, what kind of set elements are x and y?

DN : Real numbers, ma'am.

R : If x and y are real numbers, then if the coefficient is multiplied, could you do that?

DN : oh, yes, ma'am. Now, I know, ma'am. *(With optimistic expression)*

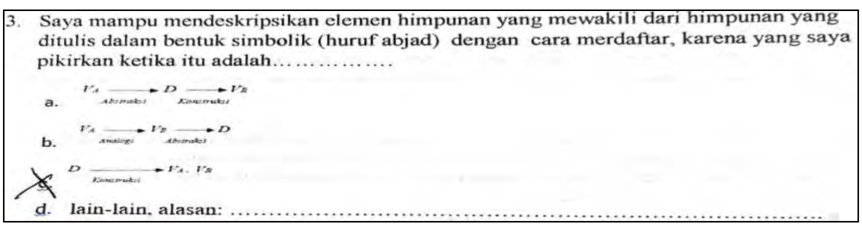
Note: R = Researcher

From the interview excerpt, it is revealed that DN depends on well-known examples without thinking about the rules that had been defined in the binary operations. Without example, DN was not able to do the task at all. The choice of path-2 is in accordance with the answers of the interview which shows that DN was unable to recognize the set element structure (or SSE-2) and unable to recognize non-standard binary operations (or SSE-3).

From the answers of the assignments, both IS and DN chose the same path, but the results of the work were different. IS was able to do the assignment correctly and knew the reason for the answer. Through the process on path-2, IS was able to recognize the structure sense of SSE-2 and SSE-3. On the contrary, DN was unable to do the assignment at all. The results of the questionnaire and the assignment are appropriate. It shows that IS and DN were able to recognize the structure sense through the well-known structure in the examples of appropriate questions. Thus, DN, without examples, was unable to construct unfamiliar structures. DN could not abstract definitions. This could be the cause of the inability to construct the structure senses of SSE-2 and SSE-3.

c. *The investigation of Structure Sense Process with Path-3*

The following figure shows the answer of SE and SN subjects who use path-3 based on the result of the questionnaire.


3. Saya mampu mendeskripsikan elemen himpunan yang mewakili dari himpunan yang ditulis dalam bentuk simbolik (huruf abjad) dengan cara merdaftar, karena yang saya pikirkan ketika itu adalah................

a. $V_A \xrightarrow{\text{Abstraksi}} D \xrightarrow{\text{Konstruksi}} V_B$

b. $V_A \xrightarrow{\text{Analogi}} V_B \xrightarrow{\text{Abstraksi}} D$

c. $D \xrightarrow{\text{Konstruksi}} V_A, V_B$

d. lain-lain, alasan: ....

Figure 8. The screenshot of SE and SN's answer of the questionnaire

Question: I am able to describe the set element which represents the written set in the form of listing symbols (letters), because I think that ....

The choice of questionnaire statement options by SE and SN subjects on the path-3 means that in describing the structure of the set elements, they made a list of symbolic form (alphabetical letters) to abstract definitions. Thus, SE and SN could construct finite element set structures from well-known structures and unfamiliar structures. SE and DN made logical deduction from well-known definitions. The results of the questionnaire were clarified with the results of the work assignment, which can be seen in the following figure.

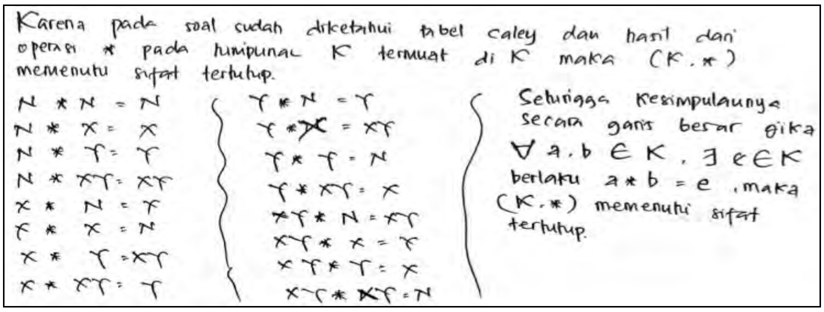


Figure 9. The screenshot of SE's work

Translation: Because the Cayley table has been known, from the question, and the result of * operation of the K set has been contained in K, so (K, *) meets the closed property.

Form the SE's work, it shows that all elements had been operated, but the choice of the set elements at the beginning was not clearly written. The answer was in accordance with the closed nature of all finite elements of $K = \{N, X, Y, XY\}$, but the result of the operation was not clearly written. If it is correlated to the SE's choice of path-3, it means that SE could construct the structure of a limited set of elements with binary operations defined in the *Cayley* table through the definition of its operation. The interpretation of the answers of questionnaire and assignments was verified by the results of interviews with SE. The following is the interview excerpt:

R : Why didn't you choose path-3 when you knew finite set elements?

SE : Because the definition of the set and the definition of the operation are already clear, ma'am.

R : Really?

SE : Yeah, The definition of set elements and binary operations are defined in the Cayley table, ma'am.

R : From your work, why didn't you write down the set elements at the beginning of the work?

SE : I immediately took two set elements to operate, ma'am.

R : Why?

SE : Because I think direct operation has represented the selected element, ma'am.

R : What about the randomness of all elements of the set?

SE : Oh, I forgot it, ma'am. *(smiling)*

Note: R = Researcher

From the answers of the assignments, questionnaires, and interview excerpts, it is revealed that SE, to recognize the structure of the number set elements, was no longer dependent on the example. SE could abstract the definition to recognize the structure of the set elements with binary operations defined in the Cayley table. However, from SE's work, he was unable to fully write the randomness of the set of elements and the results of the binary operations contained in *K* set. So, SE was able to recognize the structure sense of SSE-2 and SSE-4 by using path-3.

The next is the screenshot of SN's answer. It is presented in the following figure.

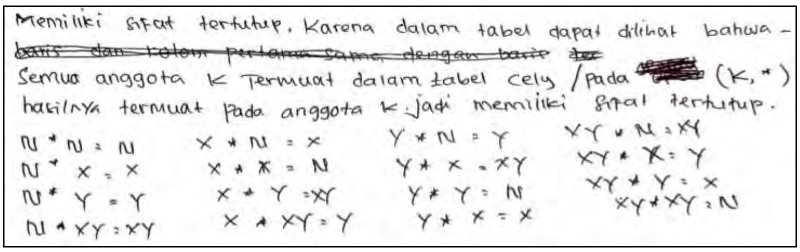


Figure 10. The screenshot of SN's work

Translation: It has closed properties, because it can be seen, from the table, that all members of K are contained in Cayley table at (K,*) and the results are contained in in the members of K, so, it has closed properties.

From Figure 10 above, it is revealed that from SN's work, when it is correlated with the SSE-2 category, it can be inferred that SN could describe the selected set of elements and able to make manipulation to show the closed properties. SN's answer was in accordance with the well-known set, but the element chosen does not represent the randomness of set elements. The interpretation of the questionnaire answer and assignment was verified by the result of interview with SN. The following is the interview excerpt with SN:

R : Why did you choose path-3 when you knew the K set element?

SN : Because I immediately saw the definition of the set and its binary operations, ma'am.

R : Why didn't you write down the element selected for the operation process?

SN : Because I only see the focus of set elements on Cayley table as the definition of binary operations.

R : What about the randomness of all elements of the set and the results of operations?

SN : I have operated all elements of the K set, ma'am.

R : Are you sure that by operating all the elements can guarantee the randomness of all elements of the set and the results of operations?

SN : *(smiling),* I'm not sure, ma'am.

Note: R = Researcher

From the interview excerpt with SN, it is revealed that the results of the assignment were in accordance with the answer of the questionnaire. SN's answer is correct, but the collection of K elements was not written at the beginning of the process. So, it does not represent yet a guarantee of the randomness of set of elements of the well-known set. SN had been able to abstract the definition of binary operations without seeing examples of the same structure. So, the answer of the questionnaire was in accordance with the results of the assignment and the results of the interview. Thus, it can be concluded that the process used by SN in recognizing structural sense was by using path-3, but in fact SN had not been able to write the structure of the set elements and the structure of the results of binary operations. If it is seen from the interview excerpt, SN subject was able to recognize the structure sense of SSE-2 and SSE-4 by using path-3, but the work of the subject's tasks has not been able to describe completely.

From the answers of questionnaire by SE and SN subjects, it is revealed they used the path-3 to recognize the structure senses of SSE-2 and SSE-4 from the characteristics of the third type of task. Thus, SE and SN were able to abstract the definition of K sets and binary operations to recognize the structure of binary set elements and operations presented in the Cayley table. However, SE and SN had not been able to describe the structure of mathematical properties or objects on the set elements and binary operations completely. So, both SE and SN knew the structure sense of SSE-2 and SSE-4 through the process of logical deduction (or using path-3). SE has a complete description of the task answer but SN's answer was incomplete.

Discussion

The subject of RN was already familiar with the structure sense of SSE-2 but not familiar yet with SSE-3 with the introduction process using path-1. With path-1, DA subjects was already familiar with SSE-2 and SSE-3, but the selected set of real numbers were not written down for reasons of inaccuracy. The difficulty of RN and DA subjects using path-1 shows their dependence on well-known structures. The dependence of relevant examples can help the subject in working on the task, but can create cognitive barriers to learning further material. This dependence also influences the conceptual knowledge, involving the understanding of basic arithmetic facts, mathematics ideas, and the procedures of solving problems, which causes the inability to interpret and improve questions understanding in the tasks (Tekin Sitrava, 2018).

The difficulty of RN and DA subjects in using path-1 corresponds to the lack of understanding of structural ideas in arithmetic (Linchevski & Livneh, 1999, Novotna & Hoch, 2008;) which can be shown from RN subjects in carrying out binary operations at $5x - 6y$, RN was unable to write down the results of its operations. It corresponds to the existence of old ideas that are lost in learning new ideas (Novotna et al., 2006), the results of interviews with RN show that RN was unable to remember old ideas of "multiplication of 5x" and "multiplication of 6y" contained in in a set of real numbers, and the results if its subtraction are contained in a set of real numbers. Thus, RN has not been able to recognize the structure of the set elements that are hidden in "$5x - 6y$" (Hoch & Dreyfus, 2004). So, RN still has cognitive barriers (Oktac, 2016) in recognizing SSE-3. Such a situation can interfere the process of constructing or the process of abstracting definitions in non-standard binary operations. The process in path-1 represents the abstraction of specific properties of one or more mathematical objects to form the basis of a new definition of abstract mathematical objects (Novotna et al., 2006).

The choice of IS and DN in using path-2 shows that the steps they took in recognizing unfamiliar structures were firstly by extracting the structure of the well-known set elements through analogy, then by abstracting the definition of the set. IS was able to recognize the structural sense of SSE-2 and SSE-3, while DN did not work on the task at all for the reason that the subject was unable to remember the appropriate examples. DN had a high tendency to depend on previous examples, so DN was unable to describe through work assignments.

The process experienced by DN is not able to make transition as suggested by Simpson and Stehlikova (2006) (as cited in Novotna et al., 2006) i.e., the work starts from the structure of the sample to work abstractly involving the complicated sequence of the following shifts:

1. Looking at the elements in the set as objects where the operation acts;
2. Looking at reciprocal relations between elements in the set as the consequences of operations;
3. Looking at the signs used by the teacher in defining abstract structures as abstraction of objects and operations, and looking at the names of relationships between signs as names for the relationship between objects and operations;
4. Looking at the other sets and operations as examples of general structures and as prototypes of general structures; and
5. Using a formal system of symbols and property definitions to obtain consequences and looking that the property inherent in the theorem is the property of all examples.

Subjects who use path-2 in the process of recognizing structural sense still rely on examples of suitable questions. This is also in accordance with the relationship between the definition and examples such as the explanation of Oktac (2016) that if a student focuses on the important qualities in the definition to meet new examples, then it should only focus on the things that are important, so it can greatly reduce student cognitive tension.

Whereas Simpson and Stehlikova (2006) suggest using two sample structures as in the approach of learning Abstract Algebra, i.e., 1) a concept definition is presented, with the aim that students see examples as examples that differ from general definitions, or 2) through the study of the example they sought to achieve generalization. The first approach implies working at a higher level of abstraction from the start, while the second approach is seen as being more pedagogical in terms of facilitating student understanding (Skemp, 1971, as cited in Oktac, 2016).

The process of recognizing the structure sense of SSE-2 by SE and SN was by using path-3. It means that they can describe the applicability of closed properties to the set {N, X, Y, XY} with binary operations defined in the *Cayley* table (as a category of structure sense SSE-4) although the description of the structure of SSE-2 and SSE-4 is incomplete in the task work. Thus, the subjects had used a logical deductive thought process; this is in accordance with the transition to advanced mathematical thinking by simultaneously in the conceptualization of a person's pre-formed thoughts and new ideas based on definition and deduction (Dubinsky et al., 1997; Novotna et al., 2006; Oktac, 2016).

SE and SN are able to abstract definitions to recognize unfamiliar structures through the process described by Harel and Tall (1989) which occurs when the subject focuses on certain properties of a given object and then considers these properties separately from the original. According to Titova, (2007) structures that are not well recognized, the abstraction process fails. The role of logic in the process of abstraction is very necessary for the process of recognizing mathematical structures (Durand-Guerrier et al., 2015) including the structure sense in group prerequisite material. Wasserman (2014) explains that if a student has a good understanding of arithmetic properties, it helps to make algebraic reasoning. Wasserman (2017) also asserts that the interaction between arithmetic operations and the extension of a set of numbers is often a source of structure in their own set, thus, making this knowledge potentially useful for conveying ideas in mathematical development.

Furthermore, the ability of the subjects, SE and SN, was in line with Harel and Tall (1989). Harel and Tall (1989) provide stages in formal abstraction that lead to mathematical definitions through: (a) all arguments applied to the closed property are also applied to all other examples possessed by the abstracted properties, (b) After the abstraction has been made, by focusing on the abstraction properties and ignoring the others, the abstraction must involve less cognitive tension. These two factors make formal abstraction a powerful tool for experts, because the cognitive reconstruction involved can cause great difficulties for students (Harel & Tall, 1989).

Transitions that occur in the process of recognizing the structure of mathematical properties or objects in the stages of student thinking that use at least three paths (Novotna et al., 2006) are very necessary. Especially, path-1 and path-2 can help reduce the cognitive load of students; it can be a recommendation to recognize the structure of mathematical properties or objects (Oktac, 2016). However, path-3 must still be taught or trained to students to reduce cognitive burden on further mathematical material.

RN subject was familiar with the structure of SSE-2 but did not know SSE-3 yet. While DA was already familiar with SSE-2 and SSE-3, however, the randomness of set of real numbers was not written down for reasons of inaccuracy.

IS, through the process on path-2, was able to recognize the structure senses of SSE-2 and SSE-3. However, DN was unable to do the assignment at all. The results of the questionnaire and the results of the assignment are in line, which reveal that IS and DN, to recognize the structure sense, were through the well-known structure in the examples of appropriate questions. Thus, DN, without examples, is unable to construct unfamiliar structures. DN did not carry out the process of abstraction from definitions at all. This is because of the inability to construct the structure senses of SSE-2 and SSE-3.

Both SE and SN know the structure sense of SSE-2 and SSE-4 by going through the process of logical deduction (or using path-3). SE has a complete description of the task answer but SN's answer is incomplete.

Conclusion

The process of structure sense through path-1 shows a tendency of being able to recognize unfamiliar structures through a well-known structure and assisted with the process of abstracting definition, even though the process of abstraction has not been fully described.

The process of structure sense through path-2 shows a tendency of being able to recognize unfamiliar structures by extracting well-known structures, then abstract definitions. The students recognize the structure sense of SSE-2 and SSE-3 categories with the help of examples of the same structure, but if there is no similar example, the students will experience cognitive barriers. The case of such students shows incompleteness in recognizing well-known structures. The failure of students to recognize the same structure can be attributed to the lack of mental structures needed and the construction of objects through the encapsulation mechanism cannot be achieved properly (Arnon et al., 2014).

The process of structure sense through path-3 shows a tendency to recognize both the well-known and unfamiliar structures by means of the process of abstracting definition. The tendency of subjects to abstract definitions results in being able to connect the components of the definition by focusing on the properties/important things to meet the new structure. This can help reduce the cognitive tension of students to learn further mathematical material.

Suggestions

The limitation of this study was the students still depend on similar examples in recognizing structure sense of the unknown mathematics properties. Since there are several more stages to recognize the structure sense in the structure of mathematical properties/objects, it is suggested that in order to help reduce the cognitive burden of students, the logical deductive way should be the main thing to be introduced to students in preparation for learning further mathematical material.

References

Arnon, I., Cottrill, J., Dubinsky, E., Oktac, A., Roa Fuentes, S., Trigueros, M., & Weller, K. (2014). *APOS Theory - A Framework for Research and Curriculum Development in Mathematics Education*. Springer.

Apsari, S.A. (2015). Pre-algebra learning by using Visualized Pattern Tracing to develop algebraic thinking ability of grade V students in primary schools. In N. M. Pujani, I. M. Kirna, I. G. N. A. Suryaputra, D. M. Citrawathi & I. G. Suwekwn (Eds), *Proceedings of National Seminar: FMIPA UNDIKSHA V* (pp. 199-204). Universitas Pendidikan Ganesha.

Creswell, J. W. (2017). *Research design (Qualitative, Quantitative, and Mixed Method Approaches)*. Learning Library.

Dubinsky, E., Dautermann, J., Leron, U., & Zazkis, R. (1994). On learning fundamental concepts of group theory. *Educational Studies in Mathematics*, *27*(3), 267-305.

Dubinsky, E., Dautermann, J., Leron, U., & Zazkis, R. (1997). A reaction to Burn's "What are the fundamental concepts of group theory? *Educational Studies in Mathematics*, *34*(3), 249–353.

Durand-Guerrier, V., Hausberger, T., & Spitalas, C. (2015). D efinitions et exemples: pr erequis pour l'apprentissage de l'algebre modern [Definitions and examples: prerequisite for learning modern algebra]. *Annals of Didactics and Cognitive Sciences/ Annales de Didactique et de Sciences Cognitives*, *20*(1), 101-148.

Harel, G., & Tall, D. (1989). The general, the abstract, and the generic in advanced Mathematics. *For the Learning of Mathematics*, *11*(1), 38–42.

Hoch M., & Dreyfus T. (2004). Sense in high school algebra structure: The effect of brackets. In M. J. Honies & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (pp. 49-56). Bergen University College.

Hoch, M., & Dreyfus, T. (2005). Students' difficulties with applying a familiar formula in an unfamiliar context. In H. L. Chick, & Vincent, J. L. (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (pp. 145-152). PME.

Junarti, Sukestiyarno, Y. L., Mulyono, & Dwidayati, N. K. (2019). The profile of structure sense in abstract algebra instruction in an Indonesian Mathematics education. *European Journal of Educational Research*, *8*(4), 1081-1091. https://doi.org/10.12973/eu-jer.8.4.1081

Jupri, Al & Sispiyati, Dan. (2017). Expert strategies in solving algebraic structure sense problems: The case of quadratic equations. *Journal of Physics: Conference Series*, *812*, 012093. https://doi.org/10.1088/1742-6596/812/1/012093.

Linchevski, L., & Livneh, D. (1999). Structure sense: The relationship between algebraic and numerical contexts. *Educational Studies in Mathematics*, *40*(2), 173–196.

Mason T., Stephens M., & Watson A. (2009). Appreciating mathematical structure for all. *Mathematics Education Research Journal*. *21*(2), 10-32.

Novotna, J., Stehlikova, N., & Hoch, M. (2006). Structure sense for university algebra. In J. Novotna, H. Moraova, M. Kratka, & N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (pp. 249-256). PME.

Novotna, J., & Hoch, M. (2008). How structure sense for algebraic expressions or equations is related to structure sense for abstract algebra. *Mathematics Education Research Journal, 2*(2), 93-104.

Oktac, A. (2016). Abstract algebra learning: Mental structures, definitions, examples, proofs and structure senses. *Annals of Didactics and Cognitive Sciences/ Annales de Didactique et de Sciences Cognitives*, *21*(1), 297 - 316.

Simpson, A., & Stehlikova, N. (2006). Apprehending mathematical structure: a case study of coming to understand a commutative ring. *Educational Studies in Mathematics*, *61*(3), 347-371.

Skemp, R. R. (1971). *The psychology of learning mathematics*. Penguin.

Sugilar, H. Kariadinata, R., & Sobarningsih, N. 2019. Symbol spectrum and sense of mathematical structure of Madrasah Tsanawiyah students. *Journal of Mathematics Education*, 4 (1), 37-48.

Tekin Sitrava, R. (2018). Prospective Mathematics teachers' knowledge of basic algorithms. *European Journal of Educational Research*, 7(3), 513-528. https://doi: 10.12973/eu-jer.7.3.513

Titova, U.S. (2007). *Understanding abstract algebra concepts* [Doctoral dissertation, University of New Hampshire]. University of New Hampshire Digital Archive. https://scholars.unh.edu/dissertation/362

Van der Klis, W.B. (2017). *Brackets and the effect on algebraic expertise* (Unpublished master's thesis). Utrecht University.

Wasserman, N. H. (2014). Introducing algebraic structures through solving equations: Vertical content knowledge for K-12 mathematics teachers*. PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies*, *24*(3), 191-214. https://doi.org/10.1080/10511970.2013.857374

Wasserman, N. H. (2017). Exploring how to understandings from abstract algebra can influence the teaching of structure in early algebra. *Mathematics Teacher Education and Development*, *19*(2), 81 - 103.