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FOR WHAT PURPOSE DO THE STUDENT TEACHERS USE DGS? A QUALITATIVE STUDY ON THE CASE OF CONTINUITY

Research Article

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Abstract

Although numerous studies have investigated how technology affects academic achievement, very few have focused on the purpose of the use of technology in mathematics education. This current study examines how student teachers (STs) benefit from GeoGebra as one of the Dynamic Geometry Software (DGS) while solving continuity problems. In order to have deeper insights and a better understanding of the intended purposes, a case study research design was adopted for this study. Participants in the study were seven mathematics STs. Six open-ended problems were used to collect data. Three themes were found to be relevant for understanding how STs use DGS in a problem solving process: visualize, verify, and calculate. The paper also shows the potential misconceptions of the STs.

Keywords: Dynamic Geometry Software, GeoGebra, problem solving, aim of use, continuity

1. Introduction

In today's warp-speed world, the amount of information is rapidly increasing day by day. In the direction of this change, curricula are constantly revising. In new curricula and education policies, it is recommended to integrate the technology into real classrooms and teacher education institutions. On the other hand, integration of technology into mathematics classes is a key role in the success of education programs (Baki, 2002; Lee & Hollebrands, 2008). The idea of technology can transform students' classrooms into mathematical labs where they can discover mathematical relations and make assumptions, generalizations, and experimental corrections lies behind the integration of technology into mathematics classes. According to the National Council of Teachers of Mathematics [NCTM] (2000) standards, students can develop deeper understanding of mathematics in a technology-enriched classroom environment.

Similarly, various mathematics education researchers have pointed out that technology-enriched learning environments have positive effects on students' mathematical abilities (e.g., Choi, 2010; Tall, 2002; Wenglinsky, 2000). Especially the developments and changes experienced in the field of information technologies for the last two decades have brought important opportunities for this learning culture. The emergence of DGS in mathematics education can be considered as one of the milestones. Research has shown that DGS plays an important role in problem solving, logical assumption-making, and improving generalization skills (Baki, 2006; Camargo, Samper, & Perry, 2007; Healy & Hoyles, & Laborde, 2001; Mariotti, 2000; Marrades & Guitierrez, 2000). DGS contributes to the problem-solving strategies by making geometric measurements, establishing dynamic constructions, examining different situations with the dragging option, and reaching generalities (Fey, Hollenbeck, & Wray, 2010; Olive et al., 2010). For this reason, the integration of DGS into classes helps students in the discovery of relationships among mathematical concepts and solving mathematical problems. Some researchers highlighted that, besides the development of these skills, DGS can make important contributions to students' transition from experimental evidences to formal proofs (Camargo, Samper, & Perry, 2007; Marrades & Gutiérrez, 2000). Hoyles and Jones (1998) have examined the influence of the Cabri Geometry software on the proof skills of students who were using DGS. As a result of the study, it has been determined that students can draw conclusions about the characteristics of geometric shapes after they have drawn the required geometrical figures, and made connections among them. In addition, students were able to explain the accuracy of these relationships with the help of the DGS. In similar studies, the potential of DGS has been revealed by how it increases proof ability through the opportunity to test and validate students’ ideas by giving immediate feedback (Camargo et al., 2007; Mariotti, 2000; Marrades & Guitierrez, 2000).

Hohenwarter and Fuchs (2004), encoders of GeoGebra software, indicated that the DGS presents opportunities to the students, such as in establishing assumptions, making calculations, testing hypotheses, and making generalizations, or of using it as an instructional tool by allowing them to experience the abstraction process. Güven (2006) stated that the function of DGS is not only to justify a mathematical situation’s relevance, but also to establish a bridge between the trial and error strategy and finding formal proofs. Similarly, Baki (2006) emphasized that the DGS makes a contribution to students’ understanding of a particular situation and discovery of different features, and through giving opposite examples and ideas of how to prove by using its features such as dragging, creating tables, and providing feedback. Considering all of these claims, we can conclude that the DGS is a tool which contributes to understanding mathematical ideas, making a plan and implementing it, creating solutions and reaching generalization as well as providing problem solving.

1.1. Technology and Problem Solving

Technology in general, and the DGS in particular, affect students’ problem solving process and directs them to use different problem solving strategies (Healy et al., 2001). This can be explained in the context of Schoenfeld's factors affecting problem solving as follow. Schoenfeld (1992) identified the factors that affect problem-solving skills in four components: resources, heuristics, control, and belief systems. Resources are the mathematical knowledge that the individual has (phenomena, data, facts and definitions). Heuristics are the strategies that an individual uses to solve the problem, such as examining special cases. Control is the process of deciding whether or not these solutions are correct when obtaining intermediate solutions while solving problems. Belief system is the attitudes and beliefs that the individual has toward a given problem. Although each of these steps, as described by Schoenfeld (1992), has a separate design, the heuristics step is becoming more prominent in the problem solving process. This stage offers students the opportunity to apply different problem solving strategies by enriching the learning environment via DGS features such as calculation, processing, guessing, and graphic drawing.

Although Schoenfeld (1992) expressed these four components when the use of technology in education was not as widespread as it is today, it can be said that the heuristics have developed and changed by means of the use of technology in the problem solving process. Particularly the use of visual and experimental approaches in problem solving can be seen as an opportunity based on mathematical discussions. Technological tools also help students to observe, manipulate, predict, test and explain an observed expression, and measure geometric shapes. It also provides opportunities such as experimenting, observing mathematical relationships, controlling predictions, proving, correlating the results of paper-pencil-solved problems, visualizing, providing effective feedback, and makes learners feel the need to prove mathematical inferences. In this context, DGS has a considerable prominence in developing students' problem-solving strategies in teaching mathematics and geometry (Barrera-Mora & Reyes-Rodriguez, 2013; Kuzle, 2012).

1.2. Continuity

The axiomatic structure of mathematics suggests that continuity is an important concept for topology that forms a large part of advanced mathematics, and helps us to understand the basics of mathematics, such as derivative and integration (Cornu, 1991). In addition to formal descriptions of this concept, there are concept images (Çekmez, 2013) that are originated informally within individuals' minds. These mental elements will not necessarily be in harmony with the nature of the concept (Tall & Vinner, 1981), since the degree of harmonisation will vary with time, with various variables such as experience, observations, and classroom practices (Rösken & Rolka, 2007). That is, the concept images possessed may be inconsistent with the concept itself (see Table 1). Informal descriptions which are far from the formal definition can provide the basis for misconceptions (Bezuidenhout, 2001; Tall & Vinner, 1981). One of the sources of informal descriptions stems from everyday uses of the given concept. According to Gough (2007), although mathematics is technically a language, it is not natural because it is a human product such as other languages. However, this unnatural language is always taught or learned via a natural language. In this regard, a daily life type of use of a concept can lead to issues in learning its mathematical form (Gough, 2007). One of these definitions is the concept of continuity we use both in daily life and in mathematics. In street/everyday? language, the term of continuity is mostly used instead of "without interruption" or "non-stop". The most common misconception about continuity in relation to this definition is that it is non-stop, with no gap, no fracture, or no disconnection. Students with this misconception say that the function is discontinuous by looking at the whole of the graph when a graphical form of a function is given and they are asked to determine continuity at any point (Baştürk & Dönmez, 2011; Tall & Vinner, 1981).

Table 1. *The formal definition of continuity (quoted in Tall, 2002, p.116)*

Given a statement which is a three-level quantification, such as the definition of continuity of F at x_0, $\forall\varepsilon > 0, \exists\delta > 0 \ni \forall x \in \text{domain}(F), \lvert x - x_0\rvert \leq \delta \Rightarrow \lvert F(x) - F(x_0)\rvert \leq \varepsilon$

The belief that each equation should be perceived as a function or that there must be a rule for every function is often a misconception of the function (Aydın & Köğce, 2009; Breidenbach, Dubinsky, Hawks, & Nichols, 1992; Vinner & Dreyfus, 1989). Similar misconceptions have also been seen when continuity at the critical points of piece-wise functions are being questioned. Some studies have shown that students with this misconception expressed the continuity when the given piecewise functions were discontinuous at critical points (Bezuidenhout, 2001; Seldon, Seldon & Mason, 1994). Besides this, some studies revealed that the students' difficulties in understanding continuity concept reason from a failure of making connections between continuity and limit concepts (Açıkyıldız, 2013; Bezuidenhout, 2001).

A number of teaching practices and materials are used to solve the misconceptions encountered by individuals. Among them, the use of DGS is seen as very useful for geometry and some calculus concepts due to its potential to give individuals experiences and visualizations of these concepts, and it has the added benefit of being easy to access and free. DGS, which is generally used in teaching geometry, can also be used for the calculus concepts that can be represented geometrically (Biza & Zachariades, 2007). When the relevant literature is examined, it is seen that many of these studies have investigated the effects of learning by means of DGS (e.g., Choi, 2010; Dikovic, 2009; Filiz, 2009; Karakuş & Peker, 2015; Kepçeoğlu, 2010). Among the studies related to computer-aided mathematics instruction, Doerr and Zangor (2000) examined the purpose for which teachers and students used graphic calculator to make mathematics meaningful in their lessons. Similarly, Goos et al. (2000), in their longitudinal studies to restructure the roles of teachers and students in technology-enriched classrooms with graphics calculators and overhead projection, revealed four roles of technology as master, servant, partner and extension of self. In another study, Maddux and Johnson's (2005) Type I and Type II classifications were remarked on. According to Maddux and Johnson (2005), the use of technology in class is either a direct result of a problem or an examination of the big ideas possessed. In other words, Type I focuses directly on the result and is about well-known problems. However, Type II is related to making conjectures and justification and using the mathematical idea behind the situation. Different from the studies given above, GeoGebra as one of the DGS was emphasized in the current study and the purposes of this software were examined in terms of continuity topic. One of the main reasons for focusing on GeoGebra software is that this software is widely used in computer-aided mathematics courses in teacher education programs (Çekmez, 2016; Çekmez & Güler, 2019) since its interface is plain and it is free software (Çekmez, 2013). First aim of conducting this research is that continuity is one of the subjects where challenges and misconceptions are common even among undergraduate students (e.g. Bezuidenhout, 2001). Second aim is to reveal the purpose of the use of GeoGebra which has the potential to be effective in addressing student challenges. Although different studies in the literature have examined the purposes of using different technological tools, some of them summarized above, it can be said that these technological tools, for example graphic calculator, are not now widely used in the second decade of 21th century. Besides, the idea that the purposes of using different technological tools will be different has been one of the basic assumptions of the current study. Briefly, this paper aims to illustrate how mathematics STs benefit from DGS in solving the problems of continuity, which is one of the concepts of calculus.

2. Method

The case study method was used to systematically examine how mathematics STs benefitted from DGS in continuity-based problem solving. Thus, it was aimed to examine STs' intentions to use the DGS in the problem-solving process in-depth. According to Yin (2009), a case study is a research method that is used when there is more than one piece of evidence or data source and the boundaries between the case and the environment in which the investigated entity resides and the environment in which it resides are not clear. An in-depth answer to the question of why and how to deal with special case studies is sought (Rowley, 2002).

STs who were involved in the study had already been taught GeoGebra before the implementation. In the context of this study, the STs were reminded of the use of GeoGebra by making some practices for 2 hours during the course, and they were shown how to create different type of functions, such as piece-wise, trigonometric, quadratics before the clinical interviews were conducted. Finally, clinical interviews were organized in such a way as to be individualized on different dates for each person.

2.1. Research Design

The design of the research was composed of three stages, as given in Figure 1. First of all, the literature was reviewed in order to determine the students' misconceptions about the continuity. Considering the learning difficulties and misconceptions in the literature, we developed a data collection tool. On the other hand, the research group was determined in voluntary basis. In the next stage, the STs were reminded of the applications of the GeoGebra software for 2 hours, and they were presented some of its basics and applications (in a general sense) such as menus and algebraic functioning, piece-wise function, function defined in a certain interval, and how to construct trigonometric functions. In this context, first of all, clinical interviews were carried out individually by taking different interview dates. Finally, interview dates were appointed for each of the participants and then clinical interviews were conducted.

Figure 1. Steps leading to the current research design

2.2. Participants

The participants of the research are composed of seven middle school mathematics STs in their fourth school year term in a state university. STs were selected based on their GPAs, and specifically on their General Mathematics and Calculus course grades. The reason for categorizing them with respect to their grades was to ensure that the STs had the theoretical background to solve the problems they encountered about the continuity. Since, when the middle school mathematics teacher education curriculum is examined, it is seen that the concept of the continuity was discussed within the scope of "General Mathematics" and "Calculus I" courses. In addition, in the course they took during their first term, "Graphic analysis", they were introduced to GeoGebra software and the course was enriched with problem solving activities. This background that STs possessed influenced the selection of the working group. As a result, the GPAs of two STs were chosen to be CC and below, 3 students within the range of CC and BB inclusively, and 2 students over BB (In the Turkish higher education system, the grades from C to A go from lower to higher.)

2.3. Data Collection

In the scope of the study, six open ended problems were included to form a data collection tool. When these problems were being prepared, firstly, the students' misconceptions about the continuity were determined from the related literature, and then, the studies on these misconceptions were investigated in detail. These problems were covered under three misconceptions themes. While the first theme involves the idea that the functions are to be a single piece of a graph, the second is that the functions are expressed by a single formula, and the third is about the definition of the continuity. In this regard, questions 1, 3, and 4 are related to the first theme, questions 2 and 6 are related to the second theme, and question 5 concerns the third theme. When the questions are examined, the first question is intended to determine what it means to be continuous in a given interval and how its representation is interpreted. In addition, we aimed to examine the steps of reaching the result with drawings using GeoGebra in this problem. In the second question, participants were required to interpret the graph of a function, and make connections between limit and limit-continuity, as well as continuity within a certain interval. The third one was prepared in a way to examine how participants identify the continuity at an undefined point. Different from the other problems, the drawing of the function at the given point includes a mistake when it's drawn with GeoGebra. Therefore, it was seen as important to investigate the views of STs toward DGS including this problem. Similar to the third one, the fourth problem was related to questioning the continuity at an undefined point. The purpose of preparing the fifth question was to investigate what STs think about the limit value of the function at a certain point and the continuity at the related point. The last question of the test was prepared so as to inquire about the piece-wise function. That question was about analyzing the limits of the function and of a given point, and the continuity at the same point. Here the problems provided opportunities for us to observe the role of GeoGebra in solving these problems. During the preparation phase of these six problems, one expert and three researchers were interviewed. In addition, some changes were made to the problems by applying them to different students before the main study.

During the collection of the data, hour-long individual interviews were conducted and recorded with the permission of the participants. In addition, field notes were taken by the researchers to picture how DGS affected their problem solving processes.

2.4. Data Analysis

When the data were analyzed, students' work- sheets were examined. In addition, the audio-recordings and observation notes taken during the interviews were also examined, and the stages of using the DGS for the solution of the questions were investigated by three researchers. Findings were presented under the themes established within the consensus of all of the researchers. The students' written responses were then translated from Turkish to English, while maintaining the essence of their meanings. In addition, the participants were coded as STX ("X" being a variable for a number assigned to each participant), and the interviewer researcher was coded as R to preserve their anonymity in observation of the ethical standards for research.

3. Results

In this study, we examined mathematics STs GeoGebra usage - GeoGebra is one of the DGS related to the continuity. In this context, it was observed that while STs did not use GeoGebra for the question that described the continuity theme; they used the software for the theme of the idea that the graphs of continuous functions are composed of a single part, and the idea that continuous functions are expressed by a single formula. STs using GeoGebra have generally been using it to *visualize*, *verify*, and *calculate* the given functions. The findings below are based on these usage patterns.

3.1. The Use for Visualization

In the questions 1, 3, and 4 which were prepared considering the misconception that the graphics of the continuous functions consist of a single part, it has been seen that STs used GeoGebra for visualization purposes. One of the participants who used the software for visualization purpose was the ST coded as ST5.

When I look at the graph using GeoGebra, I see that $\lim_{x \to 1^+} f(x) \neq \lim_{x \to 1^-} f(x)$. So the function is discontinuous. We cannot get the point in the domain, because it is undefined at zero. As Murat said, the function is discontinuous since we can see that the graph does not consist of a single piece. Murat is right.

For any point except zero, the right and the left limit values are equal, and the function is continuous at those points. In this case, Mehmet is right.

Figure 2. ST5's response to Question 1

Regarding Figure 2 (in which the answer to the first question is included and investigated), it is seen that the ST tried to reach the conclusion by looking at a graph of the function. A clinical interview was conducted during the implementation to examine it in order to understand in which stage the participant used GeoGebra,

ST5: I want to draw a graph before I solve the question. Can I draw?

R: Of course, as you wish.

(Participant drew the graph using GeoGebra.)

ST5: When I look at the graph [and] approach to 0 from the right or the left, 1 / x becomes infinite. In the meantime, the limit and continuity at a specific point are not mentioned, but the definition interval is. So, according to the graph, Mehmet's solution is correct.

As seen in the dialogue above, ST5 had benefitted from the graph drawing feature of the software when solving the question. ST5 first showed the right and left limits on the graph by hand, then took a point on the graph and calculated the limit value according to its motion. Therefore, it is to say that ST5 effectively used the dragging and other features of GeoGebra. In this context, the ST used GeoGebra for *visualization* purpose. Similarly, ST4 used it in the same way to solve the first problem, as drawn in Figure 3 and as in the dialogue given below.

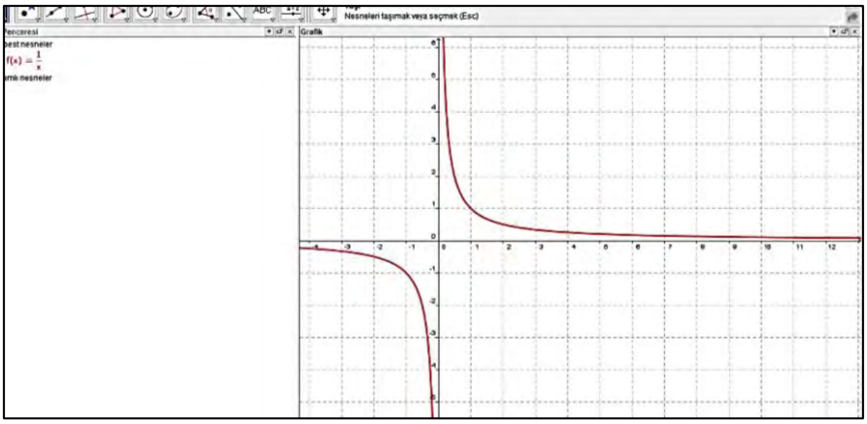


Figure 3. A screenshot of the software used by ST4 in Question 1

(ST5 draws the graph of the function as in Figure 3 and makes comments by dragging a point.)

ST4: It approaches to +∞ and -∞ at zero. It approaches to +∞ as x approaches from the right side and to -∞ from the left. So there is no limit of function at this point because the right and left limits do not have the same value.

R: You made a very nice comment for limit. How would you comment on the continuity?

ST4: If a function has a value equal to the limit at that point, we say it is continuous at that point. Now let's look for x = 2. Values are equal to each other. For example, when we look at the graph, we get the same value from the right and left for point three. Then it is continuous for R- {0}. So Mehmet is right.

As seen in the dialogue above, ST4 had drawn the graph of the function first, and then examined whether the function was continuous. When examining the continuity, ST5 selected a random point on the graph and moved it. In this way, the GeoGebra software was used for *visualization* purpose before talking about the limit of the function.

3.2. The Use for Verification

In the first question that aims to examine participants' attributed meaning of the continuity in a given interval and to investigate their interpretation, STs generally have difficulties in presenting their expressions algebraically. For example, ST1's interview script while solving the first problem before using the GeoGebra software was as follows:

ST1: Murat's view here is discontinuous in the definition interval. That's right. Murat's opinion is correct. Mehmet says that the definition is continuous in the definition interval. Mehmet thinks of it wrongly. Murat is right ... well… it is not defined for any x number in the domain ... I think I am confused ... Yes, yes, but can I have a look at the graph from the computer?

As we have seen in the above interview script, we can say that ST1 did not pay attention to the definition interval of the function at the beginning while examining the continuity. The ST, who realized the false inference, wanted to look at the graph of the given function in GeoGebra. After looking at the graph, ST1's conclusion shifted:

ST1: This function is not defined at the zero point, as we can see in the graph, but it is defined at any real number except the zero point. Now I see clearly that any real number except zero is absolutely continuous.

It is seen that ST1 explained the continuity while using GeoGebra for both the purposes of verification and visualization. Similarly, ST7 tried to interpret the continuity of the function in a given interval using GeoGebra. In Figure 4, ST7's response to the first question is presented.

Question 1
Murat: I think the f(x)=1/x function is discontinuous at any point in its domain because when it is drawn, the graph isn't composed of a single piece.
Mehmet: No, Murat! This function is continuous at any point of its domain set.

Who is right? Justify your response and support Mehmet or Murat's idea. You are free to use GeoGebra.

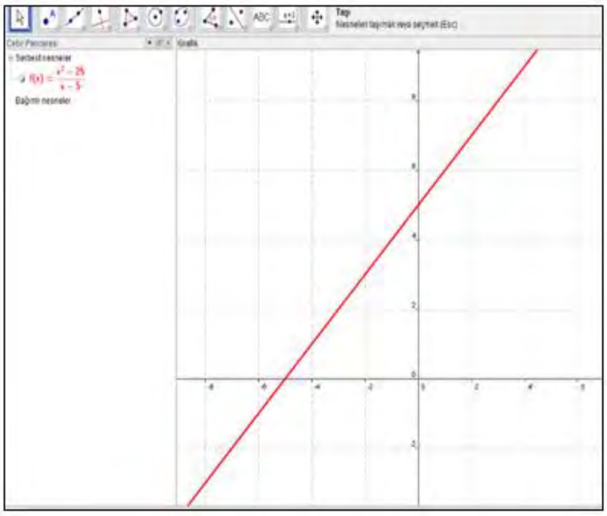
For the function in the given range, the limit from right side = the limit from the left side = the value of the function in the related point. Therefore, the function is continuous. There is no limit at $x=0$

Figure 4. ST7's response to Question 1

During the interview, ST7 stated that he/she knew the graph of the function $f(x) = \frac{1}{x}$ before, but couldn't remember it. Afterwards, ST7 drew the graph of the function in GeoGebra and then defined the interval where the function is continuous. In this context, it is to say that the software used *visualization* and *control* purposes.

While some STs have used GeoGebra software for verification purposes, they have encountered a shortage of software:

When the "f(x) = $\frac{(x^2-25)}{(x-5)}$" function is drawn by GeoGebra, a linear line $y = x + 5$ is produced. As a result, since the graph of the function is also a linear graph, it becomes a continuous function in all intervals of the domain. Some STs were aware of the error when they were asked whether their solution or the software was correct, but some STs believed that the software was correct. One of the exemplary STs for this situation was ST3 (see Figure 5).



Q3. Let f function be f(x) = $\frac{(x^2-25)}{(x-5)}$ .Examine the continuity of the function at the x=5 point. You are free to use GeoGebra.

$\lim_{x\to5^+} f(x) = \lim_{x\to5^+} \frac{x^2-25}{x-5} = \frac{0}{0} \to 2x = 2\cdot5=10$

$\lim_{x\to5^-} f(x) = \lim_{x\to5^-} f(x) = \frac{2}{0} \to 2x=10$

$f(x) = \frac{\cancel{(x-5)}\cdot(x+5)}{\cancel{x-5}} = (x+5)$

$f(5)=5+5=10$

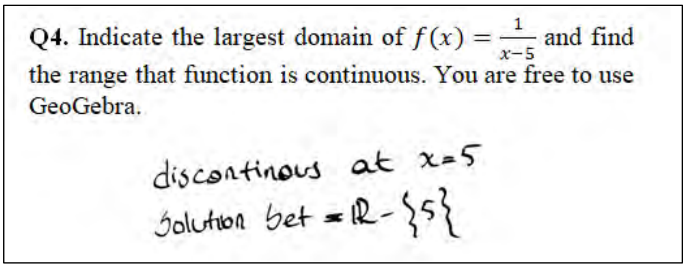
Figure 5. The image of the function in GeoGebra and ST3's answer

As seen in Figure 5, ST3 found a 0/0 uncertainty at the beginning, but afterwards they got the graph of the function in GeoGebra. The ST, who saw that the software drew a line on the graph, completed his solution shown on the right side of Figure 5. Therefore, ST3 used

GeoGebra for verification purposes. Although, unlike ST3, ST5 found the value and the limit of the function at five points without simplification and reached the correct result in this way. They stated that the graph is wrong in the software, and sometimes the DGS is wrong, and that the software may be mistaken.

3.2. The Use for Making Calculations

Some of the questions in the study were related to finding the value of the continuity. Some STs have tried to find the limits of the function directly by writing a function to GeoGebra because it is hard to find continuity by calculating directly. During this stage, they tended to use GeoGebra to make calculations. Figure 6 shows the response given by the participant, ST2.

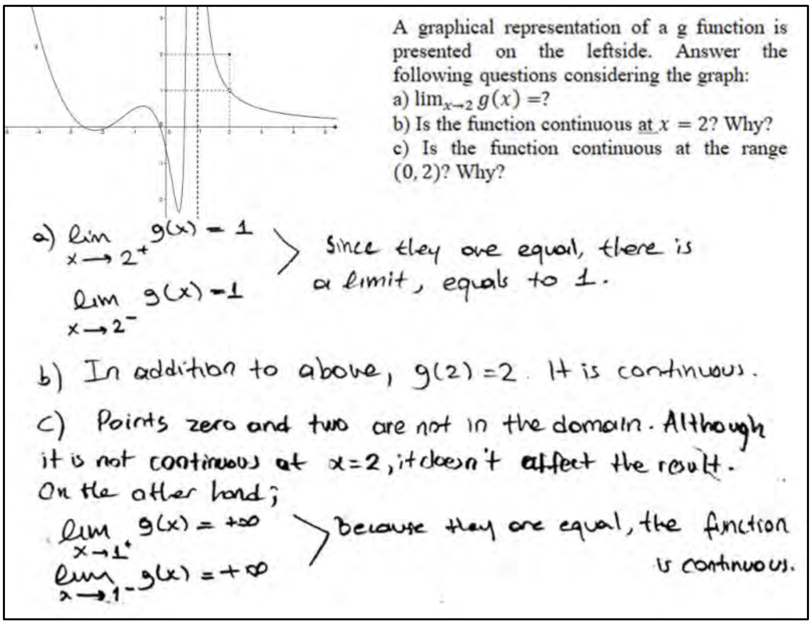

Q4. Indicate the largest domain of $f(x) = \frac{1}{x-5}$ and find the range that function is continuous. You are free to use GeoGebra.

discontinous at $x=5$
Solution set $= \mathbb{R} - \{5\}$

Figure 6. ST2's answer to the fourth question

In Figure 6, it is seen that the ST noted the interval that function is continuous by subtracting the value that made the function undefined. Here, ST2 first drew the graph using GeoGebra software and then determined the interval where the function was defined and continuous. Additionally, it can be said that ST2 had used the software effectively.

In some questions, the STs examined the continuity of the function without using the software. For example, the second question was to interpret the graph of a given function and to interpret the concepts of the continuity in a certain interval with the relation to the limit-continuity provided. In general, the STs had difficulties to comment on whether a given function would be continuous when the domain was not given. Some STs' responses were as follow:


A graphical representation of a g function is presented on the leftside. Answer the following questions considering the graph:
a) $\lim_{x\to 2} g(x) = ?$
b) Is the function continuous at $x = 2$? Why?
c) Is the function continuous at the range $(0, 2)$? Why?

a) $\lim_{x\to 2^+} g(x) = 1$, $\lim_{x\to 2^-} g(x) = 1$ Since they are equal, there is a limit, equals to 1.

b) In addition to above, $g(2)=2$. It is continuous.

c) Points zero and two are not in the domain. Although it is not continuous at $x=2$, it doesn't affect the result. On the other hand;
$\lim_{x\to 1^+} g(x) = +\infty$, $\lim_{x\to 1^-} g(x) = +\infty$ because they are equal, the function is continuous.

Figure 7. ST4's response to Question 2

Figure 7 shows ST4's answer to the second question. Here, the ST has expressed the relationship between the limit, continuity, and the value of the function at a given point clearly. In this context, ST4 seemed to construct the concept of the continuity quite well. On the other hand, ST6 had reached the same result. However, they reached the correct solution by looking at its neighboring reference point at x=2, and therefore, they used a different method than the other participants did. Similarly, the ST correctly solved the fifth question, as in Figure 7, without the need to use the software. ST4, on the other hand, answered the question incorrectly. When their response was analysed and detailed, it was seen that although the ST was good at interpreting the relation between the limit and the continuity, they were unsuccessful to make connection among the limit value and indefinability, as in Figure 7.

4. Discussion and Conclusion

This research was aimed to investigate the purpose of the use of GeoGebra software, which is a DGS for solving continuity problems - these are one of the calculus concepts of mathematics STs. The obtained results have showed some clues about the purpose of using the software, and at the same time it was found that the STs had some misconceptions about limit and continuity concepts.

In the first question of the data collection tool used within the scope of the research, the STs were given a function with domains and a set of rules, and they were asked to discuss the continuity of the function. When investigating their solution processes, it was remarkable that a large part of them wanted to draw a graph in GeoGebra instead of algebraically solving it. As Mastorides and Zachariades (2004) pointed out, since the concepts of limit and continuity are difficult concepts to understand, a direct algebraic inference method is complex for them, so they want to visually investigate the behavior of the function instead. This situation was interpreted that STs used the dynamic software for visualization purposes. Yet another finding was that some STs wanted to draw their chart during clinical interviews, especially before they solve the question. Another result in the study was similar to Alqahtani and Powell's (2006) approach to solving teachers' geometry problems, in the way of how teachers approach to the solution of the questions using the dragging feature of the technology. In other words, similar to their study, it was seen that some of the STs drew a graph of a given function when finding its continuity, and then they found an arbitrary point on the function and investigated the values of this point in different parts of the function. Therefore, the results obtained from STs using GeoGebra for the purpose of visualization were put into practice.

In the other question in the data collection tool, although the domain and the rule of the function were given, it was seen that the STs preferred to solve the question by drawing a graph on software instead of solving the question algebraically with another expression. In the literature, algebraically finding the limit and the continuity of the given functions are very common difficulties (Beery, 1975; Bridges, 2007; Clement, 2001; Keele, 2008; Nair, 2010; Williams, 1991). In this study, it was observed that STs who encountered similar situations benefited from the visualization function of the GeoGebra software by graphing the function. Thus, the STs found an opportunity of interpreting the given functions and the continuity by visualizing them through the software.

In this study, it was concluded that pre-service teachers used technology for visualization, verification and calculation purposes in solving questions related to continuity. Similarly, Doerr and Zangor (2000) found that teachers and students used graphic calculator for five purposes as computational tool, transformational tool, data collection and analysis tool, visualizing tool and controlling tool in order to make mathematics meaningful in their lessons. It supports the results obtained from this study. Goos et al. (2000), in their study, aimed to restructure the roles of teachers and students in classes enriched with graphics calculators and overhead projection, found the roles of technology for teachers and students as “master, servant, partner and extension of self”. From this point of view, it can be said that different technological instructional materials are used for different purposes. It is thought that these purposes adopted by STs will guide researchers in the integration of technology into classes.

In the first question of the data collection tool used within the scope of the research, it was expressed that STs tended to draw graphs in a computer environment, since it was difficult to solve the problem algebraically. Although some participants were able to express the steps to be followed for the algebraic solution of the problem first, they did not reach the end by completing these steps with the paper-pencil method. For example, one of the STs expressed the steps to be taken in writing, but even though they tried to draw the graph on paper, they stated that they were unable to and that they were not sure about their result. They went on to check their answers by drawing a graph with the help of the GeoGebra software. Similar participatory behaviors had also been observed in Hollebrands and Okumuş's (2017) study investigating teacher candidates’ process of solving the minimum distance between two opposing corners (the object’s diagonal line) of a cube and of a rectangle prism with the help of the DGS. Here, STs first made some estimation for the desired point in the DGS, and then tested whether the inference was correct with the software’s help. Taking this into consideration, it can be said that the STs first hypothesized, tested their inferences, then, used DGS in the last step to verify/prove them. Similar examples are found in the literature. For example, Monaghan, Sun and Tall (1994) found that the use of the Derive Program from Computer Algebraic Systems in the experimental study of the effect of teaching the concept of limit is used by the students to control their program solutions. In another study, Açıkgül and Aslaner (2015) investigated geometric space problems solved by STs using the paper-pencil method and DGS, and at the end of the research when the two processes were compared, it was stated that the STs had hypothesized, tested and generalized differently using the paper-pencil method vs. using the software. Similar to these studies in the literature, the results obtained from the present study have shown that STs can be considered to typically use DGS deductions as evidence for verifying their answers to the questions.

Although there was no problem in the scope of the research, another finding of the study was that a ST was blind to the program. Even though the teacher verbally said that the function at the given point had $\frac{0}{0}$ uncertainty, after drawing the function in GeoGebra, they saw that the point of interest was included in the domain and range of the corresponding point because of openness of the software, and that the function was defined at that point. As a consequence of this situation, it can be said that software may also be mistaken. Another result obtained from investigated STs' answers was that they used the DGS as a means of calculation. Some of the questions in the study were also related to the existence of a continuity at a point in a function. Since some of the student teachers thought that continuity of a function was difficult to calculate, they typed the expression of the function into GeoGebra and directly calculated the limits of these functions by means of the software.

5. Limitations

Although the results obtained from this study contain conclusions for the purpose use of the GeoGebra software, there are important limitations to be considered. The most important limitation of the study is that it was carried out with a small number of participants. Although the study does not aim for a generalization of its results, it should be taken into consideration that different results can be obtained in the wider participant group. Second, the results are limited to the continuity questions in the data collection tool. Various purposes can be achieved in different calculus subjects. Similar designs can be made for different topics and concepts in future research.

6. Conflict of Interest

The authors declare that there is no conflict of interest.

7. Ethics Committee Approval

The authors confirm that the study does not need ethics committee approval according to the research integrity rules in their country.

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