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GeoGebra for learning and teaching: A parallel investigation

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In the study reported on here, we investigated the effects of the use of dynamic geometry software (DGS) (i.e., GeoGebra) on learners' learning and a teacher's beliefs. The learners and teacher involved in the study were from a high-poverty, rural high school in South Africa. We compared grade 11 learners ($N = 56$) who used GeoGebra in the context of learning circle geometry with learners who experienced geometry through traditional lecture-based instruction. Participating learners were from classes in a public school located in the rural Umkhanyakude district of KwaZulu-Natal, Republic of South Africa. Results showed that learners using GeoGebra were more successful at solving problems and justifying their statements, while the other learners provided a limited justification for their answers. In a parallel and complementary investigation the teacher's attitudes toward using GeoGebra as an instructional tool were considered qualitatively. Results showed that even in high-poverty, rural settings where the availability of technological resources are limited, the use of GeoGebra affected learners' learning and had positive effects on the teacher's beliefs regarding teaching and learning.

Keywords: Euclidean geometry; GeoGebra; performance; theorems and proofs

Introduction

Due to rapidly changing global educational settings, human resources that create the economic backbone for all cultures, heedfully respond to the various aspects of education; these include but are not limited to instructional and learning approaches and materials (Hohenwarter & Jones, 2007; Jezdimirović, 2014). To this end, educational technologies have played a transformational role in teaching and learning worldwide.

This transformational perspective requires that innovative modalities of knowledge production are examined in teaching and learning. As a result, South Africa has implemented and started to invest hugely in transforming its education sector to achieve effective teaching and learning. The transformation is by blending technologies in teaching and learning leading to a discernible effort in actualising the relevance and application of educational technologies to generate a more productive and technologically driven knowledge economy (Department of Basic Education, Republic of South Africa, 2015; KwaZulu-Natal Department of Education, 2014; Ndlovu, 2014). For this purpose, the use of educational technologies such as Blackboard and Moodle, as course management systems, and GeoGebra as a subject-specific application have been integrated in South African schools to enhance and augment the current educational setting (Department of Basic Education, Republic of South Africa, 2015; KwaZulu-Natal Department of Education, 2014; Ndlovu, 2014; Ndlovu, Wessels & De Villiers, 2013). Lessons learned from rural and even peri-urban settings may well empower transformative considerations applicable to other settings around the world, which may have far-reaching implications.

Given the local and international calls for Information and communications technology (ICT) integration in education, there has been a recent surge in the enhancement of teaching and learning of geometry with the aid of DGS like GeoGebra (Mnguni, 2014; Ndlovu, 2014; Yildiz, Baltaci & Demir, 2017). International research has demonstrated that, among urban pre-service teachers, the use of GeoGebra can play a significant role (e.g., Hohenwarter & Jones, 2007; Yildiz et al., 2017). However, there is still a lack of research in the context of rural South African, where ICT use is limited due to challenging access to regular and affordable internet connection (Mnguni, 2014; Ndlovu, 2014; Yildiz et al., 2017). However, a crucial benefit of GeoGebra is that it does not require internet connectivity to operate. It can be downloaded on a laptop or external drive and then uploaded to as many school computers as desired, ready to be used by learners.

Additionally, this means that it is cost-effective, so funding is usually not a challenge. Drawing from the works of Mnguni (2014), Ndlovu (2014), and Yildiz et al. (2017), much of the extant literature has considered effects of the use of GeoGebra on learners, teachers, or pre-service teachers. However, few studies have simultaneously considered the parallel effects on learners and their teacher(s).

With this study, we sought to extend the literature by investigating the parallel effects of the use of GeoGebra on learners and teachers in a rural setting. Attention, in part, would be focussed on the works of Balacheff (2010); Chan and Leung (2014); Department of Basic Education (2010); De Villiers (2009); Mariotti (2000, 2002); Mnguni (2014); Ndlovu et al. (2013); Venema (2013) and Vilardi and Rice (2014). Attention to this work shall be related, but not limited, to GeoGebra for learning and teaching and teaching geometry with technology contextualised in a rural setting.

Literature Review

Learners typically rely on rote learning when it comes to solving demanding problems in mathematics topics such as trigonometry, Euclidean geometry, and probability (Habree, 2009). De Villiers (2009) reports that mathematics pedagogy is generally marked with traditional, teacher-centred instructional modes, emphasising learner recall of information. Similarly, due in part to many considerations, many teachers encourage rote learning over inquiry-based and learner-centred approaches, teacher-learner discussions, and limited applications of a DGS such as GeoGebra (Chan & Leung, 2014; De Villiers, 2009; Ndlovu et al., 2013). Thus, the teaching of Euclidean geometry theorems is traditionally recognised by an emphasis on rote memorisation, lacking focus on conceptual understanding.

Countering this instructional traditionalism, for learners to develop appropriate mathematics understanding, the reintroduction of Euclidean geometry in South Africa's Curriculum and Assessment Policy Statement requires instruction to be more conceptually-based with less attention to rote learning (Ndlovu, 2014). However, Ndlovu et al. (2013) report that most teachers' competence is a central issue in efforts regarding the reform of geometry teaching practices. Since teachers are challenged to understand the pedagogical value and uses of DGS, Ndlovu (2014) calls for appropriate scaffolding to address this gap.

From 428 reports of DGS use in the learning of mathematics, Chan and Leung (2014:323) note nine quasi-experimental studies eligible for meta-analysis and conclude that, while DGS-based instruction produces a positive and large effect size, "further research should be conducted to investigate the impact of DGS-based instruction in various settings." This recommendation for the need of a wider investigation of the effects of the use of DGS has been reported by other researchers as well (e.g., Balacheff, 2010; Mariotti, 2000, 2002). While numerous studies have demonstrated learner growth through the use of dynamic math computer environments, fewer studies have investigated the effects of teachers using GeoGebra in teaching their learners in rural settings (e.g., Artigue, 2010; Balacheff, 2010; Drijvers, 2012; Goos, Soury-Lavergne, Assude, Brown, Kong, Glover, Grugeon, Laborde, Lavicza, Miller & Sinclair, 2010; Ruthven, Hennessy & Deaney, 2005, 2008). However, the rural educational environment poses some unique characteristics (e.g., limited instructional resources, limited availability of professional development for teachers, and a reduced pool of teachers interested in working in this environment) and opportunity to investigate ideas which may be more applicable to non-rural settings than recognisable at first glance.

Recently, Chimuka (2017) and Ogbonnaya and Chimuka (2017) have helped to address the significant dearth of research specifically investigating DGS use in rural schools. They have recognised that, even in high-poverty, rural schools in South Africa, the use of DGS can have a strong impact on learners' learning in the context of circle geometry theorems. This confirmed De Villiers' (2009) assumption that a lack of high achieving teachers in high-poverty, rural areas may be remedied by teachers and learners using technology and DGS in the classroom. Indeed, De Villers seems to imply that a less able teacher can make appropriate instructional use of dynamic geometry. However, the findings of Stols, Ferreira, Pelser, Olivier, Van der Merwe, De Villiers and Venter (2015:1) may contradict this notion. They found that South African teacher participants (N = 22), when provided with a sufficient technological platform appropriate for instructional use, were thwarted by their perceptions of inadequacy. They report as follows:

> With regard to effort expectancy, participating teachers found the use of technology overwhelming, resulting in a need for further training. No evidence was found of social influence affecting the participants' acceptance of the technology. The participants proved to have access to sufficient equipment. However, their perceptions of their limited skills weighed heavier than external facilitating conditions. As a result, participating teachers were hesitant to utilise technology in their teaching.

In this study we sought to complement and connect the works of Ogbonnaya and Chimuka (2017) and Stols et al. (2015). The former focused primarily on the effect of learner learning through the use of DGS. The latter considered teachers' attitudes and uses of instructional technology. We simultaneously considered learners' results from, and teacher attitudes regarding the use of DGS in the context of circle geometry theorems in a high-poverty, rural high school in South Africa, in which there was a lack of sufficient computing technology, software and applications, limited technological skills and experience among the teachers and learners; consistent electricity supply, and a lack of access to even online [free] applications. In particular, we investigated how GeoGebra impacted both learners and teachers in some high-poverty, rural high schools. We considered the teachers' challenges in using GeoGebra and their need for scaffolding, and learners' learning and attitudes with respect to using GeoGebra.

It is important to clarify that in the geometry curriculum of most South African rural schools, GeoGebra is not specifically denoted as the DGS of choice. Nevertheless, to evaluate the effects of a DGS on teaching and learning, GeoGebra was chosen because it was free and could run offline. Therefore, in this study, DGS and GeoGebra are treated synonymously.

Theoretical Framework: Teaching Geometry with Technology

In the context of employing technology in instructional settings, the Cognitive Theory of Multimedia Learning (CTML) plays a pivotal role in analysing and informing approaches to instruction (Mayer, 1997, 2002, 2009; Mayer & Moreno, 2003; Mnguni, 2014; Venema, 2013, Vilardi & Rice, 2014). Key features of CTML include the input of information from the external world into the cognitive structures, the cognitive processing of this information, and the externalisation of information from the mind to the environment. More specifically, the CTML recognises that (a) there are two distinct auditory and visual channels for processing information, (b) each channel provides limited affordances to the learner and (c) learning incorporates interacting (via filtering, selecting, and organising) information through prior knowledge. This leads to the understanding that learners process only a finite amount of information in a channel at a time, construct mental representations of new information, use short-term, working, and long-term memory, integrate words, pictures, and auditory information. Since the classroom employment of GeoGebra in teaching and learning in this study were novel to both the learner and teacher participants, the construct associated with the CTML seemed well suited through which to investigate, particularly the teacher reactions to employing a DGS in the teaching and learning of circle geometry theorems.

Some argue that the use of technology in mathematics teaching is often marked with various challenges such as teachers' lack of content understanding, lack of adequate technological resources, internet connectivity, etc. (Jezdimirović, 2014). Conversely, the use of DGS enables learners with more opportunity for visualising geometric concepts, which often accommodates average and below-average learners. Research (e.g., Hanna, 1998; Sinclair & Jones, 2009) provides evidence that through the use of visualisations learners can move from empirical, visual descriptions of spatial relations to more theoretical abstract ones (Christou, Jones, Pita-Pantazi, Pittalis, Mousoulides, Matos, Sendova, Zachariades & Boytchev, 2007). These understandings support the urgency to incorporate DGS in Euclidean geometry classrooms to provide a rich learning environment, critical thinking skills and comprehensive understanding of learners' learning experiences requisite to being successful learners (Akkaya, Tatar & Kağızmanlı, 2011; Habree, 2009; Hohenwarter & Fuchs, 2004; Jezdimirović, 2014).

Study Objective

The aforementioned background establishes that learners' use of DGS to facilitate visualisation has been investigated for decades, the use of DGS contributes to learner learning, and that there is a dearth of research regarding the use of DGS in high-poverty, rural settings. Thus, we sought to extend the literature by investigating the parallel effects of the use of GeoGebra on learners and teachers in a rural setting. To do so, this study employed a test of statistically significant difference between post-test results of both control and experimental groups of learners. In parallel, we investigated whether the use of GeoGebra in the classroom affected teacher beliefs regarding the teaching and learning of geometry.

This study investigated the effects of using GeoGebra to teach Grade 11 learners Euclidean geometry theorems and proofs regarding the following circle theorems:

- In a circle, the degree measure of a central angle is equal to the degree measure of its intercepted arc.
- In a circle (or in congruent circles), two central angles are congruent if and only if the respective intercepted arcs are congruent.
- The measure of an inscribed angle in a circle is equal to half the measure of its intercepted arc.
- Inscribed angles that intercept the same arc (or congruent arcs) are congruent.
- In a circle (or in congruent circles), chords are congruent if and only if their respective arcs are congruent.
- The perpendicular bisector of a chord contains the centre of the circle.
- If a line through the centre of a circle is perpendicular to a chord, then it bisects the chord and its arc.
- If two secants intersect outside a circle, the measure of the angle of intersection is half the difference of the measures of the intercepted arcs.
- The measure of an angle formed when a chord intersects a tangent line at the point of tangency is half the measure of the arc intercepted by the chord and the tangent line.
- If a secant and a tangent line intersect outside a circle, the measure of the angle formed is half the difference of the measures of the intercepted arcs.
- If two tangent lines intersect outside a circle, the measure of the angle formed is half the difference of the measures of the intercepted arcs.
- If a quadrilateral is inscribed in a circle, the opposite angles are supplementary.

This study also sought to recognise changes in teacher beliefs regarding the teaching and learning of geometry.

Methodology

Research regarding Learner Learning

Participants and procedures for learner research

A total of 56 learners from Grade 11 – and their respective classroom teacher – were selected from a classroom in a high-poverty, rural public school situated in the Umkhanyakude district of KwaZulu-

Natal. The research regarding the learners employed a non-equivalence experimental sample design with non-random assignment where pre-tests and post-tests were treated and compared. One of the study researchers together with two other Grade 11 mathematics teachers working at the same school planned the research pre- and post-tests and content, pedagogy, and activities for all lessons involving a total of four weeks of instruction. All three of these educators had extensive experience with GeoGebra. Pre- and post-test items were taken or revised from previous assessments items used by these teachers. While the pre- and post-tests were not precisely examined for validity and reliability, the expertise of the researcher and the two teachers, along with the later expertise of the classroom teacher who both experienced the curriculum and participated in the grading of learner work, introduced some assurance of validity and reliability regarding test items.

None of the Grade 11 teachers were involved in instruction of the learners during the four weeks. To eliminate bias and unfair treatments, the two groups were taught the same content of Euclidean geometry at the same pace. The GeoGebra lessons were tailored for experimental learners to be able to use the software, do geometric constructions, modify constructions, do the "drag test," explore their correctness, and answer questions. Their control group counterparts created paper-and-pencil constructions.

In a lecture-based style similar to that previously used by the classroom teacher (i.e., teacher lecturing, learners copying notes from the chalkboard, limited questioning of learners, and little discussion among learners), the researcher who participated in the assessment and curriculum design taught the entire class of control group and experimental group learners for two weeks prior to the dissemination of the pre-test. These lessons did not cover the material associated with this study. Doing this ensured that, when segregated, both the control and experimental groups would have similar prerequisite learning experiences.

Using the results of the pre-test administered to all 56 learners, and previous mathematics performance assessments, the learners were randomly separated into two similar ability groups with nearly equivalent numbers of high, medium, and lower achieving learners. The two groups of learners had their academic schedules altered such that each day the control group class was taught the hour before the experimental group class. All participating learners signed participant agreement forms and willingly agreed to the altering of their schedules. Then, for two weeks (10, 1-hour classes) learners in the control group (N = 28) were taught circle theorems and proofs throughout the study with a traditional, lecture-based teaching approach by the same researcher who previously taught for two weeks. During the same two weeks (10, 1-hour classes), the same researcher also taught the experimental group (N = 28) using 10 activities involving GeoGebra. Notably, for a total of four weeks, the classroom teacher was not involved in the instruction of the classes. Rather, he observed instruction and practiced all of the learning activities. All of this instruction covered examples and proofs of the theorems posed above.

To orient learners in the experimental group to the use of GeoGebra, they were directed through a set of introductory tasks. This allowed for the exploration of different GeoGebra menu options as well as investigations of tutorials and presentations built into the GeoGebra platform. Therefore, since the experimental group investigated the use of GeoGebra through these introductory tasks, they experienced less than the full 10 hours of instructional time regarding circle theorems.

Instrumentation

The class was given the pre-test in the form of multiple-choice questions, and results on the pre-test were recorded. The mathematical concepts on the pre-test included the basic understanding of inscribed angles in a circle, chords and tangents, terms like "bisection" and "perpendicular lines" and concepts such as the Pythagorean Theorem and congruency. The questions covered topics taught in Grades 10 and 11 Euclidean circle geometry and applications from the Grade 12 exit examination. The pre-test and post-tests each consisted of six multiple-choice items. For each item, learners had to include explanatory work to ensure that their answer selection was based on reasoning and not simply an act of uninformed selection.

A post-test (similar to the pre-test) was administered simultaneously to both groups after ten days of teaching and learning in the content of Euclidean geometry circle theorems. The post-test was used to check any possible effects of GeoGebra on the learners' understanding compared to a traditional teaching approach. Elements from the pre- and post-tests are provided in Appendix A.

The assessment tasks were developed based on the ability to work out problems and choose the correct item that corresponded with given statements. These particularly included concepts such as opposite angles of a cyclic quadrilateral about being parallel, complementary, supplementary and perpendicular. Learners were directed to figures such as those in Figure 1 to answer particular questions.

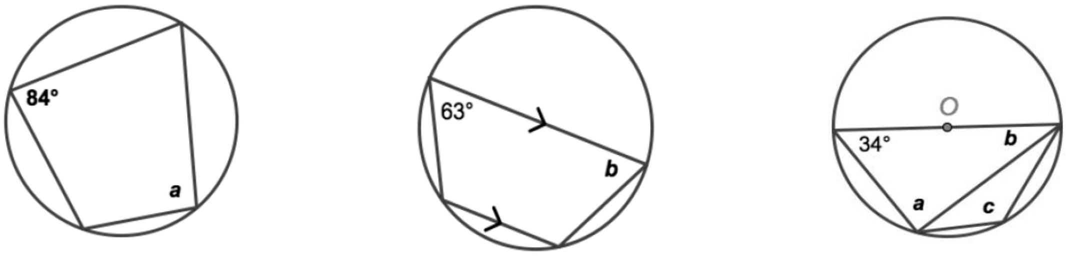

Figure 1 Cyclic quadrilateral

Data analysis

After the learning and testing activities, both sets of tests were collected and scored together by the teaching researcher and the classroom teacher. Test data was analysed using quantitative statistical techniques leading to descriptive analyses, including the means and standard deviations and a parametric comparison *t*-test. The *t*-test was used to test whether a statistically significant difference existed between the control and experimental groups at the beginning and end of the study. This was done primarily by comparing the mean scores of the pre-test and post-tests of both groups.

Investigation regarding the Classroom Teacher's Beliefs

Notably, the inquiry into the respective classroom teacher's beliefs was purposively denoted an investigation rather than research. The informal, quasi-case study approach employed in this study fell short of the detail and precision needed to be considered rising to the stature of a designed qualitative case study. The intention of the design of this investigation was simply to determine whether the teacher's beliefs changed throughout the study.

The classroom teacher had fifteen years of classroom teaching experience. The classroom teacher was interviewed before, during, and after the learner interventions by the researcher who also taught the learners. These interviews were transcribed, analysed and synthesised to investigate themes. During the learner GeoGebra activity sessions, the teacher agreed to perform all activities attempted by the learners under identical time, class, and location constraints. No professional development in the use of GeoGebra preceded the teacher's participation in the activities, and the teacher received no additional instruction in the use of GeoGebra. Ideas discovered from these interviews are reported later.

Findings

Learner Results

After the pre-test was administered to both groups, the calculated mean of the control group was 4.4167, and that of the experimental group was 5 out of 15 possible points. The standard deviations were 2.1088 and 2.3355, respectively.

Table 1 Summary of unpaired independent *t*-test results of groups (pre-test)

Group	*M*	*SD*	*t*	Sig (2 tailed)
Control	4.4167	2.1088	1.96	-0.6422
Experimental	5	2.3355		

Table 1 provides evidence that the means of control and experimental groups were not significantly different at $p < 0.05$. The pre-test results show that there was no statistically significant difference between groups. This means that at a given point in time, the two groups achieved similarly on the content at the beginning of the study. This result serves as the basis to compare both the experimental with the control teaching approaches to determine whether one approach had a greater impact on learners' performance than the other.

After the instructional interventions, the post-test administered to both groups were scored, recorded and analysed. The mean of the scores for both groups increased. The mean for the experimental group increased from 5 to 7.75 while that of the control group increased from 4.4167 to 5.333 (Table 2), at the *t*-value of -3.1659.

Table 2 Summary of unpaired independent *t*-test results of groups (post-test)

Group	*M*	*SD*	*t*	Sig (2 tailed)
Experimental	7.75	2.0057	1.97	-3.1659
Control		5.333	1.7233	

Note. *t*-value significant at $p < 0.05$.

The means of both control and experimental groups were significantly different at $p < 0.05$. This shows that both groups achieved at different levels after the post-test. This indicates that the treatment (GeoGebra) may have impacted the learners' learning and understanding of this topic. To investigate further, paired independent *t*-tests were performed to identify differences, as indicated in Table 3.

Table 3 Summary of paired independent *t*-test results of both groups

Post-test, Pre-test scores				
Group	*M*	*SD*	*t*	Sig (2 tailed)
Experimental	2.75	1.33	2.019	-8.1447
Control	0.9163	1.342	2.019	-2.5606

Note. *t*-value significant at $p < 0.05$.

The means of both groups using paired independent *t*-tests were significantly different at $p < 0.05$. Although there were improvements in justification of the previously mentioned Euclidean circle theorems for both groups, the higher mean difference for the experimental group confirms a higher improvement than that of the control group.

In summary, the findings via the independent *t*-tests reveal that both the experimental and control groups performed similarly on the pre-test (Table 1). However, the statistical difference in the post-test results of both groups revealed that the difference between the groups was statistically significant, with the experimental group performing better than the control group (Table 2). When paired, sample *t*-tests for post-tests and pre-tests results were significant, and both groups performed better at the end of the lessons, as illustrated in Table 3, with the GeoGebra treatment group performing better than the group taught through the traditional teaching approach. This indicates that in this study and for this population of participants, learners learned better with GeoGebra incorporated as a learning tool. These results comport with the findings of Chimuka (2017) and Ogbonnaya and Chimuka (2017) regarding another high-poverty, rural school in South Africa.

Classroom Teacher's Results

The following are quotes from interviews with the classroom teacher before, during, and after the research project. These quotations have been edited for readability and clarity.

Before any research activities or lessons

"*We have some access to computers in the school, but we rarely use technology in maths. We don't know how to do much with maths technology. I have used Excel a little and calculators, but nothing else. We were never trained to use technology, and I'm not sure that I know enough to help learners. I guess that it scares me some.*"

"*I have heard of GeoGebra in the past, but I have only seen it demonstrated once for the classroom. It was at a conference. The presenter went so fast that I couldn't learn anything. And I did not have computers to try. I think that I left more confused than before the presentation.*"

"*I teach traditionally. I do a lot on the chalkboard, and we give them many notes. I develop problems for every geometry theorem. I think that the learners struggle with geometry. They find some concepts to be difficult and they have much difficulty with proofs.*"

"*Circle theorems seem most difficult to my learners. I think that there are a couple of reasons. Circle theorems use all the theorems from previous topics like triangles – particularly similar triangles. Also, circle theorems come at the end of the curriculum. I think that many teachers rush through circle theorems to get to other subjects. So, they go through it quickly. I don't think that the learners get it deeply. I don't want to speak badly of other teachers, but I think that some rush through circle theorems because they, too, have difficulty with them.*"

"*I have doubts that only 10 lessons using GeoGebra will have much effect on the learners. It just doesn't seem like enough.*"

"*I confess that I am uncomfortable knowing that I will be doing the GeoGebra activities with the learners. I don't know any GeoGebra, and we don't use that much technology.*"

During the research activities (after six of the 10 activities)

"*These activities are much easier than I thought they were going to be. I mean, solving the maths in the activities challenges me some, but applying the tools in GeoGebra is much easier than we thought it was going to be. I am starting to like using the software.*"

"*I think that I am learning a lot. I thought I knew this material. But I think that the activities are revealing gaps in what I know. They make me go beyond my notes, and I have to apply what I know in new ways. I think that using GeoGebra is helping me visualise the theorems we are using. I mean, I understood them before, but GeoGebra makes the figures come alive and gives meaning to the verbal theorems.*"

"*The learners seem to be doing very well. They take on to GeoGebra much faster than I do. But they take on all technology much faster than I do.*"

After all the activities

"*I am shocked. Really. I have learned so much. I see these geometry ideas better than ever before. I can't believe how only 10 activities have helped me connect some of these geometric ideas for the first time. I guess that I never fully realised how the circle theorems are all applications of triangle theorems. I want to do more of these activities. I can't believe that I've been teaching without technology.*"

"*The words in the theorems now make more sense. I knew the theorems before because I could prove and use them. But now, GeoGebra lets me see figures and numbers change dynamically. Is it too strong for me to say that I can now experience the theorems?*"

"*At first, I didn't even know if I would be able to learn in this way. But now, I think that I could teach using GeoGebra. I would probably still need to start with activities made by others. But I think that I could move beyond that and develop my lesson activities.*"

"*As I did the activities and observed the communication and collaboration among the learners using GeoGebra, I saw that they were having the same experience as I was, only more so. They discussed the dynamic mathematics they were experiencing. Their communication was far more sophisticated than the other group. They even dared to independently experiment with ideas and test to ensure that their interpretations of the verbal theorems were correct. I had never seen my learners so collaborative, communicative, and inquisitive. I could see how the dynamic nature of GeoGebra allowed them to connect ideas between the verbal state-*

ments and the figures and construct their understanding."

"*I am excited for my learners. This is a great learning tool. But, I confess that I am also excited for myself and the potential for both my learning and my teaching.*"

Discussion

Learner Results

The results from this study suggest that GeoGebra-based mathematics lessons can be effectively used in a high-poverty, rural teaching and learning environment and have a positive impact on learners' learning in respect of the content and context of this study. The GeoGebra lessons provided opportunities for learners to verify mathematical relationships and conditions by exploring, observing, and justifying multiple Euclidean geometric properties. As recounted by both Hohenwarter and Jones (2007) and Yildiz et al. (2017) in previous research, this enabled the learners to further check and prove all features dynamically with immediate feedback (Habree, 2009; Venema, 2013; Weber, 2013), supporting the notion that the effective use of technology as a learning tool can shift learner experiences from the mere memorisation of facts to the actual understanding of information (Akkaya et al., 2011). Furthermore, results may indicate that educational technologies, as suggested by Department of Basic Education, Republic of South Africa (2015), KwaZulu-Natal Department of Education (2014), Ndlovu (2014), and Ndlovu et al. (2013), may be supportive and transformative to learners and teachers. This may be all the more imperative in resource-limited rural contexts.

From the perspective of the Cognitive Theory of Multimedia Learning (CTML) (Mayer, 1997, 2002, 2009; Mayer & Moreno, 2003; Mnguni, 2014; Venema, 2013; Vilardi & Rice, 2014), the use of GeoGebra allows learners to synthesise auditory and visual information and more efficiently interact with new information. GeoGebra facilitates the construction of new and concretised cognitive representations.

Unlike traditional teaching approaches where learners only draw shapes that many times are inaccurate and distorted and focus less on their specific conditions, the experimental learners used angularly precise GeoGebra constructions to prove and observe relationships under particular conditions. Thus, learner learning transitioned to exploring and proving theorems with less emphasis on drawing and memorising particular conditions.

Unlike the work of Hohenwarter and Jones (2007) and Yildiz et al. (2017), which was based on pre-service teachers and in urban settings, this study's findings corroborate those of Christou et al. (2007), Jezdimirović (2014), and Ogbonnaya and Chimuka (2017), that the use of GeoGebra can improve learners' abilities to filter, select, organise, and interact with new ideas and integrate notions expressed in various representational forms into novel cognitive structures. As such, the use of GeoGebra may be valuable to rural mathematics learners by creating conditions where learners can develop higher-order and more analytic thinking.

The experimental group learners did well in terms of the understanding of Euclidean geometry theorems and proofs compared to the control group, as was evidenced by their performance on the test. The lessons created in the GeoGebra environment appeared useful and enhanced learner reasoning and understanding of the content taught. Thus, supporting Ronan's (2008) findings, in this study the use of GeoGebra seemed to promote learners' logical reasoning and abstraction. In fact, despite the learners having only limited exposure to technology, it appears that GeoGebra had a positive effect on learner learning of Euclidean cyclic geometry.

GeoGebra proved to be user-friendly with a few minor technical challenges that were easily resolved through tutorials. The software encouraged a learner-centred approach to learning where the learners were motivated to see what would happen next (Agyei & Benning, 2015; Bos, 2009). While not discernible in the learners' data, and only recognised in the teacher's responses, the use of GeoGebra also encouraged learner engagement in, and communication regarding, their interactive experiences with the application. This further confirms that the use of GeoGebra had far greater implications than simply regarding the learning of lessons content. The experimental group of learners seemed to have a greater ability to solve Euclidean geometry problems, and their confidence and willingness to conjecture, experiment, and verify were improved.

Classroom Teacher's Beliefs

In this study, through the brief transcripts provided, the changes in the teacher's beliefs were apparent. In respect to learner learning, the teacher progressed from mistrusting what could be accomplished through GeoGebra activities to being a true devotee after her interaction with the software and observing her learners' interaction with the software. Through the lens of the CTML (Mayer, 1997, 2002, 2009; Mayer & Moreno, 2003; Mnguni, 2014; Venema, 2013; Vilardi & Rice, 2014), as with the learners, the teacher reported the value of the use of GeoGebra in helping to interpret, analyse, synthesise, and interact with ideas as well as to develop cognitive structures integrating ideas across representations.

Notably, the progression in the teacher's beliefs was the product of only 10 hours of teacher participation in GeoGebra activities. The teacher received no instruction in the use of GeoGebra beyond what the learners received. The teacher was not responsible for designing the curriculum or lessons that were used. While we believe the change in this teachers' beliefs is significant, more rigorous analysis should be completed through future research. This is particularly true in that it seems at odds with the findings of Stols et al. (2015) who recognised a reluctance among participating teachers to adopt the use of instructional technology.

Results from the investigation of the participating teacher's beliefs may have implications toward a beneficial way to support teachers' implementation of new technologies. Rather than extensive professional development with the hopes of converting teachers into technological experts prior to the instructional use of the technology, it may be sufficient to allow teachers to begin interacting with the technology alongside their learners; this can then lead to scaffolding through which the teacher eventually learns to design and implement lessons using the technology. Notably, the knowledge and skills associated with teaching using a technological tool differ greatly from using the tool as a learner. Nevertheless, teachers using technology alongside their learners may be considered a minimal investment in respect to time and resources to produce effects similar to professional development and provide teachers with initial insight into moving from traditional teaching to more learner-centred, technologically intensive investigations.

Conclusion

Based on the findings in this study, even rural, high-poverty learners can make gains in content knowledge and motivation when using GeoGebra. The difference in the mean of the test results indicate that there was a statistically significant difference between post-test results between the control and experimental groups. The difference in the scores shown in the mean values of the groups provided the researchers with a reason to justify the continued use of GeoGebra to teach Euclidean geometry in rural schools rather than limiting it to more developed urban and suburban classrooms.

In addition to the previously mentioned statistical results, the researchers also casually observed that learners participating in GeoGebra investigations were highly engaged, interactive and motivated. This may suggest that GeoGebra may have far-reaching implications for learner learning of Euclidean geometry. It may further indicate that multimedia and technological tools can and should be integrated into mathematics instruction, especially for rural schools, leading to possibly enhance learners' higher-order learning skills. Recalling that this study employed only minimal uses of GeoGebra and produced positive results among the high-poverty, rural learners involved, it can be hypothesised that more extensive exposure through well-planned investigations using GeoGebra could accentuate learning and may have the potential to accelerate learner learning by engaging them in meaningful tasks that encourage them to take more risks when learning, particularly when using GeoGebra.

As previously discussed, the classroom teacher involved in this study had only minimal previous involvement with GeoGebra. Nevertheless, this limited exposure led to appreciable, positive changes in the teacher's beliefs regarding the use of GeoGebra as a tool for teaching and learning. This demonstrates that teachers' beliefs are not fixed and can change. As this was the case through limited interaction with GeoGebra in this study, we wonder how more extensive interaction would affect teacher beliefs. More investigation needs to occur to know if these beliefs are sustainable, how to sustain them, and how to support teachers as they navigate successful lesson design and incorporation of this tool in their classrooms.

Research Limitations

The positive results of this parallel investigation on learner learning and teacher beliefs through the use of GeoGebra in the investigation of circle theorems was based on lessons that were carefully designed using GeoGebra activities. We do not claim that haphazard use of GeoGebra would produce such results – nor even that it would not produce negative results to either the learner or the teacher.

Due to the sample size of rural learners used as respondents in this study, selection based on a non-random sampling technique, the researchers caution the generalisation of the findings to all rural mathematics teaching scenarios. Additionally, the teacher perspective is based singularly on one teacher's experience. This makes generalisation quite problematic. However, the evidence is promising and additional studies should be designed to further investigate or replicate these findings.

Authors' Contributions

MM collected data. MM and AB wrote part of manuscript. AB and MJB wrote part of the discussion and results. AB, MJB and DW reviewed the final manuscript. AB handled all technical details.

Notes

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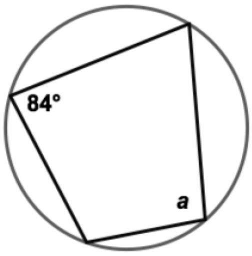
Appendix A

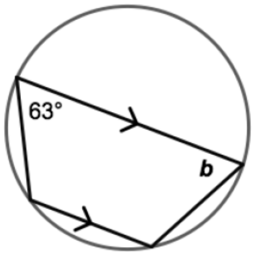
Work out the following problems and choose the correct item that corresponds with the statement.

1. The opposite angles of a cyclic quadrilateral are:
(a) parallel (b) complementary (c) supplementary (d) perpendicular

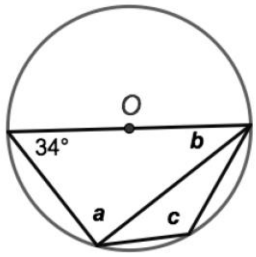
2. Refer to the figures below.
2.1. The measure of ∠a is
(a) 42º (b) 84º (c) 96º (d) 168º

2.2. The measure of ∠b is
(a) 63º (b) 117º (c) 126º (d) 36º

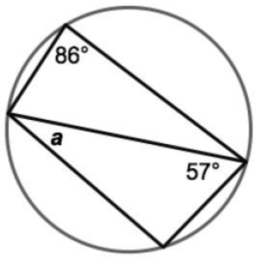




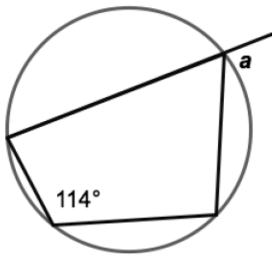
3. Referring to the adjacent figure, the measure of ∠b is
(a) 34º (b) 146º (c) 90º (d) 68º



4. Referring to the adjacent figure, the measure ∠a is
(a) 94º (b) 58º (c) 51º (d) 29º



5. Referring to the adjacent figure, the measure of ∠a is
(a) 114º (b) 66º (c) 33º (d) 228º



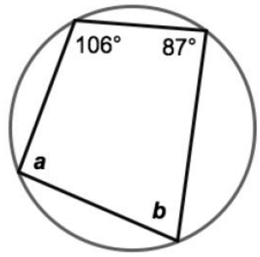
6. Refer to the adjacent figure to answer the following questions.

6.1. The value of a is

(a) 87º (b) 74º (c) 93º (d) 106º

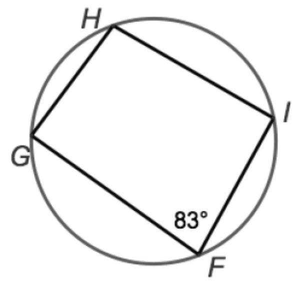
6.2. The value of b is

(a) 74º (b) 87º (c) 93º (d) 106º



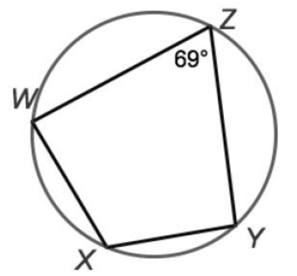
7. Using the adjacent figure, the measure of ∠GHI is

(a) 83º (b) 97º (c) 63º (d) 166º



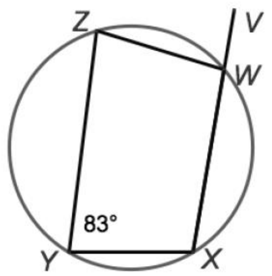
8. Referring to the adjacent figure, the measure of ∠WXY is

(a) 69º (b) 138º (c) 34.5º (d) 111º



9. Referring to the adjacent figure, the measure of ∠ZWV is

(a) 97º (b) 83º (c) 166º (d) 194º



10. Using the adjacent figure, the measure of ∠ADC is

(a) 54º (b) 108º (c) 72º (d) 144º