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STUDENTS' POSITIONING AND EMOTIONS IN LEARNING GEOMETRIC DEFINITION

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Abstract

The purpose of the present paper is to study the positions and emotions of grade 7 students who work with technology to learn geometry. This consideration of students' emotions is socially based, which makes it necessary to use a socially-based theoretical framework in order to study them. One such theory is the discursive analysis framework suggested by Evans, Morgan, and Tsatsarony, which is utilized in the present paper to analyze the positioning and emotions of fifteen groups of grade seven students who utilized technology to investigate the circle topic. The findings show that the group leaders took their positions through knowledge, action, initiation, persistence and meta-processes, while the followers of directions took their positions by accepting the group leader's requests. What most distinguished the collaborator was the communication with the other members of the group. Furthermore, the insiders used pronouns that indicated their inclusion. The results show that technology nurtured students' positive emotions as a result of nurturing their positioning throughout the investigation of the circle topic.

Keywords: Discursive Analysis, Students' Emotions, Students' Positions, Geometry, Technology

Abstrak

Tujuan dari makalah ini adalah untuk mempelajari posisi dan emosi siswa kelas 7 yang bekerja dengan teknologi untuk belajar geometri. Pertimbangan emosi siswa ini berbasis sosial, yang membuatnya perlu menggunakan kerangka teori berbasis sosial untuk mempelajarinya. Salah satu teori tersebut adalah kerangka kerja analisis diskursif yang ditemukan oleh Evans, Morgan, dan Tsatsarony, yang digunakan dalam makalah ini untuk menganalisis posisi dan emosi lima belas kelompok siswa kelas tujuh yang menggunakan teknologi untuk menyelidiki topik lingkaran. Hasil penelitian menunjukkan bahwa pemimpin kelompok mengambil posisi mereka melalui pengetahuan, tindakan, inisiasi, kegigihan, dan meta-proses, sedangkan pengikut arahan mengambil posisi mereka dengan menerima permintaan pemimpin kelompok. Yang paling membedakan kolaborator adalah komunikasi dengan anggota kelompok lainnya. Selanjutnya, anggota tim menggunakan kata ganti yang menunjukkan inklusi mereka. Temuan ini menunjukkan bahwa teknologi memelihara emosi positif siswa sebagai hasil dari memelihara posisi mereka selama penyelidikan topik lingkaran.

Keywords: Analisis Diskursif, Emosi Siswa, Posisi Siswa, Geometri, Teknologi

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Students' positions and emotions in educational contexts are attracting the attention of educational researchers in the last two decades (Evans, Morgan, & Tsatsaroni, 2006; Harré & van Langenhove, 1999), where the interest in these two educational aspects is due to the positions' and emotions' influence on other aspects of students' learning. For example, students' positions influence their educational identity (Hickey, 2011), levels of their learning engagement (Hickey, 2011), their interpersonal, intergroup and intrapersonal actions and interactions (Harré & van Langenhove, 1999), and, as a component of the social aspect of learning, students' cognition and behavior (Daher, 2013). On the other hand, students' emotions attract the attention of mathematics educators because of their relation with students' achievement in mathematics (Kleine, Goetz, Pekrun, & Hall, 2005), students' experiences in learning mathematics (Sumpter, 2010), mathematical sophistication in problem solving (Bradford & Carifio, 2007) and the carrying out of assignments with tools (Thiel, 2008). In the following, we describe in more detail the roles of positioning and emotions in mathematics education.

Harré and van Langenhove (1999) suggested positioning as a more dynamic and flexible form of social role than identity. Conversations are on-going discursive practices in which participants' roles are subject to change as conversations evolve. Participants in conversations take on certain roles, such as speaker, active or passive listener, opponent of the issue being discussed, and so on. However, the participants may not keep the same role from the beginning to the end. So, the participants assume different kinds of roles during the conversation, where these roles change according to the context and to power relations inside the conversation. Considering changes in participants' roles, it seems quite relevant to use "positioning" in order to describe the dynamics of the discursive practice.

Positioning has been used in mathematics education as a tool to analyze teaching as well as learning processes and identities of students and teachers. Esmonde (2009) considers access to discourse practices as related to access to positional identities, where this access places mathematics students as authoritative and competent members of the group as learners of mathematics. The two previous discursive aspects; access to discourse practices and access to positional identities, constitute students' opportunities to learn mathematics.

Langer-Osuna (2015) investigated a 9th grader positioning within his group and patterns of his engagement in mathematics project-based learning. In addition, the student's shift of positioning was examined across the academic year. This shift was performed through the utilization of classroom resources to serve both project-related and social functions in the process of interaction with peers. Yamakawa, Forman and Ansell (2005) found that the teacher, through revoicing and through commenting on students' characteristics - as courage, persistence, and flexibility, positioned specific students as active members of the classroom community. On the other hand, Skog and Andersson (2015) used positioning as an analytical tool for investigating teachers' identities. They found that institutional conditions may constrain the possibilities to deepen and develop identities of becoming mathematics teachers. They concluded that power relations and the becoming teacher's positioning in social settings are critical for understanding becoming teachers' identities.

Emotion is a fundamental element of the affective domain (Hannula, 2004; McLeod, 1992). McLeod (1992) identified three constructs of affect in mathematics education: beliefs, attitudes and emotions. He described emotions as the most intense and least stable. Hannula (2004) pointed at emotions as related to personal goals, which means that the achievement or the success or failure to achieve personal goals affects emotions positively or negatively. Moreover, the appropriate management of emotions could result in effective thinking rather than constrain it (Antognazza, Di Martino, Pellandini, & Sbaragli, 2015). Furthermore, Bradford and Carifio (2007) say, based on the literature that emotions can organize, focus, disrupt, distract or energize problem solving which would influence one's mathematical powers. The previous description indicates that emotions can serve cognitive as well as meta-functions in managing students' problem solving.

Different theoretical qualitative frameworks were suggested to analyze students' emotional expressions during their mathematical learning. DeBellis and Goldin (2006) suggested considering affect as an internal representational system. Roth and Radford (2011) suggested considering emotions as produced through the system of activity. A third framework utilizes the reversal theory approach (e.g., Lewis, 2013) that connects emotions with motivation. A fourth framework is the discursive framework as named by Evans, Morgan and Tsatsaroni (2006). This framework relates emotions to positions taken by students in the learning group, and is grounded in social semiotics and discourse theory with sociological, semiotic and psychoanalytic perspectives. We adopt in the present research the fourth framework, as we assume that emotions are socially based. Doing so, we consider emotions as connected with power relations in the group of learners. Here, students' discursive positions and emotions are claimed dialectically during the classroom discourse when they study geometric concepts in the presence of technology, specifically GeoGebra. Understanding students' social and emotional activity enables to better understand the cognitive and meta-cognitive aspects of their learning of mathematics (Daher, 2013). One way to get engaged in this understanding is through the discursive analysis framework.

The discursive analysis framework considers emotions and cognition as related, and emotions as socially organized phenomenon that participates in constructing and maintaining students' social identity (Evans, 2000; Evans, Morgan & Tsatsaroni, 2006). The analysis of learners' mathematical emotions and positioning, according to this framework, takes into consideration positions available to the mathematics learners through their learning practices, where those positions enable and constrain learners' emotions, and thus these emotions are considered as shaped by power relations in a small group or the whole-class situation. Evans, Morgan and Tsatsaroni (2006) describe two phases in the analysis method of students' positioning and emotions: the structural and the textual phases. The structural phase identifies, using Bernstein's sociological approach, pedagogic discourse positions available to mathematical learners in a specific educational setting. The textual phase examines the use of language and other signs in learners' interaction and describes the positions taken up by them. At this phase, the analysis focuses on indicators of emotions, like excitement and anxiety, linked to participants' positions. We describe below the discursive analysis basis and phases.

Discursive analysis is related to discourse. A discourse is a system of signs with which the participants can construct social meanings and identities, experience emotions, and relate actions. The first phase of discursive analysis, as described above, is the structural phase that examines the positions taken by the learners in the discourse. Positioning is defined as a process where a participant claims one of the positions made available by the discourse at a specific context. This results in the mutual influence of the individual and the social, where the social setting constrains the individual, but the individual keeps a degree of agency that enables the positioning in available or produced positions. According to this framework, a person's identity comes from repetitions of positions and related emotion (Evans, 2006).

Evans, Morgan and Tsatsaroni (2006) describe the positions taken care of in the structural analysis: Helper and seeker of help; where the helper is positioned as more powerful, collaborator and single worker, director of activity and follower of directions; where the follower is less powerful, evaluator and evaluated, and insider and outsider. Moreover, thee participant can claim more than one position, if in one discourse or in different discourses.

Evans (2003) says that it is not clear to what extent the above positions are associated with the criteria of the official classroom discourse or with discourses in which the students participate outside the classroom. The present research assumes that these positions could be found in groups' learning of mathematics, and in particular, when students learn mathematics using technology. It is so because technology is a tool that could empower students and help them position themselves as independent learners. This empowerment is the essence of students' positioning, as them become insiders to the group learning and also expected to be accompanied by positive emotions as pride and content.

The structural analysis helps examine the participants' emotions since positioning affects these emotions. Moreover, positioning is not permanent, not completely determined, nor freely chosen, where participants are constrained and enabled by their personal histories and the discursive resources available to them (Evans, Morgan & Tsatsaroni, 2006). Arriving at the participants' positions is the first step towards looking for the characteristics of these positions using different indicators. It is done in the textual phase of the discursive analysis which has two functions (Evans, 2006): (a) showing how positions in social interaction are actually taken up by the participants, and (b) providing indicators of emotional experience. Indicators of emotional experience can be divided into (I) those understood within the institutional and/or wider culture, drawing on the everyday culture of participants (verbal expression of feeling); behavioral indicators (facial expression, tone of voice); use of particular metaphors (e.g. a student claiming to be "coasting" in mathematics), and (II) indicators suggested by psychoanalytic theory, as indicators of defense against strong emotions like anxiety, surprising error in problem solving, behaving strangely (as laughing nervously), and denial (e.g. of anxiety).

Few attempts have been made to study students' emotions as related to their positioning. These attempts were made mainly by those who developed the framework and by the author of this paper and colleagues (e.g., Daher, Swidan & Masarwa, 2017). The present study is concerned with examining students' positioning and emotions in various groups of students. In Daher (2015), we analyzed only one group's learning when they utilized technology to investigate the circle topic. In the present study, we analyze the positions and emotions of fifteen groups of grade seven students who utilized technology to investigate the same topic. Doing that, we keep in mind the results of our previous studies, especially the factors that affect students' positioning and emotions when coming to learn mathematical topics. Two such factors are the type and functions of the group leaders, as well as the type of their processes (cognitive, meta-cognitive, etc.).

Becta (2009) identified six major ICT potentialities for learners to utilize in learning mathematics:

learning from feedback, observing patterns, seeing connections, exploring data, teaching the computer, and developing visual imagery. Researchers elaborated how technology assists students in building mathematical understanding as they interact with its tools (Noss & Hoyles, 1996; Nisiyatussani, et al. 2018; Nurwijayanti, et al. 2019). These tools facilitate the development of different and sophisticated mathematical ideas than when working with static representations (Heid & Blume, 2008; Noss & Hoyles, 1996). There are many different technological tools that students can use to learn mathematics, and each of these tools is likely to shape students' learning in a slightly different way. Assuming that technology offers new ways for students to understand mathematics, it is necessary to examine how students' learning is shaped by their uses of technology. In the present research, we examine how technology affects students' positioning and emotions. In addition to the previous potentialities of technology, it can provide opportunities for students to collaborate in learning mathematics, where students' collaboration in groups has attracted the attention of researchers in mathematics education (Hoyles & Sutherland, 1989; Noss & Hoyles, 1996, Septia, et al. 2018).

Working in groups provides students with assistance from each other, where stronger students can help those who are struggling (Webel, 2010). This assistance could indicate asymmetric relations between students, where some students take on an "expert" role, while their peers accept their authority (Esmonde, 2009). Moreover, dialogue and interchange in groups encourage mathematical critical thinking (Meyers, 1986) and the development of understanding (Schoenfeld, 1991). Furthermore, Students' experiences with collaborative learning formats, as part of their experiences in the mathematics class, can affect their identities as learners of mathematics (Boaler, 2002).

In addition to the above, technology assists the collaborative learning of mathematics. Baya'a, Daher & Mahagna (2017) reported positive effects of the collaborative use of GeoGebra on the development of seven graders' concept images of the angle, especially the dynamic one. Moreover, Naftaliev (2017) reported that 13- and 14-year-old students explored collaboratively mathematical models in electronic diagrams and developed shared knowledge of the abstract representations regarding the phenomena. In the present study we analyze students' positioning and related emotions when they collaborate to learn geometric definitions by utilizing technology.

The positioning and emotions phenomenon is a complex one which has different relationships that we tried to verify in previous research but on a small scale. Here we look at a larger scale that involves various groups in order to examine the relationships that Figure 1. The present research tries to verify this relations (Figure 1) and to shed light on them.

Emotion is distinct, but inseparable from cognition (Evans, Morgan, & Tsatsaroni, 2006). Furthermore, positions are one aspect of the social climate of learning, which influence students' emotional behavior (Daher, 2013). This influence is little studied from a discursive standpoint that combines the social and the emotional using different methodological methods. This standpoint will be used in the present research to analyze the properties of the mathematical discourse, in terms of positions and related emotions, when middle school students consolidate their knowledge about the circle topic in the presence of technology. It will be done looking at the learning of fifteen groups, which would shed more light on power and other social relations between mathematics students learning in a group, as well as their associated emotions, when they learn a geometric topic in the presence of technology.

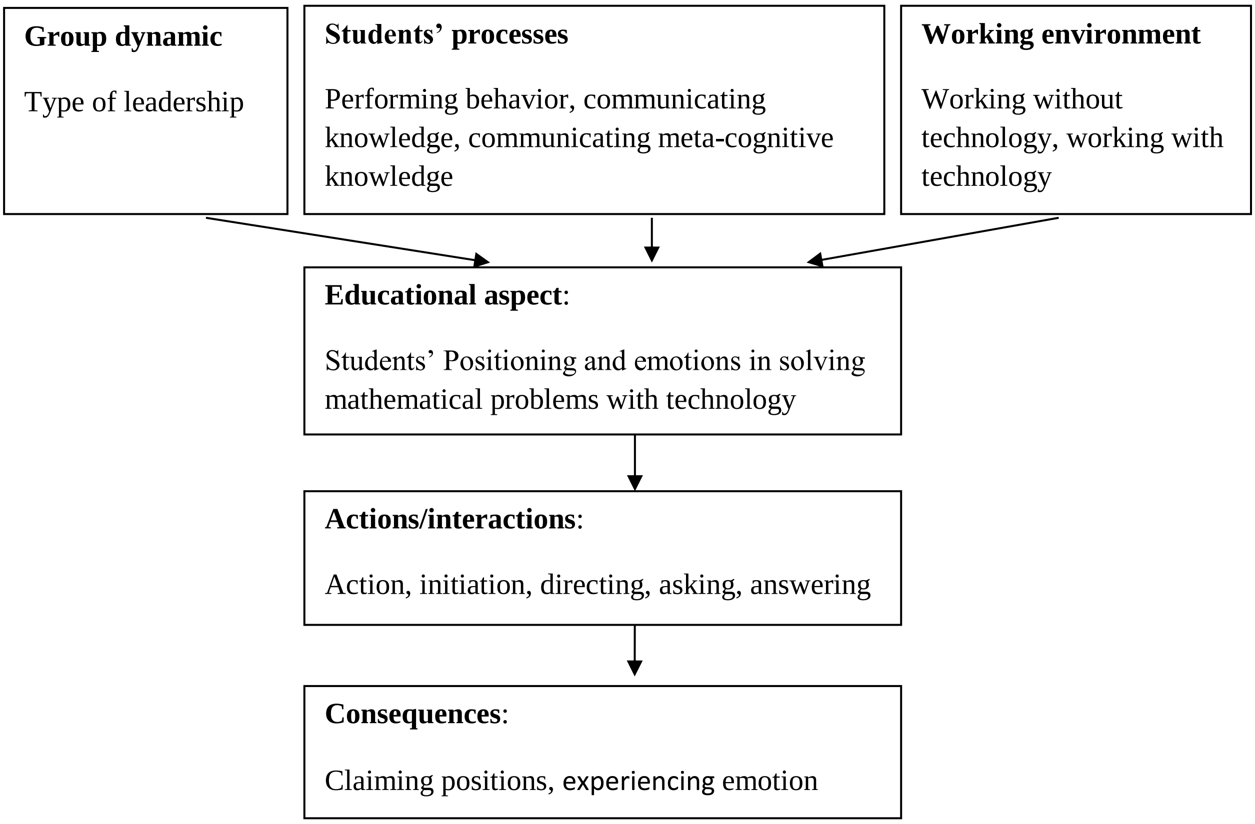

Figure 1. Conjectured relationships associated with the positioning and emotions phenomenon

This goal meets calls from mathematics education researchers to move beyond distinguishing between positive and negative emotions (Martínez-Sierra & García González, 2014). Martínez-Sierra and García González (2014) argue, depending on Lewis (2013), that this has not been done because it is more difficult to have a solid basis for qualitative results concerning emotions than for other affective constructs. At the same time, it is methodologically easier to study affective phenomena quantitatively, especially emotions phenomena; what lessened the study of this phenomena qualitatively. Here we attempt to study qualitatively students' emotions by associating their positive and negative emotions with the positions that they claim in their group learning. In previous studies, we studied one group's participants' positioning and emotions while using GeoGebra to learn geometric definitions (Daher, Swidan, & Shahbari, 2015; Daher, Swidan, & Masarwa, 2017). Here we look at the positioning and emotions of various groups, which makes it possible to reach more general conclusions than in our previous attempts. In addition, this would enable arriving at the comparison of the positions and emotions in groups of different characteristics; in the present study, these characteristics are related to the leadership style in the group. Therefore, this study describe the positions characterize that students undertake when they learn geometry with technology and the characteristics of students' emotions in each of the positions that they undertake when they learn geometry with technology.

METHOD

The present research is part of an ongoing research that examines middle school students' positions and emotions, when they study mathematics with technology. These students work in groups of 2-4 students each, where the participation in the research, the number of members in a group, as well as its formation, were decided by the students themselves. The only condition was that the group consists of 2-4 students each. Our use of the 2-4 member groups agree with group settings described by previous researchers. These researchers described 2-4 member groups to be appropriate for group setting in the mathematics classroom (See, for example Gillies, 2003; Webb, Nemer, Chizhik, & Sugrue, 1998). Furthermore, this agrees with other researchers' report about the size of groups that participated in their research (See for example Lan, 2007; Swidan & Yerushalmy, 2014).

The present research reports students' positions and emotions in fifteen groups of seventh grade students who used GeoGebra to improve their knowledge of the circle topic, where each group consisted of 2-4 students. These students were between 12 and 13 years old, coming from different socio-economic status and different mathematics ability. Some of the groups were all males, all females or mixed.

Examining the group leaders, some groups had one leader, while other groups had two leaders. Having one or two leaders depended on the characteristics of the members themselves, where, as described above, the formation of the group was decided by the students themselves. Moreover, analyzing the leaders' discourse in the group, five types were found: the one leader collaborates with the other members, the one leader does not collaborate with the other members, the two leaders collaborate with the other members, one leader collaborates and the other does not, and neither of the two leaders collaborates with the other members. In addition to the previous description, according to the theoretical framework utilized in this research, positions are not fixed, but regarding the mathematical context in which the students were engaged, these positions did not change, but the students could have other positions in other groups. In addition, the learning experiences of the students could result in change in their positions in other mathematical contexts.

The theoretical saturation of sampling, according to the grounded theory (Glaser, 2001), was followed to ensure the appropriateness of the sample for the present research goals. This theoretical saturation is "the conceptualization of comparisons of these incidents which yield different properties of the pattern, until no new properties of the pattern emerge" (ibid, p.191). In our case, the pattern was the leadership types.

We used two collecting tools (observations and interviews) to examine the group learning of grade seven students of the circle topic. The fifteen groups, who participated in the research, were videoed and at the end of each lesson, the members of each group were interviewed regarding their positions and emotions during learning. The learning of each group was videoed using a computer program that captured the footage in two different windows, one each for the computer screen and for the student. The interviews were semi-structured and held with each student individually. Examples on the interviews' questions are: What was your role in the group during your learning the circle topic? What did you feel while collaborating in the group to define the center of the circle? Why did you have this feeling?

We analyzed the two types of collected data using the discursive analysis suggested by Evans, Morgan, and Tsatsaroni (2006), and which was presented above. First the videos transcripts were analyzed, and then the interviews transcripts and afterwards the two analyses were combined. Using this method, all the transcribed data were coded independently by two coders; one of them is the author and the other is a colleague in the educational department at the university where the author works.

The validity of the research analyzing procedure was guaranteed by our analysis method which ensured the theoretical saturation. This theoretical saturation was due to our taking account of all the possible values of leader's type in the group, which ensured that no new group with different leader's type (category) would appear (Strauss & Corbin, 1998).

Lincoln and Guba (1985) say that no validity exists without reliability, so the ensuring of validity also ensures reliability. This means that following theoretical saturation maintains not only the validity of the research procedure but also its reliability. Further, two experienced coders (one of them the author) coded the participants' positions and emotions, searching for the participants' position and emotion types. The agreement between the coders (Cohen's Kappa coefficient) (when satisfied) ensures the reliability of the qualitative coding.

Cohen's kappa (Cohen, 1960) was used as a measure of agreement between the two raters. Inter-rater reliability for the observation data was 90.7% for positions and 89.1% for emotions. For the interviews data, it was 88.8% for positions and 89.3% for emotions. The high percentages of agreement suggest a satisfactory reliability for each type of data. Following is a detailed description of the categorization made in the present research of students' positions and their emotions.

We describe the categorization of students' positions in the learning groups, which utilizes the description in Esmonde (2009).

1. Leaders and followers: group members were coded leaders when they were frequently deferred to (mathematically) and who were often granted authority to decide whether the mathematical work of the group was correct. A leader participant is accompanied by a follower participant. With no follower, there is no one to defer to the leader and to take up his or her ideas. A participant was coded as a follower when he or she was instructed by a leader and accepted these instructions.
2. Solitary workers: group members were coded solitary workers when they went through periods of individual work during their work to define geometric concepts. The distinguishing characteristics for solitary workers were a tendency for working individually at least part of the learning, not asking for help when needed, and refraining from help to other members who needed help with or without requesting assistance.

3. Collaborating participant: group members were coded collaborative participants when group members worked together, as for example completing an idea together. Collaborative participants contributed together to arrive at a geometric definition. Evidence for claiming the collaborative participant position was the group member's putting more than one idea forward for discussion, or if several people jointly arrived at a geometric definition.
4. The ‘helper’ participant: members were coded as helpers when the mathematical talk was asymmetrically organized, and a participant instructed other participants about what to do. In contrast with the collaborative participant, a 'helper' was characterized by the uncritical uptake of ideas. Evidence for claiming the 'asking for help' position was when the participant oriented his/her actions towards obtaining an answer from a more expert participant.

The categorization made in the present research of students' emotions utilized the multimodal analysis described in Sakr, Jewitt and Price (2016). This analysis attended to a range of multimodal aspects of the students’ emotions, as the use of language explicitly referring to an emotion (e.g., “happy,” “sad”); the use of a gesture or movement associated with an emotion (e.g., a hand raised in the air to indicate accomplishment or content); the presence of a facial expression that suggests an intensity of feeling (e.g., a furrowed brow). These three means of evaluating emotions; language, gestures and facial expressions, lessened the possibility of mistakenly recognizing a behavior as related to an emotion or an action as related to more than one emotion.

In the analysis, students' positioning were connected with their related emotions. We did that utilizing the categories described above of the two previous categorizations of positions and emotions. The connection was done utilizing the form represented in Table 1.

Table 1. The form for analyzing students' positioning

Raw	Participant	Conversation/ [action]	Position	Emotion
25	Haya	How can we set the radius of the circle? [Her facial expressions showed she was worried]	Asking for technical help	Worry
26	Janan	We extend a line from the center to the line of the circle. [She was sitting comfortably. Her voice tone showed she was happy to help]	Helper	Comfort, Happiness
28	Haya	[Pointing at the circumference on GeoGebra interface] To the circumference. [Her facial expressions showed she was content to correct Haya, while her voice tone showed she was enjoying doing so]	Leader	Content Enjoyment

RESULTS AND DISCUSSION

Grade seven students worked with GeoGebra to improve their knowledge about the circle and its components. Doing that, the members of each group took different discursive positions: Group leader, follower of directions, collaborator, insider and outsider. Students' positions and emotions associated with them are described below, with examples from the groups' work.

Group Leader

In nine of the fifteen groups, one student took the position of the group leader, where in three of them, the leader was authoritative; not a collaborator, while in the other six groups, the leaders were collaborators. Moreover, in six of the groups, two students took the position of the group leader. Table 2 details the discursive functioning of these two leaders in terms of their collaboration with the other group members, as well as the collaboration between them.

Table 2. Discursive positioning of the leaders in the two-leader groups

The group	Collaborating with the other group members	Collaborating with themselves
First group	Two of them	Collaborating
Second group	Two of them	Not collaborating
Third group	One of them	Collaborating
Fourth group	One of them	Not collaborating
Fifth group	Neither of them	Collaborating
Sixth group	Neither of them	Not collaborating

In all the groups, the group leader who collaborated with the other members of the group directed the work of the group towards performing specific mathematical actions. In addition, he/she manipulated the applet, initiated mathematical actions and participated actively in the mathematical discussions. Moreover, in all the groups, the group leader who did not collaborate with the other group members tried to solve the problem alone, manipulating the applet and writing down the answers to the activity questions.

In one of the collaborating-leader groups, the leader (Let's call her Haya), as Transcript 1 shows, initiated the exploration of the group (1), by telling the group's members (she and two other members) that they should follow the directions of the activity (1 and 5), and by using GeoGebra to drag the circle. Then she addressed Janan and Rana (the other two members of the group), and started to discuss the circle's center, but soon the conversation turned to be about the chord (6-11), the diameter (6-11), the secant (12-17) and the tangent (12-17).

Note: When describing the learning events, silence for m moments is denoted by [..m..].

1	Haya	The circle's center is ….
2	Janan	It is the point lying in the middle of the circle.
3	Haya	The middle …
4	Janan	It is the center.

5	Haya	(again): The circle's center is ….
6	Janan	Every chord that passes through it becomes a diameter.
7	Haya	A diameter? [..15..] What is a diameter?
8	Janan	It is this that passes through the circle.
9	Haya	It is this that passes through the center and the circle.
10	Janan	It is a line that passes through any part of the circle.
11	Haya	If it passes through the center it becomes a diameter [Haya uses the mouse to drag the circle and watch how the diameter and radius change] … the secant is like … it intersects the circle in two points.
12	Rana	The tangent surrounds the entire circle [Rana and Janan were looking at GeoGebra interface).
13	Janan	(Vehemently): Yeh [Haya dragged the tangent repeatedly].
14	Rana	The tangent is this that touches the circle.
15	Janan	(Again vehemently): It does not intersect the circle. It touches the outer line. [Haya continues to drag the tangent].
16	Janan	(Looking at GeoGebra interface with interest): When the secant touches the circle it becomes a tangent.
17	Haya	The secant is like … it intersects the circle in two points.

Transcript 1: The leader functioning

Haya claimed the position of the group leader, but generally the group worked as collaborators more than a leader and two followers. The collaboration occurred through a asking questions and answering them, in an attempt to agree on the definitions of terms associated with the circle. Haya directed the activity by persisting in asking questions and the utilization of GeoGebra to generate new examples of the circle and its elements. Haya's questions and actions led the group to improve their discourse regarding the concepts associated with the circle. These positions were accompanied by positive emotions, where the facial expressions of the group members showed that they enjoyed learning with GeoGebra on their own. This learning enabled them to improve, on their own, their knowledge of the circle topic, which empowered them, making them content and happy [interview]. Table 3 describes the functions of the collaborating leaders in the groups.

In the two-leader group, where the two leaders collaborated between themselves and collaborated with the other group members, the two leaders performed the functions in Table 3, where these functions were interchangeable between them. Following are some leadership styles, which give us a picture of how the individual leaders functioned and claimed leadership. Haya, a group leader, claimed authority mainly by action and knowledge through working with the applet, asking questions about the definition of concepts related to the circle topic, correcting the other two members' mathematical statements or actions and writing the 'assumed correct mathematical statements'. Wafa, another group leader, claimed authority by initiation (as asking question), action (as reading the activity requirement), requesting action (as requesting to manipulate the applet), and knowledge. Yara, a third group leader, claimed authority mainly by action and persistence (going back again and again to define a concept related to the circle topic). Sewar, a fourth group leader, claimed authority mainly through action and giving orders (as requesting to answer a question in the activity), as well as through demonstrating knowledge of geometry.

Table 3. Functions of a collaborating leader-Haya, Wafa, and Yare as leaders

The function type	The function in the specific context	Examples
Directing the learning	Giving orders	Haya: Wait till I finish writing.
Planning	Stating the next step in the group's work	Haya: Now we should work with the applet, and follow the directions in the activity.
Demonstrating knowledge	Correcting another member's mathematical statements or action	Haya: to draw the tangent, we do not draw a straight line totally outside the circle. It should touch it.
Evaluating through asking questions	Asking mathematical questions	Haya: What is a diameter?
Evaluating through rejecting other's statements	Rejecting mathematical statements	The same example in 'Demonstrating knowledge'
Requesting action	Requesting to manipulate the applet	Wafa, to Halim: Can you please drag the end point of the diameter to see what happens to it.
Initiation	Starting the discussion, summarizing the group work or discussion	Wafa: this dragging means that the diameter is the longest chord in the circle.
Doing	Controlling the work with the applet	Haya dragged the circle to get new circles and new measures of the diameter and radius.
Discussing	Participating and directing the discussion of the group's members	Haya: How can we set the circle radius? Janan: we draw a line from the center. Janan: no, a radius. Haya: a radius, how can we set a radius? Janan: we draw a line from the center of the circle to the line. Haya (pointing at the circumference in the applet): to the circumference
Writing mathematical statements	Writing the properties of geometric objects	After asking the group about the definition of the circle's center, and not getting an answer that satisfied her, Haya wrote on the paper what she thought the definition of the center: a point located in the middle of the circle
Persisting	Yara requested again and again that the group should define the circle center	Yara: We should go back to define the center, the first time we didn't finish to do that.

The work of Haya, as a leader of her group, made her enjoy the learning of the circle topic, as she declared in the interview: "I enjoyed that my questions led the group to understand the circle topic". This is the case of the rest of collaborating group leaders. The collaborating leaders had also negative emotions as a result of their discursive positioning. This is the case Sewar who said that she was uncomfortable because the other group's members did not collaborate with her [interview]. Moreover, the collaborating leaders had negative emotions because of personal characteristics or previous experience; i.e. as learners and not as a result of their discursive positioning. For example, the work of Haya with the computer made her a little uncomfortable, because she was used to stand and walk while she studied, as she admitted [interview]. In addition, Wafa; another collaborating leader, was a little uncomfortable with the activity as a result of being uneasy with geometric activities [interview]. The previous description substantiates that students' processes, the type of leadership and the working environment (here with technology) impact students' positioning and emotions.

Follower of Directions

The follower of directions generally followed the directions of the group leader towards performing specific mathematical actions. In the group of Haya, Rana and Janan, the last two took the position of the follower of directions. Doing so, as Transcript 2 shows, they answered the questions of the leader of activity (35, 36, 38), repeated her statements (40, 41), and nodded with their head as a sign of agreeing with the leader of the activity, or with each other (41).

34	Haya	What is the radius of the circle?
35	Rana	A line from the center of the circle …
36	Janan	The radius is a line that goes from the center to the circumference.
37	Haya	[Reads the next question]: What is the diameter of the circle? [The group members looked at the diameter that they drew].
38	Janan	It is a straight line that passes through the circle's center.
39	Haya	It is the chord that passes through the circle's center.
40	Janan	Yes, a chord that passes through the circle's center.
41	Rana	[Nodded with her head, also repeating Haya's statement]

Transcript 2: The follower functioning

Rana behaved as a follower. At the same time, Janan not only tried to answer the leader's questions and agree with her (40), but also contributed to building the definitions (36, 38). Table 4 describes the functions of the followers in the groups.

Sometimes the followers of directions expressed a negative emotion for being called back to engage in learning, as when the follower of directions did not enjoy engaging in geometric problems. This call was performed by the leader of the group, which shows that leader's actions impact the other group members' positioning and emotions. In addition, all the members' processes impact their positioning and, as a result, their emotions, which agrees with the theoretical framework assertion of the relationships between students' processes, positions and emotions.

Table 4. Functions of the followers of directions in Haya's group- Rana and Janan as followers

The function	The function in the specific context	Examples
Answering questions	Answering mathematical questions	Haya: what is the center of the circle? Janan: It's the point in the middle of the circle, and each chord that goes through it becomes a diameter.
Performing requested actions	Performing geometric calculations	After Haya explained how to find the circumference of the circle, she asked Rana to do the calculation for a circle in the applet on the paper, which Rana did.
Repeating statements	Repeating statements related to the influence of dragging the circle on its circumference and area	Haya: what happens to the circle when we drag it? Janan: the center does not stay the center. Haya: sometimes it gets larger and sometimes smaller. Janan: sometimes larger and sometimes smaller. Rana: when you enlarge it gets bigger.
Nodding with the head	Nodding to agree with a mathematical claim	Haya: there are infinite number of radii Rana: (nodding with her head): Yes.

Collaborator, Insider, and Outsider

The collaborating members in a group participated to internalize the topic of the circle. To do that, as Transcript 3 shows, they tried to refer to each other sayings (40), building upon each other sayings, where each one built on the precedent's statement (41-45)

39	Haya	[Reads the next question]: What is the relation between the chord and the diameter?
40	Rana	The relation is ... [..3..] the diameter is a chord.
41	Haya	The diameter is in origin a chord passing through the circle's center.
42	Janan	If it does not pass through the center it will be a chord, but if it passes through the center it will be a diameter.
43	Haya	The diameter is a straight line that starts at the circle's circumference and passes through the circle's center.
44	Rana	It ends at the second side of the circle. [Rana's and Janan's facial expressions showed satisfaction]
45	Haya	[Wrote]: the diameter is a straight line that starts at the circle's circumference, passes through the center and continues to the other side of the circle.

Transcript 3: The collaborator functioning

Working as collaborators led the group to agree on one of the definitions of diameter of the circle. Here, Haya was the one to ask questions (39) and write answers (45). Rana and Janan, as contributors to the knowledge building of the group, increased their power, which resulted in their satisfaction, as appearing in their facial expressions (44).

Table 5 describes the collaborator's functions in the groups. It includes the type of the collaborator's function, the function in the specific context, and an example from the groups' work. These functions were specifically fulfilled in two groups with collaborating leaders.

Table 5. Functions of the collaborator

The function	The function in the specific context	Examples
Asking and answering questions	Asking and answering a question about a geometric object that could be related to another	Wafa: the diameter is the segment that passes through the circle's center. What about the chord? Yara: The chord passes in any part of the circle.
Requesting and agreeing to do actions	Requesting and agreeing to investigate the consequences of changing a geometric object	Wafa: change the center to see what happens. [Yara dragged the center of the circle, while the group members watched the consequences].
Agreeing with another member's statement, with or without withdrawal/ modification	Improving a geometric statement about a geometric object	Haya reads: what can you say about the radius? [Silence] Rana: it is half of the diameter. Haya: it is half the length of the diameter. Janan: it is half the length of the whole diameter.
Looking together at the computer screen	Watching the consequences of a geometric action	Haya: the question asks what happens to the circle when we drag it….. Let us drag it. [The three girls looked at the computer screen, watching what happens to the circle.

The collaborating group leader, as in the case of Haya, referred to the sayings of the other group members less than they referred to hers, which could mean that she attempted to encourage the participation of her group members in the geometric activity. Moreover, the collaboration of the group members varied. For example, one member of Haya's group (Janan) collaborated more than the other member (Rana), indicating that she (Janan) was more an insider than the other member. Different reasons influenced the participation of the collaborators, where the educational setting, personal characteristics, previous experience and leadership styles were the most apparent reasons behind the collaborators' discursive positioning and associated emotions. More elaboration on the issue follows.

The relatively little collaboration of Rana could have occurred due to two reasons. First, she had some difficulty in her learning due to previous experience (coming newly to her present school from a school that did not teach enough geometry), and second due to personal characteristics (being weak in English; the language of the applet). When interviewed, Rana said: "it is uncomfortable to work with applets in English, because my English is poor". This means that the educational setting (applets in English) affected negatively Rana's learning emotions. From the other side, Rana felt content and happy because her participation in the activity was facilitated by the group work and by the use of technology, which made her understand the topic. In the interview, she said: "I am content because our work together and GeoGebra helped us understand all about the circle, while we do not understand geometry when the teacher teaches us. Here we knew what to do". Rana's use of the pronoun 'we' indicates that she considered herself an insider. This probably suggests that Rana will become more of an insider in future group work than she was here for she liked this work. Regarding the use of technology, the work with the applet made Rana curious to study the circle topic. In the interview, she said: "the work with the applet made me curious to learn the circle topic … the applet enabled us to experiment with the circle". The use of the pronoun 'me' indicates that Rana wanted to describe the curiosity as her own experience, while her use of the pronoun 'we' indicates that the applet facilitated the work of the whole group, which supported them being collaborators. Thus, technology potentiality to support students' experimenting encouraged the inclusion of the group members in the activity, which resulted in their positive emotions as content. This conclusion is supported by the declaration of Sana (an insider) in the interview: "in the sixth grade I did not understand the circle topic. In this activity, I felt content because the applet made us understand it". The use of the pronoun 'I' probably indicates Yara's intention to talk about a special experience she went through. All the previous argument proves the influence of the educational setting on available students' positioning and related emotions.

In one group, the three members were not collaborators, probably because the leader of the group was not a collaborating leader, which was represented in her little reference to the sayings of the other two members, trying to answer the activity questions alone. This situation could indicate two possibilities: (1) the leader of the activity was not used to working in a group, or/and (2) the leader of the activity pre-evaluated negatively the mathematical work of the other two members. In the described group, the leader used only singular first and second person pronouns, when communicating in the group, to refer to herself and each of the other members respectively. This little collaboration made the members feel uncomfortable. When interviewed, Salim (a member of the described group) said: "I did not feel comfortable in the group … Sewar was dominant and Sandra did not participate". Probably, Sewar and Sandra, being outsiders, made Salim feel an outsider too. This positioning resulted in a negative feeling; specifically being uncomfortable.

Here too, the previous description substantiates the hypothesis that students' processes and the type of leadership impact students' positioning and emotions. Furthermore, students' processes were, in this case, in mutual relationship with the learning environment that included technology.

It was our intention in the present research to characterize the positions and related emotions of seven grade students who work with technology to learn geometric definitions. Below, we discuss the research results through discussing what impacted each position taken by the members of the various groups, as well as the emotions associated with each position.

Group Leaders

The research results show that action was the main means to claim authority in all the groups, while knowledge was so in twelve out of the fifteen groups. Knowledge was acknowledged as means of claiming authority in other studies (see for example Evans, Morgan & Tsatsaroni, 2006), but in our case action was as important as demonstrating knowledge. A distinguished part of the group's action was done utilizing GeoGebra to notice geometric objects' behavior, geometric patterns and geometric relations. So, the presence of technology could explain the phenomenon that action turned to be the main means to claim authority in the present research. These findings point at technology as a main participant in students' social as well as cognitive learning of mathematics, where the behavioral aspect (students' work with technology) influenced the social aspect (their positioning), and as a result the affective aspect. Here, we can say that the work with technology (the behavioral aspect) affected students mathematical processes (the cognitive aspect), which affected their positioning (the social aspect) and, as a result, their emotions.

In addition to action and knowledge, initiation and persistence also helped maintain the position of the group's leader, where these behaviors support maintaining leadership (Bachiochi, Rogelberg, O'Connor, & Elder, 2000) and a successful student's life in general (Hirschy & Wilson, 2002). Furthermore, the two behaviors could be related to the 'task oriented skills' described as part of effective leadership (Bachiochi, et al. 2000). Other studies reported metacognitive processes as impacting the leaders' claim of their position (e.g., Daher, Anabousy, & Jabarin, 2018).

Only part of the group leaders admitted that they enjoyed being group leaders. The other leaders did not express emotions that could be related to their position as leaders. From the other side, some group leaders expressed negative emotions that could be related to this positioning, like being uncomfortable in leading the group's learning. In some cases, this lack of comfort is due to the other group members being not collaborators. Thus it could be claimed that the leading processes or events determined the emotions of the leader.

Other causes of the lack of comfort of the group members were personal characteristics as learners, though these causes could have been overcome if the learner was not a group leader who needs to act in a specific way like staying in front of the computer to manipulate the applet. This learner, if she was not a leader, would have acted in a way that resulted in her comfort. For example Haya would have stood and walked while she studied. Thus, we can say that the leading functioning, when contradicting with the personal characteristics could result in the negative emotions of the leader. Furthermore, we can say that personal characteristics moderated the relation of positioning and emotions. For example the inability to stay in front of the computer made the leader's emotions negative; these emotions that could have been positive, due to the leader being in control.

It can be concluded, from the above, that students' behavioral processes (manipulating the circle with technology), their cognitive processes (observing and being attentive to the change in the circle and its elements), the type of leadership and the working environment (technology-based) impacted students' positioning and emotions. The research results also indicate that the personal characteristics could also impact students' emotions while learning mathematics.

Followers of Directions

To realize the position of the follower of directions, the group member accepted the group leader request for action or for answering geometric questions. Tsatsaroni, Evans, and Morgan (2007) say that the relation leader-follower of directions makes implicit the hierarchical nature of the relationship between transmitter and acquirer. This is true generally, but the collaborating leader lessens the hierarchical nature of the relationship between her and the other group members, towards a more equal relation. This lessening of the hierarchical nature of the relationship between the leader and the follower could result in positive emotions of the follower for it empowers the follower, causing him/her to feel confident and content (Daher, Swidan, & Masarwa, 2017).

Sometimes the follower of directions had a negative emotion for being called back to the group learning. In the present research, this happened for personal characteristics as not liking to engage in geometry learning; an emotion held by other school students (Sunzuma, Masocha & Zezekwa, 2013). Nevertheless, if the same problem was related to real life and specifically to students' life, there is greater chance that the learner, in our case follower of directions, would be willing to engage with the problem, as students are enthusiastic and curious to work with such problems (Daher, 2012). So, the present research joins the call of researchers (e.g., Treacy & O'Donoghue, 2012) for integrating real life and authentic activities into students' learning of mathematics.

Collaborator, Insider, and Outsider:

What mostly distinguished the collaborator were the communication with the other members of the activity and the mutuality of relations with them, where this mutuality was represented in students' engagement with dualities like asking – answering questions related to the definition of the circle concepts and requesting – performing actions related to working with GeoGebra. This communication regarding the geometric concepts resulted in the mutuality in relations between the group's members, which helped maintain common ground (e.g. mutual understanding of how to go on discovering the definition of the circle's concepts) necessary for the success of the group's work (Cornelius & Boos, 2003).

Collaborators expressed positive emotions (as being content) as well as negative emotions (as being uncomfortable), where these emotions were influenced by different factors related to previous mathematical history and learning processes. More specifically, what made the collaborators uncomfortable were: previous mathematical history (here not being strong in geometry), and one property of the educational setting (the applets being in English). From the other hand, another property of the educational setting (learning in a group) made the collaborators content and happy. Furthermore, a third property of the educational setting (working with an applet) made the members of each group curious to collaborate in investigating and thus internalizing the circle topic, and, as a result, made the collaborators content and happy. It seems that two properties of the educational setting; working with an applet and the characteristics of the activity (here, being clear and gradual), encouraged the collaborators to participate in investigating the circle topic, and as a result they understood this topic. These two aspects of students' learning (working with technological tools and the properties of the activity) are reported to have influence on students learning (Baya'a & Daher, 2010). Moreover, the work with technology supported the participants in their geometric processes, and as a result, at arriving at the geometric definitions caused the collaborators to be happy and content for their activity and achievement. Thus, it could be said that understanding the geometric concepts mediated positioning (as collaboration) and positive emotions, as content and satisfaction.

Pronouns and use of verbs, as imperatives, used by the group's member, indicated whether he/she was an insider. Being an insider or outsider influenced the dynamic interchange during the progress of the group's discourse, as pronouns are "one of the main factors in maintaining a good interchange in a conversation activity" (El Saj, 2012). The use of the singular first person pronouns by the leader of one of the groups in her communication with the other members reflected the slowness of the defining process of the circle concepts. The previous implies that the collaborator's position and emotions were impacted by four factors: students' processes, leadership type, the learning environment and personal characteristics.

CONCLUSION

The current research utilized the discursive analysis framework to analyze the positioning and emotions of groups of seventh grade students while working with technology to investigate the circle topic. The research findings indicate that the students took the following positions in the groups: group leader, follower of directions, collaborator and insider and outsider. The group leaders took these positions by demonstrating knowledge, performing actions, initiation and persistence, while the followers of directions took this position by following the request of the group leader to perform actions and by answering questions. What mostly distinguished the collaborator were the communication with the other members of the activity and the mutuality of relations with them, where this mutuality was represented in the students' engagement with dualities like asking/answering. Moreover, the insiders' language included pronouns that indicated their inclusion in the group. Furthermore, it can be claimed that technology generally nurtured the participants' positive emotions towards their learning of geometry, for it allowed the participants to investigate mathematical relations on their own. To be more specific, the group leaders utilized GeoGeobra to lead the group into understanding the mathematical concepts, and thus into claiming their positioning by manipulating the geometric object, which indicates the role of the use of technology in leading the learning of the group. At the same time, GeoGebra encouraged the participants to be collaborators. This happened due to the support of the technological tool for the group members towards discovering mathematical relations, which made the participants value their being collaborators and, at the same time, have positive emotions about geometric learning. The previous argument shows that students' positions and emotions are impacted by the group dynamics, by the group processes; including their geometric processes, by the learning environment - here technology-rich environment.

In light of the above, in some groups whose members work together to learn mathematics, leaders could impact negatively the equality between the members in the group, which could impact negatively the engagement of some group members because their opportunity to perform geometric processes lessens. This situation of inequality in the group could result when the leader is not a collaborating one. This happened in some of the groups that we studied. The teacher needs to observe the group's working in order to suggest specific suggestions that improve the collaboration of the group, as well as trying to move the less collaborating leader into a more collaborating one. Thus, it is the present research conclusion that teachers need to encourage students to show initiation and persistence, for they help maintain a successful student's life in general and a way of maintaining leadership, in our case mathematics education leadership. This could be done by making teachers aware, through workshops, of values as initiation and persistence. Moreover, teachers should plan and prepare educational environment that are full of potentialities for students' actions and interactions, where some of these potentialities are exploration activities and tools that support the exploration of mathematical concepts and relations. These actions and interactions help maintain positive positions for students, and as a result, positive emotions towards mathematics and doing mathematics. Furthermore, technology can be means of encouraging students' being collaborators and insiders and thus have positive emotions. In addition, teachers, who work with technology, need to pay attention to students' working with the technological tool, so to encourage every member to manipulate the mathematical objects which the tool addresses. This would encourage their being collaborators and insiders. Taking that into account, it is important that the teacher does not impose positions on the students, because this intervention could impact the group dynamics negatively, which could impact negatively all the group's learning of mathematics. Nevertheless, the teacher should try to encourage equality between the group's members through encouraging their collaboration. It needs to be done smoothly.

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