ACTA DIDACTICA NAPOCENSIA
Volume 12, Number 2, 2019 - DOI: 10.24193/adn.11.2.10

INFLUENCE OF DRAWING AND FIGURES ON SECONDARY SCHOOL STUDENTS' ARGUMENTATION AND PROOF: AN INVESTIGATION ON PARALLELOGRAM

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Abstract. In this article, we wish to explore the influence of the figure of the drawing on the argumentation of students who are involved in a proof task. It is about analysing the knowledge that students associate with parallelograms and the interactions between students and drawing. Our research is based on both Toulmin's model and Vinner's concept image and concept definition. After decomposing down the students' arguments, we analyse the data in order to identify their origin and the element of the concept image mobilized in argument. Our data suggest that the students' personal concept definitions do not correspond to the formal definition of the figure, the drawing causes a conceptual change in the students' personal concept definition. The data resulting from the abusive interpretation of the drawing is found in both the students' argumentation and proof.

Keywords: argumentation, proof, drawing, figure, concept image and concept definition.

1. Introduction

Research in mathematics education has helped to bring out some cognitive aspects that emerge when students interact with drawings and figures. In particular, the distinction between drawing and figure in geometry have been studies. Parzysz (1988) reveals the existence of a conflict between what students know about a figure and what they see on the drawing. Other researchers highlight that some of the students' difficulties may stem from the teacher's use of so-called "prototype" drawings (Coppé, Dorier, & Moreau, 2005; Gobert, 2007; Laborde, 1994; Souvignet, 1994). Indeed, because students tend to build their knowledge by using their perception, the use of "prototype" drawings by teachers can induce didactic obstacles among students (Walter, 2001). Previous studies showed that the use of drawings in geometric problems may be difficult for students to understand, especially those that provide guidance on problem-solving strategies (Elia & Philippou, 2004). Research has also shown that definitions constructed by students during their schooling about figures often conflict with formal definition about them. These definitions are the manifestation of their concept image (Vinner, 1983).

In parallel, research in mathematics education (Garuti, Boero, & Lemut, 1998; Mariotti, 2000, 2001; Pedemonte, 2002), focusing on the comparison between argumentation and proof in mathematics, revealed that there is cognitive continuity between the two processes, called *cognitive unity*. In solving open problems, the students may produce argumentation to justify their conjecture. The hypothesis of *cognitive unity* is that to construct a proof, the students can organize the arguments previously produced to justify the conjecture into a logical chain (Garuti, Boero, & Lemut, 1998; Pedemonte, 2002). Pedemonte (2002) observes that there are two kinds of continuity that may be observed: continuity in the referential systems and structural continuity. Some shows that, during the exploration phase developed to look for a resolution strategy, argumentation is produced. The arguments produced in argumentation are probably reused during the proof.

Received March 2019.

Cite as: Tchonang Youkap, P., Njomgang Ngansop, J., Tieudjo, D. & Nchia Ntam, L. (2019). Influence of Drawing and Figures on Secondary School Students' Argumentation and Proof: An investigation on Parallelogram. *Acta Didactica Napocensia*, 12(2), 133-144, DOI: 10.24193/adn.11.2.10.

Several researches have been conducted on the impact of drawing in the writing of proof in geometry, it has focused on students' productions. But research on the influence of drawing and figure on students' argumentation in the exploration phase preceding the proof writing phase remains little known. Some authors have shown the conflict between drawings and figure in geometry, while other have parallel work on the cognitive continuity between argument and proof, but to the best of our knowledge no work has linked the effect of drawings to argumentation and proof.
This research provides researchers and teachers with information on the influence of drawing and figure on student reasoning and will be useful to them in choosing which drawings to use in developing geometry problems that could lead to mathematical proof learning. Secondly, it shows the need take into consideration students' prior conception of the figure, as it will impact their argumentation and proof.

Our objective in this article is to highlight the interactions between students and objects studied in geometry at secondary school (drawing and figure) in a problem-solving situation that leads to proof of the nature of a figure. We take into account the following elements, the influence of drawing in the students' argumentation and proof, the part of concept image mobilized by the students and its influence on their argumentation and proof.

In the next section, we present the framework of the studies, we describe the notion of drawing and figure in mathematics education. We also present the description of argumentation and proof in mathematics education. We continue by presenting Toulmin's and Vinner's models that we adopted, other to discuss later our methodology (Tall & Vinner, 1981; Toulmin, 2003; Vinner, 1983). After this, we show our data analysis concerning the influence of drawing and figure on the student's argumentation and proof. We complete this paper with some final remarks.

2. Theoretical framework

2.1. Drawing and Figure in Geometry Education

In many African countries, geometry is the branch of mathematics that is generally chosen to introduce students to deductive reasoning. The objects studied in geometry in these countries evolve as we progress through schooling. In geometry at the secondary school, students start by studying drawing as a physical object, then they study the figure as a geometric object that will serve as a support to poof. This transition generates some didactic obstacles (Laborde, 1994; Parzysz, 1988; Walter, 2001). Therefore, it is necessary to have a definition of each of these objects in order to differentiate them (Parzysz, 1988). Parzysz defines the drawing as the material trace on the sheet of paper. The figure is the theoretical object represented by the drawing. The figure is composed of related geometrical objects. It is the figure that is generally described in the statement of a geometry problem. Laborde (1994) considers the figure as an element of geometrical reality and the drawing as the model that describes and interprets the figure. We consider the figure in this article as an ideal object with all the elements of the theory that can describe it. Parzysz (1988) shows that there is a conflict between what the student knows about the geometrical object and what the drawing supposed to represent that object shows to be seen. Taking this conflict into consideration is an aid to the teacher in getting students to build their knowledge. The research identified some functions of the drawings associate of the texts of geometrical problems: decorative functions, representative functions, organizational functions and informative functions (Elia & Philippou, 2004). In our view, the drawings, that has an organizational function does not promote the development of creativity in students. Walter (2001) identifies some problems related to the use of drawing in a geometry problem. According to her, the predominance of the figure may hide the need of proof. The drawing can interfere with the student's reasoning, this would be due to the difficulty a student has in apprehending it, analysing it and deciphering it. The use of freehand drawings is not a solution to high school drawing problems, as its use poses other problems (Gobert, 2007). However, drawing remains an important element in geometry that has to be taken into account. We cannot do without it.

2.2. The Concept Image and Concept Definition

The concept image and concept definition model were developed by Vinner (1983) as a theoretical framework that guides the researcher in understanding the student's mental process. Vinner argues that there is a conflict between the structure of written mathematical definitions or statements or concepts and the cognitive process of acquiring the concepts. We use concept image and concept definition in these articles to describe students' mental processes about figures and drawing which represent it.
The concept-image is a concept that is used to describe the total cognitive structure of an individual, associated with a given concept, it includes all mental images and properties of objects, theorems as well as the processes that are associated with it. This may not be consistent and have aspects that are very different from the formal definition of the concept (the definition accepted by the mathematical community). When a concept is mentioned or when we solve a task in relation to a concept, our memory is stimulated and something is mentioned. However, what is mentioned is rarely only the formal definition of the concept, but rather, a set of visual representations, images (drawings in geometry), properties of objects associated with the concept, theorems related to the concept or experiments. This set constitutes the concept-image. Various studies report that individual concepts image differ from formal theory and contain factors that cause cognitive conflicts. The student can memorize the definition of a figure, which he produces when it is requested. This verbal definition that can be memorized and repeated by the student is called by Vinner concept definition, it is a set of words used to specify this figure, and it is related to the figure as a whole. It can also be the student's personal reconstruction of a definition. In this particular case, these are words that the student uses to explain his or her own concept-image (evoked).
We believe that the Vinner's model will help us to describe and interpret the influence of figure and drawing on student reasoning during the problem solving. When a geometric problem is proposed to students with drawing on which he shows proof the nature of a figure, his concept image is evoked, he can compare the information on drawing with the mental image of the figure. We believe that in a student's argumentation, the statement use is funded on their concept image. This element of the concept image may have been activated by the figure to which it is attached or by the view of a configuration on the drawing that resembles the student's mental image associated with the figure.

2.3. Argumentation and Proof in Mathematics Education

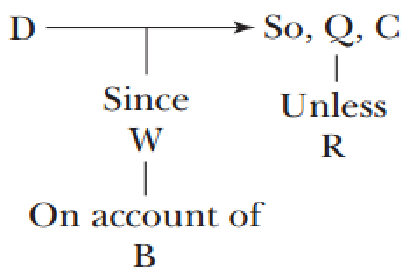
Argumentation and proof are identified by mathematical researchers as two types of reasoning. Balacheff (1988) defines reasoning as an intellectual activity that is not completely explicit in manipulating data or acquiring information to produce new information. According to Pedemonte (2002), argumentation and proof that are a central part in mathematical activity, has to be defined and characterized.

In ICMI Study 19, Durand-Guerrier et al. in Hanna and de Villier (2008) describe an argument as a written or oral speech conducted according to common rules, and aimed at a mutually acceptable conclusion of a proposal whose content or truth is the subject of debate (Hanna & de Villiers, 2008). According to Duval (1991), argumentation is a reasoning that aims to convince an interlocutor of the truth of a statement. When this discourse is accepted in a community, argumentation becomes proof, and if this community is the mathematical community, it is called mathematical proof. These various contributions show that mathematical proof can be considered as a particular argumentation. Understanding the relationship between argumentation and proof by the teachers could help them to develop student skills in the practice of this activity.
Mariotti (2001) believe that the practice of argumentation can lead to the learning of the mathematical proof. Argumentation is part of the mathematician's work (Pedemonte, 2002). Thus, there is continuity between argumentation and mathematical proof, known as *cognitive unity*. Cognitive unity is a process analysis tool that allows highlighting the potential of certain problematic situations. This is particularly true when problems are used to introduce learners to mathematical proof. According to cognitive unity hypothesis, the conjecture is usually produced by the learner at the end of the argumentation process. The arguments resulting from this phase are organized to build a mathematical proof of the statement, which thus becomes a theorem. From our point of view, cognitive unity can also be observed when students solve problems that do not necessarily lead to the production of a

conjecture. We believe that during the solution of the open problem that leads to the proof of an assertion, the student is involved in an exploratory activity during which argumentation is produced and the arguments can be reused, restructured and reorganized in the proof phase. One way to compare these two processes to identify continuities and ruptures is to decompose them using Toulmin's (2003) model.

We use Toulmin's model in this article to analyse the arguments within the students' argumentation. The analysis of the data and warrant will allow us to describe the influence of the drawing on their argumentation and proof. With the help of this model, we will identify the origin of the argument data, the warrant used by the students and the backing that support the warrant. Indeed, Toulmin's model is a powerful tool for comparing argumentation and mathematical proof in mathematics, there are many researchers who use Toulmin model to analyses argumentation and proof in education (Pedemonte, 2002; Tsujiyama, 2012). According to Toulmin, the argument has a ternary structure. Toulmin give this example of argument: "Harry is a British subject because he was born in Bermuda," this proposition can be analysing as follows.

Table 1. *Analysis of an argument with the Toulmin model*

<table>
<tr>
<td>Data (D): Harry was born in Bermuda
Conclusion (C): Harry is a British subject
Warrant (W): since a man born in Bermuda will generally be a British subject
Backing (B): on account of the following statutes and other legal provisions
Rebuttal (R): unless Both his parents were aliens/he has become a naturalised American</td>
<td>
</td>
</tr>
</table>

This model suggests a way to categorize data and warrants. As a result, a categorization of data in geometry can be obtained by questioning their origins. The question that can be asked is the following: where did the data for this argument come from? The possible origins are:

- the data that are part of the assumptions of the situation, it may be those contained in the problem statement and those coded on the drawing;
- the data that are deduced from the assumptions of the situation;
- data that are not part of the assumptions of the situation and are not deduced from these assumptions. These may be abusive interpretation of the relationships between sides and angles or abusive interpretation of the shape of the drawing.

To have the warrant, we answer the following question: What makes it possible to move from data to conclusion? The warrant that supports the students' arguments is part of their concept image, as we show in the following. The elements of the concept image mobilize can be divided into two categories: those that are consistent with the formal theory and those that are in conflict with this formal theory. We believe that taking this categorization into account is useful in describing the influence of drawing on student's argumentation and proof.

The definitions of the concepts we have realized in this section allow us to differentiate them. The articulation between Vinner's model and Toulmin's model will allow us to describe the influence of drawing, figure on the students' argumentation and proof.

3. Methodology

This qualitative study examined how drawing and figure influence student's argumentation and proof in problem solving.

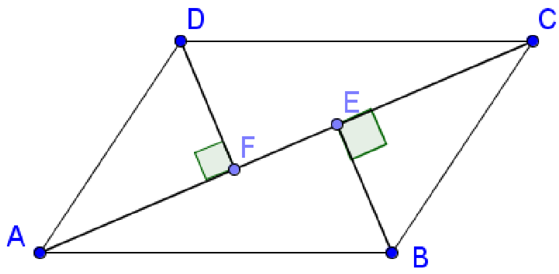
3.1. Participants

The participants in the study are 12 students of 14–16 years old. They are in Form 5 attending a school in Yaoundé, Cameroon. These 12 students were chosen among 30 volunteer students during the 2018–2019 school year. These are students who have experience in Euclidean geometry. The proof is supposed to be part of their culture for having practiced it and observing the teacher practicing it in their geometrical classes. Some of these students are considered to have a good level of mathematics while others have an average level. The students were asked to agree to solve a problem in Euclidean geometry. Students in this sample studied geometrical figures such as quadrilaterals and triangles in previous classes. The theorems necessary to solve the problems were taught to these students and were sufficiently reinforced in the exercises and lessons.

3.2. Data Collection

The participants in the study were observed in a problem-solving situation. We conducted an experiment with students during which we recorded their discussion and collected their written production. We provided the students with a sheet containing a Euclidean geometry problem, and we used a tape recorder to record the students' argumentation. The problem consists of a statement and a drawing that illustrates the text, the drawing here plays two functions: representative the drawing represents all or part of the content of the problem statement; informative function the drawing gives essential information for the resolution of the problem; the problem is based on the drawing.

The problem proposed for the experiment is the following:
In the ABCD parallelogram, the straight line (DF) and (BE) are perpendicular to the straight line (AC). Can we say that the quadrilateral DFBE is a parallelogram?



The participants know that each statement has to be proved as stated in the didactic contract. We chose to associate drawing with the problem statement for the following reasons: we want to observe the interactions between students and the drawing; we want students to have the same drawing; we want to avoid that students represent false drawings that may complicate the solving problems; we want students to concentrate on argumentation and proof. However, the drawing is not complete, it is up to the student to complete the quadrilateral DFBE.
The experiment took place in the evening, after school hours. In addition to that, the recordings are of acceptable quality during this time of the day because students are already gone to their house. The fact that the students worked in groups led them to verbally interact. This make easy to access their strategies and arguments. The students proceeded in two phases to solve the problem, the first phase consists of argumentation and the second phase consists of producing proof. The students' argumentation was recorded. The teacher and a researcher were present in the room, they did not interfere in solving the problem.

3.3. Data analysis

The recordings of the students' argumentation were transcribed and translated from French into English. The argumentation and proof of students were analysing following four codes: personnel concept definition of the students about the parallelogram; drawing and student resolution strategy; drawing and students' argumentation; drawing and continuity or gap between argumentation and proof.

According to the categorization we have made above, the data can have several sources, the analysis of this data makes it possible to understand the influence of the drawing in the students' arguments. Students' argumentation and proof can be compared to observe cognitive continuity or cognitive gaps in argumentation and proof. We will also examine the backing of the students' arguments. This will ensure that the students evoked concept image is consistent with the formal theory about the figure.

4. Results

In student solving process, we analyse students' a priori personal concept definitions about geometric objects. In the students' arguments, we identify the origins of the data, we also analyse the warrants of the students' arguments make hypotheses about their backing. We then observe the effects of the drawing and the figure inside the cognitive continuity analysis between argumentation and proof.

4.1. Personnel Concept Definition of the Students about the Parallelogram

Students' copies may differ from one group of students to another. For example, to answer the question of the problem, two students, Nono and Kenne, begin by explaining how the parallelogram is represented. They propose a definition of a parallelogram, then they check that the figure satisfies definition they have mutually accepted. The definitions formulated are discussed in order to reach a consensual definition. The definitions proposed in the group may not be the definitions accepted as formal definitions of the figure. The attributes contained in the students' definitions do not allow describing the parallelogram and excluding some particular cases of the parallelogram.

Table 2. *Students' personnel concept definition about parallelogram*

Student activity	**Analyses**
Nono: First of all, what is it? *Kenne: it is a quadrilateral with four two-by-two equal sides;* *Nono: no, which has four sides with equal straight line contain that sides here;* *Kenne: which has four sides with supports that are two to two equal and parallel;* *Nono: this means that the side here is parallel to it, and the side here is parallel to it;*	Students are looking for a definition associated with the parallelogram. They identify the relationships that the sides of a quadrilateral must satisfy the parallelogram properties for the sides (two by two equal and parallel sides). The attribute proposed here is "to have two equal sides with parallel lines contain those sides", we think they could have specified that these are the opposite sides. However, students try to make themselves understood by indicating on the drawing relationships mentioned in their definition.

The statements that students propose to describe the parallelogram are considered to be students' personal concept definition. The definition of a parallelogram provided by the students is in conflict with the formal definition. We think that the students, who have constructed this personnel concept definition, forgot that the described sides have to be opposite sides. From our point of view, when the personal concept definition of the students from the figure is in conflict with the formal definition, there can create conflict between the information the students see on the drawing and their concept image about the figure. This phenomenon cool has consequence on students' proof.

4.2. Drawing and Student Resolution Strategy

The students work on the drawing and the interpretations made of it guide them towards resolution strategies. The analysis of the students' copies shows that they have tried several strategies to proof the nature of the quadrilateral. For example, they try with equal vectors, they try congruent triangle to deduce equal sides. It has been observed that students abandon resolution strategies for the following reasons: the data from the visual inspection on the drawing did not correspond to the constraints of the concept image mobilized; the students could not deduce the concept image mobilized constraints from the data perceptible on the drawing; the data from the visual inspection are not sufficient to implement these strategies. Let us observe Amba and Njeteji argumentation below.

Table 3. *Influence of drawing on students' strategy*

Students' argumentation	Analyses
Amba: no, a parallelogram is a figure that has two sides equal two by two ... so DC is equal to AB and AD is equal to BC, but CB does not have the same length as DC *Njeteji: But....* *Amba: if we try to make a small figure here, we'll see that they don't have the same length* *Njeteji: but even the square is a parallelogram…* *Amba: no* *Njeteji: so, the square is not a parallelogram?* *Amba: a parallelogram is a geometrical figure that has two parallel sides two by two;*	Njeteji gives an incorrect definition of parallelograms and this creates a conflict between what he knows and the visual observation of the drawing (where ABCD is a parallelogram). *Argument* D: ABCD is a parallelogram C: DC is equal to AB, AD is equal to BC, but CB is not equal to DC… W: since a parallelogram is a figure that has two equal sides two by two. The Warrant is supported by the students' knowledge about the parallelogram, which is wrong. This conflict gives rise to a new personal concept definition that is also wrong.

The observations made on the drawing show that the quadrilateral ABCD does not verify the first description of the parallelogram provided by the students. This conflict leads the students to renounce this personal concept definition. Students struggle to find an acceptable description of the parallelogram; it would come from a personal reconstruction of the definition. This personnel concept definition is in conflict with the formal definition of the parallelogram. The drawing provokes conceptual change of student personal definition of a parallelogram. We can imagine that this is happening because the student has a confidant of the drawing giving by the teacher.

4.3. Drawing and Student's Argumentation

The students also validated the nature of the parallelogram by visual observation, this is the case with Ndondi and Kenmogne. They first completed the drawing and then checked their agreement between the image in the drawing and their idea of a parallelogram. Once this verification has been done, they engage in a search for a theoretical validation strategy.

Table 4. *Influence of drawing on students' argumentation*

Students' argumentation	Analyses
Ndondi: if I draw the drawing, here it can be a parallelogram and it can also be a diamond. *Kenmogne: it can be a rectangular parallelogram and a triangular parallelogram; yes, it is a parallelogram, because DE... If you connect DE [D to E] and FB [F to B] now it is a parallelogram.* *Ndondi: it is a parallelogram;*	For Ndondi, the drawing obtained looks like both a parallelogram and a diamond. *Argument* D: if I draw [DE] and [BF], the drawing looks like a parallelogram C: DFBE can be a parallelogram W: since, if a quadrilateral takes the shape of a parallelogram then the quadrilateral can be a parallelogram. The data evokes an experience, and the warrant is an element of *Kenmogne*'s concept image, an intuition that comes from the perception of the drawing.

It can be observed that the students implement an empirical control and a visual control to deduce the nature of the quadrilateral DFBE. The drawing is used here as a support for reasoning. It is assumed that to demonstrate that a quadrilateral is a parallelogram, the visual perception of the drawing should correspond to the mental image that students associate to the parallelogram. We can see that the drawing evokes two figures in the students, the diamond and the parallelogram. Although a diamond is a particular parallelogram, there is no information on the drawing to specify the type of parallelogram

represented. We assume that this is a superficial interpretation of the drawing, and that the hierarchical relationship between the parallelograms is not stabilized in these students.
The students look at the parallelogram ABCD, implement visual control the relationships between its sides, and control visually that both sides of the DFBE satisfy these relationships. The pair who took part in this exercise was Ngono and Keneka. The analysis of the argumentation of these students is contained in the table below.

Table 5. *Influence of drawing on students' argumentation*

Students' argumentation	**Analyses**
Ngono: hence the parallelogram ABCD where (AB) is parallel to (DC) and (AD) parallel to (BC); *Keneka: so, we can only pose like that, they are parallel (DE) is parallel to (FB) and (EB) is parallel to (DF), so it is a parallelogram.*	*Argument* D: In the parallelogram ABCD, we see that (AB)∥(DC) and (AD)∥(BC), we also see that in the DFBE quadrilateral (EB)∥(DF) and (DE)∥(BF); C: DFBE is a parallelogram; W: since if in a quadrilateral, the opposite sides have parallel supports then this quadrilateral is a parallelogram. The data in this argument is a mixture of information derived from the hypothesis of the situation and information from the visual control on the drawing.

Analysis of the students' argumentation reveals that they have observed visually the parallelism of the supports on opposite sides of the ABCD parallelogram proposed by the teacher. Then they carried out a visual control to verify the parallelism between the opposite sides of the DFBE quadrilateral, which allowed them to conclude that this quadrilateral is a parallelogram. It can be assumed that the visual inspection made it possible to construct a concept image that served as a basis for the warrant used in the students' proof. The relationships between the sides of the DFBE quadrilateral are the result of an abusive interpretation of the drawing because no information represented on the drawing makes it possible to establish a direct relationship between the sides of this quadrilateral.

4.4. Drawing and Continuity or Gap between Argumentation and Proof

The analysis of the students' argumentation and the students' proof allows us to compare them. The data from some of the arguments were directly used in the students' deduction steps. Thus, we were able to observe the data resulting from the abusive interpretation of the drawing both in the students' arguments and in the deduction step of the proof produced. We also observe that students have difficulty to use symbolique representation of figures. Let us give an example of proof of students.

Table 6. *Influence of drawing on students' proof*

Students proof	**Analyses**
From our point of view, the DFBE quadrilateral is a parallelogram because we observe that on the ABCD parallelogram the lines DC∥AB and DA∥BC from where DF∥EB and DE∥FB then the DFBE quadrilateral is a parallelogram.	D: ABCD is a parallelogram; DC∥AB and DA∥BC; DF∥EB and DE∥FB; C: DFBE is a parallelogram; W: since by considering two quadrilaterals, if the relationships between the opposite sides of one are the same in the other, then its quadrilaterals are of the same nature. The data of this argument come from visual control of the relation between the sides of the two parallelograms. This relation can be deducing from the hypotheses of the problem. The Warrant comes in part of the concept image of the student, it does not correspond to the theory of parallelograms. We also observe that the way that student writes symbolically the straight line cannot be interpreted as straight line. DC∥AB **should** be **writing** like this (DC)∥(AB).

It can be seen that the students used a visual check to establish the relationships between the sides of the parallelogram. This is a mistake because in the context of proof only data contained in the hypothesis of the situation or those resulting from recycling by deduction are allowed in proof. An attempt to explain this phenomenon may be the following: students begin by learning geometry using visual perception and experience before gradually moving on to deductive reasoning, these modes of validation percolate into student activity and stand as an obstacle to learning mathematical proof. The warrant mobilized here is not part of the theory of parallelograms. We can imagine that the concept image associated with the parallelogram is not sufficiently constructed by the students.

It is also observed in the students' proof that the symbols used to represent geometric figures do not always correspond to these figures. It is assumed that the part of the students' concept image associated with symbolism is not sufficiently constructed. This could be due to the poor construction of these objects by the students, but it is also possible that this phenomenon could be due to the negligence of the teachers who do not sufficiently mention these representations.

When we compare the findings made in the students' proof with those made in their argumentation, we see that the proof is directly derived from their argumentation. The data of students' argumentation and proof have the same origin. The arguments mentioned in the argumentation are reused in the proof. The symbols used to represent the configurations of the figure are the same in the argumentation and in the proof. We can imagine that the students' argumentation is the support of their proof. The influence of the drawing on the proof comes directly from the students' argumentation phase. It can also be seen that some of the drawings actions on the students' arguments are not perceptible in the proof they produce. For example, the strategies that were mentioned by the students, and which were then rejected because the information allowing their use, were not perceptible on the drawing.

5. Conclusion and Discussion

The objective of this article was to understand in depth the influence of drawing and figure on the students' argumentation and proof. To achieve this objective, we conducted an experiment in which we invited students to solve a problem that leads to the proof of a statement.

The students' argumentation and proofs were analysed by articulating Toulmin's model and the concept-image and concept-definition developed by Vinner (1983). The analyses of the students' protocols show that the drawing occupies a very important place in the students' argumentation. The students' interpretation of the drawing guides them towards a resolution strategy.

To prove that a quadrilateral is a parallelogram, students mobilize their personnel concept definition that allows them to describe the parallelogram. Our analyses show that this personal concept definition does not often correspond with the formal definition of the parallelogram. We believe that the failure of the proof would come from the fact that students' knowledge about the figure is superficial and ambiguous, leading to cognitive conflict as Tall and Vinner (1981) point out. This conflict could arise from the mismatch between the information from the interpretation of the drawing and the concept image evoked.

Students use data from their perception of the drawing. This is information that is not represented by the code on the drawing. These results are consistent with those obtained by Souvingnet (1994). We can imagine that drawing is imposed on students, the visual perception of the drawing active in the student a mental image associated with this type of drawing. It can also be assumed that for students the drawing is considered a physical object and not a figure. It can be assumed that this know-how is part of the student's concept image construct in previous classes where drawing was studied through visual perception and experience. Indeed, we have also observed that for some students the proof is produced from abusive interpretations of the drawing, which makes them fail. This abusive interpretation of the drawing is thought to be a cause of error for students (Laborde, 1994; Walter, 2001).

The results of the analyses of the comparison between argumentation and proof revealed cognitive continuities between argumentation and proof. Indeed, the influence of drawing and figure on students' argumentation is the same on students' proof (Mariotti, 2001; Pedemonte, 2002).

At the end of our analyses, we found that students interact with drawing and figure in their activities. They use drawing to find a strategy of resolution, and to validate their personnel concept definition about the figure. They also use the information on drawing as data in their arguments. The adequacy between the students' personal concept image about the figure and the theory of this figure is a prerequisite for the success of proof activity. The data that have to be used in proof activity are those, which come from hypotheses of problems, or those, which are deduced from theses hypothesis. As a point of view, students should be thought to use only theses data.

This study has limitations insofar as the students' transcripts do not allow us to see the actions carried out by the students on the drawing, the hesitations. Moreover, the problem proposed to the students was not intended to build new knowledge. The number of participants is small and does not allow the results to be inferred with certainty. One perspective for this research work could be to repeat the experiment with a large sample. The experimental situation could be a didactic situation that aims to construct a definition or a theorem.

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