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A Course in Mathematical Modeling for Pre-Service Mathematics Teachers

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Abstract

The United States' Common Core Standards for Mathematics give a prominent role for mathematical modeling in the K-12 curriculum. This has implications and creates challenges for teacher preparation programs. We describe one program's attempt to meet these challenges with a course in mathematical modeling for pre-service secondary mathematics instructors. The course was co-developed and co-taught by a mathematics educator and an applied mathematician and implemented as a design experiment. The students in the course, all mathematics majors, experienced growth in dispositions yet also faced mathematical challenges that likely extend beyond modeling contexts.

Keywords: mathematical modeling, teacher preparation, quantitative reasoning

Introduction

Mathematical Modeling is one of just six conceptual categories for high school in the United States' Common Core State Standards for Mathematics (CCSSM) and is one of the eight CCSSM mathematical practices which span all of K-12 mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). These standards are adopted in 41 of the United States ("Standards in Your State," 2019), however there is reason for concern about teachers' preparation for implementing mathematical modeling in their classrooms. It may be rare for teacher preparation programs to even introduce students to mathematical modeling (Doerr, 2007; Lingefjärd, 2007a). Moreover, programs that do currently include or wish to develop instruction in mathematical modeling may be hindered by the lack of a robust research base about best practices, both for teaching mathematical modeling and for preparing teachers to teach modeling.

The elevated status of mathematical modeling in the curriculum, both as a practice and as a conceptual category, requires many secondary teacher preparation programs to adapt. Herein, we report on a design experiment in which the authors, a mathematics educator and an applied mathematician, co-designed and co-taught an undergraduate course in mathematical modeling for mathematics majors intending to be secondary teachers (N=9), herein referred to as pre-service mathematics teachers (PSMTs). In this first iteration of our design experiment, we were guided by a very broad research question about the nature of PSMTs' dispositions for engaging in and teaching mathematical modeling both before and after the course. Students reported that the course contributed to their growth as mathematical modelers and as future teachers of mathematical modeling, however their work throughout the semester suggests that a one semester course in mathematical modeling may not be sufficient preparation to meet the expectations of the CCSM.

Perspective

The Guidelines for Assessment & Instruction in Mathematical Modeling Education (GAIMME) described mathematical modeling as a process used to answer "big, messy, reality-based questions" (Garfunkel & Montgomery, 2016, p. 7). The process begins with identifying a problem and ends with reporting results. In between, the mathematical modeler makes assumptions; defines variables; refines the original question; develops and implements models; and analyzes the outputs of the model. This all transpires in a non-linear, often cyclic manner. The messiness, openness, and time requirements of authentic mathematical modeling tasks present an array of both pedagogical and conceptual challenges for students, teachers, and teacher preparation programs.

Some of the challenges for learners of mathematical modeling are documented by Thompson (2011) in his description of mathematical modeling as emerging from quantitative reasoning, which serves as grounding for several nontrivial abilities that are essential to mathematical modeling. Foundationally, the ability for quantitative reasoning allows a student to conceptualize a situation quantitatively. Extending this, covariational reasoning is needed for students to make sense of dynamic situations in which quantities vary in relation to each other. The ability to generalize, in the context of mathematical modeling, allows a student to represent these relationships. Thompson describes a mathematical model as a generalization "of a situation's inner mechanics—of 'how it works'" (p. 51).

Doerr (2007) noted that the pedagogical knowledge for teaching modeling is distinctive and she enumerated some specific pedagogical tasks for teachers of mathematical modeling, among them: choosing and adapting modeling tasks; anticipating and evaluating students' varied strategies; and helping students make rich mathematical connections. This description of a teacher's role in supporting mathematical modeling is largely echoed in the GAIMME report (2016), which also devotes considerable attention to the challenge of assessing mathematical modeling. Unfortunately, there is a dearth of research which investigates the development of the pedagogical knowledge teachers need for teaching modeling. Indeed, Doerr observed that "how teachers acquire this knowledge… remains an open question for researchers" (p. 77).

Doerr (2007) also describes a mathematical modeling course for pre-service teachers (N=8) which she developed and taught. In her course, students read about the modeling cycle and they engaged in and reflected on the modeling processes. She suggested that pre-service teachers engage in a variety of modeling tasks that require explanations, justifications, and reflection. Zbiek (2016) designed and taught a course for a similar audience. She focused on productive beliefs and corresponding unproductive beliefs about teaching and learning mathematical modeling. For example, it is productive to believe that mathematical modeling is a messy process as opposed to the unproductive belief that problem solving should follow a clearly determined path. Through modeling and pedagogical tasks, students in her course moved toward productive beliefs, though this was accompanied by some persistent confusion about mathematical modeling. She echoed Lingefjärd's (2007b) recommendation that modeling be integrated throughout teacher education programs, not just in a single course.

The Mathematical Modeling for Teachers Course

We co-designed and co-taught the Mathematical Modeling for Teachers course from the joint perspectives of our disciplines, mathematics education and applied mathematics, and with direction from the GAIMME report. The preservice mathematics teachers in the course engaged in the modeling cycle through in-class team activities, homework/exam questions, and a final team project. From the first day of class, we were explicit about the modeling cycle and, after modeling tasks were completed, the PSMTs reflected on their work as an expression of the cycle.

We did not organize the course as instruction in a sequence of different modeling techniques. Instead, the first half engaged the PSMTs in a variety of modeling tasks in which they were to rely primarily on their existing algebraic, geometric, and statistical knowledge. This was followed by three weeks of instruction in linear programming, statistical and mathematical simulations, and some useful features of Microsoft Excel (2013) such as visualizing data, using random numbers to do simulations, making predictions with models, and solving linear programming problems. The next three weeks focused on pedagogical content knowledge such as modifying high school textbook tasks, analyzing curricular materials, and evaluating student work; some of this was foreshadowed by similar tasks in the first half of the course. The rest of the course was devoted to final projects by teams of PSMTs in which they identified a question, developed a model, reported on the model, and connected their work to the CCSSM. Throughout, we adjusted instruction according to what we perceived to be difficult parts of the modeling process for the PSMTs. For instance, many PSMTs lacked facility with generalization, as described by Thompson (2011), so we focused on this piece of the modeling process with some matching activities in which students linked equations to verbal scenarios and linked the structures of equations to scenarios.

Rather than using a textbook, we developed, adapted, and curated tasks for the students. Students did readings from the GAIMME report and from teacher-focused articles about mathematical modeling. An often-used resource was the set of high school textbooks used by the local school district which claimed to be aligned with the CCSSM. The books labeled questions as “Modeling with Mathematics” within each problem set. However, to borrow phrasing from the GAIMME report (2016), the tasks would more aptly be described as “traditional word problems or textbook applications where all of the necessary information is provided and there is a single, known, correct answer” (p.28). This echoes Meyer’s (2015) analysis of two different supposedly CCSSM-aligned textbooks — tasks labeled as “modeling” rarely required students to model. The local textbooks were valuable both as illustrations of some of the curricular challenges our PSMTs would face as teachers and as a source of tasks for students to analyze and modify.

Methodology

We approached the development and implementation of the Mathematical Modeling for Teachers course as a design experiment in which course design and theory development are “iterative and interactive” (Schoenfeld, 2006, p. 198). Herein, we report on the first iteration of the course; our research goals were to: 1. identify emergent themes related to the mathematical modeling preparation of teachers, and 2. generate hypotheses to be tested in future iterations. Our data are 1. notes from weekly planning meetings between the researchers/instructors which included both planning and reflections on the progress of the course; 2. student-generated artifacts from the course (homework, projects, exams); and 3. an end-of-course survey. Seven of the nine PSMTs were undergraduate students in a Bachelor of Science (BS) program in Mathematics, Option in Mathematics Education. The remaining two PSMTs had already completed the BS program and were taking the course out of interest. All of the students had completed Linear Algebra and a first course in proof, most with coursework beyond that.

The different sources of data required different investigatory methods. Emergent themes were identified as we systematically and collaboratively reviewed the notes from planning meetings and student work from throughout the semester. Selected student work on two tasks from the course were iteratively coded to highlight emergent ideas related to the foundational quantitative reasoning skills described by Thompson (2011) and to the types of pedagogical knowledge for teaching modeling enumerated by Doerr (2007). The survey data were analyzed through the use of summative statistics.

Results

The end-of-course survey indicated that the students/PSMTs found the course to be worthwhile. They reported that their knowledge of mathematical modeling increased and that they were excited to teach mathematical modeling. On a scale of 1 (excellent) to 5 (poor), seven of nine PSMTs rated their pre-course knowledge of modeling as 4 or 5. At the end, all nine rated their understanding of mathematical modeling as 1 or 2. They also reported growth on more affective and pedagogical dimensions. All nine PSMTs either agreed or strongly agreed with statements about improved mathematical communication skills, their ability to identify and implement opportunities for mathematical modeling, and their excitement to teach modeling. All but one (who was neutral) agreed, but did not strongly agree, with the statement that they feel comfortable converting textbook word problems into modeling tasks.

As the instructors, our perceptions of the course align with the PSMTs' report of their growth in knowledge of mathematical modeling. However, there were also several challenges related to the PSMTs' mathematical and pedagogical knowledge for teaching mathematical modeling. Though not a primary cause of the challenges, our decision to begin the course with a formal discussion of the modeling cycle (as presented in GAIMME and CCSM) had some unfortunate side effects. We chose to present the cycle early because, throughout the course, we asked the PSMTs to reflect on the modeling process and connect it to their work. Though, early discussions of the modeling cycle may have been more meaningful if delayed until after they had rich mathematical modeling experiences as referents. Indeed, on the survey, most of the PSMTs strongly agreed that their "understanding of mathematical modeling really came together with the final project." However, our main concern with early discussions of the modeling cycle stems from several instances in which PSMTs unproductively looked to the modeling cycle for quasi-procedural guidance as they worked on modeling tasks. This may have hindered their appreciation of the messiness of authentic modeling and created a perception that there is a single correct way to approach modeling tasks.

We hope that, by delaying the explicit naming of the components in the modeling process, the focus can be shifted from the steps in the cycle to the development of the PSMT's productive mathematical modeling problem-solving dispositions (e.g., those enumerated by Zbiek, 2016) and quantitative reasoning skills. A prominent and recurring challenge was our frequent need, as instructors, to encourage the PSMTs to examine more specific cases in order to develop a general model. We came to call this an examination of a "toy model." It is noteworthy that this leveraging of the specific/concrete (e.g., a toy model) in order to arrive at the general/abstract is not explicitly part of the modeling cycles described in the CCSSM or in the GAIMME report. Without instructor intervention, the PSMTs often became mired in premature attempts to define appropriate variables and develop an abstract model. Our notes indicated that this was a persistent challenge throughout the semester, the PSMTs needed support in generating specific cases, noticing patterns, and then making suitable generalizations. These are all skills that are based on quantitative reasoning as described by Thompson (2011).

The PSMTs' work with abstraction often betrayed a lack of comfort connecting verbal and symbolic representations. In a linear programming exam question, adapted from a local school district's quarterly Algebra II exam, we asked students to write a sentence or two that aligns with the inequality $8m + 4d \leq 64$ in the following context:

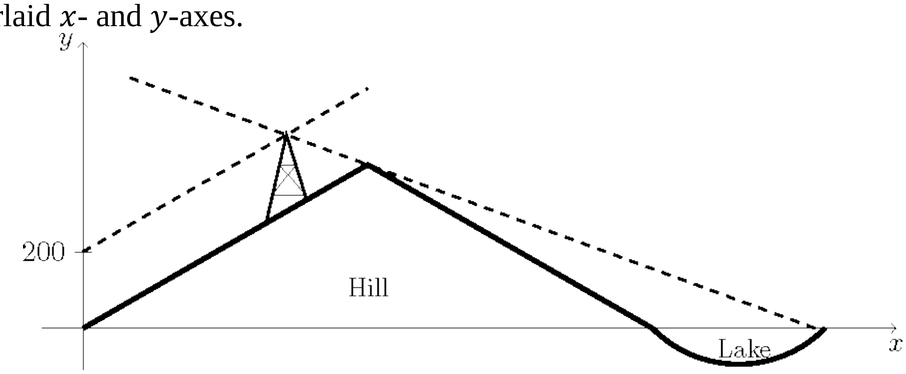
> Baking a tray of muffins takes 8 cups of milk and 6 cups of flour. A tray of donuts takes 4 cups of milk and 6 cups of flour. A baker has 64 cups of milk and 60 cups of flour. He makes $10 profit per tray of muffins and $8 per tray of donuts.

Six of the nine PSMTs made errors with units. For example, one read m and d as labels rather than quantities. She wrote, “The total max amount the baker needs is 64 cups to make 8 muffins (8m) and 4 donuts (4d).”

We also observed challenges with more pedagogically focused tasks. For example, the PSMTs’ attempts to modify textbook problems to create authentic mathematical modeling tasks often resulted in tasks that were not open enough or were imprecisely stated. As a homework assignment, we asked the PSMTs transform a textbook problem into a mathematical modeling task. They each chose one of two problems from a locally used textbook, each of which was labeled as “Modeling with Mathematics”. Both provided a population growth context and asked students to write the equation of an exponential growth function and make a prediction using that function. Neither started out “big, messy, and unmanageable” as described in the GAIMME report as a characteristic of authentic modeling tasks. Instead, they fit the GAIMME report’s description of traditional textbook applications “where all of the necessary information is provided and there is a single, known, correct answer” (Garfunkel & Montgomery, 2016, p. 28)

In four of the eight PSMT responses, all of the necessary mathematical information was provided though one of these required (nonmathematical) research about the context in order to make a decision. Two PSMTs provided or asked students to find population data and then model it and make a prediction. These PSMTs did add some messiness to the problem, yet it still did not feel big or unmanageable. Two other PSMTs changed the problem in a way that essentially parameterized previously known values. Though some of the transformed tasks were improvements, none met the standards for a mathematical modeling task established in the GAIMME report.

Other pedagogical tasks posed challenges that were likely not exclusive to the context of modeling. Of note, on an exam we asked students to make sense of and recommunicate a hypothetical high school student’s linear model for a scenario that our students had already modeled (geometrically) during an in-class activity. We provided the student’s description of her coordinate-based strategy for a modified version the Over the Hill task (Slowbe, 2011) available from the National Council of Teachers of Mathematics. The goal was to figure out where, on the western face of a hill, to place a cell phone tower so that the signal can reach the shore of a lake on the east side of the hill. The PSMTs had collaboratively worked on and solved a more general version of the problem in class, though all had used geometric strategies without coordinates. On the exam, they were provided with the picture below, adapted from the Over the Hill task, on which we overlaid x- and y-axes.



We provided the following hypothetical student's description of her strategy based on provided measurements.

> We decided to assign coordinates. We made the bottom, west corner of the mountain $(0, 0)$ so the peak of the mountain is at $(1400, 800)$. So the slope of the line through those two points is $\frac{4}{7}$. If we slide that line up 200 feet it will go through the point $(0, 200)$ and the tip of the tower. If we can also find the equation for line that goes through the east corner of the lake and the tip of the mountain, we can find out how far the tower should be built from the western corner of the mountain.

First, we asked the PSMTs to describe the units of $\frac{4}{7}$, hoping for an answer like "vertical feet per horizontal foot." We anticipated a common error for students to say that the slope is unitless, dividing feet by feet. Though, we felt like the complexity of the question was authentic to what may be found in a secondary classroom. Of the nine students who responded, only one correctly identified the units of the slope. Three said that the slope has no units, three said the units were "feet", and two gave answers that were hard to categorize but incorrect.

Next, we asked the PSMTs to use the student's strategy to

> Find an equation you can solve, the solution to which is the horizontal distance from where the tower should be built to the bottom the west side of the mountain
> .

To do so, students needed to define another line through given points at the mountain's peak and the eastern edge of the lake. They then needed to find an equation, the solution to which is the x-coordinate of the intersection of the lines. Only one of the eight PSMTs who answered the question did so correctly (the same one who got the previous part correct). Although, three found a correct system of equations but did not find a single equation to solve. One PSMT used an entirely different strategy to find a correct equation and two others attempted geometric strategies but never wrote an equation. Another defined new variables and produced an equation that had no discernable meaning in the context.

Discussion

We have briefly documented three examples PSMTs' difficulties engaging with mathematical modeling tasks in which they were to approach the task either as student or as (future) teacher. These were exemplary of what we, as instructors, observed throughout the semester both in team and individual work. The examples lend credence to Thompson's (2011) claim that quantitative and covariational reasoning skills are essential to the mathematical modeling process; students need to be able to notice patterns and mathematize in order to be successful at modeling. During class, we were able to support the PSMTs with timely scaffolds and by creating supportive working groups. However, removing these supports during individual exams resulted in work that raised additional doubts about the PSMTs' readiness to teach mathematical modeling.

The existence and persistence of these challenges could perhaps be used to support the call for greater integration of mathematical modeling throughout teacher preparation programs, not just in a single course**.** Though, if we accept Thompson's (2011) view that mathematical modeling depends on quantitative and covariational reasoning, one could also argue that an increased focus on these foundational abilities is both merited and more feasibly implemented across a mathematics or mathematics education program. This is not to say that engagement in mathematical modeling cannot be done in service of developing those reasoning abilities (e.g., see Swan, Turner, Yoon, & Muller, 2007). But our experience illuminated that teaching mathematical modeling as envisioned in the GAIMME report and finding (or developing) good modeling tasks requires time and expertise. We faced instructional challenges even in an environment with two instructors for a class of nine mathematics majors.

Furthermore, the challenges students faced related to generalizing (e.g., translating from verbal or specific scenarios to symbolic representations) and quantitative reasoning (e.g., working with units) may not be detected by assessments in more computationally-focused lower-division courses. In fact, our decision to start the course by formally discussing the modeling cycle may have encouraged the PSMTs to seek more familiar formulaic approaches. Building foundational reasoning abilities and mathematical dispositions for mathematical modeling in prerequisite courses would yield impactful benefits throughout the students' undergraduate careers and their careers as teachers. We also hypothesize that it is beneficial to delay formal introduction of the modeling cycle until after productive mathematical modeling experiences can be referenced.

Despite these observations, students were open to the culture shift required of mathematical modeling. They saw value in it and expressed willingness to do it as teachers. As Zbiek (2016) found in her study, it is possible to enact productive shifts in students' attitudes toward mathematical modeling. Though, obstacles remain to implementing mathematical modeling in secondary schools as envisioned in the CCSM and in the GAIMME report. Many of the PSMTs' challenges, as these novice teachers begin their teaching careers, will be compounded by textbooks which do not support authentic modeling. We have documented PSMT difficulties with both mathematical modeling tasks and tasks related to the teaching of modeling, though we view these challenges as opportunities to improve the Mathematical Modeling for Teachers course. Perhaps more importantly, we hope they frame an examination of students' experiences throughout our teacher preparation program.

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