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Solving word problems in mathematics: An exploratory study among Fijian primary school teachers

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Abstract

Word problems have been well recognised for their pedagogical value in mathematics teaching and learning. The authors of this paper examined Fijian primary mathematics teachers' mathematical knowledge by analysing 34 teachers response to three word problems. The teachers were enrolled in a primary mathematics teaching methods course at a Fijian university. These teachers had a minimum of two years of teaching experience and an official undergraduate qualification from a teacher training college. One of the word problems was a nonsensical one, while the other two word problems contained some technical defects. The overall intention was to explore whether in-service primary teachers could identify the word problems as nonsensical or problematic, and if so, how do they apply their judgment based on real world knowledge when faced with non-standard word problems. Findings suggest that in-service primary teachers do not apply real world mathematical knowledge when solving word problems. This research provides an insight into the lack of mathematical knowledge of a small sample of in-service teachers with respect to identifying real world mathematical knowledge. Some awareness on the pedagogical potential of such word problems are also evident in this study.

Keywords

Mathematical knowledge; teacher knowledge; word problems; problematic items

Introduction

In 2000, the National Council of Teachers of Mathematics (NCTM) of USA published *Standards* that rest on six principles guiding the vision for school mathematics. Two of the principles are directly related to teaching and learning of mathematics:

- Teaching: Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
- Learning: Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

Both principles have strong reference to the way mathematics should be taught, with 'understanding' being the key term. Teaching and learning with 'understanding' means accommodating the different levels of mathematical knowledge. According to Dossey, McCrone, Giordano, and Weir (2002),

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mathematical knowledge can be divided into three related areas: concepts, procedures, and problem solving. When students deal with concepts, they are learning "what something is?" (Dossey et al., 2002, p. 48), for example, knowing that a triangle is a figure having three sides. Students show conceptual understanding when they are able to "use concepts and their representations to discuss or classify mathematical objects". In other words, conceptual understanding is used to compare and contrast objects, as well as to form interrelationships between concepts and principles. Students exhibit procedural knowledge when they "select and apply procedures correctly" (Dossey et al., 2002, p. 49). The third area of mathematical knowledge is problem-solving. Problem-solving requires students to recognise situations, abstract their core structure, model the relationships involved, manipulate those relationships, and communicate the results. The Australian Curriculum (Australian Curriculum, Assessment and Reporting Authority, 2012) recognizes four important strands in mathematical proficiency. These include understanding, fluency, problem-solving and reasoning. The National Research Council (NRC, 2001) acknowledges that no single term fully captures all aspects of mathematics such as mathematical "expertise, competence, knowledge and facility in mathematics" (p. 5). NRC proposes an umbrella term "mathematical proficiency" to capture what it means to successfully learn mathematics. Mathematical proficiency includes five important strands which are interwoven and interdependent. These are:

- Conceptual understanding—comprehension of mathematical concepts, operations and relations;
- Procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence—ability to formulate, represent and solve mathematical problems;
- Adaptive reasoning—capacity for logical thought, reflection, exploration, and justification;
- Productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (p. 5).

The Fiji Islands National Curriculum Framework (Ministry of Education, 2007) also makes parallel references to the key aspects of mathematical proficiency. For example, with respect to numeracy skills, the National Curriculum Framework (2007) notes that learners should be able to "use a range of mathematical skills in a variety of contexts" (p. 36). The role of the teachers is to facilitate mathematical learning in authentic learning contexts. Based on the different strands of mathematical proficiency, the National Research Council (NRC, 2000) makes explicit recommendations for teaching for mathematical proficiency. Developing proficiency in mathematics teaching is directly linked to the five intertwined strands of mathematical proficiency outlined above. For example, parallel to the c*onceptual understanding* strand, the NRC notes that teachers must have conceptual understanding of the core knowledge required in the practice of teaching. Furthermore, as expressed under sub strands *strategic competence* and *adaptive reasoning*, teachers must also be able to represent mathematical problems in real life contexts and be able to reason logically. Teachers' understanding of mathematical word problems, including mathematical modeling and problem solving, could be seen as one of the important areas of these strands of teaching for mathematical proficiency (Haylock & Manning, 2014).

The aim of the small study reported here was to highlight Fijian in-service primary mathematics teachers' understanding of three word problems, ascertaining whether these teachers could apply real world knowledge when confronted with a mathematical situation, and if so, how do they do it. One of the three word problems was an infamous 'nonsensical' question, while the other two items were 'problematic' (P-items) (Verschaffel, Greer, & de Corte, 2007). Problems of this type often do not have a solution at all or could have multiple solutions. Such problems have been referred to as non-standard problems (Daroczy, Wolska, Meurers, & Nuerk (2015). While research findings indicate that students face significant difficulties handling such word problems, it was expected that teachers would be able to pick out these limitations and comment on the appropriateness of the word problems accordingly. The study involved 34 in-service mathematics teachers' who were enrolled in an undergraduate degree level teaching methods course at a Fijian university. The research question which guided the study was as follows: whether, and if so how, do in-service primary mathematics teachers apply real world knowledge when confronted with nonsensical and problematic mathematical situation? This intent was accomplished by analysing 34 in-service primary teachers' responses to a workshop activity consisting of three word problems. Because teachers' own conceptual knowledge is extremely vital for teaching for mathematical proficiency, this study intended to inquire into teachers' understanding of word problems by examining how teachers worked out the answers to three simple yet problematic word problems related to number sense. Teachers' responses provided several insights into teachers' understanding of the word problems.

The study is important because it adds to our understanding of how teachers judge the appropriateness of simple, number related, word problems. Although the mathematics education community is aware of primary mathematics teachers' own limited knowledge and understanding of mathematics (Maher & Muir 2013; Roche & Clarke, 2013; Haylock & Manning, 2014), this study provides a fresh insight into teachers' knowledge related to word problems. What we do know is that the majority of the students' have difficulties when faced with mathematical word problems of such nature (Verschaffel, et al., 2007). As pointed out by Verschaffel et al., (2007), pupils generally tend to "exclude real world knowledge and realistic considerations when confronted with these P-items" (p. 587). However, there are limited studies on teachers' understanding of nonsensical and problematic word problems. Often times, researchers have relied on using conventional but correct mathematical items to probe teachers' mathematical proficiency (Hill, Sleep, Lewis, & Ball, 2007). On other occasions, researchers have asked teachers to write their own mathematical problems (Roche & Clarke, 2013), or comment on students' solutions (Maher & Muir, 2013) to gain an insight into teachers' mathematical knowledge. Apart from adding to our current understandings of teacher knowledge in general, this study could open up prospects for further studies involving non-standard items, an area which Daroczy et al. (2015) see as important in enhancing mathematical understanding. Exploring teacher knowledge using such contrasting means would be useful, not only in understanding what teachers know, but also exploring interesting pedagogic implications for fallacious mathematical items. Furthermore, understandings gained from such studies would be useful in assessing teachers knowledge of selecting or developing good items, given the fact that a teacher develops approximately, on average, more than fifty classroom tests per year (Moss, 2013).

Research on teacher knowledge involving in-service teachers remains infrequent (Roche & Clarke, 2013). This study, therefore, brings us additional insights into how Fijian primary teachers interpret nonsensical and problematic word problems. There have been limited studies on teacher knowledge reported from similar contexts. The findings would be relevant for the mathematics teacher training community, given the scarce local literature available on teachers' understanding of word problems.

Categories of teacher knowledge for effective teaching

According to National Research Council (2000), effective teaching can be a general term that describes teaching which leads to students attaining mathematical proficiency. NRC sees three important players in effective teaching—teachers, students and content interacting in meaningful contexts to ensure effective mathematics teaching. They note:

> The effectiveness of mathematics teaching is a function of teachers' knowledge and use of mathematical content, of teachers' attention to and work with students, and of students' engagement in and use of mathematical tasks. Effectiveness depends on enactment, on the mutual and interdependent interaction of the three elements—mathematical content, teacher, students—as instruction unfolds. (p. 9)

The NRC provides a useful categorisation of teacher knowledge. The first area of teacher knowledge they mention is 'mathematical knowledge'. Mathematical knowledge includes "knowledge of mathematical facts, concepts, procedures, and the relationships among them; knowledge of the ways that mathematical ideas can be represented; and knowledge of mathematics as a discipline—in particular, how mathematical knowledge is produced, the nature of discourse in mathematics, and the norms and standards of evidence that guide argument and proof." (NRC, 2000, p. 371) According to NRC (2000), such knowledge is the "cornerstone of teaching for proficiency" (p. 372) and it is important for any discussion on improving student learning. Two questions are central to this domain of teacher knowledge—first, what mathematics teachers need in order to teach effectively, because "you cannot teach what you don't know" (NRC, 2000, p. 373). The second question is that once they know the mathematics themselves, how they are going to use it in classroom practice. (NRC, 2001). NRC's definition of 'mathematical knowledge' covers Shulman's (1986) categories of mathematical content knowledge as well as pedagogical content knowledge. Shulman (1986) introduced the term "pedagogical content knowledge" as a "special kind of teacher knowledge that intertwines content and pedagogy" (Hill, Sleep, Lewis, & Ball, 2007, p. 122). Such knowledge, including what Ma (1999) called a "profound understanding of fundamental mathematics", is based on the fact that 'knowing mathematics' and 'knowing mathematics for teaching' are separate things. As pointed out by Hill et al., (2007), teaching mathematics is not about simply "knowing mathematics in front of students."

The second area of teacher knowledge for effective teaching is knowledge of students. According to NRC (2000), when discussing the issue of teachers' knowledge of students, two things come to mind. First, teachers must know their students as individuals, their attitudes, preferences, likes and dislikes, strengths and limitations and any other general characteristics worth knowing, (such as social, cultural, and economic), and academic backgrounds of their students. The second issue which teachers must also be aware of is the general structure of students' learning, thinking, and development. Then, teachers must know how a particular mathematical learning and thinking develops and what errors or misconceptions students bring to their classrooms and what understandings and misconceptions they inherit in their new classrooms. An excellent example of this type of knowledge of learners is given by NRC (2001) and this is related to teaching of the sign of equality (=). Many elementary school children see the equals sign as something which requires a definite calculation to end the mathematical statement. For example, given the number sentence $8 + 4= \_\_ + 5$, many students would put 12 in the blank (NRC, 2001).

The final related component of teacher knowledge is knowledge of classroom practice. This branch of teacher knowledge is something which is not as simple to define because of its broadness. It includes knowing what to teach, how to organize the teaching, and how to assess students learning. It includes other skills such as organizing and managing classrooms and resources. According to NRC (2000), knowledge of classroom practice is gained through experience. It must be noted that no specific area of teacher knowledge is more important than others. The current study was focused on teachers' mathematical knowledge.

Impact of teacher knowledge

Is teachers' mathematical knowledge linked to students' achievement? According to NRC (2000), the simple answer is yes, based on the assumption that teachers must know, and know well, what subject matter they propose to teach. This simple argument of "you cannot teach what you don't know" leads us to think that instructional practice is a function of teachers' mathematical knowledge. For example, one cannot expect a teacher to adequately explain a given concept in mathematics when he or she has little knowledge of the concept. This argument is consistent with research findings in this area of teacher knowledge. National Research Council (2001) noted that "teachers with relatively weak conceptual knowledge of mathematics tended to demonstrate a procedure and then give students opportunities to practice it. Not surprisingly, these teachers gave the students little assistance in developing an understanding of what they were doing. When teachers did try to provide a clear explanation and justification, they were not able to do so. In some cases, their inadequate conceptual knowledge resulted in their presenting incorrect procedures. Studies such as Ma (1999) also confirm that many primary teachers generally lack a conceptual understanding of mathematics which they are supposed to teach. Although many other studies such as Peressini, Borko, Romagnano, Knuth, and Willis (2004), Fennema and Franke (1992), and, Ball, Lubienski, and Mewborn (2001) recognize that teacher knowledge is a major determinant of what teachers do in their classroom; they argue that mathematical content alone is not sufficient. The works of Shulman (1986, 1987) also pointed out that knowledge of mathematical content alone was not sufficient for effective teaching.

There remains a conflicting relationship between 'teachers having more mathematics' and 'their students' achievement'. One of the reasons for this inconsistency could be related to the complex nature of teacher knowledge (Cooney, 1999). Deciding 'how much' mathematics a teacher needs to know could be another problematic area (Davis & Smitt, 2006). According to Maher and Muir (2013), studies involving only the content domain may not tell us enough about what teachers need. It would be worth considering other domains of knowledge to give a more relevant picture of teachers' knowledge and its impacts in terms of students' achievement. The old assumption teaching mathematics by knowing mathematics or being a mathematician is no longer relevant (Sowder, 2007). Furthermore, using a standardized summative assessment to measure students' achievement could also be a source of inconsistency. In other words, linking teacher knowledge to examination results would only provide limited evidence of impact of teachers' knowledge on students' achievement. This study specifically aims to shift the focus away from the mathematical content domain, by giving a particular focus to applying mathematical knowledge in real world situations.

Primary teachers' mathematical knowledge

Maher and Muir (2013) mention that concerns about pre-service primary mathematics teachers' mathematical knowledge is well explained in the literature. For example, they discuss findings which strongly point out the weak conceptual understandings of fractions amongst 20 percent of the pre-service primary teachers in one of the studies (Stacey et al., 2001, as cited in Maher & Muir 2013) and a strong tendency of pre-service primary teachers in adopting a procedural or instrumental approach to teaching mathematics, where getting the correct answer by repeating the standard procedure is encouraged. This is evident in the lack of reasoning teachers are able to give, for example, for the multi-digit multiplication operations.

Maher and Muir (2013) investigated mathematical knowledge of twenty final year pre-service teachers using a written test, followed by interviews with seven of the pre-service teachers. The purpose of the interview was to achieve a deeper understanding of these teachers on what the student did wrongly. In addition, the interviews would suggest how the pre-service teachers could help the students correct the mistakes. In other words, the interviews provided insights into pre-service teachers' pedagogical content knowledge. The authors looked at the pre-service teachers' test scores and their interview data to explore possible links between knowledge of mathematics and pedagogical content knowledge. The findings suggested that pre-service teachers had a weak understanding of multiplication, with only six out of the twenty providing a correct response to a three-digit by three-digit multiplication problem. Pre-service teachers generally had difficulties in explaining how they arrived at the correct answer, showing a lack of conceptual understanding of multiplication. Many pre-service teachers "would not have been able to help the student with developing an understanding" (Maher & Muir, 2013, p. 85) of the multiplication process.

In a similar study, Roche and Clarke (2013) explored primary teachers' mathematical knowledge about division. They used an open-ended item called Division Stories with 378 primary school teachers from Victoria, Australia. The aim of the study was to examine how practicing primary school teachers would apply the two different models of division and how they would write appropriate representations of simple division scenarios. Such an aim could be seen as comparable to the aim of the current study given that representation of division could be expected to model some real world knowledge of mathematics. The findings with respect to this particular aim were that teachers lacked a thorough understanding of division. The authors predict that these teachers "may not be well equipped" (p. 274) to develop a profound understanding of division in their students.

Research in the area of teacher knowledge has often focused on special curricular sub-domains of mathematics curriculum such as multiplication, division, fractions or decimals. Studies of this nature reported in Verschaffel et al. (2007) confirm that primary mathematics teachers often make the same conceptual mistakes that the students would be expected to make. Verschaffel et al. make the following remarks based on one of the longitudinal studies:

> At the end of the three year training program, the overall test performance had become substantially better, although there were still reasons to be seriously concerned about the readiness of some student teachers to teach mathematics to elementary school children, especially with respect to their modeling and applied problem-solving skills. (2007, p. 606)

Hill et al. (2007), in their review also cite studies that utilised mathematical tasks and interviews to gain deeper understanding of teachers' mathematical knowledge. Hill et al. (2007) claim that the predominant finding of similar studies was that some pre-service teachers do not know the mathematics they will teach. This study utilized a similar mathematical task to explore teachers' understanding of mathematics that they are supposed to teach.

Mathematical modeling, problem solving and word problems in mathematics

Word problems provide a special means for developing conceptual understanding, strategic thinking, and adaptive reasoning—the three important areas for effective mathematics teaching and learning. These three stands of mathematical proficiency could be viewed as strongly intertwined with the idea of problem solving and modeling in mathematics. In other words, teachers and students go through the processes of modeling and problem solving in order to achieve conceptual understanding, strategic thinking, and, adaptive reasoning (Haylock & Manning, 2014; Roche & Clarke, 2013; Verschaffel et al., 2007). Verschaffel et al. (2007) claim that word problems have been an important part of mathematics curriculum and have been used for purposes such as applying formal knowledge and skills to real world situations, developing students' general problem solving skills, and building a conceptual understanding of mathematical procedures.

Haylock and Manning (2014) distinguish mathematical modeling from simple model making by arguing that mathematical modeling is the "process whereby we use the abstractions of mathematics to solve problems in the real world" (p. 54). Furthermore, a good mathematical problem is one "in which we have some givens and we have a goal, but the route from the givens to the goal is not immediately apparent" (p. 56). Both the processes of modeling and problem solving are closely linked and could be seen as complimenting each other. The basic teaching and learning ideas behind the two processes include understanding the problem, setting up a method of solving the problem, including setting up a mathematical model, and checking the solution by interpreting the solution back in the real world situation. These processes are not limited in use to word or story problems only, however, we do use a lot of word problems in primary mathematics when carrying out mathematical modeling and problem solving.

With respect to students' understanding of word problems, Verschaffel et al. (2007) note that many students do not use the ideas discussed above when confronted with mathematical word problems. They provide the following summary of students' actions:

> The student glimpses the problem, quickly deciding what calculations to perform with the numbers given in the problem statement and then proceeds with these calculations without considering any alternatives even if no progress is made at all. (p. 586)

The authors' mention that this lack of sense making to determine if it is "sense-making" when confronted with word problems becomes overt when they are given nonsensical and problematic items to solve. For example, when lower elementary children were asked the nonsensical question: 'There are 26 sheep and 10 goats on a ship. How old is the captain?" a majority of the students were ready to supply an answer (Baruk, 1985; cited in Verschaffel, 2007, p. 587). Verschaffel et al., (2007) review similar findings when students were presented with items which contained technical defects. Such items have been called problematic items (P-items).

Such a "suspension of sense making" (Verschaffel et al., 2007, p. 587), albeit to a lesser degree, has been noted in prospective primary school teachers' problem solving behaviour as well (Verschaffel et al., 2007). This lack of sense making could be a result of a belief that every problem has a single correct solution, which could be obtained by carrying out the arithmetical operation, and that it is acceptable to ignore any real world knowledge of the mathematical situation (Verschaffel et al., 2007). A recent review on word problems (Daroczy et al., 2015) suggests that these problems still persist. Many prospective primary and secondary teachers prefer to apply formal rules of algebra and arithmetic when confronted with word problems without understanding the problems, failing to realise that the solution may be even simpler. In other words, these teachers fail to *understand the problem*, a crucial first step in the problem solving process. A common belief promoted by teachers and mathematics textbooks is that every problem is solvable. Furthermore, there could be specific linguistic or numerical complexities which prevent teachers from making sense of such word problems (Daroczy et al., 2015).

The present study explores these misconceptions in a Fijian setting. This study has the potential to contribute to the limited literature on teachers' sense making of word problems with respect to real world knowledge, especially when the word problems themselves contain flaws. The following section describes the research methodology.

Methodology

This study used mathematical tasks to explore in-service primary mathematics teachers' mathematical knowledge. The main aim of the study was to explore the degree to which in-service primary teachers could identify nonsensical and problematic mathematical tasks with respect to real world knowledge.

Participants

The participants in the study were 34 in-service primary school teachers who were enrolled in a primary mathematics teaching methods course at a Fijian University. There was equal number of male and female teachers. The teaching methods course was offered in the distance and flexible mode. This meant that the majority of the participants were teaching on a full-time basis during the duration of the course. These teachers had access to the online learning platform, and there were a limited number of scheduled face-to-face sessions during the semester as well. Although the exact number of years of teaching experience could not be estimated, it could be inferred that all the participants had at least a minimum of two years of teaching experience at primary school levels. This 'two-years of full time teaching' is an official requirement for enrolment into the Bachelor of Education (Primary) programme. All the teachers had a Diploma in Education from a teacher training college. The Diploma in Primary programme offered by colleges in Fiji has no specific mathematical content courses which is consistent with many other universities across the South Pacific region. Primary teachers have to rely on the mathematics content learnt at the senior secondary level. It would be rational to assume that some of these teachers would not have had success with secondary mathematics. Primary schools in the Fiji Islands provide Years 1–8 of formal schooling.

Instruments

As part of the requirements of the course, these teachers had to participate in a number of face-to-face workshop (tutorial) sessions. In the first workshop session, these teachers were asked to participate in this study by providing answers to the three word problems contained in the workshop activity sheet. They were allowed approximately fifteen minutes at the start of the workshop session to do the activity individually. There was no specific preparation of teachers for this task since it was understood that they were already familiar with the application of real world knowledge. The first author, who was also the course coordinator, was in-charge of this workshop activity. The first word problem was a nonsensical one, while the other two word problems contained some technical errors. The activity contained the following:

> Arithmetic word problems constitute an important part of the mathematics program at the primary school level. Try solving the following word problems and comment on the appropriateness and usefulness of the following word problems for upper primary classes.
>
> Problem 1: There are 26 sheep and 10 goats on a ship. How old is the captain?
>
> Problem 2: Bruce and Alice go to the same school. Bruce lives at a distance of 17 kilometres from the school and Alice at 8 kilometres. How far do Bruce and Alice live from each other?

Problem 3: John's best time to run 100 meters is 17 seconds. How long will it take him to run 1 kilometre?

The three word problems were directly taken from Verschaffel et al., (2007). The three problems have been traditionally utilised for checking pupils understanding of mathematics based on real world scenario. The first problem has been widely used to judge elementary school students understandings of mathematics. The other two problems are examples of problematic items (P – items). These problems require a judgment based on real-world knowledge and not any simple application of mathematical operations (Verschaffel et al., 2007). For example, in problem 2, the exact location of Bruce's and Alice's houses, and the school, are unknown, and it is unrealistic to say how far the two live from each other. This problem is open to many different interpretations. In problem 3, the difficulty when judged from a real-world perspective is that it is almost certainly impossible for John to run at a constant speed over the given distance. However, there is evidence of a few people (seasoned athletes) having run a thousand meters in less than 170 seconds (see for example: https://en.wikipedia.org/wiki/1000_metres_world_record_progression). It was assumed that in-service teachers would easily judge problem 1 as unimportant and irrelevant. It would be reasonable to expect in-service primary teachers to confidently categorise problems 2 and 3 as defective with respect to real world knowledge. The question statement did not explicitly state that the items were defective; it did ask the in-service teachers to 'try solving', and 'commenting' on the appropriateness of the three items.

The written answers and explanations were collected and analysed using a two-way classification suggested by Verschaffel et al., (2007). In other words, a teacher's response was classified as 'non-realistic reaction' if the teacher's answer sheet contained no evidence of any common sense based on real-life knowledge. A teacher's response was categorized as 'realistic reaction' if the answer provided had taken into account any real world knowledge, or a non-realistic answer was supported by a statement indicating that the teacher was aware of the problematic nature of the items. While the sample size remained relatively small, teachers' responses to these items provided sufficient insight into primary in-service teachers reasoning into real world mathematical word problems. The results are presented below:

Results and discussion

Findings of the study are presented for each of the items in the workshop exercise. Each response was read by both the authors, and responses were classified as realistic or otherwise based on the authors' mutual agreement. Each activity sheet was given a number, starting from one up to number 34. This ID is reflected beside the in-service teacher's quotations that are used in this section.

Teachers' response to the nonsensical item

As assumed previously, in-service primary mathematics teachers would easily pick out the nonsensical nature of this item. Only 22 of the 45 in-service teachers (65%) gave responses which could be classified as a 'realistic reaction'. These teachers did not write any numerical answer to this question. Instead, they wrote comments stating the irrelevancy of the item or questioning the usefulness of the item. For example, as one teacher wrote, "What am I supposed to calculate? How old is the captain? The problem cannot be solved as there is/are no data to help determine the captain's age" (Teacher 34). Another teacher had made a similar remark,

> Information is not clear in this problem. What does the number of sheep and goats has to do with the captain's age. Either it should have how many life stock are there or the date of birth of the captain should be given, or something else. This problem is inappropriate. (Teacher 32)

Another teacher noted the inappropriateness and commented that the students could "add up the # of sheep and goats i.e. 26 + 10 = 36 for the captain's age" (Teacher 28). For Teacher 7, this word problem contained nonsense information.

Of the 22, four of teachers (18%) knew that the problem was inappropriate but stated that the item could still be used to test how alert the students at upper primary school are when they attempt word problems. One of the teacher's responses was "This problem tests students understanding or comprehension skills in figuring out that no answer could be produced from the data available" (Teacher 14). Another wrote, "It could be used as a brain teaser to help them think" (Teacher 21). Teacher 20 wrote that it would be useful to help diagnose the learning difficulty that many students have when it comes to solving word problems. The teacher argued that many students "just see the numbers and start adding or subtracting". In summary, 22 out of the 34 teachers (65%) were able to explicitly point out the error in the question. This group of teachers had met our expectations as teacher educators by showing not only that the item was inappropriate but also suggesting that it could be used sparingly to tap on students thinking.

Ten out of the 34 in-service teachers (29%) gave responses which indicated a 'non-realistic reaction" to the nonsensical item. The reasons provided by the ten teachers were often related to the fact that numbers in any given situation could be manipulated to get to an answer. They also noted a view that any word problem containing some numbers could be used in mathematics, and that word problems of any nature are very challenging. For example, one of the teachers wrote that "this is trick question where the sheep and goat relate to the age of the captain which is 36 years old…it means the same thing as adding up numbers to find the final amount" (Teacher 2). Another teacher wrote "the problem is useful because sometimes we just ask remembering questions rather than giving thinking questions (Teacher 30). For Teacher 19, it represented a "very challenging question which requires a lot of thinking". Two of the teachers left this item blank. The findings, much to the surprise of the researchers, suggest that a significant number of Fijian in-service teachers have a very narrow understanding of simple, nonsensical mathematical word problems. Their interpretations of the nonsensical item are no different from any lower primary students' understandings, as suggested by studies on learners' interpretations of same word problems reviewed in Verschaffel et al., (2007). This lack of understanding of mathematical word problems became apparent in the analysis of the problematic items.

Teachers' response to the problematic items

Item 2

Only three out of the 34 primary teachers (9%) gave a response which was classified as a realistic reaction to item two. These teachers saw that there could be many different possibilities of the locations of the two students and the schools. For example, Teacher 23 wrote that

> We don't know the direction that Alice and Bruce use to travel to school. If they come from the same direction, then we can use the differences between the distances travelled to school, i.e., 17 – 8 = 9 km. If they come from opposite directions, then we can add the directions.

Another teacher gave a similar descriptive scenario whereas the third teacher drew two diagrams to show the different directions stated by Teacher 23. These responses show us a reasonable understanding of the problem in a real world situation. None of the three teachers were able to geometrically situate the two students and their school in the numerous locations to conclude that the item was problematic and not recommend it for use. For example, none of the teachers used a non-linear or a two-dimensional representation of the problem. Their responses showed that they could only see the school in two different possibilities—either in a corner or in the middle of a horizontal line. However, their supposedly limited interpretation of this problem could be due to the reason that they would have taken this situation from a primary school student's perspective. It must be noted that primary school teachers do not teach higher-order coordinate geometry concepts in primary schools.

The majority (31) of the in-service primary teachers' responses (91%) were scored as non-realistic response. This group of teachers gave one common answer to this problem. All of them felt that this problem was appropriate for the upper primary class without realising that there could be multiple possibilities. Also, these teachers failed to mention that the information given in the question was insufficient to locate the three venues. They went ahead with solving the problem, giving an answer of nine kilometres as the distance between the two houses. All the teachers in this group saw this problem as a linear or 'number line' arrangement in one direction only—with the school in one corner (left) and the two houses on the other (right); or vice-versa.

As previously mentioned, such an understanding is reflective of a primary school student's understanding of the situation. It could be said that the majority of the primary school teachers are accustomed to the mathematics they teach in that they gave only an inadequate real worldview of the problem.

Item 3

A similar pattern of thinking was observed in the analysis of the final problematic item. This item was structurally sound. The only limitation of this item was that John could not continue running at a constant speed over a longer distance. Out of the 34 in-service teachers, only four of them (12%) were able to pick out this real world applicability of the item. These four responses were scored as realistic reaction. For example, as Teacher 20 wrote:

> Students will be able to solve this question with much ease but I believe that this question is not appropriate in real life context as human's speed cannot be constant through one kilometre. I believe that we will give our students the wrong picture.

Another teacher stated: "the problem here is appropriate but we all know that John is not a machine and wouldn't maintain his speed. If it was a car, then it would be appropriate" (Teacher 10).

Thirty of the teachers (88%) saw this item as appropriate. They solved the item by converting the units and using proportion method to arrive at an answer of 170 seconds. These teachers did not realize that John will not be able to maintain a constant speed over one kilometre. Findings of the study are summarised in the table below.

Table 1. Summary of findings

Item	Realistic reaction	Non-realistic reaction
1. There are 26 sheep and 10 goats on a ship. How old is the captain?	Twenty two teachers (65%) able to identify that this item cannot be solved. Four teachers (12%) said that the item can be used sparingly.	Ten teachers provided an answer—the captain is 36 years old. Two teachers (6%) did not attempt this item.
2. Bruce and Alice go to the same school. Bruce lives at a distance of 17 kilometres from the school and Alice at 8 kilometres. How far do Bruce and Alice live from each other?	Three teachers (9%) gave a reasonable explanation by offering two different solutions. They said that the problem was not appropriate.	Thirty one teachers (91%) solved this problem by doing 17 - 8 to give 9 as the answer.
3. John's best time to run 100 meters is 17 seconds. How long will it take him to run 1 kilometre?	Four teachers (12%) were able to point out that the problem does not represent a real world situation.	Thirty teachers (88%) said that this problem was appropriate. All of them gave an answer of 170 seconds.

Conclusion

The aim of the study was to explore whether Fijian in-service primary teachers apply real world knowledge when solving nonsensical and problematic word problems. The teachers' responses to the nonsensical item showed that almost one third of them could not work out the meaninglessness of this item. This could suggest that even teachers are carried away by the desire to perform arithmetical operations blindly, without making sense of the mathematics in a given item. They were happy to provide an answer and some even explicitly stated that the problem was a good one.

The analysis of teachers' responses to the problematic items revealed that the majority of the teachers continued to exhibit a lack of understanding of good word problems, which could be used to showcase real world examples of mathematics. Only a small number of teachers could provide a limited insight into conceptual understanding, strategic competence and adaptive reasoning. None of the teachers demonstrated a complete understanding of Item 2. For Item 3, only four of the teachers correctly pointed out the weakness in the item with respect to real world knowledge. It could be inferred that the majority of teachers see any word problem containing a few numbers as a good word problem. They quickly begin to manipulate these numbers to get to an answer. Such a behaviour has been noted in elementary school- aged children and in some prospective teachers as well (Verchaffel et al., 2007). It could be argued that the teachers who had difficulties in recognising the nonsensical and problematic aspects of the word problems would face a range of difficulties in interpreting other types of conventional word problems. These teachers may also face difficulties in developing their own word problems for use in teaching or assessment situations. These conjectures could be further explored in future studies.

One of the limitations of this study is related to the nature of the data collection. The data in this study was limited to in-service teachers' written responses to a workshop activity. While this has benefits in terms of eliciting valid individual responses, unlike situations where teachers could have copied someone else's responses, it could be possible that teachers felt obliged to provide a definite answer, given that this was their first face to face session with their course coordinator. Another plausible limitation could have been related to language problems, which would have prevented some of the teachers to fully understand the word problems, as Daroczy et al. (2015) claim that the linguistic complexities related to word problems could become severe for those who use English as a second language. For the majority of teachers in this study, English would be their second preferred medium for communication. The authors believe that future studies must take into account any linguistic differences and difficulties. A final limitation of the study is linked to the triangulation of data. Teachers written responses did provide an adequate insight into their mathematical thinking in relation to real world knowledge. However, the study could have benefited by utilising follow-up interviews to probe further into teachers' understandings and misconceptions. It would have been useful to observe some actual teaching lessons related to the study's focus to yield an even pronounced comprehension of a teacher's knowledge with respect to real world word problems. Such a methodology would help elicit possible pedagogical benefits of word problems, including nonsensical and problematic items. Such nuanced methods could be explored in future studies. Another area worth exploring would be problematic and nonsensical items which are non-word problems.

The mathematics education community is well aware of the problems of primary mathematics teachers' lack of mathematical knowledge. This study paints a largely deficit picture of primary mathematics teachers' knowledge. However, the common notion of 'we cannot teach what we do not know ourselves' (NRC, 2001; Ma, 1999) necessitates us to accept that this could be one of the leading challenges of Fijian mathematics education community. This leads to the key implications of the findings for the current study—an urgent need for in-service primary teachers to strengthen their mathematical knowledge. This could be achieved by teachers' self-assessment and reflection on their regular practices. Also, any teacher professional development programme must include elements of the self-reflective process. According to NRC (2000), such knowledge is the "cornerstone of teaching for proficiency" (p. 372). In-service primary teachers need to be assisted in strengthening not only their mathematical content knowledge, but also their pedagogical content knowledge. The fact that these teachers are already in the service means that such assistance must come in the form of continued professional development of primary mathematics teachers. For example, practicing teachers could be given more opportunities to engage in analysing word problems and exploring the pedagogical potentials and limitations of such problems. What seems alarming is that the teachers in the current study already have several years of teaching experience and held some form of formal undergraduate qualifications. The findings, although based on a small sample of in-service teachers, could imply that Fijian primary school teachers' knowledge of word problems is limited and is an area worth investigating further. Such investigations must consider teachers' development and use of word problems at the classroom level, in both teaching as well as in assessment contexts.

References

Australian Curriculum, Assessment and Reporting Authority. (2012). *The Australian curriculum: Mathematics.* Sydney, NSW, Australia: Author.

Ball, D. L., Lubienski, S.T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (pp. 433–456). New York, NY: Macmillan.

Cooney, T. J. (1999). Conceptualizing teachers' ways of knowing. *Educational Studies in Mathematics*, *38*, 163–187. https://doi.org/10.1023/A:1003504816467

Daroczy, G., Wolska, M., Meurers, W. D., & Nuerk, H. C. (2015). Word problems: A review of linguistic and numerical factors contributing to their difficulty. *Frontiers in Psychology, 6*, 348. https://doi.org/10.3389/fpsyg.2015.00348

Davis, B., & Simmt, E. (2006). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics*, *61*, 293–319. https://doi.org/10.1007/s10649-006-2372-4

Dossey, J. A., McCrone, S., Giordano, F. R., & Weir, M. D. (2002). *Mathematics methods and modeling for today's mathematics classroom: A contemporary approach to teaching grades 7–12.* Pacific Grove, CA: Brooks/Cole**.**

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147–164). New York, NY: Macmillan.

Haylock, D., & Manning, R. (2014). *Mathematics explained for primary teachers*. London, England: Sage.

Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing teachers' mathematical knowledge: What knowledge matters and what evidence counts. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 111–156). Charlotte, NC: Information Age.

Ma, L. (1999). *Knowing and teaching elementary mathematics. Teachers understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum

Maher, N., & Muir, T. (2013). "I know I have to put down a zero, but I'm not sure why": Exploring the link between pre-service teachers' content and pedagogical content knowledge. *Mathematics Teacher Education & Development, 15*(1), 72–86.

Ministry of Education. (2007). *The Fiji Islands National Curriculum Framework.* Suva, Fiji: Author.

Moss, C. M. (2013). Research on classroom summative assessment. In J. H. McMillan (Ed.), *Sage handbook of research on classroom assessment* (pp. 235–255). Thousand Oaks, CA: Sage. https://doi.org/10.4135/9781452218649.n14

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics. Reston, VA: Author.* Retrieved from www.http://my.nctm.org/standards/document/chapter2/index.htm

National Research Council. (2000). *How people learn: Brain, mind, experience, and school.* Washington, DC: National Academies Press.

National Research Council. (2001). Adding it up: Helping children learn mathematics. doi:0.17226/9822

Peressini, D., Borko, H., Romagnano, L., Knuth, E., & Willis, C. (2004). A conceptual framework for learning to teach secondary mathematics: A situative perspective. *Educational Studies in Mathematics*, *56*, 67–96. https://doi.org/10.1023/B:EDUC.0000028398.80108.87

Roche, A., & Clarke, D. M. (2013). Primary teachers' representations of division: Assessing mathematical knowledge that has pedagogical potential. *Mathematics Education Research Journal, 25*(2), 257–278. https://doi.org/10.1007/s13394-012-0060-5

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4–14. https://doi.org/10.3102/0013189X015002004

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1–22. https://doi.org/10.17763/haer.57.1.j463w79r56455411

Sowder, J. T. (2007). The mathematical education and development of teachers. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp 157–223). Charlotte, NC: Information Age.

Verschaffel, L., Greer, B., & de Corte, E. (2007). Whole number concepts and operations. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 557– 628). Charlotte, NC: Information Age.