
Numeracy as critical thinking

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**Abstract**

Adult learners preparing for university level study have varying degrees of prior experience, confidence, ability, motivation and need when it comes to mathematical tasks. The development of numeracy skills is a core aspect of university preparation, with many university courses requiring a certain level of mathematical literacy.

The standard model of numeracy support in enabling programs essentially replicates what students learned (or failed to learn) in high school. For adult learners with previous negative experiences, this approach can be intimidating, while others perceive high school mathematics as irrelevant and unnecessary to their chosen field of study. Adult learners require an entirely new approach to numeracy support.

Our approach has been to turn the standard model on its head, by recognising numeracy as a form of critical thinking that requires much more than an ability to manipulate numbers. The University of Tasmania Academic Numeracy unit utilises Watson’s (2009) Four Resource Model of Numerical Literacy to focus on critical evaluation of information, in both everyday and academic situations. The development of higher-order critical thinking skills forms the core of our unit, with explicit mathematical instruction provided in service of situating and critiquing real research. This approach has allowed us to develop critical numeracy skills in a way that is relevant, meaningful and valuable to all learners, whatever their future academic endeavours.

Keywords: numeracy, critical thinking

**Introduction**

The importance of numeracy skills in the workforce and daily life is becoming an increasingly pertinent issue. The relevance of mathematics is receiving attention both in Australia (Marginson, Tytler, Freeman and Roberts, 2013; Office of the Chief Scientist, 2014) and worldwide (Breiner et al., 2012) as STEM (Science, Technology, Engineering and Mathematics) education is seen as essential to support the modern knowledge based economy (Abbott-Chapman, 2011). As such, there is an expectation that university graduates will be both literate and numerate, with numeracy skills being increasingly recognised as a key outcome of university study. Deputy Vice-Chancellor of the University of Technology Sydney Shirley Alexander says, “We think that to produce graduates that are really going to contribute to society every single graduate needs to have numeracy” (Alexander, 2016).

There is a growing recognition that numeracy skills form an important precursor to successful study at the university level. Concerns about school leavers’ level of mathematics preparedness for entering STEM disciplines is garnering high levels of focus. Sydney University has recently reversed a
decision to waive mathematics pre-requisites, and from 2019 will reinstate 2-unit mathematics as a prerequisite for entry into 62 degrees (Alexander, 2016), including economics, commerce, engineering and IT, psychology, pharmacy, veterinary science and science. These courses have an indiscussable mathematical component. However, an understanding and ability to think critically about numerical concepts and information is important for students in all academic areas (Gazit 2012). Indeed Australian Prime Minister Malcolm Turnbull has recently suggested that Maths/Science should be a prerequisite for any university study. “More universities are requiring and they should all require in due course that mathematics or science should be a prerequisite school subject to have completed to go onto university.” (Bagshaw, 2016).

The University of Tasmania Tertiary Numeracy Enquiry (Skalicky et al., 2010) audited the level and type of mathematics and numeracy in undergraduate courses across the university and the level of numeracy needed to successfully complete an undergraduate course. It was conducted in response to staff concerns about a mismatch between numeracy requirements of University courses and students’ ability to meet these demands. Findings emphasise the need for numeracy skills across all disciplines, with nearly every School reporting that data representation, analysis, and interpretation were specifically required for their courses. Academic staff also identified concerns about the critical thinking and problem solving abilities of students, which is consistent with findings at other higher education institutions in Australia (e.g., Chapman, 1998; Galligan & Taylor, 2008; Kemp, 1995) and internationally (e.g., Gill & O’Donoghue, 2006), as well as at the pre-tertiary level (Office of Tasmanian Assessment, Standards and Certification, 2008).

**Development of the Academic Numeracy unit**

The University of Tasmania Academic Numeracy unit was developed in response to the findings of the Tertiary Numeracy Enquiry, to ensure all enabling course students had a base level of numerical literacy, regardless of their future area of study. The focus of the core, semester-long Academic Numeracy unit is a critical thinking approach to build numerical literacy.

Pre-degree Programs at the University of Tasmania is the entry level course for students who have not traditionally considered tertiary education (including first-in-family, mature-aged, low socio-economic background, those not meeting entry requirements and students with health or learning issues). This course acts as an enabling, pathway program for students who wish to enter bachelor level study and equips adult learners with skills needed to successfully transition to degree-level study.

Pre-degree Programs adult learners have different and diverse mathematical backgrounds and abilities, and varying levels of confidence and motivation. Many are intending to undertake non-STEM degrees (Education, Nursing, Arts, and Social Sciences etc.) and may not see the need to study mathematics. The student cohort is non-traditional, 27% aged over 30 years, 24% last studied mathematics more than 10 years ago, and 30% have not studied mathematics beyond Grade 9/10. Approximately 13% of the students are from culturally and linguistically diverse backgrounds. Many students have disengaged from study (particularly mathematics) earlier than year 10. In Tasmania, overall tertiary participation of 14.3% is well below the national average of 18.8%, with some regions, such as the Cradle Coast being well below average at 8.7% (Australian Bureau of Statistics, 2011). Many students enrolling in Pre-degree mathematics programs have either not attempted or not done well in college mathematics for a variety of reasons, including lack of ability, interest, motivation or perceived relevance, ability, unsuitable teaching techniques and prior negative experiences.

**Background - what we used to do and why we needed to change**

Previous entry-level mathematics units in the University of Tasmania Pre-degree Program were essentially a replication of high school mathematics with traditional topics (number, algebra, geometry,
and statistics) delivered in traditional ways. However, the traditional content and delivery was not producing good results: students lacked confidence, their liking of and ability in mathematics was not increasing and many did not see its relevance to non-STEM areas of study. Many students objected to the fact that the unit replicated high school mathematics and they felt that they had been there, done that!

The challenge was to design a curriculum that was significantly different to the standard unit. We needed to meet a number of requirements, in a way that resolved multiple, somewhat competing, constraints. The new unit would:

- Prepare maths-inclined students for further mathematics study
- Prepare nursing/paramedic practice/education pathways students for mathematics entry requirements
- Prepare students entering non-mathematics disciplines for general academic success in first year units
- Be unlike high school mathematics courses
- Achieve all of the above while catering to a non-traditional student cohort with vastly diverse mathematical backgrounds and abilities.

Additionally, we were keen to improve our student’s attitudes towards mathematics by building our student’s confidence in their mathematical abilities and fostering in them an understanding and appreciation of the relevance of mathematics in their lives.

**The critical thinking approach**

In designing Academic Numeracy, we were well aware of a larger responsibility to our students: for many of our students, this may be their last formal mathematics instruction. What should a “last chance” mathematics unit focus on? How much mathematics is “enough” (for academia, for life)?

High school mathematics typically divides mathematics into specific topics: number and algebra, measurement and geometry, statistics and probability (Australian Curriculum, Assessment and Reporting Authority, 2016). Rather than mirror these topics, we focus on developing skills that go far beyond being able to do calculations and solve word problems. We realised that mathematical ability is in fact a form of literacy, in which critical thinking performs a crucial role. Numerical literacy involves making sense of information that contains mathematical concepts and expressions, making connections with other ideas and contexts, and creating meaning and knowledge. In short, it involves being able to think critically about numerical information. Critical thinking, then, is the both the foundation and the goal for the Academic Numeracy unit. The unit aligns with Jane Watson’s Four Resource Model for critical numeracy (Watson, 2009), which in turn is based on a framework for critical literacy (Freebody & Luke, 1990). The Four Resource Model (Figure 1) highlights the components of “de-coding, meaning-making, using and analysing” within the critical thinking framework. The application of this framework in the Academic Numeracy unit involves engaging our students in understanding, analysing, evaluating and making decisions about everyday and academic issues that involve mathematical concepts.

The critical thinking approach makes the unit very different to previous high school courses our students may have encountered. With the development of higher-order critical thinking skills at the core of the unit, the unit is immediately relevant and valuable to all of our students, whatever their chosen field of study.

To meet entry requirements for STEM and other pathways, we recognised the need to provide explicit mathematical instruction within the unit. However, the diversity of our student cohort discouraged a traditional approach. One consideration that was paramount in our unit design was the
The kind of maths that people are taught at school focuses on algebra and calculus, which they hardly ever use later in life. … You use statistics all the time – for the weather forecast or calculating your income. And whether you’re talking about it with other academics or in the pub, these are topics that matter to people.

(Roser, quoted in Cumming 2015)

**Table 1.** Watson’s Four Resource Model for Critical Numeracy (2009)

<table>
<thead>
<tr>
<th>Decoding</th>
<th>Meaning-making</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terminology</td>
<td>What do I know?</td>
</tr>
<tr>
<td>Maths ideas and key concepts</td>
<td>What is the text about</td>
</tr>
<tr>
<td>Different ways numbers are used and represented</td>
<td>How do mathematical concepts make sense and help me understand this context?</td>
</tr>
<tr>
<td>Key mathematical processes and procedures</td>
<td>What is confusing or misleading?</td>
</tr>
<tr>
<td>Are there other possible meanings?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Using</th>
<th>Analysing</th>
</tr>
</thead>
<tbody>
<tr>
<td>How are the numbers or mathematical concepts significant of useful?</td>
<td>Is it true? What is the evidence? Are mathematical concepts used appropriately?</td>
</tr>
<tr>
<td>What is the purpose of the text</td>
<td>Is it logical and consistent? Is it researched appropriately? From a reputable source?</td>
</tr>
<tr>
<td>How might the text be used to promote different viewpoints?</td>
<td>Is it fair? Different views, values, perspectives or types of research.</td>
</tr>
<tr>
<td>What are the possible applications?</td>
<td>What is missing?</td>
</tr>
<tr>
<td>What are the likely impacts?</td>
<td>How does it position me?</td>
</tr>
<tr>
<td>How would I use this text and what decisions would I make based on it?</td>
<td></td>
</tr>
<tr>
<td>Am I now thinking about issues and mathematical concepts differently?</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 1.** Watson’s Four Resource Model for Critical Numeracy (2009)

With this in mind, the Academic Numeracy unit centres around critical thinking about statistical research, using both everyday and academic examples, with explicit mathematics instruction provided as needed, to clarify, situate, and critique actual research. In this way, students develop the mathematical skills needed for entry requirements, alongside higher-order critical thinking skills that are of value to all students regardless of their future academic endeavours.

**How we do it? The structure and focus of our unit**

Forefront in the minds of staff in the Academic Numeracy unit is the nature of the cohort of students. The Australian Association of Mathematics Teachers (AAMT, 1998) discusses the importance of a grounded appreciation of context and this is particularly important for our adult student cohort who, as noted above, may have disengaged from high-school mathematics, have poor mathematics’ attitudes and do not see the relevance of algebra and trigonometry to their lives. Kemp and Kissane (1990) consider that “If students have not learned how to use mathematics across the curriculum unless explicitly asked to do so, it is unlikely that they will use mathematics beyond the mathematics classroom” (p. 117). Additionally, Maciejewski (2012) and Clute (1984) found that students with high
math anxiety benefited from an approach based on explicit instruction. To address this issue, the Academic Numeracy unit provides sequenced skill-based, computational exercises aligned with in-context use of those mathematical skills to make sense of everyday and discipline specific research. The focus is to engage students in using mathematical tools to critically evaluate and make meaning of mathematical information using critical thinking skills. Students are encouraged to consider mathematics as a way of thinking and communicating with direct relevance to their lives, future study and career aspirations. Further details are provided in the sections below.

The unit structure employs principles of constructive alignment (Biggs & Tang, 2007), with components structured around Watson’s Four Resource Model for critical numeracy (Watson, 2009). Opportunities for students to engage in self-directed study develops their understanding and skills from different entry points. Additionally there are mechanisms for self-checking their understanding and formal assessment processes that engage with both numerical literacy and critical thinking.

Integral to the sequential development of skill and concept is the linkage between different aspects of the unit. Each component (lectures, tutorials, open learning resources and assessment tasks) focuses on aspects of Watson’s Four Resource Model (2009) and provides recursive and iterative opportunities for students to develop and check on construction of knowledge, skill level development and the ability to apply number skills to critical thinking processes. Within this framework, opportunities are provided for students to revisit earlier tasks, and pursue additional work at their own level and pace (Figure 2).

Figure 2. Relationship between components of Academic Numeracy highlighting the recursive and iterative nature of opportunities for students to develop skills.

**Lectures**

Lectures utilise all four parts of Watson’s (2009) Resource Model for critical numeracy with content based around statistical literacy. Lectures provide opportunity to develop understanding of terminology and number skills (de-coding), while placing mathematical concepts within a meaningful and relevant context (meaning-making). Lectures analyse how numbers and mathematical concepts promote particular viewpoints and allow predictions and decisions to be made (using). Students are encouraged
to examine evidence, assess assumptions and perspective, and evaluate and reflect on their own position (analysing). Throughout the lectures, students work through problems, engage in individual and group activities, self-check understanding and clarify and develop concepts and skills with peers and staff.

The statistical focus of the unit is not typical - statistical calculations form only a small part of the broader notion of statistical literacy. We aim to develop informed citizens who recognise the value of statistics and large-scale population research in providing insights into current issues, where content and delivery emphasises the importance of data in telling a convincing story about the world. The use of popular media as a source of text and data engages students in the four domains of Watson’s Four Resource Model (2009). Graphs, tables, case studies and evidence presented in legal cases is used to engage students in numerical concepts relevant to everyday life. This is where the mathematics is in everyday life! We try to bring in topics that matter to people, and the research around these topics that is based on statistics (not algebra and calculus).

**Example 1: Climate Change.**

To introduce the topic, students consider data from current and recent newspapers highlighting heatwaves, droughts, floods etc. Long-term data in the form of reports, tables and graphs from the Australian Bureau of Meteorology and other reputable sources builds the story. Students then examine their own perception of climate change, with discussion regarding what needs to be known to make informed decisions. Students are challenged to undertake meaning-making and use data to critically evaluate and analyse the concept of climate change. Previous skill development in percentages and ratios are used in context to support understanding and interpretation of the statistics and graphs.

**Example 2: Mandatory bike helmet legislation.**

A mandatory bike helmet law lecture provides an example of integration of critical thinking skills and mathematical skills developed in the unit within the context of Watson’s Four Resource model (2009). Initially the lecture involves students engaging in Watson’s concepts of meaning-making, using and analyzing, by examining their personal views and considering the wider context of mandatory helmet laws. Discussion centers around interpreting research and popular articles that promote differing viewpoints regarding helmet laws. Class discussion culminates in developing ideas about what types of data would be required to assess the effect of mandatory helmet laws.

Additional web-based research into the effectiveness of bike helmets leads to examination of an original research article by Dinh et al. (2013). The research results are discussed and interpreted using statistical concepts that have been developed within the unit (percentages, ranges, demographic data, sample selection, correlations and causation etc.) This involves the process of integrating de-coding with using, meaning-making and analyzing to integrate mathematical skills in an example of a broader context.

**MiniMaths**

Within the formal lectures, “MiniMaths” comprises a short numeracy skills focussed component (part of Watson’s (2009) de-coding) to introduce or reinforce numerical concepts required to engage in critical thinking. This explicit mathematics instruction supports students and prepares for further study in the other critical numeracy domains. Varying backgrounds made it essential to include this component, while delivery also needed to be sensitive to students already skilled in basic numeracy concepts. The focus of the unit is on developing numerical critical thinking and the inclusion of the MiniMaths component is innovative as it is provided for students to develop mathematical skills at their own pace, at their own level.

Students are presented with a concept (such as order of operations, rounding, directed numbers, decimals, drawing graphs) in a short focused mini-lecture, followed by an opportunity to explore and self-assess their understanding. Additional self-study opportunities are provided, including relevant
self-paced open learning resources (Khan Academy clips and guided lessons through the web-based program MathsOnline).

Students can work through suggested MathsOnline lessons to develop skills introduced in MiniMaths. Students with lower level entry skills can search for and work through lessons appropriate to their individual level at their own pace in privacy. Students with advanced level skills can extend and expand their knowledge with access to higher-level lessons and questions. Through a process of self-assessment of their skill and confidence in the MiniMaths lesson, students can do as much or as little as they individually need.

**Tutorials**

Tutorials engage students in Watson’s (2009) domains of using, meaning-making by examining texts and research articles to identify mathematical concepts and make sense of the material in context, utilising the terminology and mathematics concepts covered in the lectures and “MiniMaths” components. For example, students are asked to design and conduct a short research study to identify average signature length. As a tutorial group, students develop the research question, design an experiment, collect and analyse the data by performing basic analysis of central tendency, drawing tables and graphs to identify distribution of data to draw conclusions. In other tutorials, students are involved in Watsons (2009) domain of analysing to assess whether mathematical concepts are being used appropriately by examining the evidence provided in texts and to understand their own position.

Each weekly tutorial provides the support that links with the MiniMaths, weekly quiz, tutorial exercise and numerical component that is introduced in the statistical research component of the unit.

**Weekly online quizzes**

Students engage in self-checking their skill development with low weighted weekly online quizzes related to the MiniMaths topic. The quizzes are focused on Watson’s (2009) domain of decoding to provide development of underpinning numeracy skills which are then applied in critical thinking contexts. Additionally, the MiniMaths and quiz weekly topic forms part of the following week’s tutorial which enables tutors to check on skill development and provide students with specific and focussed support.

**Open Learning Resources (OLRs)**

To support student development, open learning resources are used to provide self-paced learning support for students. These OLRs provide opportunities to engage students in all domains of Watson’s (2009) Four Resource model by exposing students to examples of data representation and research studies that highlight skills or extension topics covered in the lectures.

**But where’s the maths?**

The choice of topics within the unit are selected by the current and future needs of the students to understand and interpret research results within their non-STEM degree context. The focus is on conceptual development but with some explicit instruction in

- Statistical literacy and data analysis - research design, variables, graphs (drawing and interpretation), tables, calculations of central tendency, normal and skewed distribution
- Proportional reasoning (as research results are often expressed using percentages and ratios)
- Algebra (as a way of formalising patterns and linked to relationships between variables)
- Finance (for personal numeracy)

Statistical literacy is a core component and is the focus of the first six weeks, as well as the overarching theme. Although there are some calculations of measures of central tendency, this is done
only within the context of evaluating statistical research, with the emphasis on critical thinking about what the research results are telling us. The development of number facts is integrated with statistical literacy by discussing aspects such as sample size and figures as raw numbers and as percentages or ratios. The focus is on understanding the figures in context to interpret and find the significance of the results.

**Student perspective**

Many students report that they find the unit surprising and challenging. The focus of the unit is very different to what they were expecting or their previous experiences with mathematics, resulting in changes to their perceptions.

Overall my view and attitude towards learning Maths have dramatically changed in direct response to what I have learned this semester from the unit.

Part of our concerns about the original content and structure of the previous Academic Numeracy unit was that it mimicked high school mathematics and our aim for the new unit was to challenge students to think differently and apply mathematics in a context to which they had not previously been exposed. For some students the challenge is unexpected. Based on previous study of traditional mathematics topics students expect that they will be able to easily complete the assessment tasks. However, the critical thinking focus and the requirement for meaning-making, using and analysing produces a more demanding unit.

I found that I had to work in order to comprehend the concepts.
The Assignments were challenging but very enjoyable to partake in.

Students respond positively and develop confidence with mathematics, with many seeing the value in critical thinking about numerical information and the importance of developing skills in the decoding domain to support higher order thinking.

Learning how to critically evaluate information with a mathematical view. Given that I was a less than successful student in high school many years ago, I have large gaps in my mathematical knowledge. This unit has helped to fill these gaps with things such as percentages, fractions. The ability to utilise mathsonline as an additional source of education which is tied in with the weekly lessons and helps clarify and reinforce what has been taught is also of great benefit.

Particularly relevant for the cohort of students is the increase in confidence and reduction in maths anxiety. Many students report that they are less fearful of mathematics.

At the beginning, I was very nervous about taking this unit but now I feel much more confident in mathematics than I could ever imagine.
I had really struggled with my maths at high school and developed a negative relationship and I was extremely nervous about starting this course. I can now quite comfortably say that I really enjoyed this course, and was eager to see my results… I am currently looking into trying the next level up.

Students find that they transfer the skills learned in the classroom to their everyday life, talk to their families about the research studies and learn to see the relevance of mathematics to their career aspirations.

I found myself challenging myself when I was offered an opportunity to investigate numerically in day to day life, previously I would have let it go.
I am pleased that I am able to use the things we learnt in real life as well, such as calculating percentages, thinking critically about information presented by media sources, and also tax.

The unit requires students to read research articles and write responses explaining their answers using supporting evidence from the research and/or graphs. Ackland (2014) argues that literacy may be an additional barrier. This may be true if the assessment tasks are primarily a procedural mathematics
problem dressed up (e.g. a word problem). However, we design assessment to be primarily a higher order conceptual task (with perhaps some procedural mathematics required to fully appreciate it) and utilise current topics and issues to engage students and promote the relevance of mathematics to their lives and study. For example, assessments relate to changes in learner driver speeds, health issues (heart attack rates, dental costs, autism, and happy people getting more done at work) and current finance issues.

One of our original concerns was how to provide learning opportunities that addressed individual student entry points. Our answer was to provide the short MiniMaths lecture each week to introduce the topic, provide links to open learning resources and MathsOnline lessons and then give students an opportunity to self-check progress with weekly quizzes. This allows students to develop their own understanding from their individual base level.

I found the MathsOnline lesson to be very helpful and a really useful resource to cement the techniques learnt in the lectures and tutorials.

The material and resources were particularly interesting and helpful in assisting me to grasp various numeracy concepts. The additional online mathematics aids were excellent and being able to access so much information at my own pace lead to my improvement in the subject.

The University of Tasmania student feedback system (eVALUate) provides quantitative and qualitative responses from students. Students from 2015 and 2016 indicate high levels of overall satisfaction with 96% of responding students indicating overall strong agreement or agreement in favour of the unit and learning opportunities provided. Specifically students respond positively to the following unit evaluation questions in relation to the unit.

- The learning experiences helped me achieve the learning outcomes - 96% agree or strongly agree
- The learning resources in this unit help me to achieve the learning outcomes - 96%
- The assessment tasks in this unit evaluate my achievement of the learning outcomes - 97%
- I am motivated to achieve the learning outcomes in this unit - 93%
- I make best use of the learning experiences in this unit - 92%
- I think about how I can learn more effectively in this unit - 91%

**Authors’ reflection**

Development of the unit has been a challenging and rewarding experience. For those involved, the focus on critical thinking about numbers has been a process that has required a change in the way we think about mathematics and what it means for students. Rather than focusing on process as a goal, we have been constantly looking for relevant and interesting examples that provide an opportunity to integrate number skills. This provides students with a reason to develop number skills that promotes an understanding of the concepts that underpin the examples.

It has been challenging to align the two strands and still ensure that we meet the requirements for students who are progressing to non-STEM degrees, while providing an engaging unit that aims to alter the attitudes of many students to mathematics

**Reflection of staff unit teaching staff**

Staff involved in the unit have identified that the critical thinking approach in this unit is desirable for adult learners, as being able to critically analyse a piece of research or even a newspaper article for soundness will serve them well both in university as well as generally in life. The focus on critical thinking and analysis using mathematics has the effect of producing two groups of students: one group
loves the approach and the other is wary and perhaps confused as to why there is not more standard mathematics in the unit. Staff find that the critical thinking approach has the welcome effect of “levelling the playing field”: Students who are comfortable with calculation-based tasks still find the critical thinking aspects challenging and rewarding, whereas students who are initially daunted by standard mathematics find that our approach allows them to engage with mathematical thinking in a way that has been previously inaccessible to them. Most of our students come to appreciate that critical thinking is the cornerstone of numerical literacy, and understand that the skills they learn in this unit will stand them in good stead throughout their future study.

Summary

The development and delivery of Academic Numeracy as a mathematics unit with a focus on critical thinking has been a challenging and rewarding process for staff that has required us to think differently about mathematics education. We have developed a unit that engages adult learners in thinking critically about mathematical information, while providing underpinning explicit mathematics instruction to enable students to appreciate and utilise mathematics across their preferred discipline area and within their lives. Responses from students have been generally positive, with some students surprised at their ability and enjoyment of mathematics. By being cognisant of the diverse entry levels of skills we have created a unit that allows self-paced development in procedural mathematics alongside higher order critical thinking skills to provide a rich and relevant learning experience for all of our students.

This was my favourite subject…. I finally understood and ‘got’ numeracy.

References


