I'm proud to be a toy teacher:

Using CRA to become an even more effective teacher

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The role of manipulative materials in mathematics can be a contentious one. It seems that in the mind of some teachers, manipulatives are fine in the early years but have no place in a rigorous maths classroom as the students get older. It is a topic worth our consideration.

When I started teaching a very long time ago, I was talking to a secondary mathematics teacher who disparagingly called primary school mathematics teachers 'toy teachers', in reference to the use of materials in the teaching and learning program. Although this was meant disapprovingly, the research is quite clear that in fact, all teachers should be toy teachers!

In 2003 John Hattie wrote about the difference between the 'expert' teacher and the 'experienced' teacher and identified five major dimensions of the expert teacher that differentiates him or her from the experienced teacher. Hattie wrote that expert teachers can: "identify essential representations of their subject; guide learning through classroom interactions; monitor learning and provide feedback; attend to affective attributes; and influence student outcomes" (2003, p.6). It can be argued that the thoughtful use of manipulative materials can facilitate the use of all five of these dimensions. Other research around effective teachers (e.g. Charalambous, Hill & Mitchell, 2012) highlights that effective teachers provide quality instruction. Quality instruction is instruction which is focussed, and requires teachers to make decisions (Hill & Ball, 2009; Hill et al., 2008; Sullivan, 2011). One major decision to be made is the place of procedural knowledge and conceptual knowledge in the teaching of mathematics (Rittle-Johnson & Schneider, 2015; Star & Stylianides, 2013). It is my experience that it is the goal of most teachers to teach in a way that allows students to develop conceptual understanding. This seems to suggest two really important considerations that need to be discussed: what it means to teach for conceptual understanding; and how this might best be achieved within the busy classroom.

In defining conceptual knowledge, it is probably best to consider what is meant by the term 'concept'. One definition of a concept is "a mental representation that embodies all the essential features of an object, a situation, or an idea. Concepts enable us to classify phenomena as belonging, or not belonging, together in certain categories" (Westwood, 2008, p. 24). Later, consideration will be given to the idea of how we produce these mental representations through the use of manipulative materials. According to the work of Jerome Bruner (1966), concepts are developed through a progression. This progression starts with an 'enactive' stage where learning should involve concrete experiences. The second stage is the 'iconic' stage, the stage in which pictorial representations and other graphic representations are employed, before the final stage, the 'symbolic' stage. The symbolic stage is when abstract symbols and notation are suitable for conveying meaning to the learner. Bruner's seminal work still has currency today and underpins the practice of Concrete-Representational-Abstract (CRA). Originally envisaged as a graduated sequence of instruction for working with students with learning difficulties (Strickland & Maccini, 2013), CRA (also referred to in the literature as CPA, Concrete-Pictorial-Abstract) proved to be an effective strategy for all students to gain an understanding of the mathematics concepts/skills needed to be learned (Agrawal & Morin, 2016).

Concrete-Representational-Abstract (CRA)

Concrete (manipulative) materials

The use of concrete (manipulative) materials is built on the belief that developmentally it is advantageous for students to be allowed to move from the concrete to representational (pictorial) and then to the abstract

(Goonen & Pittman-Shetler, 2012), the argument being that through touching, seeing and doing, students are enabled to gain deeper and lasting understandings of mathematical concepts (Mutodi & Ngirande, 2014). Concrete materials are materials which can be experienced through senses of sight, touch and/or sound. These materials are used to create a physical, external representation that stands for a mathematical idea, in order to eventually develop an internal representation.

Research (Reimer & Moyer, 2005; Sowell, 1989) indicates that concrete materials have a positive effect on learning and can promote positive classroom behaviours. Students who use concrete materials in their mathematics classes usually outperform those who do not, something true across all year levels, ability levels and topics (Sowell, 1989). What the research makes clear is that benefits occur when the concrete material is appropriate to the topic. The materials must be thoughtfully selected and must stimulate students' thinking. Simply providing students manipulative materials to play with is not going to gain the same benefits. The use of concrete materials is a strategy by which teachers can make a lesson more engaging through providing a hands-on experience, which allows the teacher and learners to break away from the traditional classroom setting and instructional style (Merriam & Brockett, 2011; Mutodi & Ngirande, 2014). Very importantly, manipulative materials allow students an effective way in which to represent their thinking, in a manner which the teacher then can explore further with the student, and enables the teacher to determine if there are any misconceptions in the student's understanding.

Unstructured and structured manipulative materials

Manipulative materials may be either unstructured or structured. Some examples of unstructured manipulative materials are counters, stones, buttons, and pop-sticks. Unstructured materials could be considered as materials that are not designed for mathematical purposes. Materials, such as pattern blocks and base-ten blocks (sometimes known as Diene's Blocks and sometimes as Multi-base Arithmetic Blocks—MAB), are examples of structured materials. Even though structured materials are built for mathematical purposes, the materials alone are not enough for learning to occur. However 'obvious' the mathematics might seem to the teacher, it is essential that the mathematics be articulated for the students. For example, just because we, as teachers, see the inherent place value understandings in MAB, does not necessarily mean that students see those understandings.

One of the most powerful ways to encourage a conceptual understanding of place value is the act of bundling and unbundling pop-sticks. As an unstructured manipulative, pop-sticks do not suggest place value, the power is in the way that they are used. In some sense the familiarity of pop-sticks, the lack of novelty, helps in the teaching and learning process. In the first instance the children can use the pop-sticks as aids for counting up to ten and bridging through ten. Coupled with the use of a place value board (Figure 1) the idea of trading 10 'ones' for 1 'ten' can be developed. One way of doing this is through an activity where the children generate numbers through rolling dice or using a spinner and then adding that number of pop-sticks to their place-value board. Once they have more than nine pop-sticks in the ones column they then have to make a collection of ten, wrap an elastic band around that collection (Figure 2), and place it into the tens' place.

The action of collecting 10 'ones' and physically making 1 'ten' and putting it in the appropriate place is extremely powerful. The making of the collection and the visual sense of its magnitude, along with the discussions those actions can facilitate, are all important in developing the understanding.

This understanding can then be extended to wrapping 10 lots of ten into one bundle of 100, again giving a sense of the magnitude of each place and also starting to

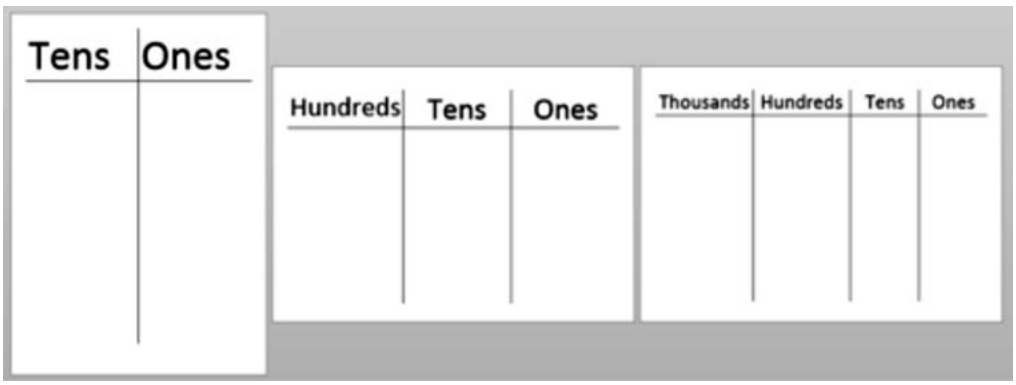

Figure 1. Place value boards.

hint at the multiplicative nature of place value (every place moving to the left is ten times greater than the one to its right).

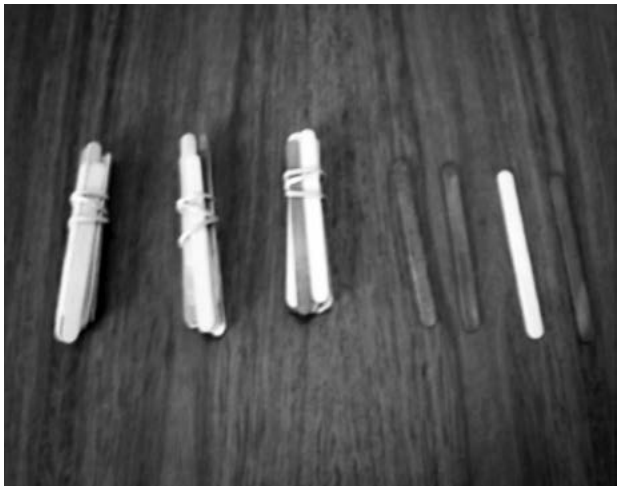

Figure 2. Bundling pop-sticks.

Once the idea of bundling has been established, the notion of unbundling is then introduced. To do this the children are asked to model a number, for example 32. When they roll the dice they then subtract that number from 32. If they roll a number greater than three, they will need to unbundle a bundle of ten to be able to execute the action. Although not the primary focus at this point, the action of the bundling and unbundling is a very strong illustration of what is often hidden in the execution of the algorithm (Figure 3). For instance 32 subtract 6:

$$\begin{array}{r} 2\ 1 \\ \cancel{3}\ 2 \\ -\ \ 6 \\ \hline \end{array}$$

Two take 6 cannot be done, cross out the 3 (unbundle one set of 10) and make it 2 (keep two bundles of 10) and put one (one unbundled set which makes 10 ones) in the ones column to make 12. 12 subtract 6 is 6 and the 2 in the tens column gives a total of 26.

Figure 3. A subtraction algorithm.

The transparency of what is actually happening when we cross out the 3 and make it 2 and then move the 1 to the ones place has a physical manifestation in the pop-sticks. The 2 is not really 2, it is 2 tens, and the 1 is not really 1, it is 10 ones. This transparency will be greatly enhanced through explicit teaching and getting the children to notice the connections between the actions and the algorithm. In other words, the pop-sticks will be nowhere near as successful at promoting understanding, without the direct intervention of the teacher.

There is a point where pop-sticks become less useful in the process and where structured materials such as MAB can be employed. When making numbers such as 623 the materials are already structured in a way that allows for access to the quantities without the distraction of having to bundle the appropriate large amounts (six 'flats', two 'longs' and three 'ones'). Conceptual rather than procedural understanding in using these materials is based on the students already understanding the relationship between each of the different denominations, for instance, that a flat is worth 100 and is 10 times greater in quantity than the long. Because the materials are structured, and therefore 'pre-bundled', their properties are often not as obvious as we would like them to be and these properties are a little more difficult to explore as the materials cannot be unbundled. Explicit teaching about the materials is required before the mathematics can be properly exploited.

Representations (pictorial)

After students have shown mastery (and by mastery, mostly we are looking for signs of conceptual understanding) towards the mathematical task through the use of concrete manipulatives, the teacher uses the same procedures of model, guide, and practice during the second phase, the representational phase, of the process. This phase involves illustration of the mathematical process by using pictures to represent objects (which in turn represent numbers) and is used until the students again demonstrate mastery.

This second phase of the CRA process involves the use of visual representations to help bridge the gap in conceptual development between the use of concrete materials and the eventual aim of the process, the use of abstract mathematical notations (numbers and symbols). The representational phase can be characterised by the use of pictures of the manipulatives, such as pictorial representations of attribute blocks, MAB, counters, pop-sticks, tally marks or hundreds charts.

For instance, $16 + 6 = 22$ using pop-sticks can be represented as in Figure 4. Or even more abstractly, representing the pop-sticks as drawn lines or marks.

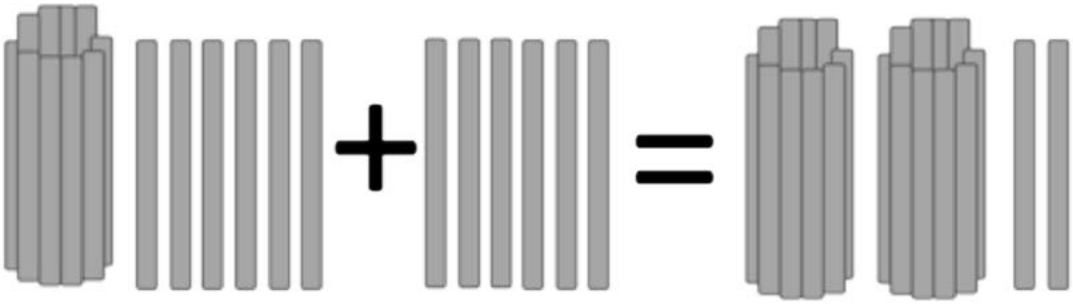

Figure 4. Pop-sticks equation example.

Abstract

The final step in the process is the move from the representational to the abstract through adoption and use of symbols and numbers. Both research and teachers' own experience indicates that the transition from the representational to the abstract stage is the most challenging aspect of the CRA sequence (Strickland & Maccini, 2013). This stage is challenging because students are required to generalise their understandings in succinct and collectively agreed upon ways. Abstract manipulation of the mathematics is the ultimate objective, and this manipulation should be conceptual rather than procedural. As the emphasis in the concrete and representational phases is on conceptual understanding, both contribute to the fluency and automaticity required to operate in the abstract phase.

Virtual manipulatives

The proliferation of technology in the form of computers, tablets (in all their various forms) and internet access, has brought the use of virtual manipulatives into most classrooms and into the hands of students, many of whom are far more adept than their teachers, at accessing and manipulating technology devices. Virtual manipulatives, as with concrete manipulatives, offer a variety of learning opportunities if thoughtfully used.

There is a good deal of research (e.g. Moyer-Packenham & Westenskow, 2013) regarding the efficacy of Information and Communication Technologies (ICT) use in classrooms, which, on the whole, shows improved learning outcomes through their use. Moyer-Packenham, Salkind and Bolyard (2008) state that dynamic virtual manipulatives are unique in that they offer a visual image such as a pictorial model but can be manipulated like a physical model. They further report that as some virtual manipulatives contain links between enactive, iconic and symbolic notations, their potential for increased mathematically meaningful action for users is increased. Again, the warning which comes with physical manipulative materials, that the manipulative in no way guarantees learning will happen, is still pertinent here. Teachers carefully helping students make connections between what is hoped to be learned and the materials is essential.

Using manipulative materials provides additional sources of brain activation that does not occur when virtual manipulatives are employed (Klahr, Triona, & Williams, 2007) and it can be argued that developing conceptual understanding is better served where physicality is involved over virtual manipulation (Zacharia, Loizou, & Papaevripidou, 2012). According to Zacharia, Loizou, and Papaevripidou (2012), who were working with kindergarten children, it appears that under certain conditions, physically manipulating materials might even be a prerequisite for learning. This finding about the need for physical manipulation fits the 'grounded/embodied cognition' position about learning. The grounded/embodied cognition position is the idea that knowledge comes from a dynamic interaction between a person and their physical world (Barsalou, 2008; Smith & Gasser, 2005). Using manipulative materials also fits with the idea that the brain works best by processing information from multiple modalities (e.g. movement, spoken words) rather than an individual modality (Chan & Black, 2006). Anyone who has conducted a mathematics lesson knows that manipulative materials encourage children to not only physically handle the materials, but also to talk about them.

The use of manipulative materials in the mathematics classroom is important. They serve the purpose of being engaging and tangible, which can allow conceptual development to occur. It is difficult to talk about abstract situations, whereas when concrete materials are in front of the student, they can manipulate them, discuss them, and use them to illustrate their understanding. Manipulative materials can be toys and a distraction from the learning, unless teachers make some important decisions. These decisions are: thoughtfully selecting the materials to make sure they support and enhance the mathematical understandings required; spending time with students to help them understand the mathematics that the

materials are illustrating; and asking thoughtful questions where the students can use the materials to illustrate developing understanding. In a busy classroom, manipulative materials afford teachers the opportunity to be effective, expert teachers, who are working towards developing conceptual understanding of mathematics in their students,

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