Five principles of educationally rich mathematical games

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Mathematical games are widely used in the primary classroom; however, not all games are equally valuable. How might teachers decide which specific games to introduce? The authors present five principles of educationally-rich games to support teachers to address this issue.

Introduction

As educators, we enjoy playing and teaching mathematical games. We spend many hours lost in conversation attempting to create new, educationally-rich games for our students. Once an idea for a game germinates, we play it ourselves and continually refine it until we feel it is classroom ready. After introducing the game to our students, we tinker with it some more. We modify the rules based on student feedback, and observations. This cycle of invention, feedback, and refinement, maintains our energy and motivation to keep attempting to develop new games for our students. However, are our newly created games truly original? Our 'new' mathematical games may be simply built on the principles of prior games.

Despite our creative endeavours, so-called new mathematical games tend to be a derivative of mechanisms and representations that are used within existing activities. Gough (2004) argued it is highly unusual for mathematical games to be truly original, as they tend to cycle through a similar set of processes and ideas. We argue that an understanding of these underlying commonalities can further assist us and other educators with both the creation of new games, as well as the evaluation of existing games. This invites the question: What principles do educationally-rich mathematical games have in common? The purpose of this article is to shed light on this question by presenting five principles of educationally-rich mathematical games that emerged from our experiences as game designers, classroom teachers and a review of relevant literature.

Principle 1: Students are engaged	Mathematical games should be engaging, enjoyable and generate mathematical discussion.
Principle 2: Skill v luck	Mathematical games should appropriately balance skill and luck.
Principle 3: Mathematics is central	Exploring important mathematical concepts and practising important skills should be central to game strategy and gameplay.
Principle 4: Flexibility for learning and teaching	Mathematical games should be easily differentiated to cater for a variety of learners, and modifiable to cater to a variety of concepts.
Principle 5: Home–school connections	Mathematical games should provide opportunities for fostering home-school connections.

Figure 1. Principles of educationally-rich mathematical games.

These five principles are not intended as definitive criteria for designing or evaluating mathematical games. Instead, our intent is to support teachers to critically evaluate games for classroom use through providing a usable framework to stimulate professional discussion and guide decision-making. Examples of mathematical games are provided to illustrate these five principles.

Principle 1: The mathematical game engages students

Mathematical games should be engaging, enjoyable and generate mathematical discussion.

Games are often viewed as an effective way of engaging students in mathematics through creating positive learning environments, enhancing student motivation and generating mathematical discussion (Bragg, 2006; Oldfield, 1991). Monroe and Nelson (2003) suggested that providing opportunities for social interaction and meaningful mathematical dialogue will increase enjoyment and engagement with mathematics. Indeed, there is evidence that lessons built around game-based activities result in students spending more time on task and generate more student-to-student mathematical dialogue compared with traditional mathematics lessons (Bragg, 2012a). As enjoyment, engagement and mathematical discourse are central benefits derived from playing mathematical games, it is important that game play unlocks these three aspects. We argue that if students are not engaged in the game, then an alternative task should be offered which addresses the same mathematical concepts attended to in the game. Games that students are not motivated to play are unlikely to generate positive learning outcomes.

How can a teacher evaluate if a game meets this first principle? We would suggest that if your students are on-task, enjoying themselves and the hum of classroom chatter is predominantly mathematical, then it is likely that the game meets this first principle. While a game may be engaging and enjoyable, it is prudent to not repeat the game excessively; even novel teaching approaches can induce boredom if over used (Bragg, 2012a).

Principle 2: Skill and luck are balanced

Mathematical games should balance skill and luck.

Mathematical games need to provide a balance between skill and luck to sustain the interest and engagement of students. Gough (1999) argued that activities relying solely on luck do not warrant being classified as games, as player choice and interactivity should be inherent in any game. Snakes and Ladders and Bingo are examples of what Gough classified as a 'pseudo-game' and a 'luck race' (2001, p. 14). Gough acknowledged that such activities may have educative value (e.g., exploring early counting concepts and number recognition), however, this educative value is substantially improved through introducing an aspect of player choice. Greedy Pig, for example, evolves a 'luck race' into a game requiring an application of the conceptual understanding of probability through the introduction of a poison number (Gough, 2001).

By contrast, games that are solely skill-based often allow more mathematically-able students to dominate. This can be de-motivating and disengaging, as the outcome of the game is effectively known to students before play begins. In our experience, perpetual losing or winning dampen student interest.[1] Consequently, games should include a sufficient element of luck to give all students a reasonable chance of winning (Badham, 1999). Sometimes this luck aspect can be simulated through role-reversal or turn-taking, in games where there is an asymmetry between the players' chances of winning depending on whether they play first or last. A simple rule of thumb is to allow the loser of the previous round to go first or last in the

Nearest to the Gnarly Number

Materials: Playing cards

Choose a Gnarly Number, for example 100.

Deal 5 communal cards. For example, 7, 4, 9, 3, 7.

Player 1 uses two of these cards to make a 2-digit number. For example: 43.

Player 2 makes a 2-digit number from the remaining three cards. For example: 77.

Discard the remaining card. Deal 5 new communal cards. For example: 2, 8, 9, 5, 1.

Player 2 uses two cards to make a 2-digit number to add to their first number. For example: 21.

Player 1 makes a 2-digit number from the remaining three cards. For example: 58.

Players sum their numbers together.

Nearest to the Gnarly Number wins.

In our example, Player 1 (101) beats Player 2 (98).

Player 2 made a mistake. What cards should they have selected on their second turn?

Figure 2. Nearest to the Gnarly Number game (Russo, 2017).

1. It is worth noting that this idea has universal applicability. For example, research has continually demonstrated that without some degree of reciprocity, competitive play amongst mammals cannot be sustained (Pellis & Pellis, 2017).

subsequent round, depending on which role yields the greater advantage (see Figure 2 for an example where Player 2 has the advantage).

Ensuring that luck plays a role in the outcome of the game provides all students with the opportunity to experience both winning and losing outcomes. More importantly, the capacity to lose and win a game with grace is critical to a young person's social and emotional development, and should be modelled and reinforced.

Principle 3: Mathematics is central

Exploring and practising mathematical concepts and skills is central to game strategy and gameplay.

Games evoke a competitive edge that keeps the learning environment energised and sustains student engagement. However, as Gough (1999) reminds us, the competitive aspect of a game can potentially detract and distract from the mathematics. Consequently, educationally-rich mathematical games require students to focus on the underlying mathematical concepts as an integral component of game strategy. Buchheister, Jackson, and Taylor (2017) stated that games should "directly align to planned mathematical goals" (p. 8), whilst Swan (2004) argued that "the game needs to have a clear purpose and the mathematics behind the game needs to be clearly defined" (p. 7). Games can be used both to provide opportunities for practising particular skills and concepts or exploring new mathematical ideas.

Nearest to the Gnarly Number is an example of a motivating game that provides opportunity for students to practise a key skill or concept (e.g. addition) (see Figure 2). Computationally fluent players focus on game strategy, generating a range of potential addition facts as they attempt to calculate both their own and their opponent's optimal play and choose cards that maximise their opportunity of winning. Importantly, although computationally fluent players are advantaged by being able to focus on game strategy, the winner of the previous round is disadvantaged by becoming Player 1 in the subsequent round. Thus, Nearest to the Gnarly Number is a clear demonstration of a game that simultaneously meets Principle 2—the balancing of skill and luck, and Principle 3—ensuring the mathematical ideas being explored are central to gameplay.

Although generally the emphasis is on practice and consolidation, games can be used to explore new ideas. The game Over and Under (see Figure 3) introduces students to the concept of dependent probability. Although independent and dependent events are not formally included in the curriculum until Year 10, this game has been played by Year 5 students as an effective and fun way of providing initial exposure to these concepts. The authors advocate exposing students to mathematical concepts at earlier stages than introduced in the curriculum, particularly in playful and informal contexts, such as through games, investigations, and children's literature to develop numerate thinking (Russo, T., 2018). Having this early exposure and initial concept development means that students have an informal foundation on which to base their later formal learning.

Principle 4: Flexibility for learning and teaching

Mathematical games should offer differentiation to cater for diverse learners, and be modifiable to cater to a variety of concepts.

Games should be modified to be optimally challenging for students, and, ideally, lend themselves to seamless differentiation. Buchheister, Jackson and Taylor (2017) argued that well-designed games can be adapted to have multiple entry points, and provide all students with opportunities to reason mathematically and think strategically. For example, some students might benefit from using manipulatives and more concrete representations (e.g., 10-frames), whilst other students will require rules or playing materials (e.g., dice) to be modified to increase the level of challenge.

Over or Under

Materials: Playing cards and 10 counters.

Deal A to 10 of a given suit to each player, discard the other cards. Each player fans the cards in their hand.

GAME 1 (with replacement):

Player 2 takes a card at random from Player 1's hand (e.g., a 7) and places it on the table. Player 1 then takes a card at random from Player 2's hand, stating whether the card will be 'over or under' the card on the table. If correct, Player 1 gets a counter; otherwise, Player 2 gets a counter.

Players pick up their cards and put them back in their hand. A new round begins, with players switching roles. Play continues until all counters are exhausted, and the player who has collected the most counters wins.

GAME 2 (without replacement):

Identical to Game 1, except the cards are left on the table at the end of each round. This allows students to explore how the probability of an event is dependent on previous outcomes.

Figure 3. Over or Under game.

Learning the rules of a given game is a substantial investment of instructional time, whilst disagreement about rules can be a barrier to mathematical objectives (Badham, 1999). Consequently, it is prudent to consider using the same mechanism, representation or objective for a variety of games. For example, we have used the same gameboard (hundred chart) and game objective (recording three different 3-in-a-rows) for several different games to explore a variety of concepts including: place value, addition and subtraction, and multiplication (Russo, 2015; Russo, J., 2018)

Sometimes, a single game in its original design can simultaneously deliver a broad range of benefits. Clarke and Roche (2010) argued that the pressure of the crowded curriculum means that games that can simultaneously address multiple important mathematical ideas in an engaging and enjoyable manner are particularly valuable to classroom teachers. They explored how the game they present Colour in Fractions can be used to explore several interrelated ideas and skills, including: equivalent fractions, using fractional language, understanding improper fractions, adding fractions, problem solving, visualisation and probability.

Principle 5: Home-school connections

Mathematical games should provide opportunities for fostering home-school connections.

During our years as primary school practitioners in Australia, we have become aware of the gap between the strong desire of many adult carers to connect with their children's mathematical learning and their perceived ability to do so. This gap causes anxiety for adults who wish to better understand and support their child's mathematical development. The barriers to this home-school connection with mathematical learning include: limited carer understanding of the content and skills being taught, carer perceptions that some techniques have changed considerably since their own schooling and, in some cases, adult anxieties around their own mathematical understanding.

Conversely, it is common for children to read at home with their parents from a very young age, with many carers feeling relative comfort reading with children when compared with engaging in mathematical learning. Perhaps the finding around the importance of 'fun' as a driving force behind the adult-child reading connection provides a key insight into strengthening the connection between the home environment and mathematical learning (YouGov, 2015). Playing games may allow adults and children to explore mathematical ideas together in a positive context.

One of the authors is currently conducting a pilot program to use mathematical games to promote home-school connections with carers and primary students. Students learn a mathematical game at school each week. At home, students and their adult carer watch an instructional video of the game and play the game together. Preliminary results have found that this program has provided a fun and regular interaction between the carer and child within a mathematical learning context. The experience has promoted an increase in carer engagement in mathematical learning at the school.

Family Math Nights (Bofferding, Kastberg, & Hoffman, 2016) and other home-school mathematics connections have been happening for some time, often with a focus on games. A Western Australian school principal utilised the card game Numero as an engaging way for students to develop their understanding of number concepts, with the purpose of using the game in both school and home environments to help students grasp the 'building blocks' of mathematics (Drysdale & Hancock, 1999).

We contend that students playing engaging maths games with their carers can be a meaningful and fun way to strengthen the home-school connection around mathematical learning. As well as the aforementioned learning benefits of the game play itself, the numerous benefits include:

- Games can help carers more effectively understand the mathematical concepts being taught at school, particularly when conversations about mathematics are limited in some households.
- Games allow carers to better appreciate the value of reasoning strategies to support mathematical fact fluency, which may differ somewhat from rote memorisation approaches they used as children (Bay-Williams & Kling, 2014).
- Games can position students as the experts, as they can explain the gameplay and key concepts to the adults by taking on the role of teachers.
- The value of games as a teaching tool is witnessed firsthand by the carers.

Concluding thoughts

Early research revealed games to be more effective than more traditional instructional approaches in improving student achievement (Randel, Morris, Wetzel, & Whitehill, 1992). However, games are not a panacea; there is evidence that, if used in isolation, games are less effective at supporting learning retention compared with other engaging, student-centred, but more mathematically explicit, activities (Bragg, 2012b). Consequently, it has been recommended that games constitute one element of a varied mathematical program; and there is evidence that games have been found to be highly effective when used in parallel with other effective pedagogies, such as teaching mathematics with picture story books (Young-Loveridge, 2004).

Most primary classroom teachers intuit that mathematics games are valuable. Rather than focus on these potential benefits, in this article we have attempted to identify five principles of educationally-rich mathematical games (see Figure 1). These principles are informed by research findings, the insightful commentaries of our colleagues, and our own classroom experiences. We hope that classroom teachers and teacher-educators find these principles valuable when determining mathematical games to introduce into their classrooms, or when exploring designing their own games.

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