Dynamic estimation: A guided approach to refining student estimates

Enhancing estimation skills through exposing students to enumeration ('how many') problems where they are required to estimate before calculating is a critical aspect of developing number sense (Reys et al., 2012). Indeed, enhancing estimation skills has been linked with improved mental computational skills and spatial visualisation capacity (Verschaffel, Greer, & De Corte, 2007). Beyond assessing the accuracy of an initial estimate, however, estimation is rarely embedded into primary mathematics tasks, particularly for younger students. In fact, the Australian Curriculum: Mathematics does not explicitly mention the term ‘estimation’ until Year 5. Although learning and applying sophisticated estimation strategies may be beyond the capacity of many young students (Brade, 2003), research has suggested that even students in the early years of schooling can benefit from being exposed to tasks that provide them with opportunities to estimate (Siegler & Booth, 2004).

This article describes a ‘how many’ problem undertaken with a small group of Year 1 students (identified by their teachers as high-performing), in which students were given repeated opportunities to refine their estimates as additional information became available. This can be described as dynamic estimation, and is presented as a counterpoint to the idea that estimates should only occur at one point in time (i.e., before beginning a problem), typically when very little information is available to students.

Dynamic estimation can be viewed as a further attempt to make tasks undertaken in primary mathematics involving estimation more closely resemble its use in the real world. In a previous article (see Russo, 2016), I explored that, sometimes, estimation involves striving for precision under uncertainty; that is, making numerical judgements when there is insufficient information to perform a precise calculation. In the case of dynamic estimation, the emphasis is instead on refining estimates as new information comes to hand.

Although the activity as it is discussed in this article is confined to estimates in relation to a container of counters, it could easily be replicated using a more engaging, context-rich, enumeration task such as an outdoors maths lesson counting sheoak cones (Parrington & Millar, 2018).

Stage 1: Inspecting the container

To begin the investigation, the container of counters was presented to students, along with the question: “How many counters do you think there are in this container?”

Students were permitted to examine the container, shake it and weigh it in their hand before arriving at their individual initial estimate. Students put forward the following estimates:
- Justin → in the 70s
- Riya → in the 90s
- Daniel → in the 200s
- Kadira → between 100 and 200
- Jaydeep → between 300 and 500

These estimates were then recorded on the whiteboard for students to view. Interestingly, all students provided a range of possibilities, rather than an exact amount. Considering their responses collectively, the
number of counters was estimated to be in the range of 70 to 500, with a median (middle) estimate of approximately 150. This summary information regarding the first round of estimations was presented back to students.

**Stage 2: Considering other students’ estimates**

Students were then asked if they wished to change their estimates at this stage. The expectation was that the range of estimates would narrow somewhat as a given student’s thinking was influenced by the responses put forward by others. In a sense, evaluating others’ estimates with respect to their own provided students with the first opportunity to consider additional information that could shed light on the central question: “How many counters do you think there are in this container?”

During Stage 2, the following revised estimates were put forward:

- Justin → between 60 and 100
- Riya → in the 200s
- Daniel → between 300 and 400
- Kadira → between 100 and 200
- Jaydeep → between 200 and 400

It is interesting to note that all students except for Kadira changed their estimate, and that there was a tendency to narrow their range of responses, in line with expectations. The number of counters was now estimated to be in the range of 60 to 400, with a median estimate of approximately 250. Again, this summary was presented to students at the conclusion of Stage 2.

**Stage 3: Handfuls activity**

At this point, students were invited to take two handfuls of counters from the container (i.e., one in each hand). Consistent with the focus of the lesson, students were encouraged to estimate how many counters they had in their hands. Students then had to arrange their counters in such a way that they could work out how many counters they had collected without counting by ones. This aspect of the investigation mirrors Gervasoni’s (2015) ‘Handfuls’ activity, designed to focus student attention on the structural aspects of number.

Students went about structuring their handfuls of counters in different ways. Kadira and Jaydeep arranged their counters in arrays and skip-counted to calculate the total (see Figure 2–bottom). Similarly, Daniel relied on creating groups of five and Justin created a somewhat idiosyncratic structure which still facilitated skip-counting by 5s (see Figure 2–top). By contrast, Riya organised the counters by colour and then calculated the total number of counters by adding together how many of each coloured counter she had ($7 + 8 + 11 + 15 = 41$).

After the Handfuls activity, it was concluded that the container was still “a bit over half full of counters”.

**Stage 4: Post-handfuls estimates**

The group was then asked: “How many counters do you think there were in this container when it was full?” Students were also provided with the prompt: make sure you use information you gained through the Handfuls activity to help you. The group proceeded to tackle this next stage of the lesson in different ways.

Justin, Daniel and Riya decided that the best approach would be to combine all their counters from the Handfuls activity to work out how many counters they had removed from the container in total. They organised all the counters into groups of 5, “to make the counting easier”, and proceeded to skip-count. Although they encountered some confusion when skip-counting took them past 100, the group finally concluded that collectively they had taken out 180 counters (the actual number was 162).

Jaydeep and Kadira decided that there was no need to actually physically combine the counters, and that instead they could use addition strategies to mentally compute the total number of counters removed from the container. They proceeded to use a number splitting strategy (i.e., partitioning each of the totals into tens and ones, and then adding the tens and ones separately), and concluded (correctly) that they had removed 162 counters. Jaydeep decided that “there are definitely more counters left in the container than we took out” so the group would need to “double 162 and then add some more... maybe another 100”.

The group found Jaydeep’s means of framing the problem persuasive, and all proceeded down this path. Collectively, using place-value partitioning, they managed to accurately calculate that “double 162 was 324”. Adding another 100 counters meant that the group had together arrived at a revised estimate of 424 counters. All students endorsed this number as their final estimate, with the exception of Jaydeep. Jaydeep decided at
the last minute that “424 was too much” and he did not want to “have the same as everyone else”. He decided his final estimate should be 355 counters, despite being most responsible for steering the group towards the 424 total.

**Stage 5: The count (and one final estimate)**

The final stage of the lesson involved the group collectively calculating how many counters there were in the container altogether. It was put to students that the counters needed to be organised in such a manner that it would be easy to check whether their counting was accurate. Subsequently, the group decided to organise their counters into groups of five, and then into groups of 100 (see Figure 3).

![Figure 3. Students count all the counters in the container.](image)

Once the group had counted four groups of 100 counters, I quickly gathered the remaining counters in my hand, and showed it to the group (see Figure 4 – top). I then asked students to make one final estimate, before revealing how many counters I had gathered (see Figure 4 – bottom). Student estimates, having converged around a collective calculation process, dispersed somewhat again, as students disagreed about exactly how many counters my hand could hold. Riya maintained her estimate of 424, while Daniel concluded there were 419 counters. Justin and Will both reduced their estimate more dramatically to 414, while Jaydeep went in the other direction, and decided that 431 counters was “probably right”.

![Figure 4. The final leftover counters revealed.](image)

As is apparent from Figure 4 (bottom), there were in fact 22 counters in my hand, leading to a total of 422 counters. After plotting all their final responses on a number line, the group realised that Riya’s estimate was the most accurate, and were impressed that Jaydeep’s calculation during Stage 4 had generated such a precise estimate.

The lesson concluded with a teacher-led discussion about how students had changed their estimates as additional information became available. Both Justin and Riya were amused by how “wrong” their initial “guesses” were, compared with their final estimates. Students were encouraged to reflect on the extent to which they believed their estimates had become more (or less) accurate over the course of the investigation, as well as whether they had felt more (or less) confident in making their estimates.

**Concluding thoughts**

This article has outlined one particular approach for encouraging a more dynamic approach to estimation, where students refine their estimates as more information becomes available. However, an activity such as this clearly requires students to build on a range of other skills and proficiencies relevant to developing number sense that go beyond numerical estimation. For example, our Year 1 students were encouraged to focus on the structural aspects of number when counting a collection of objects (see Stage 3 and Stage 5), leading to them considering concepts such as skip-counting, arrays and place value ideas. Also, the group engaged with combining collections of objects using efficient addition strategies (see Stage 4).

I encourage teachers to develop rich, “how many?” type of investigations that allow students the opportunity to make multiple estimates over time. Such lessons can conclude through the teacher facilitating a post-activity discussion that supports students to reflect on the accuracy of their estimates over the course of the investigation. I believe that such investigations have considerable potential for honing students’ number sense, as well as normalising the process of making and revising estimates as part of doing everyday mathematics.
Bringing STEM to Life
Understanding and recognising science, technology, engineering and maths

Authors: Nicole Halton & Natashja Treveton
Publisher: Teaching Solutions, 2017, Aust., soft cover. 43 pp.

I write this review of Bringing STEM to Life through the lens of what I’d like for my daughter Chloe (21). She is nearly halfway through her Early Childhood Education degree at Flinders University in Adelaide. This book is of high relevance for early years teachers and pre-service teachers.

A strength is the engagement of the authors, Halton and Treveton, in provoking thought about STEM in relation to the Early Years Learning Framework (EYLF). Useful prompts are offered that school leaders might pose to colleagues:

- How do you feel when you hear the term STEM?
- Should we be using acronyms such as STEM, STEAM and STREAM?
- What impact do these terms have on our programs and practice?

Reference is made to a notion from Angela Hanscom (2016) about the length of time it takes for young children to enter into ‘deep play’, and the implications for how STEM experiences are planned and carried out in early childhood settings. School structures and routines are a big factor in giving young learners this opportunity. Investing in giving young children enough time to engage their curiosity in deep play is a strong challenge to teachers and schools!

Halton and Treveton make a distinction between ‘teaching STEM’ and children’s natural curiosity and observation when playing. They offer a gentle provocation about the value in providing materials for children to tinker with. The photos chosen evoke a narrative about what children could be exploring, while also offering guidance to teachers about their planning, and what they might talk about with their young students. Short lists of materials that would intrigue young students are provided.

The authors are (thankfully) strident in placing a strong marker on not over-doing the approach when offering STEM learning experiences for young learners. Providing opportunities for creativity with materials rather than doing STEM is the approach encouraged. The authors identify the nature of some systemic initiatives of STEM as being of a direct instruction approach, the antithesis of what they hope students in the early years would bring to a challenge.

Bringing STEM to Life would be a worthy addition to any staff room coffee table or library…and my daughter’s Christmas stocking! At 44 pages, it is packed with lots of practical ideas and is an easy-to-read provocation to challenge early years teachers’ thinking about STEM in a supportive way.

Highly recommended!

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