The impact of vocabulary on numeracy

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In order to solve word problems, students need to be able not just to read the words but comprehend what they mean. Paul Swan discusses three tiers of vocabulary that need to be included in the pedagogy of mathematics lessons and strategies for classroom practice.

At the outset, it must be stated that I am not a literacy expert. In my many years teaching mathematics, few things were as clearly apparent as the impact of literacy on the ability of students to solve word problems. Given that only a small portion of questions are presented without any words at all, this becomes an important area for the mathematics teacher to teach. In order to solve word problems, students need to be able to not just read the words but comprehend what they mean (Newman, 1977). To further complicate matters, the students will also need to interpret any graphics and symbols used in the problem (Deizman, 2008, Lowrie, 2010, Quinnell & Carter, 2012).

Consider, for example, the word 'annulus'. Everyone reading this article can read the word annulus, but likely very few know what it means. The reader could look up the meaning in a mathematics dictionary—but that may not help either, as dictionary definitions are written in a concise manner. The definition of an annulus is 'the area between two concentric circles', however, the reader has to know what 'concentric' means in order to understand the definition. A diagram or an analogy may be of more help. Basically, an annulus looks like a washer or doughnut, and refers to the area between the two circles (that is, the 'doughnut' itself, not the 'doughnut hole').

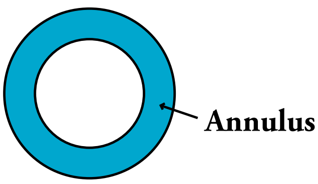

Figure 1. Annulus.

The graphic helps the reader to interpret what is being described in the question. Graphics feature in 60% to 70% of questions found in NAPLAN tests. Students who have not been exposed to graphics beforehand are likely to struggle.

Another complicating factor is that mathematical text often uses symbols. Even a 'simple' symbol such as the = sign is commonly misunderstood, and causes many difficulties for students when trying to interpret the intent of a word question in mathematics.

Further, when reading text in English, the words are read from top–left to bottom–right. However, when reading mathematics, the reader often has to stop reading part way through a sentence, look at a graphic (common examples being a graph or table), and then continue reading the sentence. This process of disrupting the reading may happen several times within a question.

Newman analysis

Newman Analysis (Newman, 1977) employs five questions or prompts which are used to determine where students are experiencing difficulty solving word problems.The five prompts are:

1. "Please read the question to me. If you don't know a word, leave it out." (Decoding, reading)
2. "Tell me what the question is asking you to do." (Comprehension)
3. "Tell me how you are going to find the answer." (Transformation)
4. "Show me what to do to get the answer." (Students are encouraged to talk aloud)
5. "Write down your answer." (Encode)

Studies employing a Newman Analysis indicate that "approximately 70 per cent of errors made by Year 7 students on typical mathematics questions were at the Comprehension or Transformation levels" (p 102, White, 2009, p. 102).

Polya (1954) listed 'Understand the Problem' as step one in his four-step process for solving problems. Many students do not spend enough time exploring the problem to be solved and hence misunderstand or misinterpret the problem and solve a completely different problem—often a harder one than was intended. The rest of this paper will focus on developing student mathematical vocabulary: the first and most fundamental step of problem solving.

The Problem Solving Approach

1. **Understand the problem**
   - **a) Vocabulary**
   - **b) Graphics**
   - **c) Interpret symbols**
2. **Devise a plan**
3. **Carry put the plan**
4. **Look back**

Figure 2. The Problem Solving Approach.

Vocabulary

Hirsch and Nation (1992) indicate that students need to know between 90% to 95 % of the words in a text in order to comprehend it. It should be noted that they were discussing general reading and not the dense technical type of reading required in mathematics and science. Vocabulary can be described in different ways. Marzano (2004) refers to 'academic vocabulary' that is used in text specific to a discipline, such as mathematics or geography. 'Hypotenuse' would be an example of a word that is used in a specific academic context; it is not an everyday word. Marzano found that teaching academic vocabulary could positively influence standardised test scores.

Beck, McKeown, and Kucan (2002) outline three tiers of vocabulary that need to be taught. Tier One refers to everyday words that students come to school with, and the words learnt in the early years. ('Everyday' words may vary depending a number of factors, such as socio-economic status, locality, and context). Tier Two words are more academic but not specific to any one discipline. Words like 'analyse' or 'evaluate' would qualify as Tier Two words. These types of words—general academic words—tend to receive less attention because they do not belong to any one discipline. Tier Three words are content-specific words, such as 'hypotenuse', that Marzano would call 'academic vocabulary'. It should be noted that in mathematics, what might be thought of as Tier One words can prove to be tricky. When applied to Venn diagrams, the use of the words 'and' and 'or' changes the meaning dramatically. Consider the phrases "25% of ten dollars" and "25% off ten dollars". 'Of' and 'off' would be considered Tier One words.

In the primary years, teachers are more likely to focus on Tier One and Tier Two words. In the secondary context, mathematics teachers have to cover a great deal of specialised content and often struggle to teach just the required Tier Three words. In addition, secondary teachers also need to teach students how to apply Tier One and Tier Two words (like 'not') to specific mathematics content. Typically, glossaries are supplied at the end of text books, or students are encouraged to search mathematics dictionaries to learn the meanings of words. Unfortunately for students who struggle, providing dictionary definitions does not help, as we have seen.

Marzano (2004) suggests six steps when learning vocabulary, none of which involve formal definitions. Initially, teachers would provide an explanation of the meaning of a word, but a formal definition is reserved for much later when the students have received multiple exposures to the word. Stahl and Fairbanks (1986) indicate students need more than just a surface knowledge of the word; solely teaching definitions does not assist students in comprehending words found in text. They also recommend that students be exposed to words at least seven times over spaced intervals for retention to occur. Authors tend to vary on the number of exposures required, but what is clear is that multiple, spaced exposures are required for retention.

The issue for teachers is how to find the time to teach the vocabulary required to understand word questions in mathematics.

Incorporating more literacy within a mathematics lesson

Teachers can use direct teaching methods involving introducing words at the beginning of a lesson, literally tuning students in to the lesson. Teachers may teach vocabulary indirectly by informal use throughout the lesson. Given how much teachers need to do within a mathematics lesson, it makes sense to use a blend of the two methods. Here the focus will be on direct methods.

Laying the groundwork for direct teaching of mathematical vocabulary

The use of direct methods implies that the teacher—or better still, the school—has a list of words that needs to be taught and learned. Literacy educators use lists such as the Oxford Word List (http://www.oxfordwordlist.com/), that contain high frequency words. Such a list is a good starting point for Tier One words.

Mathematics vocabulary word lists

There are a few such lists for mathematical vocabulary. One list, broken up according to year level and

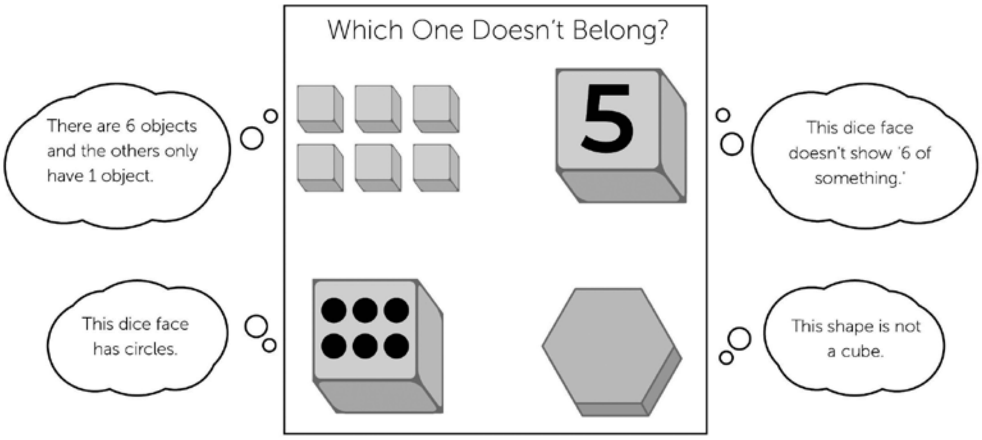

Figure 3. Which one doesn't belong?

mathematical topic, is contained in Swan and Dunstan (2018). Whatever the list, using a consistent set that spans the whole school, rather than each teacher devising their own lists, will be far more effective in teaching students all the appropriate vocabulary. Knowing the vocabulary to which the students have already been exposed to, and what should be taught at the current year level, streamlines the process of teaching appropriate vocabulary.

A list of words used in Year 3 NAPLAN tests 2010–2018 is shown in Appendix One. Even a cursory glance will indicate that there are as many words, if not more, in the geometry and measurement strand than in the other strands combined. Given the current emphasis on STEM in schools and the fact that "there is no STEM without G & M", it would make sense to directly teach these words to students in this year.

Looking further into the listing of words, teachers will notice that the vocabulary demands on younger children are quite high, and that each year level builds on the vocabulary of the preceding year levels.This means that early childhood teachers have a large responsibility to directly teach mathematical vocabulary.

Direct teaching of mathematical vocabulary

I favour using a variety of set routines when developing understanding of mathematical vocabulary. Once the routines have been taught, they may be used regularly, making the best use of time and resources. These common routines may then be compiled into a school bank, or menu, of mathematics vocabulary routines that may be applied throughout the school. Here are some examples of vocabulary routines that may be used.

Barrier games

The quintessential barrier game is called Battleships. Players sit either side of a barrier and use 'expressive' and 'receptive' language to describe and interpret positions given as coordinates. While playing this game, students will be learning geometry content (location) and reasoning. The game may be played with anything on either side of the barrier.

The same game (routine) may be played in the early years and at secondary level, making it the perfect game to go on the menu of whole-school approaches to teaching mathematics vocabulary. For a detailed explanation of how to use barrier games across a school, see Swan (2018). To maximise the value of the game, teachers will need to highlight the mathematical language they would like students to use while playing. What words do teachers expect the students to use? Teachers will require a list from which to choose words appropriate to the task. These words may be Tier Two or Tier Three words (specific content words); however, teachers may want to review the use of Tier One words within the context of mathematics. A list of prepositions, made up of mostly Tier One words is provided in Appendix Two. The key point is that given how much content teachers are expected to cover, it makes sense to integrate mathematics and language where possible. It should be noted that the use of a barrier game is not restricted to mathematics lessons; it may equally be employed in other subject areas, particularly literacy.

Which one doesn't belong?

These simple puzzles, created by Danielson (2016), are easy to learn and then create. They encourage the use of mathematical language (oral) and reasoning (the use of mathematical language). The puzzle involves four sections, each containing an image, object, word or symbol, and students have to explain why a particular component does not belong. Each of the four components might not belong for a variety of reasons. The students have to explain or justify why they believe a particular component does not belong. (Figure 3.)

Mystery bag

An object is placed into an opaque bag and a student is asked to slide his/her hand into the bag and to describe features of the object. Larger items may be placed inside a box with packing material.

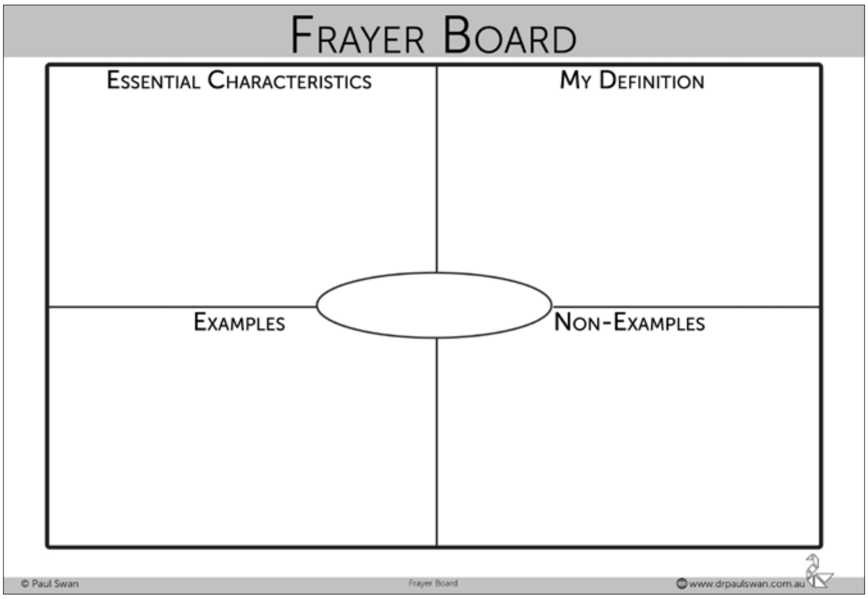

Figure 4. A Frayer Board.

Graphic organisers

There are many graphic organisers used to develop vocabulary and these may be adapted for developing mathematical vocabulary. A Frayer Board is one such example.

Diagrammatic representations of word problems

The role of diagrams in solving word problems is gaining prominence in Australia. Although first mentioned by Sawyer (1964) these diagrams are often labelled as 'Singapore maths: diagrammatic representations of word problems that are designed to help students move from comprehending a problem to translating it into a mathematical equation to be solved. These diagrammatic approaches—sometimes referred to as 'bar diagrams, strip, tape and ribbon diagrams'—will assist in comprehending questions involving operations. Polya (1954) suggested that one way to understand a problem is to draw a diagram.

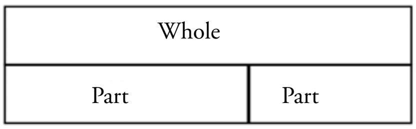

Figure 5. Bar model.

Conclusion

This article has focused primarily on the development of vocabulary as a first step in being able to comprehend and translate a word problem in mathematics. Given that students need a well-developed Tier One vocabulary and a specialised Tier Three vocabulary, it makes sense to incorporate these elements in to the pedagogy of a mathematics lesson.

At the outset, two other components of a typical word problem—graphics and symbols—were alluded to. It is beyond the scope of this article to expand on these. Readers are encouraged to look at the work of Diezmann (2008), Lowrie (2010) and Quinnell and Carter (2012) for further explanation.

References

Beck, I. L., McKeown, M. G., & Kucan, L. (2002). *Bringing words to life: Robust vocabulary instruction.* New York. Guilford Press.

Danielson, C. (2016). *Which one does not belong?* USA: Stenhouse.

Diezmann, C. (2008). Graphics and the national numeracy tests. In: *31st Annual Conference of the Mathematics Education Research Group of Australasia*, June 28–July 01, 2008, St Lucia Brisbane. Retrieved from http://eprints.qut.edu.au/18534/1/c18534.pdf

Lowrie, T. (2010). Primary students decoding mathematics tasks: The role of spatial reasoning. *Teaching Mathematics make it count: What research tells about effective teaching and learning of mathematics.* Retrieved from http:/ research.acer.edu.au/cgi/viewcontent.cgi?article=1085&context=research_conference

Hirsh, D., & Nation, P. (1992). What vocabulary size is needed to read unsimplified texts for pleasure? *Reading in a Foreign Language, 8*, 689–696.

Marzano, R. J. (2004). *Building background knowledge for academic achievement: Research on what works in schools.* Alexandria, VA: ASCD.

Newman, M.A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks. *Victorian Institute for Educational Research, 39*, 31–43.

Polya, G. 1954. *How to solve it.* New York: Princeton.

Quinnell, L & Carter, M. G (2012). Greek or not: the use of symbols and abbreviations in mathematics. *Australian Mathematics Teacher, 68*(2), pp. 34–40. Retrieved from: http://eprints.qut.edu.au/54166/2/54166.pdf

Sawyer, W.W. (1964). *Vision in elementary mathematics*. Mineola, NY: Dover Publications.

Stahl, S. A., & Fairbanks, M. M. (1986). The effects of vocabulary instruction: A model-based meta-analysis. *Review of Educational Research, 56*(1), 72–110

Swan, P., & Dunstan, D. (2018). *My word book: Mathematics.* Perth: A–Z Type.

Swan, P. (2018). *Barrier games*. Perth: A–Z Type.

White, A. (2009). Diagnostic and pedagogical issues with mathematical word problems. *Brunei International Journal of Science and Mathematics Education., 1*(1), 100–112

Appendix 1

NAPLAN ▶ YEAR 3

Lists from My Word Book: Mathematics (Swan & Dunstan, 2018)

Number and Algebra		Statistics and Probability
add(ed) / subtract(ed)	in total	certain
altogether	left	chance
amount	more	chart
arrange	more than	data
arrow points	most / least	fair
buy / sell, sold	next number	favourite
change	not	fewer
cheap, cheapest	number	flip
close, closest (to)	number line	graph
clues	number sentence	heads / tails
collect, collections	one more / one less	less, least
cost	ones place	likely / unlikely / equally likely
counting down	only	picture graph
days later	pattern	possible / impossible
different, difference	place (ones, tens ...)	spins / spinner
digit	problem	table
each	quarter (turn)	tally
equal	removed	times
estimate, best estimate	repeat (pattern)	tossed a coin
exactly	row / column	
extra	score	
fewer	second from the	
four	second oldest	
greatest/ least number	shortest / tallest	
groups of	solve	
half way, half / twice	some	
how far	tens	
how many	three times	
hundreds place	whole	

Appendix 1 (continued)

[redacted]

Measurement and Geometry		
(more / half) full	gram	powered
3D objects	grid paper	prism
area	half / quarter past (to)	rectangular (prism / pyramid)
around	half as wide	
balanced, balance (scale)	heaviest/ lightest	right hand
	heights	same size
between	hours, minutes, seconds	scales
block		seasons (names of)
cell	joining	shape (shape names)
centimetres	kilogram	shorter / longer / taller
cents	kilometres	sphere
circle	largest	square (based pyramid)
circular	layer/ layers	start / end / finish
clock, clockwise, anti-clockwise	least	symmetry / symmetrical
	left / right	
closed	length, total length	tile
container	light / heavy	time
cube	litre	top / bottom
cup	located	triangle (based prism / pyramid)
cylinder	map	
different/ same	mass	turn, turning
direction	measure, tape measure	view
East / West / North / South	metres	weighs / weight
	missing	
edges / faces / vertices	model	
floor plan / plans	months (names)	
fold / unfold	net	
front / side / top view	pattern	
furthest away from	position, positioned	

Appendix 2

Prepositions

While playing barrier games students will use specialised mathematical language. The mathematical language will be used in conjunction with prepositions. A preposition is generally used in front of nouns.

above	in
across	inside
after	into
against	near
along	on
alongside	onto
among	opposite
amongst	outside
around	over
at	past
before	round
behind	through
below	to
beneath	toward(s)
beside	under
between	underneath
by	up
down	upon
following	within
from	

Students whose first language is not English will struggle with the use of prepositions. They may need to be provided with printed support statements or vocabulary cards. Examples include: "Put a ... under, over, above, below the ..."

(from p. 9, *Barrier games,* Swan, 2018)